Incorporating Misperception into the Graph Model for Conflict Resolution

by

Yasir Aljefri

A thesis
presented to the University of Waterloo
in fulfillment of the
thesis requirement for the degree of
Doctor of Philosophy
in
Systems Design Engineering

Waterloo, Ontario, Canada, 2017

Examining Committee Membership

The following served on the Examining Committee for this thesis. The decision of the Examining Committee is by majority vote.

External Examiner	Gregory (Grzegorz) E. Kersten Professor
Supervisor	Keith W. Hipel University Professor
Supervisor	Liping Fang Professor
Internal Member	Mitali De Professor
Internal Member	Andrea Scott Assistant Professor
Internal-external Member	Carl Haas Professor

I hereby declare that I am the sole author of this thesis. This is a true copy of the thesis, including any required final revisions, as accepted by my examiners.

I understand that my thesis may be made electronically available to the public.

Abstract

A flexible hypergame methodology is designed and implemented for modeling misperceptions by participating decision makers (DMs) in a conflict having two or more DMs within the framework of the Graph Model for Conflict Resolution (GMCR). This comprehensive approach allows one or more of the DMs to have misunderstandings about the actual situation. Moreover, the methodology can account for misperceptions founded upon other misunderstandings such that different levels of misperception exist. This improved methodology can handle a DM's misperception about itself as well as its perceptions about its opponents. To accomplish this, the options or courses of action of each DM in a conflict are categorized according to various types of misperceptions that can occur either due to others or the particular DM. Furthermore, the union of all possible kinds of option perceptions creates the universal set of options for each DM, which in turn can be extended across all DMs in the dispute to generate the universal set of states or possible scenarios for the hypergame. The universal set of states permits the DMs to experience and view the dispute independently, yet allows an analyst to distinguish between the states that are commonly recognized by all DMs and those that are individually misperceived. Furthermore, DMs' preferences are expressed in a relative fashion by pairwise comparisons between any pair of states, thereby allowing the hypergame in graph form to accommodate both transitive and intransitive preference structures.

A general stability analysis procedure is developed to analyze a hypergame under any level of perception. Within this approach, two techniques are developed: one to analyze each DM's subjective game or hypergame and another to analyze and predict the equilibria for the overall hypergame. Moreover, to study the effects of DMs' misperceptions on the outcomes of the dispute, the overall hypergame equilibria are categorized based on the type of misperceptions into eight classes of equilibria. To test and refine the hypergame methodology as well to apply it in practice, three case studies are investigated. In particular, the 2011 conflict between North and South Sudan over South Sudanese oil exports, as well as the 1956 nationalization of the Suez Canal dispute, are investigated within the paradigm of a first-level hypergame in graph form, which is a decision situation in which at least one DM has a misperception about the conflict situation, and neither the DM who misperceives the circumstance nor any of the other DMs are aware of this misunderstanding. Additionally, a detailed case study about the hydropolitical conflict among the Eastern Nile Countries over the Grand Ethiopian Renaissance Dam is investigated within the structure of a second-level hypergame in graph form, in which at least one DM is aware of another DM's misperception. Interesting strategic insights found in these case studies confirm the distinct advantages of utilizing the new hypergame methodology in graph form.

Acknowledgments

I would like to start my acknowledgments to the many people who were instrumental in making this work possible, by expressing my special sincere appreciation and gratitude to my supervisors, Professor Keith W. Hipel, from the University of Waterloo, and Professor Liping Fang, from Ryerson University. I would like to thank Professor Hipel for his tireless guidance, direction setting, encouragement, support, and on top of that believing in my ability to achieve high quality research in my field of study that will add real value to the discipline of the science of conflict resolution. Professor Hipel, thank you for the wonderful years I spent learning under your supervision. I am truly fortunate that I had the opportunity to study under your guidance.

I am equally grateful and genuinely appreciative, to Professor Liping Fang for his continuous support, guidance and the long hours he spent with me reviewing and developing this research problem till its successful conclusion. Thank you Professor Fang for your great support, mentoring, and encouragement to make me able to develop these significant research findings.

I will be forever in debt to both of my supervisors as mentors and as great human beings whom I learned and gained research capabilities, depth of knowledge and problem solving abilities in many fields. They taught me how to think creatively and sensibly to address challenging problems in research and life.

Besides my supervisors, I would like to thank my PhD committee members, Professor Carl Haas and Professor Andrea Scott, both from the University of Waterloo, and Professor Mitali De, from Wilfrid Laurier University. Their valuable feedback and discussions helped me to refine and improve my thesis. I am grateful to Professor Gregory Kersten, my external examiner from the John Molson School of Business at Concordia University, for the time he spent in carefully reading my thesis and providing helpful suggestions.

I am grateful to Dr. Abul Bashar for taking the time out of his busy schedule to review and offer many constructive suggestions and helpful feedback on early versions of my work, which enhanced various mathematical definitions. He taught me how to think like a mathematician.

Also, I would like to acknowledge my colleagues in the Department of Systems Design Engineering, for their insightful discussions and valuable remarks, especially, Dr. Yasser Matbouli, Dr. Abdullatif Alwasel, and Dr. Yi Xiao.

I would like to thank Mr. Muhammad Zamin Ahmed for his support in preparing a map for the Nile River and the Grand Ethiopian Renaissance Dam. Further, I would like to extend my gratitude to Mr. Conrad Hipel for editing and proofreading many of my journal and conference papers.

I acknowledge the positive and constructive criticisms I received from anonymous referees of different journal and conferences papers, including reviewers with the IEEE Transaction on Systems, Man, and Cybernetics: Systems; Group Decision and Negotiation annual conferences; and IEEE Systems, Man, and Cybernetics Society annual conferences.

I would like to offer a special thanks to my better half, Esra Bafagih, who has been a great companion for me during my graduate studies, and also a wonderful and loving mother to our daughter, Marya. She gave me great support and encouragement during the tough times and was highly supportive at all times. Esra managed to get a Bachelor of Kinesiology degree from the University of Waterloo while still providing support and care to our daughter Marya and me. Thank you, Esra, for always being there for me. I could not have finished my PhD degree without you.

Enormous thanks go to my little princess, my daughter Marya, for her patience, her endless love, and her encouragement. Thank you, Marya! Your smiles made me forget all the worries and obstacles I faced during my education.

I am greatly thankful to my wonderful and supporting parents, Dr. Mohammed Aljefri and Mrs. Hend Aljefri, for their prayers and for everything they did for me. They have always been my true role models. My parents encouraged me to pursue my higher education abroad, and to seek education from first class universities. They directed me to be self-motivated, and to work hard to achieve success. Also, they taught me not to fear failure, as failure is the way to success. Their immense support and wisdom assisted me in finishing my PhD research. I am incredibly grateful to them for always believing in me and being there for me at all times.

I would like to express my appreciation to each and every member of my family back home for all their support, love, and motivation that encouraged me to finish my degree. Also, I would also like to thank my friend Hatem Sindi and his family for the friendship and support they provided to my family and me during our time in Canada.

Moreover, I would like to thank the Government of my home country, Saudi Arabia, and the University of Jeddah for providing me with full financial support throughout my PhD studies at the University of Waterloo. I am truly fortunate to have that scholarship in order to be able to come and pursue my PhD degree in Canada. I am also grateful to the Natural Sciences and Engineering Research Council of Canada for covering some of the publication costs associated with my journal and conference papers. Furthermore, I would like to thank the University of Waterloo for providing a great learning environment and the many individuals at Waterloo who supported me and made my years on campus so enjoyable. I will greatly miss my Canadian academic home.

Last but not the least, I would like to thank the Canadian people for hosting my family and me for so many years with lots of respect and love and in a great friendly community with a positive, friendly, and productive environment that promotes innovation and enhances one's ability for achievement. Thank you Canada.

Dedication

To my mother, Mrs. Hend Aljefri, and my father, Dr. Mohammed Aljefri, for their prayers and encouragement which always inspired me.

Table of Contents

List of Tables

List of Figures

List of Abbreviations

AHD	Aswan High Dam
BCM	Billion Cubic Meters
bpd	Barrels Per Day
CFA	Cooperative Framework Agreement
CPA	Comprehensive Peace Agreement
DMs	Decision Makers
DoP	Declaration of Principles
GERD	Grand Ethiopian Renaissance Dam
GDP	Gross Domestic Product
GMCR	Graph Model for Conflict Resolution
GMR	General Metarationality
IPoE	International Panel of Experts
JMP	Joint Multipurpose Project
NBI	Nile Basin Initiative
NBTF	Nile Basin Trust Fund

SEQ	Sequential Stability
SMR	Symmetric Metarationality
UI	Unilateral Improvement
UIL	Unilateral Improvement List
UK	United Kingdom
UMs	Unilateral Moves
UNDP	United Nations Development Program
USBR	United States Bureau of Reclamation

List of Symbols

$\succsim_i$	*DM i's* preference relations over the set of possible states S in G
$\succsim_{ii}$	*DM i's* preference relations on S_i in G_i as perceived by itself
$\succsim_{ji}$	*DM j's* preference relations on S_i in G_i as perceived by $DM\ i$
$\succsim_{kji}$	preference relations of $DM\ k$ over the states in S_{ji} as perceived by $DM\ j$ and then by $DM\ i$
$\succsim_{qkw}$	preference relations of $DM\ q$ as seen by k and then by w , a sequence of DMs, in G_{kw}
$\succ_i$	*DM i's* strict preference relations on the set of possible states S in G
$\sim_i$	*DM i's* equally preferred relations on the set of states
λ	total number of options in O
$\emptyset$	empty set
η_i	number of *DM i's* distinct Nash strategies obtained from E_i^{Nash} within G_i
η_j	number of *DM j's* distinct Nash strategies obtained from E_j^{Nash} within G_j

ε_i	number of Nash equilibrium states in G_i
ε_j	number of Nash equilibrium states in G_j
Σ^1	set of all ordered strings of DMs of length 1
Σ^2	set of all ordered strings of DMs of length 2
Σ^3	set of all ordered strings of DMs of length 3
Σ^h	set of all ordered strings of DMs of length h
Σ^{h*}	set of all strings of DMs from length 1 to length h
$\Omega_H(s_1,\ s_2)$	set of all last DMs in H in the valid sequences of moves from s_1 to s_2
$\Omega_H^+(s_1,\ s_2)$	set of all last DMs in legal sequences of unilateral improvements from s_1 to s_2
$\Omega_{Hji}(s_1,\ s_2)$	set of all last players in H in the legal sequences of UMs from s_1 to s_2 as seen by *DM j* and then *DM i* in G_{ji}
$\Omega_{Hji}^+(s_1,\ s_2)$	set of all last players in H in the legal sequences of unilateral improvements from s_1 to s_2 as seen by *DM j* and then *DM i* in G_{ji}
$\Omega_{Hjw}(s_1,\ s_2)$	set of all last players in H in the legal sequence of moves from s_1 to s_2 as seen by *DM j* and then contemplated by the string of DMs in w
$\Omega_{Hjw}^+(s_1,\ s_2)$	set of all last players in H in the legal sequence of UIs from s_1 to s_2 as seen by *DM j* and then contemplated by the string of DMs in w
A_i	*DM i's* allowable state transitions from one state to another in one step

A_{ii}	set of state transitions for $DM\ i$ as perceived by itself
A_{ji}	set of state transitions for $DM\ j$ as perceived by $DM\ i$
A_{kji}	set of state transitions available for $DM\ k$ from one state to another in S_{ji} as perceived by $DM\ j$ and then by $DM\ i$
A_{qkw}	state transitions of $DM\ q$ as seen by k and then by the string of DMs in w in G_{kw}
$CHGMR^1$	contingent hyper GMR equilibrium state for a first-level hypergame
$CHGMR^2$	contingent hyper GMR equilibrium state for a second-level hypergame
$CHNash^1$	contingent hyper Nash equilibrium state for a first-level hypergame
$CHNash^2$	contingent hyper Nash equilibrium state for a second-level hypergame
$CHNash^h$	contingent hyper Nash equilibrium state for an h-level hypergame
$CHSEQ^1$	contingent hyper SEQ equilibrium state for a first-level hypergame
$CHSEQ^2$	contingent hyper SEQ equilibrium state for a second-level hypergame
$CHSMR^1$	contingent hyper SMR equilibrium state for a first-level hypergame
$CHSMR^2$	contingent hyper SMR equilibrium state for a second-level hypergame
E	set of all equilibrium states in G
E_i	set of all equilibrium states in G_i
E_j	set of all equilibrium states in G_j
E_{ji}	set of all equilibrium states in G_{ji}

E_{ji}^{GMR}	set of all GMR equilibrium states in G_{ji}
E_{jw}^{GMR}	set of all GMR equilibrium states in G_{jw}
E_{i}^{Nash}	set of all Nash equilibrium states in G_i
E_{j}^{Nash}	set of all Nash equilibrium states in G_j
E_{ji}^{Nash}	set of all Nash equilibrium states in G_{ji}
E_{jw}^{Nash}	set of all Nash equilibrium states is in G_{jw}
$e_{\varepsilon_i}^{Nash_i}$	a Nash equilibrium state in G_i, where ε_i is the total number of Nash equilibrium states in G_i
$e_{\varepsilon_j}^{Nash_j}$	a Nash equilibrium state in G_j, where ε_j is the total number of Nash equilibrium states in G_j
E_{ji}^{SEQ}	set of all SEQ equilibrium states in G_{ji}
E_{jw}^{SEQ}	set of all SEQ equilibrium states is in G_{jw}
E_{ji}^{SMR}	set of all SMR equilibrium states in G_{ji}
E_{jw}^{SMR}	set of all SMR equilibrium states in G_{jw}
$EHGMR^1$	emergent hyper GMR equilibrium state for a first-level hypergame
$EHGMR^2$	emergent hyper GMR equilibrium state for a second-level hypergame
$EHNash^1$	emergent hyper Nash equilibrium state for a first-level hypergame
$EHNash^2$	emergent hyper Nash equilibrium state for a second-level hypergame
$EHNash^h$	emergent hyper Nash equilibrium state for an h-level hypergame
$EHSEQ^1$	emergent hyper SEQ equilibrium state for a first-level hypergame

$EHSEQ^2$	emergent hyper SEQ equilibrium state for a second-level hypergame
$EHSMR^1$	emergent hyper SMR equilibrium state for a first-level hypergame
$EHSMR^2$	emergent hyper SMR equilibrium state for a second-level hypergame
f	mapping function of a state , which is represented by a λ-dimensional column vector
G	graph model
G_i	*DM i's* subject game
g_i	mapping function of a strategy for *DM i*
G_j	*DM j's* subjective game
G_{ji}	*DM j's* game as seen by *DM i*
G_{kw}	*DM k's* subjective game as seen by w, a sequence of DMs
G_w	the graph model as seen by w, a sequence of DMs
$g_1^{s_1}$	*DM 1's* strategies associated with state s_1, which is represented by m_1-dimensional column vector
$g_n^{s_1}$	*DM n's* strategies associated with state s_1, which is represented by m_n-dimensional column vector
$g_i^{*Nash_i}$	set of *DM i's* Nash strategies obtained from E_i^{Nash} within G_i
$g_i^{e_1^{Nash_i}}$	*DM i's* strategy obtained from the equilibrium state $e_1^{Nash_i}$ in E_i^{Nash} within G_i
$g_j^{*Nash_j}$	set of *DM j's* Nash strategies obtained from E_j^{Nash} within G_j

$g_j^{e_1^{Nash_j}}$	*DM j's* strategy obtained from the equilibrium state $e_1^{Nash_j}$ in E_j^{Nash} within G_j
$g_{jw}^{*Nash_{jw}}$	set of Nash strategies of $DM\ j$ in G_{jw}
$g_{jw}^{e_{1jw}^{Nash_{jw}}}$	strategy of $DM\ j$ as viewed by w that is attained from $e_{1jw}^{Nash_{jw}}$ in G_{jw}
H	subset of DMs in N
h	level of hypergame ($h \geqslant 1$)
H^0	zero-level hypergame
H^1	first-level hypergame
H^2	second-level hypergames
H^3	third-level hypergame
H^h	h-level hypergame
H_i^{h-1}	*DM i's* $(h-1)$-level hypergame
HE^{1GMR}	set of hyper GMR equilibrium states for a first-level hypergame
HE^{2GMR}	set of hyper GMR equilibria for a second-level hypergame
HE^{hGMR}	set of hyper GMR equilibria for an h-level hypergame
HE^{1Nash}	set of hyper Nash equilibrium states a the first-level hypergame
HE^{2Nash}	set of hyper Nash equilibria for a second-level hypergame
HE^{hNash}	set of hyper Nash equilibria for an h-level hypergame
HE^{1SEQ}	set of hyper SEQ equilibrium states for a first-level hypergame
HE^{2SEQ}	set of hyper SEQ equilibria for a second-level hypergame

HE^{hSEQ}	set of hyper SEQ equilibria for an h-level hypergame
HE^{1SMR}	set of hyper SMR equilibrium states for a first-level hypergame
HE^{2SMR}	set of hyper SMR equilibria for a second-level hypergame
HE^{hSMR}	set of hyper SMR equilibria for an h-level hypergame
i, j, k, p, q	DMs in a conflict
m_i	total number of options for $DM\ i$
N	set of DMs
n	number of DMs
N_i	set of DMs as seen by $DM\ i$
N_{ji}	set of DMs as seen by $DM\ j$ and then as contemplated by $DM\ i$
N_{jw}	set of DMs as perceived by $DM\ j$ and then by w, a sequence of DMs
N_{kw}	set of DMs as perceived by $DM\ k$ and then by w, a sequence of DMs, within G_{kw}
O	set of options for all DMs
$\hat{O}^1$	universal set of options for a first-level hypergame
$\hat{O}^2$	universal set of options for a second-level hypergame
$\hat{O}^h$	universal set of options for an h-level hypergame
O_i	*DM i's* set of options in G
$\ddot{O}_i^1$	*DM i's* universal set of options for a first-level hypergame
$\ddot{O}_j^1$	*DM j's* universal set of options for a first-level hypergame
$\ddot{O}_i^2$	universal set of options of $DM\ i$ for a second-level hypergame

$\ddot{O}_i^h$	universal set of options of *DM* i for an h-level hypergame
O_{ii}^I	set of *DM i's* options imagined by itself
O_{ij}^I	set of *DM i's* options that are imagined by *DM j*
O_{ipq}^I	set of options of *DM* i that are imagined by *DM* p as perceived by *DM* q
O_{iw}^I	set of *DM i's* options that are imagined by w, a sequence of DMs
$o_{\bar{k}}^i$	*DM i's* $\bar{k}^{th}$ option
O_{qkw}^I	set of *DM q's* options that are imagined by *DM* k and then by w, a sequence of DMs
O_{ii}^M	set of *DM i's* options that are misunderstood in meaning by itself
O_{ij}^M	set of *DM i's* options misunderstood by *DM j*
O_{iw}^M	set of *DM i's* options that are misunderstood in meaning by w, a sequence of DMs
O_{ipq}^M	set of options of *DM* i that are misunderstood in meaning by *DM* p as perceived by *DM* q
O_{qkw}^M	set of *DM q's* options that are misunderstood in meaning by *DM* k and then by w, a sequence of DMs
O_i^R	set of *DM i's* correctly perceived options that are recognized by all DMs in N.
$O_i^{\bar{R}}$	set of *DM i's* options that exist in reality, but are misinterpreted by *DM* i
O_{ii}^R	set of *DM i's* options that are correctly identified by *DM* i itself

O_{ij}^R	set of *DM i's* options that are correctly identified by *DM* i itself and recognized by *DM* j
O_{iw}^R	set of *DM i's* correct options that are correctly considered by w
O_{ipq}^R	set of *DM i's* actual options that are considered by *DM* p as seen by *DM* q
O_{qkw}^R	set of *DM q's* correct options that are correctly considered by *DM* k and then by w, a sequence of DMs
O_{ii}^U	set of *DM i's* options that are unknown to itself
O_{ij}^U	set of *DM i's* options that are unknown to *DM* j
O_{ipq}^U	set of options of *DM* i that are unknown to *DM* p as perceived by *DM* q
q'	opponent of q in N
$R_H(s_1)$	set of all UMs from s_1 by one or more DMs in H through a legal sequence of moves starting from s_1
$R_H^+(s_1)$	set of UIs from s_1 by a group of DMs, $H \subseteq N$ and $H \neq \emptyset$
$R_{Hji}(s_1)$	set of all UMs from s_1 by any number of DMs $\in H$ over a legal sequence of moves beginning from s_1 as seen by *DM* j and then *DM* i in G_{ji}
$R_{Hji}^+(s_1)$	set of all UIs from s_1 by any number of DMs $\in H$ over a legal sequence of moves beginning from s_1 as seen by *DM* j and then *DM* i in G_{ji}
$R_{Hjw}(s_1)$	set of possible moves available from s_1 by any DMs in H via a valid sequence of moves as seen by *DM* j and then contemplated by w, a sequence of DMs in

$R^+_{Hjw}(s_1)$	set of UIs available from s_1 by any DMs in H via a valid sequence of moves as seen by *DM* j and then contemplated by w, a sequence of DMs
$R_q(s_1)$	set of *DM* q*'s* unilateral moves starting from s_1
$R^+_q(s_1)$	unilateral improvement from s_1 by *DM* q
$R_{qi}(s_1)$	set of *DM* q*'s* unilateral moves starting from s_1 as seen by *DM* i
$R^+_{qi}(s_1)$	set of *DM* q*'s* unilateral improvements starting from s_1 as seen by *DM* i
$R_{qjw}(s_1)$	set of *DM* q*'s* UMs from the initial state s_1 as seen by *DM* j and then contemplated by w, a sequence of DMs
S	set of states or scenarios for a dispute
$\hat{S}^1$	universal set of states for a first-level hypergame
$\hat{S}^2$	universal set of states for a second-level hypergame
$\hat{S}^h$	universal set of the states for an h-level hypergame
S^{GMR_q}	set of all GMR stable states for *DM* q in G
S^{Nash_q}	set of all Nash stable states for *DM* q in G
S^R	set of states that are correctly perceived by all DMs in N
S^{SEQ_q}	set of all SEQ stable states for *DM* q in G
S^{SMR_q}	set of all SMR stable states for *DM* q in G
s_1, s_2	states or possible scenarios for the conflict
S_i	recognizable set of states for *DM* i
$S_i^{GMR_{qi}}$	set of all GMR stable states for *DM* q as seen by *DM* i in G_i

S_i^H	set of states that are hidden to $DM\ i$ in its game
S_i^I	set of states that are imagined by $DM\ i$ itself
$S_i^{I,M}$	set of states that are imagined and misunderstood by $DM\ i$ itself
S_i^M	set of states that are misunderstood by $DM\ i$ itself
$S_i^{Nash_{qi}}$	set of all Nash stable states for $DM\ q$ as seen by $DM\ i$ in G_i
S_i^P	set of states that are correctly perceived by $DM\ i$ itself and possibly by some of its opponents but not all of them
$S_i^{SEQ_{qi}}$	set of all SEQ stable states for $DM\ q$ as seen by $DM\ i$ in G_i
$S_i^{SMR_{qi}}$	set of all SMR stable states for $DM\ q$ as seen by $DM\ i$ in G_i
S_{ji}	recognizable set of states for $DM\ j$ as seen by $DM\ i$ in G_{ji}
$S_{ji}^{GMR_{pji}}$	set of all GMR stable states for $DM\ p$ as seen by $DM\ j$ and then $DM\ i$ in G_{ji}
$S_{ji}^{Nash_{pji}}$	set of all Nash stable states for $DM\ p$ as seen by $DM\ j$ and then $DM\ i$ in G_{ji}
$S_{ji}^{SEQ_{pji}}$	set of all SEQ stable states for $DM\ p$ as seen by $DM\ j$ and then $DM\ i$ in G_{ji}
$S_{ji}^{SMR_{pji}}$	set of all SMR stable states for $DM\ p$ as seen by $DM\ j$ and then $DM\ i$ in G_{ji}
S_{kw}	set of states perceived by $DM\ k$ as contemplated by w in G_{kw}
$S_{jw}^{GMR_{qjw}}$	set of GMR stable states for $DM\ q$ in G_{jw}
$S_{jw}^{Nash_{qjw}}$	set of Nash stable states for $DM\ q$ in G_{jw}
$S_{jw}^{SEQ_{qjw}}$	set of all SEQ stable states for $DM\ q$ in G_{jw}

$S_{jw}^{SMR_{qjw}}$	set of all SMR stable states for $DM\ q$ in G_{jw}
$SCHGMR^1$	self-contingent hyper GMR equilibrium state for a first-level hypergame
$SCHGMR^2$	self-contingent hyper GMR equilibrium state for a second-level hypergame
$SCHNash^1$	self-contingent hyper Nash equilibrium state for a first-level hypergame
$SCHNash^2$	self-contingent hyper Nash equilibrium state for a second-level hypergame
$SCHNash^h$	self-contingent hyper Nash equilibrium state for an h-level hypergame
$SCHSEQ^1$	self-contingent hyper SEQ equilibrium state for a first-level hypergame
$SCHSEQ^2$	self-contingent hyper SEQ equilibrium state for a second-level hypergame
$SCHSMR^1$	self-contingent hyper SMR equilibrium state for a first-level hypergame
$SCHSMR^2$	self-contingent hyper SMR equilibrium state for a second-level hypergame
$SHGMR^1$	a steady hyper GMR equilibrium state for a first-level hypergame
$SHGMR^2$	a steady hyper GMR equilibrium state for a second-level hypergame
$SHNash^1$	a steady hyper Nash equilibrium state for a first-level hypergame
$SHNash^2$	a steady hyper Nash equilibrium state for a second-level hypergame

$SHNash^h$	a steady hyper Nash equilibrium state for an h-level hypergame
$SHSEQ^1$	a steady hyper SEQ equilibrium state for a first-level hypergame
$SHSEQ^2$	a steady hyper SEQ equilibrium state for a second-level hypergame
$SHSMR^1$	a steady hyper SMR equilibrium state for a first-level hypergame
$SHSMR^2$	a steady hyper SMR equilibrium state for a second-level hypergame
$STHGMR^1$	stealthy hyper GMR equilibrium state for a first-level hypergame
$STHGMR^2$	stealthy hyper GMR equilibrium state for a second-level hypergame
$STHNash^1$	stealthy hyper Nash equilibrium state for a first-level hypergame
$STHNash^2$	stealthy hyper Nash equilibrium state for a second-level hypergame
$STHNash^h$	stealthy hyper Nash equilibrium state for an h-level hypergame
$STHSEQ^1$	stealthy hyper SEQ equilibrium state for a first-level hypergame
$STHSEQ^2$	stealthy hyper SEQ equilibrium state for a second-level hypergame
$STHSMR^1$	stealthy hyper SMR equilibrium state for a first-level hypergame
$STHSMR^2$	stealthy hyper SMR equilibrium state for a second-level hypergame
$UCHGMR^1$	unsteady contingent hyper GMR equilibrium state for a first-level hypergame
$UCHGMR^2$	unsteady contingent hyper GMR equilibrium state for a second-level hypergame
$UCHNash^1$	unsteady contingent hyper Nash equilibrium state for a first-level hypergame
$UCHNash^2$	unsteady contingent hyper Nash equilibrium state for a second-level hypergame

$UCHNash^h$	unsteady contingent hyper Nash equilibrium state for an h-level hypergame
$UCHSEQ^1$	unsteady contingent hyper SEQ equilibrium state for a first-level hypergame
$UCHSEQ^2$	unsteady contingent hyper SEQ equilibrium state for a second-level hypergame
$UCHSMR^1$	unsteady contingent hyper SMR equilibrium state for a first-level hypergame
$UCHSMR^2$	unsteady contingent hyper SMR equilibrium state for a second-level hypergame
$UHGMR^1$	an unsteady steady hyper GMR equilibrium state for a first-level hypergame
$UHGMR^2$	an unsteady steady hyper GMR equilibrium state for a first-level hypergame
$UHNash^1$	an unsteady steady hyper Nash equilibrium state for a first-level hypergame
$UHNash^2$	an unsteady steady hyper Nash equilibrium state for a second-level hypergame
$UHNash^h$	an unsteady steady hyper Nash equilibrium state for an h-level hypergame
$UHSEQ^1$	an unsteady steady hyper SEQ equilibrium state for a first-level hypergame
$UHSEQ^2$	an unsteady steady hyper SEQ equilibrium state for a second-level hypergame

$UHSMR^1$	an unsteady steady hyper SMR equilibrium state for a first-level hypergame
$UHSMR^2$	an unsteady steady hyper SMR equilibrium state for a second-level hypergame
$USTHGMR^1$	unsteady stealthy hyper GMR equilibrium state for a first-level hypergame
$USTHGMR^2$	unsteady stealthy hyper GMR equilibrium state for a second-level hypergame
$USTHNash^1$	unsteady stealthy hyper Nash equilibrium state for a first-level hypergame
$USTHNash^2$	unsteady stealthy hyper Nash equilibrium state for a second-level hypergame
$USTHNash^h$	unsteady stealthy hyper Nash equilibrium state for an h-level hypergame
$USTHSEQ^1$	unsteady stealthy hyper SEQ equilibrium state for a first-level hypergame
$USTHSEQ^2$	unsteady stealthy hyper SEQ equilibrium state for a second-level hypergame
$USTHSMR^1$	unsteady stealthy hyper SMR equilibrium state for a first-level hypergame
$USTHSMR^2$	unsteady stealthy hyper SMR equilibrium state for a second-level hypergame
w	an ordered string of decision makers in a hypergame
x_1, x_2,..., x_{14}	*DM i's* options

Chapter 1

Introduction

Conflict is pervasive for individuals, organizations, nations, to name but a few. The Graph Model for Conflict Resolution (GMCR) (Kilgour et al., 1987; Fang et al., 1993) is a comprehensive methodology that systematically models and analyzes real-life disputes under the assumption of complete information (i.e., stakeholders have the same understanding of each other's courses of action, strategies, potential scenarios, and preferences). This consideration of completely shared perceptions among decision makers (DMs) may not always be true in reality. In fact, many conflicts are found to have an inconsistency of perceptions among DMs, which may not only alter the outcomes of the conflicts but also lead to surprising results. For example, in the Eastern Nile countries (Egypt, Ethiopia, and Sudan) dispute over the Grand Ethiopian Renaissance Dam (GERD), on April 11, 2011 the Ethiopian government surprised both Egypt and Sudan by violating 1929 and 1959 agreements and started the construction of the GERD project within the Blue Nile River without obtaining Egypt and Sudan's approval. To avoid any direct and harsh confrontation with Egypt and Sudan, Ethiopia announced its surprise decision to build the dam while Egypt and Sudan were preoccupied by critical political situations. Egypt was in the middle of the Egyptian revolution that began on January 25, 2011; and Sudan lost the southern region of the country due to the independence of South Sudan, which occurred in 2011. In Chapter 6, the GERD dispute, which is strategically modeled and analyzed within the paradigm of a hypergame for capturing misperceptions in graph form, expressed itself in several rounds at distinct points in time. In fact, because hypergame or games of misperceptions arise so often in practice, a key objective of this research is to design a comprehensive hypergame methodology within GMCR and refine this methodology by applying it to three actual disputes including the GERD controversy.

1.1 Motivation

Among the formal ways to model and analyze misperception in conflicts, the Bayesian (Harsanyi, 1967, 1968a,b) and the hypergame (Bennett, 1977, 1980; Takahashi et al., 1984; Hipel et al., 1988; Wang et al., 1988, 1989) approaches are two widely used platforms. These two approaches are different, and each has its strengths and weaknesses in modeling and analyzing real-world conflicts. The Bayesian approach is a quantitative methodology that models conflicts with incomplete information by assigning a probability distribution to its uncertain parameters, whereas the hypergame approach is a qualitative platform that considers inconsistency of perceptions among DMs by constructing a group of games, each of which represents a particular DM's viewpoint of the conflict circumstance. This framework allows DMs to formally represent and utilize misperception about their opponents' options, strategies, scenarios, and preferences. DMs' preferences in the hypergame are usually constructed by using an ordinal ranking of scenarios or states which means that classical hypergame analysis is designed for employment with only transitive preferences.

However, a number of questions can be raised. Which types of option misperception are encountered in conflict models? Can a DM's misperception about itself be modeled and analyzed within the traditional hypergame structure? Will the modeling of a hypergame be informative if all perceived courses of action are collected together to make a unified set and used to formulate possible scenarios or states? Can DMs' relative preferences in the hypergame handle both transitive and intransitive preference relationships? What strategic insights can be obtained from the hypergame equilibria? Can DMs learn from the current hypergame equilibria to improve their perception for future uses? Can the hypergame analysis be incorporated into the paradigm of GMCR?

Although the GMCR methodology relies on complete information, its efficiency in modeling scenarios based on DMs' courses of action, strategies, states, preferences, as well as its strength in qualitative analysis, motivated the author to incorporate hypergames into the current graph model structure. The modeling platform of GMCR can be used to construct states for a hypergame by using the concept of an option, which is the standard way a state is defined in the option form design of a dispute. A unified set of options can be constructed for a hypergame by including all possible DMs' courses of action (correct or incorrect) in a conflict under investigation. This new design permits the buildings of a hypergame in a very general fashion and allows the potential states to be divided into five disjoint classes. Moreover, based on these disjoint sets of states, one can classify the hypergame equilibria into meaningful categories to provide better results and insights.

1.2 Objectives

The goal of this research is to allow GMCR to model and analyze not only conflicts with complete information but also disputes having misperceptions among the engaging DMs. To accomplish this goal, the hypergame theory developed by Hipel et al. (1988) and Wang et al. (1988, 1989) needs to be refined and incorporated into the GMCR framework. In summary, the research objectives are to:

- Identify the common sources of misperception in a real-life conflict.
- Formally define hypergames within the paradigm of GMCR;
- Investigate hypergames starting at the option level rather than at the higher state

level, by (1) developing the universal set of options for a hypergame, and (2) constructing the universal set of states for a hypergame;

- Allow for a particular DM to have misperceptions about itself;
- Handle both transitive and intransitive preference relationships;
- Clearly define for 2-DM and n-DM cases hypergame models at any level of DMs' perception, for which the number of DM $n \geqslant 2$;
- Precisely explain and define how to carry out stability analyses for any level of hypergame in graph form;
- Precisely show how to calculate the overall hypergame equilibria;
- Develop a procedure to classify overall hypergame equilibria into eight classes based on the types of misperception; and
- Apply the new modeling and analysis technique of the hypergame in graph form to three actual case studies.

1.3 Organization of the Thesis

This dissertation is divided into eight chapters, as shown in Figure 1.1. The first chapter describes the motivation and objectives of this thesis. The paradigm of GMCR methodology is summarized in the first part of Chapter 2, while in the second part the concept of perception, as well as games with incomplete information, are discussed. The key original contributions of this dissertation are included in Chapters 3 to 7 as explained below:

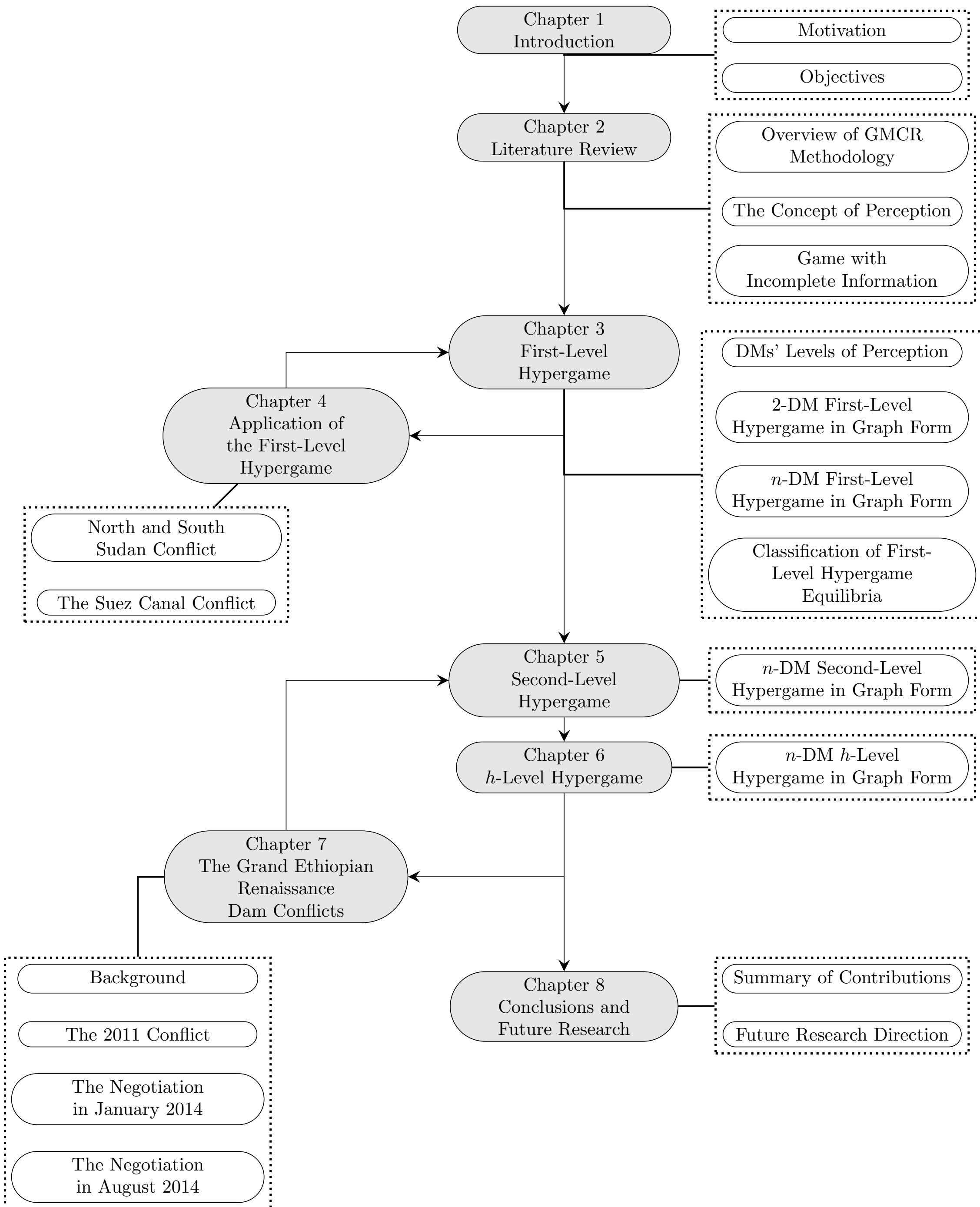


Figure 1.1: Outline of the Thesis

Chapter 3 puts forward a new modeling and analysis technique for a first-level hypergame in a dispute having two or more DMs within the framework of GMCR. This comprehensive approach consists of individual games, each of which represents a single DM's viewpoint of a conflict situation, thereby, enabling DMs to see the dispute based on their own perceptions. The chapter also includes the development of the universal sets of options and states for a first-level hypergame. Additionally, a stability analysis method is designed and implemented to calculate the equilibria for a first-level hypergame in graph form. Moreover, a classification of the first-level hypergame equilibria is developed to generate better strategic insights about the first-level hypergame situation under investigation. This foundational methodology facilitates the development of a second-level hypergame in graph form as well as the h-level hypergame, $h \geqslant 1$, in graph form presented in Chapters 5 and 6, respectively. Chapter 3 is based on three published papers by Aljefri et al. (2014a, 2015, 2017a)

Chapter 4 provides two case studies about the first-level hypergame in graph form. The first case study is about the 2011 conflict between North and South Sudan over South Sudanese oil exports; and the second one is about the 1956 Suez Canal nationalization dispute between Egypt and Britain/ US partnership. The chapter is based on the work published by Aljefri et al. (2013, 2014b, 2016a).

Chapter 5 designs a fresh concept of a second-level hypergame with two or more DMs within the GMCR model. A second-level hypergame is a decision-making situation in which each DM is playing a different game and at least one DM think that he or she possesses knowledge of the misperceptions of the other DMs. This advancement extends GMCR's usefulness by allowing it to examine not only conflicts with complete information, but also those with misperceptions. This novel technique investigates the misunderstanding of a DM about itself, its adversaries, and its competitors as contemplated by the DM. To achieve this, the sets of options and states for a first-level hypergame are extended to the universal sets of options and states for a second-level hypergame. Consequently, not only the real options and states for the dispute are included but also the misperceived ones that are thought to exist by the participating DMs. Furthermore, stability analysis procedures are introduced to analyze each DM's game or hypergame as well as the overall second-level hypergame. Moreover, a framework to classify the second-level hypergame equilibria is proposed to help an analyst to understand the source of misperceptions that exist in the dispute, the possible reactions of the DMs after they become aware of their misperceptions, and the possible ways for the DMs to successfully execute a strategic surprise. This research is largely based on the work developed by Aljefri et al. (2016b, 2017b).

In Chapter 6, a methodology is developed to capture DMs' various kinds of misperceptions in a conflict setting within the framework of GMCR. This technique models and analyzes DMs' different levels of perception in a real-life situation. It also handles a DM's misperception about itself as well as misperception about its opponents. A hypergame in graph form is a framework that consists of subjective hypergames, each of which illustrates a given player's viewpoint of the hypergame situation under investigation. Each DM's subjective hypergame is constructed in a hierarchical fashion to depict its understanding of the conflict situation and its opponents' perceptions of the situation as contemplated by that particular DM. The universal sets of options and states for the hypergame, which include all possible perceptual options and states for the conflict, are used to construct each DM's subjective hypergame. To gain strategic insights from the overall hypergame analysis, the overall hypergame equilibria are classified based on the types of misperceptions and the awareness of DMs with respect to eight classes of resolutions.

In Chapter 7, hydropolitical conflicts between the Eastern Nile countries over the Grand Ethiopian Renaissance Dam (GERD) are systematically analyzed at three points in time: just before the announcement of construction by Ethiopia on April 11, 2011, and before the negotiations in early January 2014 and late August 2014, respectively. Hypergame theory within the framework of GMCR is used to gain strategic insights into these conflicts and to ascertain the possible resolutions of the disputes. In all of these disputes, the key decision makers are Egypt and Sudan, the downstream countries, and Ethiopia, the upstream nation. The analyses demonstrate the significant utilization of strategic surprise, a decisive act in which a decision maker intentionally exercises a course of action in the dispute that is hidden to its opponents to achieve a firm outcome, in a conflict setting. The conflict investigations also show that the geopolitical and economic changes in Egypt, Sudan, and Ethiopia allow Ethiopia to construct the dam without any harsh confrontation with Egypt and Sudan. Chapter 7 is partially based on the published extended abstract by Aljefri et al. (2016c). Finally, in Chapter 8, a summary of contributions and future research directions are put forward.

Chapter 2

Literature Review

Conflict arises in almost every field of study in which people interact because individuals have different goals or value systems to satisfy when some issue occurs. Regardless of the type of dispute, it often can be described as a game by using game theoretical methods, such as metagame analysis (Howard, 1971), conflict analysis (Fraser and Hipel, 1984), and the Graph Model for Conflict Resolution (GMCR) (Kilgour et al., 1987; Fang et al., 1993; Kilgour and Hipel, 2005; Hipel, 2009a,b; Kilgour and Eden, 2010). These techniques can be used to calculate the possible equilibria or resolutions of the real-world conflict and provide valuable strategic insights. These methods are considered to be qualitative because each DM's preference between any two states is expressed in a relative fashion by pairwise comparisons (Fraser and Hipel, 1984; Fang et al., 1993; Hipel, 2009a,b). In contrast, classical game theory is interpreted as being quantitative because it uses a cardinal utility function to capture each DM's preferences among the set of possible states (Nash, 1950, 1951; Von Neumann and Morgenstern, 1944). For a summary of the aforementioned methodologies, the reader is referred to the work detailed by Kilgour and Eden (2010) and Hipel (2009a,b). Due to GMCR's simplicity and flexibility, it is widely used to model and analyze a large range of real-life conflicts. However, it models and investigates disputes only under the assumption of complete knowledge and common perception among the engaging DMs. Since the objective of this research is to incorporate misperception into GMCR, the structure of GMCR is presented next.

2.1 The Graph Model for Conflict Resolution

GMCR is a tool for systematically analyzing real-life disputes (Kilgour et al., 1987; Fang et al., 1993; Kilgour and Hipel, 2005). In GMCR, the possible compromise resolutions for the conflict are ascertained by examining the participating DMs' moves and counter-moves according to a range of solution concepts (also called stability definitions). The overall architecture of GMCR is shown in Figure 2.1, and described below (Kilgour et al., 1987; Fang et al., 1993; Kilgour and Hipel, 2005).

2.1.1 Procedures

As can be seen in Figure 2.1, GMCR consists of two modules: modeling and analysis. The modeling of a conflict starts by identifying all DMs, their options (also called courses of action), and their preference information. Next, a set of feasible states for the model is constructed. Finally, each DM's state transitions and its relative preference over the set of feasible states are identified. In the analysis module, the stability of each state for each DM is determined by using range of solution concepts. State that is stable for all DMs according to a particular solution concept constitutes a possible equilibrium for the dispute. Then a sensitivity analysis is performed to verify the robustness of the equilibria of the model. Computerized decision support systems called GMCR II (Fang et al., 2003a,b) or GMCR + (Kinsara et al., 2015b), can be utilized to model and analyze any real-life dispute having complete information.

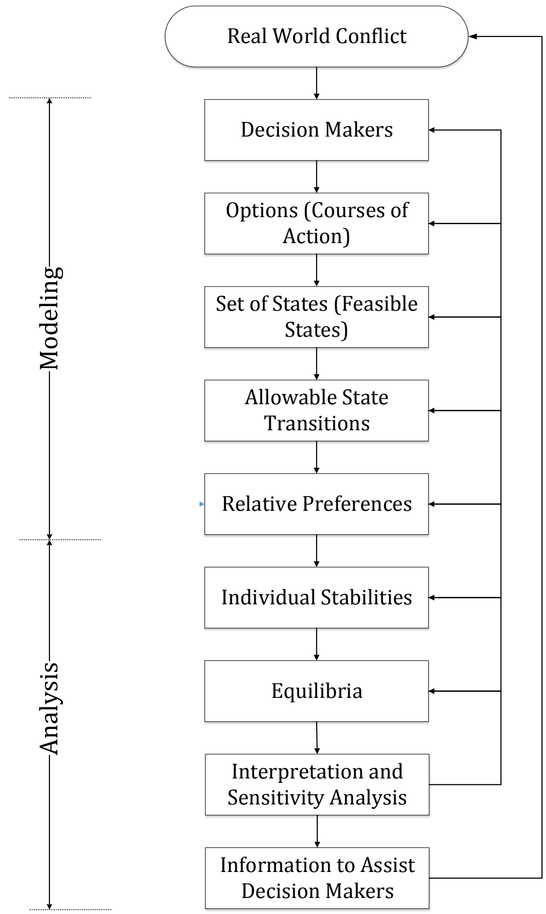


Figure 2.1: Architecture of the Graph Model for Conflict Resolution

2.1.2 Notation and Definition

The main elements of GMCR are a set of decision makers (DMs), a set of states that depict the possible scenarios of a real-life dispute, each DM's possible moves among the states, and each DM's relative preferences over the states. A directed graph is constructed to depict a DM's allowable state-to-state movement. The states represent the nodes in all DMs' directed graphs, and the arc represents the DMs' possible movements. In graph form, DMs' preferences over the set of states are given by a binary relation.

Let $N = \{1, 2, ..., i, ..., n\}$ be a non-empty, finite set that denotes the set of DMs. S is the set of states for a dispute, that denote all possible scenarios of a real life conflict and are represented by the vertices in a directed graph. A_i signifies *DM i's* allowable state transitions from one state to another in one step, which are represented by arcs in a directed graph. Lastly, the script $\succsim_i$ expresses *DM i's* weak preference relations over the set of possible states, where $s_1 \succsim_i s_2$ means that state s_1 is more preferred or equally preferred to state s_2 by *DM i*. $s_1 \succ_i s_2$ indicates that s_1 is more preferred to s_2 by *DM i*, and $s_1 \sim_i s_2$ means that s_1 is equally preferred to s_2 by *DM i*. With the notation given above, the graph model G, which represent a real-life conflict under the assumption of shared perception among DMs, can be expressed as

$$G = \langle N, S, \{A_i : i \in N\}, \{\succsim_i : i \in N\}\rangle \tag{2.1}$$

As indicted in Eq. 2.1, a basic unit of a graph model is a state. A convenient way to define a state is to utilize the concept of an option (Howard, 1971), which is the way a state is defined in option form representation of a conflict (Kilgour et al., 1987). More specifically, in a given conflict, each DM has under its control a set of one or more different options, each of which can be selected or not by the DM. When a particular DM decides upon which options to choose or not, the resulting choice is called a strategy for the DM. When all of the DMs participating in the conflict have selected a strategy, the result is referred to as a state. The definition of a state using option form is given as follows.

For each $DM\ i \in N$, the set $O_i = \{o^i_{\bar{k}} : \bar{k} = 1, 2, ..., m_i\}$ is *DM i's* set of options, in which $o^i_{\bar{k}}$ is *DM i's* $\bar{k}^{th}$ option and m_i represents the total number of options for *DM i*. Mathematically, a strategy for *DM i* can be represented by a mapping function $g_i :\ O_i \longrightarrow \{0, 1\}$, such that for $\bar{k} = 1, 2, ..., m_i$,

$$g_i(o^i_{\bar{k}}) = \begin{cases} 1, & \text{if } DM\ i \text{ selects option } o^i_{\bar{k}} \\ 0, & \text{otherwise} \end{cases}$$

Furthermore, let $O = \cup_{i \in N} O_i$ denote the set of options for all DMs. A state is mapping $f : O \rightarrow \{0, 1\}$ such that,

$$f(o^i_{\bar{k}}) = \begin{cases} 1, & \text{if } DM\ i \text{ selects option } o^i_{\bar{k}},\ for\ i =\ 1,\ 2, ...,\ n, \\ 0, & \text{otherwise} \end{cases}$$

A state is represented by a λ-dimensional column vector, where λ is the total number of options in O. Note that a typical state defined above is a vector of the form: $(f(o^1_1)$, $(f(o^1_2)$, ..., $(f(o^1_{m_1})$,..., $f(o^n_1)$, $(f(o^n_2)$,..., $(f(o^n_{m_n}))^T$. Since each option can either be selected or not by the DM who controls it, total number of states for a dispute can be mathematically calculated by 2^λ. The set of mathematically possible states is denoted and expressed as $S = \{s_1, s_2, ..., s_{2^\lambda}\}$. However, some states are removed from the model because they are categorized according to four types of option conditions to be infeasible: mutually exclusive options, at least one option, option dependence, and direct specification (Fang et al., 2003a,b). Consequently, the remaining states are considered as feasible states for the dispute. For $N = \{1, 2, ..., i, ..., n\}$, DMs' strategies associated with state $s_1 \in S$ are expressed as $g^{s_1}_1$, $g^{s_1}_2$,..., $g^{s_1}_i$,..., $g^{s_1}_n$. Thus, $s_1 = ((g^{s_1}_1)^T, (g^{s_1}_2)^T, ..., (g^{s_1}_i)^T, ..., (g^{s_1}_n)^T)^T$.

The definition in Eq. 2.1 specifies the conflict model under complete information. Hence, stakeholders are entirely aware of each other's options and preferences. That is, the zero-level hypergame, H^0, (Bennett, 1977, 1980; Takahashi et al., 1984; Hipel et al., 1988; Wang et al., 1988, 1989; Aljefri et al., 2017a,b) in graph form, which describes the game under complete information, can be represented by Eq. 2.1.

2.1.3 Stability Definitions

A range of solution concepts are defined within the paradigm of GMCR to predict the possible compromise resolution for the dispute. These stability definitions answer what-if questions in term of what can happen when DMs strategically interact using moves and counter-moves. The four solution concepts that evaluate the stability of a state for each DM are Nash stability (Nash, 1950, 1951), sequential stability (SEQ) (Fraser and Hipel, 1979, 1984), general metarationality (GMR) (Howard, 1971), and symmetric metarationality (SMR) (Howard, 1971). To perform a stability analysis for an n-DM graph model, the concepts of reachable list and unilateral improvement by a group of DMs must be defined first.

Let $H \subseteq N$, $H \neq \emptyset$, be any subset of DMs in N. For $s_1 \in S$ and $q \in N$, let $R_q(s_1)$ represents the set of *DM q's* unilateral moves (UMs) starting from s_1. Let $R_H(s_1)$ denote the set of all UMs from s_1 by one or more DMs in H through a valid sequence of moves starting from s_1. A sequence of moves by DMs in H is considered valid if no DM makes two consecutive moves. For $s_2 \in R_H(s_1)$, let $\Omega_H(s_1,\ s_2)$ denote the set of all last DMs in H in the valid sequences of moves from s_1 to s_2. The reachable list by $H \subseteq N$ can now be formalized as follows.

Definition 2.1.3.1 (**Reachable List by** $H \subseteq N$)**.** *Let $s_1 \in S$. Then, $R_H(s_1)$ can be defined as follows:*

- *If $q \in H$ and $s_2 \in R_q(s_1)$, then $s_2 \in R_H(s_1)$ and $q \in \Omega_H(s_1,\ s_2)$;*
- *If $s_2 \in R_H(s_1)$, $q \in H$, and $s_3 \in R_q(s_2)$, then*

  (a) if $|\Omega_H(s_1,\ s_2)| = 1$ and $q \notin \Omega_H(s_1,\ s_2)$, then $s_3 \in R_H(s_1)$ and $q \in \Omega_H(s_1,\ s_3)$.

  (b) if $|\Omega_H(s_1,\ s_2)| > 1$, then $s_3 \in R_H(s_1)$ and $q \in \Omega_H(s_1,\ s_3)$.

  The induction stops when there is no new state s_3 that can be added to $R_H(s_1)$ and no change from $|\ \Omega_H(s_1,\ s_2)\ | = 1$ to $|\ \Omega_H(s_1,\ s_2)\ | > 1$ for any $s_2 \in R_H(s_1)$. Any state in $R_H(s_1)$ is a UM from s_1 by H.

A state is considered as a unilateral improvement (UI) from a prespecified state by a particular DM if the state is reachable by the DM and is preferred to the initial state. The set of all UIs from a state s_1 by $DM\ q$ is referred to as the unilateral improvement list (UIL) from s_1 by $DM\ q$, denoted by $R_q^+(s_1)$. Let $R_H^+(s_1)$ denote the set of UIs from s_1 by a group of DMs, $H \subseteq N$ and $H \neq \emptyset$. Also, let $\Omega_H^+(s_1,\ s_2)$ represent the set of all last DMs in valid sequences of unilateral improvements from s_1 to s_2. $R_H^+(s_1)$ is defined as follows.

Definition 2.1.3.2 (**Unilateral Improvement List by** $H \subseteq N$)**.** *Let $s_1 \in S$. The UIL $R_H^+(s_1)$ is constructed inductively as follows:*

- *If $q \in H$ and $s_2 \in R_q^+(s_1)$, then $s_2 \in R_H^+(s_1)$ and $q \in \Omega_H^+(s_1,\ s_2)$;*
- *If $s_2 \in R_H^+(s_1)$, $q \in H$, and $s_3 \in R_q^+(s_2)$, then*

  (a) if $\mid \Omega_H^+(s_1,\ s_2) \mid = 1$ and $q \notin \Omega_H^+(s_1,\ s_2)$, then $s_3 \in R_H^+(s_1)$ and $q \in \Omega_H^+(s_1,\ s_3)$,

  (b) if $\mid \Omega_H^+(s_1,\ s_2) \mid > 1$, then $s_3 \in R_H^+(s_1)$ and $q \in \Omega_H^+(s_1,\ s_3)$.

The induction stops when there is no new state s_3 that can be added to $R_H^+(s_1)$ and no change from $\mid \Omega_H^+(s_1,\ s_2) \mid = 1$ to $\mid \Omega_H^+(s_1,\ s_2) \mid > 1$ for any $s_2 \in R_H^+(s_1)$. Any state in $R_H^+(s_1)$ is a UI from s_1 by a group of DMs H.

Now that the concepts of reachable list and UIL by a set of DMs $H \subseteq N$ have been introduced, one can formally define stability concepts in G with more than two DMs. The stability definitions put forward here are Nash stability, SEQ stability, GMR stability, and SMR stability.

Definition 2.1.3.3 (**Nash Stability**)**.** *A state $s_1 \in S$ is Nash stable (Nash) for DM $q \in N$ in $G \Longleftrightarrow R_q^+(s_1) = \emptyset$. The set of all Nash stable states for DM q in G is denoted by S^{Nash_q}.*

Definition 2.1.3.4 (**SEQ Stability**)**.** *A state $s_1 \in S$ is sequentially stable (SEQ) for DM $q \in N$ in $G \Longleftrightarrow$ for each $s_2 \in R_q^+(s_1)$, $\exists\ s_3 \in R_{N-\{q\}}^+(s_2)$ such that $s_3 \precsim_q s_1$. The set of all SEQ stable states for DM q in G is denoted by S^{SEQ_q}.*

Definition 2.1.3.5 (**GMR Stability**). *A state* $s_1 \in S$ *is general metarational stable (GMR) for DM* $q \in N$ *in* $G \iff$ *for each* $s_2 \in R_q^+(s_1)$, $\exists\ s_3 \in R_{N-\{q\}}(s_2)$ *such that* $s_3 \precsim_q s_1$. *The set of all GMR stable states for DM q in G is denoted by* S^{GMR_q}.

Definition 2.1.3.6 (**SMR Stability**). *A state* $s_1 \in S$ *is symmetric metarational stable (SMR) for DM* $q \in N$ *in* $G \iff$ *for each* $s_2 \in R_q^+(s_1)$, $\exists\ s_3 \in R_{N-\{q\}}(s_2)$ *such that* $s_3 \precsim_q s_1$, *and* $s_4 \precsim_q s_1$, $\forall\ s_4 \in R_q(s_3)$. *The set of all SMR stable states for DM q in G is denoted by* S^{SMR_q}.

Having defined the aforementioned stabilities, one can now define the equilibria for the conflict. A state is considered as an equilibrium under a specific stability definition iff it is stable for every DM under the same stability notion. Formally,

Definition 2.1.3.7 (**Equilibrium**). *A state* $s_1 \in S$ *that is stable for every DM according to a particular solution concept is an equilibrium for the game under that particular solution concept. The set of all equilibrium states in G is denoted by E.*

2.2 Perception and Conflict Analysis

Perception is the process of transforming real-life reality into a self-centered context (Rummel, 1975; Passer et al., 2011). People's perception of reality is formed based on their particular beliefs, motivations, interests, experiences, and knowledge, among other factors. Accordingly, different people sometimes have inconsistent understandings about a particular situation. People's experience of reality does not operate as a one-to-one correspondence with what occurs in the real world; instead, different perceptions are obtained by different people (Jervis, 1968, 1976; Passer et al., 2011). People's inner thought processes act as a filter that determine which aspect of a real-life situation is observed. Based on the ideas presented by Jervis (1968, 1976) and Passer et al. (2011), Figure 2.2 shows the system of perception.

When DMs have complete information about a conflict situation, the dispute is modeled and analyzed as one game. However, under misperception, the modeling and analysis of a real-life dispute become more challenging (Jervis, 1968, 1976; Passer et al., 2011). More specifically, under misperception, DMs have different views of the conflict situation and may not be fully aware of each other's options, or courses of action, and preferences (Bennett, 1977; Bennett and Dando, 1979; Wang et al., 1988, 1989). Therefore, the dispute is modeled as a collection of games, each of which depicts a focal DM's viewpoint of the circumstances. A hypergame approach is designed to examine conflicts that have some kind of misperception by one or more DMs (Bennett, 1977, 1980; Takahashi et al., 1984; Hipel et al., 1988; Wang et al., 1988, 1989). This methodology allows one to predict the possible resolutions of a dispute when DMs hold asymmetric perceptions.

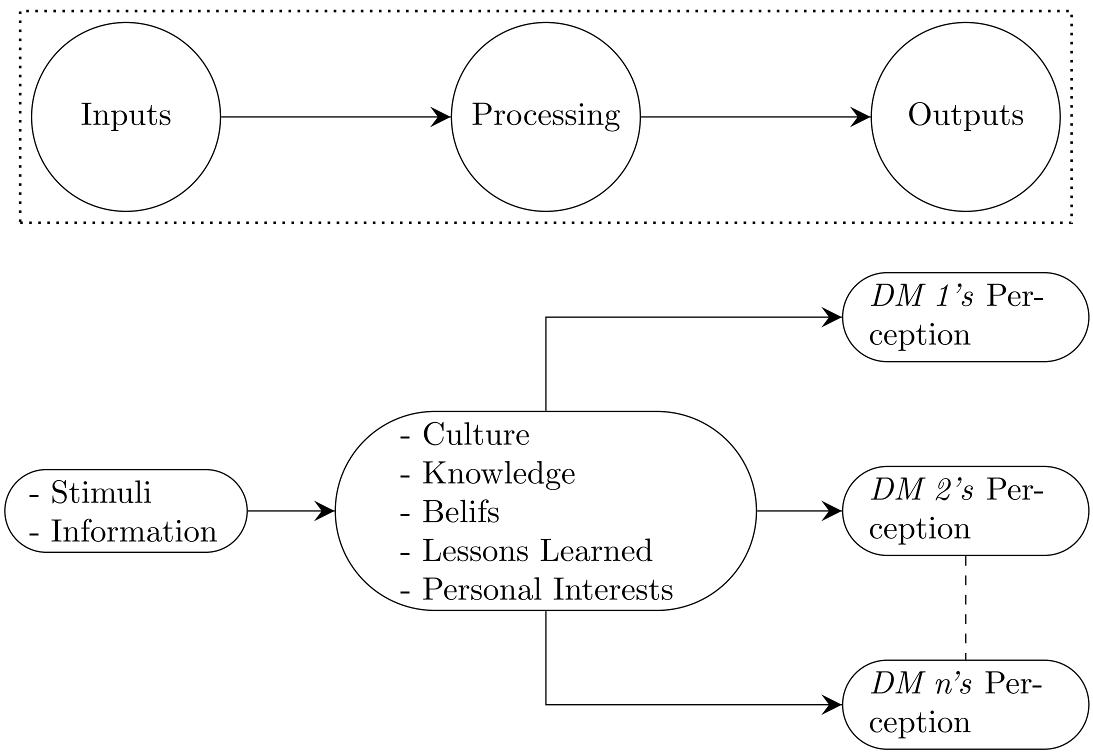


Figure 2.2: Perception System (Jervis, 1968, 1976; Passer et al., 2011)

In this thesis, a perceptual-based approach is utilized rather than a knowledge-founded method to model DMs' different understandings and interpretations of a particular situation. The reasons behind selecting the perceptual-based procedure to incorporate misperception into GMCR are as follows. First, the word perception is based on the idea of interpretation and understanding, whereas the word knowledge is founded on information, or experience one can obtain through education or training. Second, based on the top-down approach to the concept of perception that is presented by Passer et al. (2011) and shown in Figure 2.2, one can see that DMs' internal factors, such as knowledge and culture, alter their interpretation of a real-life situation which may lead to an incorrect or incomplete understanding of the circumstance under investigation. Third, in classical game theory, knowledge is regarded as the information available to DMs about a particular situation. Finally, the literature on hypergame theory starting from the work of Bennett (1977) all the way to the improved hypergame analysis developed by Wang et al. (1988, 1989), the terminologies of perception and misperception are used to label DMs' different viewpoints about a real-life situation. The study of misperception in conflict analysis is reviewed in the next section.

2.2.1 Misperception in Conflict

Classical game theory provides a set of methods for modeling and analyzing conflicts. The notion of the theory goes back to Von Neumann and Morgenstern (1944), and has been widely utilized to examine disputes in many fields of study such as economics, environmental management, energy, military science, and information security. Although game theory has been demonstrated to be a useful methodology for analyzing a range of conflict situations, there are nevertheless limitations to the approach, thereby restricting its application in modeling real-life situations. The assumption of complete information (i.e., that participants have full knowledge about the situation they are facing) has been shown to be a shortcoming by many scholars, who have pointed out the need for incorporating incomplete information into the game theory framework (Von Neumann and Morgenstern, 1944; Bennett, 1977, 1980; Takahashi et al., 1984; Hipel et al., 1988; Wang et al., 1988, 1989; Luce and Raiffa, 1957; Harsanyi, 1967, 1968a,b; Brams, 1977; Aumann and Maschler, 1995; Inohara et al., 1997; Obeidi et al., 2005, 2009; Sasaki and Kijima, 2008; Gharesifard and Cortes, 2011, 2012; Sasaki et al., 2015; Walker et al., 2013; Rêgo and dos Santos, 2015; Kuang et al., 2015). If the participants' preferences and other parameters of the game are represented by numerical payoff functions, then the game is modeled quantitatively. A significant amount of research has been conducted in this area. For example, the work of Harsanyi (1967, 1968a,b) formulated a Bayesian approach to handle incomplete information within the structure of game theory. Under the assumption of cardinal utility values and uncertain parameters modeled by probability distributions, DMs are assumed to take actions that maximize their payoffs. Drawbacks of Harsanyi's approach include high mathematical complexity and the assumption of having cardinal utility values which may be difficult to obtain in practice.

On the other hand, if the game is constructed according to the set of DMs, set of options for each DM, and each DM's relative preferences among the set of possible outcomes, then the game is analyzed qualitatively. In the direction of qualitative analysis and incomplete information, Brams (1977) and Stein (1982) developed methods that account for the role of deception and misperception for 2-DM strictly ordinal games. In their work, the possible circumstances for deception and preference misperception are introduced into the game in normal form. For instance, under asymmetry of information, the deceiver is assumed to have more information than its opponent and is trying to utilize this gap to achieve better results. However, under misperception, the real-life situation is misunderstood by each DM, leading them to face unexpected results. The work of Brams (1977) and Stein (1982) provides the primary foundation for the development of hypergame analysis.

Although quantitative analysis is widely used in modeling real-life disputes, its application has been found to be limited as it requires a large amount of numerical information regarding the game parameters which is not easy to find in real-life circumstances. Thus, this research is in the direction of qualitative analysis, and particularly focuses on improving the hypergame modeling and analysis procedures. In the next section, hypergame analysis is discussed.

2.2.2 Hypergames

Since GMCR is based on a qualitative analysis, it is appropriate to look at the hypergame which is also based on the qualitative analysis. A hypergame is a comprehensive procedure that investigates differing perception (correct, incorrect, or incomplete) in conflict. It is constructed in a hierarchical fashion to account for different levels of DMs' perception about the real-life situation. This methodology permits one to evaluate the consequences of a DM's misperception in a real-life situation. It also provides solutions for a complex decision situation that may include ambiguity (Song et al., 2009), lack of information, and asymmetry of perception among DMs. A hypergame can be used as a mediation, negotiation, or consulting tool. Furthermore, when a given DM is not sure of its opponents' true preferences and options under their control in the conflict, then a hypergame can be used to represent the conflict situation as seen by the focal DM (Rosenhead, 1989; Rosenhead and Mingers, 2001).

The notion of a hypergame was first proposed by Bennett (1977), using the normal form of the game, which was applied by Bennett and Dando (1979) to the Battle of France in 1940. Bennett (1980) extended his theory of hypergames in normal form to account for levels of misperception. However, this early research in hypergames represented in normal form only considered Nash stability as used by Howard (1971) in metagame analysis. Additionally, although a given DM could have misperceptions about himself or herself, this aspect of misperception was not pursued in detail and hence the focus was on misperception by opponents.

Takahashi et al. (1984) incorporated the conflict analysis of Fraser and Hipel (1984, 1979) into the paradigm of a hypergame to study and analyze strategic conflict and predict the possible compromise resolutions for any type of hypergame. Each DM's subjective game in a hypergame is represented in option form (Howard, 1971) and DMs' preferences among the set of possible scenarios are expressed by an ordinal ranking. Takahashi et al. (1984) approach has been utilized to model many real-life conflicts, such as the Normandy invasion of 1945, the US-USSR nuclear confrontation (Fraser et al., 1983), and the Lake Biwa conflicts (Okada et al., 1985). In addition, Hipel et al. (1988) and Wang et al. (1988, 1989) provide mathematical definitions for both the modeling and the analysis procedures for hypergame analysis, thereby extending its applicability in investigating real-life disputes. Also, various levels of a hypergame are constructed in a hierarchical order to represent DMs' order of expectation. For instance, if the participants are assumed to be playing the same game and aware of each other's options, strategies, and preferences, then the game is a simple game or a zero-level hypergame, denoted as H^0. If, on the other hand, some DMs misperceive the real-life situation, then a set of subjective games is constructed to represent each DM's viewpoint of the conflict situation. In this case, the game is called a first-level hypergame, denoted H^1. Moreover, if at least one DM is aware of its opponents' subjective games, then it will consider them in analyzing the dispute, and the game is a second-level hypergame, denoted H^2. In other words, a given DM is aware of its opponent's misperception. Furthermore, if at least one DM is aware of other DMs' second-level hypergames, then the game is a third-level hypergame, indicated as H^3. In fact, based on a DM's perception, the level of hypergame can be extended to any level of expectation. Moreover, a range of solution concepts (Nash, 1950, 1951; Howard, 1971; Fraser and Hipel, 1984) was introduced into hypergame analysis. DMs' preferences among the set of possible states are represented in an ordinal fashion. Additionally, the hypergame approach of Hipel et al. (1988) and Wang et al. (1988, 1989) can be utilized to model any finite number of DMs and options, and can be modeled in both normal and option forms. In their mathematical theory of hypergames, Wang et al. (1988, 1989) recognized that a DM could have misperceptions about himself or herself but decided not to take this into account in the development of their theory. Therefore, it only models a given DM's misperception about its opponents. For instance, a DM can misperceive other DMs' preferences, options, and strategies. Under option misperception developed by Hipel et al. (1988) and Wang et al. (1988, 1989), a DM can imagine some potential and/or unreal options for its opponents, misunderstand some of its opponents' options, and be unaware of one or more courses of action available to its opponents.

Although this improved hypergame analysis is found to be useful in modeling real-life disputes, it nevertheless has some shortcomings that may limit its applicability. For instance, in a hypergame, each DM's subjective game is modeled independently and, as a result, the states in each subjective game are defined separately. Therefore, one cannot distinguish between the states that are correctly perceived or misperceived among all DMs and those that are perceived individually. This in turn places limitations on the analysis of the model. Also, the modeling of a hypergame is only provided for ordinal preferences, which means that transitive preferences are assumed. Moreover, as just mentioned, although the hypergame procedure of Wang et al. (1988, 1989) as well as the normal form definition of hypergames provided by Bennett (1977, 1980) could have been appropriately expanded mathematically to handle in detail self-misperceptions, this was not the case. Therefore, the mathematical theory of hypergames can be appropriately extended to more explicitly account for self-misperception. In other words, the types of misperceptions that a DM can have about itself and the method used for mathematically defining them were not fully addressed in their work. Also, situations like common moves and irreversible moves cannot be modeled and graphically represented within their hypergame approach. Furthermore, hypergame analysis was not formally defined within the paradigm of GMCR, which can formally model disputes and graphically represent all DMs' possible moves and counter-moves among the set of possible states.

The classification of hypergame equilibria by Wang et al. (1988, 1989) is limited in scope. In particular, they classified the hypergame equilibria into two broad categories: hypergame preserving and destroying equilibria. These two equilibrium classes cannot specify explicitly the types of misperceptions that affect the hypergame. Also, they cannot differentiate between the equilibrium states that constitute strategic surprise and the other different equilibrium classes. Moreover, they do not take into account a DM's self-misperception as stated by them. Also, Sasaki and Kijima (2008) developed the concept of a stable hyper Nash equilibrium within the framework of Wang et al. (1988, 1989). This definition is limited to the Nash solution concept and to preference misperception.

More recently, Obeidi et al. (2005, 2009) put forward a method, within the paradigm of GMCR, to study and analyze the effect of DMs' emotions on strategic conflicts. The primary components of this technique were derived from the standard GMCR framework. Based on a particular DM's emotion, a set of feasible states in the standard GMCR approach, which models a real-life dispute under the assumption of complete information, is partitioned into the following three groups: (1) hidden states, (2) potential states, and (3) recognizable states. Hidden states cannot be observed because of the DM's negative emotion, and a DM cannot recognize potential states because of the absence of positive emotion. Lastly, if a state is not categorized as either hidden or potential, then the DM is aware of it. Because DMs do not share the same emotions, a system of integrated graph models is introduced to represent each DM's perception of the conflict situation. Although the sets of a DM's state transitions and relative preferences in the standard GMCR structure are considered to be preserved, discrepancies in recognizing the set of feasible states in the standard GMCR model alter the sets of state transitions and relative preferences perceived by the DM. Perceptual stability analysis is then used to calculate the possible compromise resolutions for the dispute.

Although the applicability of the perceptual graph method of Obeidi et al. (2005, 2009) has been tested in real-life case studies, certain limitations are associated with it and need to be addressed. For instance, this method investigates misperception caused by DMs' emotions starting at the level of a state rather than the option level. As a result, various types of option misperceptions cannot be modeled within its current structure. Since DMs' subjective games are mapped from the standard GMCR structure, only misperception of unknown real options can be accommodated within this paradigm. Also, this method classifies the standard GMCR's set of feasible states into three categories: hidden, potential, and recognized states. However, it does not provide any criteria for classifying them mathematically. Moreover, this approach assumes that a DM's sets of state transitions and relative preferences are altered only when the DM misperceives the set of feasible states of the standard GMCR. In real-life disputes, however, it is possible for a DM to perceive the standard GMCR set of feasible states correctly but still misperceive another DM's relative preferences.

In summary, all the reviewed papers studied and analyzed hypergame analysis under either a game theory framework or an improved conflict analysis. Either cardinal utility function or ordinal ranking (which restricts the hypergame approach to transitive preferences) was used to represent DMs' preferences among the set of possible states. A DM's misperception about itself was never addressed in the reviewed literature. Finally, hypergame analysis was not incorporated into the structure of GMCR.

There is a stream of publications in the literature that looked at the study of learning within the context of hypergame theory to understand how DMs' can update their perception after they become aware of the true situation. For instance, Sasaki et al. (2007), Sasaki and Kijima (2008), and Sasaki et al. (2015) introduced a new solution concept, called a stable hyper Nash equilibrium, within the paradigm of hypergame theory. The objective of the new definition is to study the effect of hypergame equilibria on DMs' perceptions. If a state is classified as a stable hyper Nash equilibrium for the dispute, then DMs' misperception are preserved and the equilibrium state is considered as a final resolution for the conflict. However, if a state is not classified as a stable hyper Nash equilibrium for the conflict, then new information may be accessible to DMs which may motivate them to escalate the situation if they can. Drawbacks of Sasaki et al. (2007), Sasaki and Kijima (2008), and Sasaki et al. (2015) approach include the following. This approach is limited to preference misperceptions (i.e other sources of misperception is not addressed). Additionally, the approach did not provide any mathematical procedure that enable DMs to update their perception about their opponents' preferences. Finally, this method is limited to Nash equilibria (i.e. other solution concepts are not taking into account). Furthermore, Gharesifard and Cortes (2011, 2012) presented a new definition of learning under perfect observation within the structure of a first-level hypergame developed by Wang et al. (1988, 1989). The new concept, designated as swap learning, allows DMs to improve their perceptions about their opponents' preferences after they realize that their perceptions about their opponents are incorrect. Subsequently, Gharesifard and Cortes (2014) utilized the notion of swap learning to model deceptive situations within a second-level hypergame of Wang et al. (1988, 1989). Within their technique, if at least one DM is aware of the inconsistency of beliefs among DMs, then the DM may take advantage of this discrepancy of information and try to deceive its opponents by revealing erroneous information to them. This approach is limited to preference misperceptions and to the Nash solution concept.

Chapter 3

First-Level Hypergame in Graph Form

Conflicts can vary from gentle differences of opinion of the decision makers (DMs) to violent confrontations (Fraser and Hipel, 1984; Fang et al., 1993; Hipel, 2009a,b). Conflicts arise when DMs hold dissimilar goals about the situation under investigation. These incompatible goals of the DMs may quickly change from being conflicts of interest to rigid hostile actions as a consequence of the (correct or incorrect) perception of the circumstances by the different DMs. Situations in which the DMs, also known as players, have a common understanding, can be modeled and analyzed as a single game using the framework of GMCR (Fang et al., 1993; Kilgour et al., 1987). This methodology, which is described in Section 2.1, investigates the possible moves and counter-moves of DMs by employing a collection of solution concepts, also called stability definitions, which replicate the ways in which humans may interact in a conflict situation (Nash, 1950, 1951; Howard, 1971; Fraser and Hipel, 1979, 1984). However, in certain real-world conflicts, the involved DMs may not have a common understanding about the actual situation. This may be due to incomplete information, the experience gained and lessons learned by a DM that alter his or her perception, which may contain misunderstandings. Thus, a conflict situation can be captured via a number of subjective games, each of which depicts the perception of a given DM about the real-life conflict. The framework of a first-level hypergame investigates a conflict situation having misperception among DMs in which no one is aware of the other DMs' misperceptions.

In this chapter, a first-level hypergame for the case of two- and n-DM ($n \geqslant 2$) conflicts is introduced for the first time within the structure of GMCR. The new modeling and analysis approach for a first-level hypergame in graph form permits one to model for the first time self misperception. The procedures also explores the common sources of misperception that may occur in real-life conflicts. It also handles both transitive and intransitive preference relationships. Additionally, refined stability definitions are introduced and implemented to calculate the equilibria for a first-level hypergame. Also, a classification of the first-level hypergame equilibria is presented to capture the source of misperception that causes the dispute as well as the way DMs are expected to behave after they become aware of their misperception in reality. This chapter is based on three publications by Aljefri et al. (2014a, 2015, 2017a)

3.1 Decision Makers' Levels of Perception in Hypergames

A game with complete information in graph form is viewed as a face-to-face confrontation. More specifically, the DMs engaging in the dispute are found to be playing the same game; the DMs see the same sets of DMs, states, preferences, and state transitions. As a consequence, the equilibria of the game are anticipated by every DM in the dispute.

Unlike a game with complete knowledge and perception among DMs, when misperceptions or hypergames exist players are not viewing the same conflict situation. Rather, they perceive different games, each of which reflects a player's viewpoint of the situation under investigation. A hypergame in graph form is designed to construct a conflict situation under different levels of DMs' perceptions. Figure 3.1 depicts the relationship between a real-life situation and the perceived conflict up to a third level of perceptions for a 3-DM hypergame.

Starting from the far left of Figure 3.1, G depicts a zero-level hypergame H^0. G is identical to the graph model defined in Section 2.1 as it expresses a conflict situation under complete information. As a result, DMs have no misperception about the conflict situation. Hence, a conflict's parameters such as the preference relations $\succsim_i$, $\forall\ i \in N$, are correctly comprehended by all DMs engaging in the dispute. $\succsim_i$ denotes *DM i's* preference relations over the set of states. The subscript, i, in $\succsim_i$ indicates the ownership of the set, which as mentioned earlier represents *DM i's* set of preferences in G. With only one subscript, the level of DMs' perception is equals to zero.

As can be seen in Figure 3.1, the design of a first-level hypergame H^1 starts by constructing each player's subjective game, each DM's subjective game is a zero-level hypergame. As a result of the asymmetry of viewpoints among DMs, each subjective game describes a particular player's perception of the conflict situation. For instance, *DM i's* subject game G_i reflects the conflict condition as contemplated by itself. That is, *DM i* perceives all the information of the dispute, such as the DMs' preferences in a subjective manner. Let $\succsim_{ji}$, $\forall\ j \in N_i$, denote *DM j's* preference relations as seen by *DM i*, where $N_i \subseteq N$ denotes the set of DMs as seen by *DM i*. There are two subscripts in $\succsim_{ji}$. The first indicates the ownership of the set; the second depicts the DM who perceives the set. Hence, in H^1, the level of perception equals 1. Within H^1, DMs are not aware of any misperception happening and they assume that their games are the actual ones.

In a second-level hypergame H^2, at least one or more DMs know that they are playing different subjective games. *DM i*, for example, will try to predict not only its opponents' parameters in its subjective game G_i but also try to predict how its opponents view of the conflict parameters. For example, in *DM j's* game as seen by *DM i*, G_{ji}, let $\succsim_{kji}$, $\forall k \in N_{ji},\ j \in N_i$, reflect how *DM i* sees *DM j's* view about *DM k's* preference relations, where N_{ji} denotes the set of DMs as seen by *DM j* and then as contemplated by *DM i*.

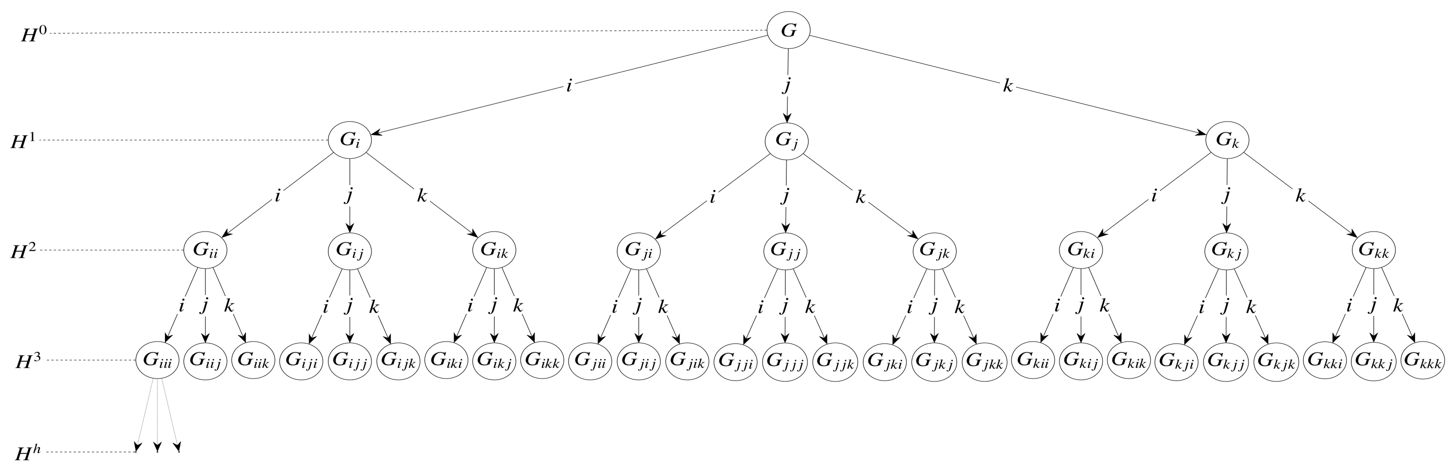


Figure 3.1: Logical Structure of a Hypergame in Graph Form

The subscript *"kji"* in $\succsim_{kji}$ has two parts: the first subscript, k, represents the ownership of the set; the other subscripts, ji, accounts for the order of DMs' perception. In H^2 the level of perception equals two. In fact, the level of a hypergame can be increased to any h-levels of DMs' perception. For $N = \{1, 2, ..., i, ..., n\}$, *DM i's* h^{th}-level of perception can be described as follows:

Definition 3.1.0.1 (***DM i's*** h^{th}**-level of Perception**)**.** *DM i's h^{th}-level of perception in an n-DM hypergame is expressed as what DM i sees DM j's $(h-1)$ level of perception of DM k's $(h-2)$ level of perception... of DM q's understanding of the conflict situation to be. Let $\succsim_{q...kji}$ stand for DM i's h^{th}-level of perception of DM q's preferences. The string q...kji contains (h+1) items and the level of perception is h.*

The five types of misperceptions that can be taken into account utilizing the hypergame framework in graph form are:

- Option misperceptions held by and about a player and its adversaries.
- Preference misperceptions held by and about a player and its adversaries.
- A lack of knowledge of the number of players engaging in the dispute.
- Wrong interpretation of the level of players' perceptions.
- Any collection of the above.

Remark 3.1.0.1. *Self misperception is permitted within hypergame theory in graph form. Hence, it is assumed that $\succsim_i \neq \succsim_{ii}$ whenever DM i misconceives its actual capability in a real-life situation. It is also assumed that a DM will maintain its misperception about itself at any level of hypergame beyond the first level. Therefore, $\succsim_{ii} = \succsim_{iiii...i}$. Moreover, it is assumed that a DM will hold its perception about its adversaries at any level of hypergame. Hence, $\succsim_{jji} = \succsim_{ji}$.*

3.1.1 Hypergame Formal Definition in Graph Form

In this subsection, the formal definition of an h-level hypergame H^h for the case of n-DM conflict in graph form is put forward. H^h represents a conflict condition in which each DM perceives a different game, and the greatest level of perception involved in the subjective games is $h-1$. For $N = \{1, 2, ..., i, ..., n\}$, the formal definition of H^h is furnished as follows:

$$H^h = \langle H_i^{h-1} : \forall i \in N,\ h = 1, 2, 3, ...\rangle \tag{3.1}$$

where, H_i^{h-1} is *DM i's* $(h-1)$-level hypergame. Further information about the modeling of the game H_i^{h-1} is provided in Subsection 6.1.3. By utilizing Eq. 3.1, a hypergame model in graph form can be written at any given level.

3.2 First-Level Hypergame with Two Decision Makers in Graph Form

In a first-level hypergame, at least one DM has a misperception about the conflict situation, and neither the DM who misperceives the circumstance nor the other DM is aware of this misunderstanding. Misperception of preferences by DMs is the basic type of misperception in a first-level hypergame. In this circumstance, at least one DM misperceives preferences in the dispute. In this instance, all DMs consider the same set of feasible states in their subjective game. Hence, under preference misperception, the set of feasible states is not sensitive to any change (Aljefri et al., 2014a) and the same integrated graph model can be used among all DMs' subjective games (Aljefri et al., 2014a). However, misperception of options by DMs alters the sets of options, states, and preferences that are perceived by each DM in its subjective game. The overall architecture of a first-level hypergame procedure with two DMs within GMCR is shown in Figure 3.2. More specifically, the proposed graph model for a first-level hypergame analysis consists of three modules: one to generate the universal set of states for a first-level hypergame, a second to model each DM's subjective game, and a third to carry out analyses, as indicted on the left in Figure 3.2.

Starting in the top part of Figure 3.2, the universal set of states for a first-level hypergame is a combination of all viable scenarios of DMs' perceptions in a conflict. States in a particular conflict circumstance are determined from the universal set of options for a first-level hypergame. In particular, a given DM's universal set of options in a first-level hypergame includes all of the options that are correctly recognized by the DM or mistakenly perceived by itself and/or its opponent. The combination of DMs' universal sets of options for a first-level hypergame generates the universal set of states for the first-level hypergame, which can then be used to specify states in each DM's subjective game. In the central part of Figure 3.2, the modeling of a DM's subjective game within the first-level hypergame structure starts by dividing the universal set of states for a first-level hypergame based on a DM's perception into two disjoint subsets, which are the collection of states that are (1) hidden to the DM in its subjective game, and (2) recognized by the DM in its subjective game. Hidden states, on the one hand, describe the states that are not considered by the DM in its game as they represent the other DM's perception of the conflict situation. Recognizable states, on the other hand, represent the states that are included in the DM's subjective game as they capture its perception of the conflict situation. Furthermore, as shown in Figure 3.2, a DM's set of recognizable states is further divided into two groups: infeasible states and feasible states. Infeasible states are the collections of states that are removed from the model because they are categorized according to four types of option conditions to be infeasible: mutually exclusive options, at least one option, option dependence, and direct specification (Fang et al., 2003a,b). Consequently, the remaining states are considered as feasible states for the dispute. Within these feasible states, a DM can cause the conflict to move from one state to another. This process is called state transition.

As explained earlier in Section 2.1.2, option form (Howard, 1971) is utilized in GMCR to encode states in a conflict model. States or possible scenarios for a real-life conflict are formed when each DM chooses its own strategy. A strategy of a DM, on the other hand, is a particular selection of options or courses of action from the set of options it controls. If there is a total of λ options across all of the DMs the number of possible states is 2^{λ}, since each option can be selected or not. As λ increases, the number of states or the size of the conflict will become quite large. In practice, analysts or users of GMCR methodology tend to over-specify a conflict by having too many options. Suppose, for example, a decision maker has two options it controls. Doing nothing can be represented by both options not being taken. However, a user may feel more comfortable in having a third option called "do nothing". Because this will result in having more infeasible states to remove before carrying out a stability analysis, exactly the same stability results will be obtained for the more complicated model as the simple one.

As depicted in the bottom portion of Figure 3.2, the analysis module is further composed of a standard stability analysis and a first-level hypergame analysis, which is followed by an informative classification of the first-level hypergame equilibria. The standard stability analysis is utilized to ascertain the equilibrium states for each DM's graph model. Then a first-level hypergame analysis is conducted among all DMs' games, by taking the Cartesian product of all DMs' strategies arising from their equilibrium states within their subjective games, to ascertain the equilibrium states for a first-level hypergame. Finally, the first-level hypergame equilibria are further investigated and classified into eight categories to study the effect of DMs' misperception on the outcome of the conflict.

The outline of this section is as follows. First, the modeling procedure for the universal set of options for a first-level hypergame with two DMs in graph form is addressed in Section 3.2.1. After that, the universal set of states for a first-level hypergame with two DMs in graph form is constructed in Section 3.2.2. Moreover, the modeling and analysis of a first-level hypergame with two DMs in graph form are put forward in Section 3.2.3.

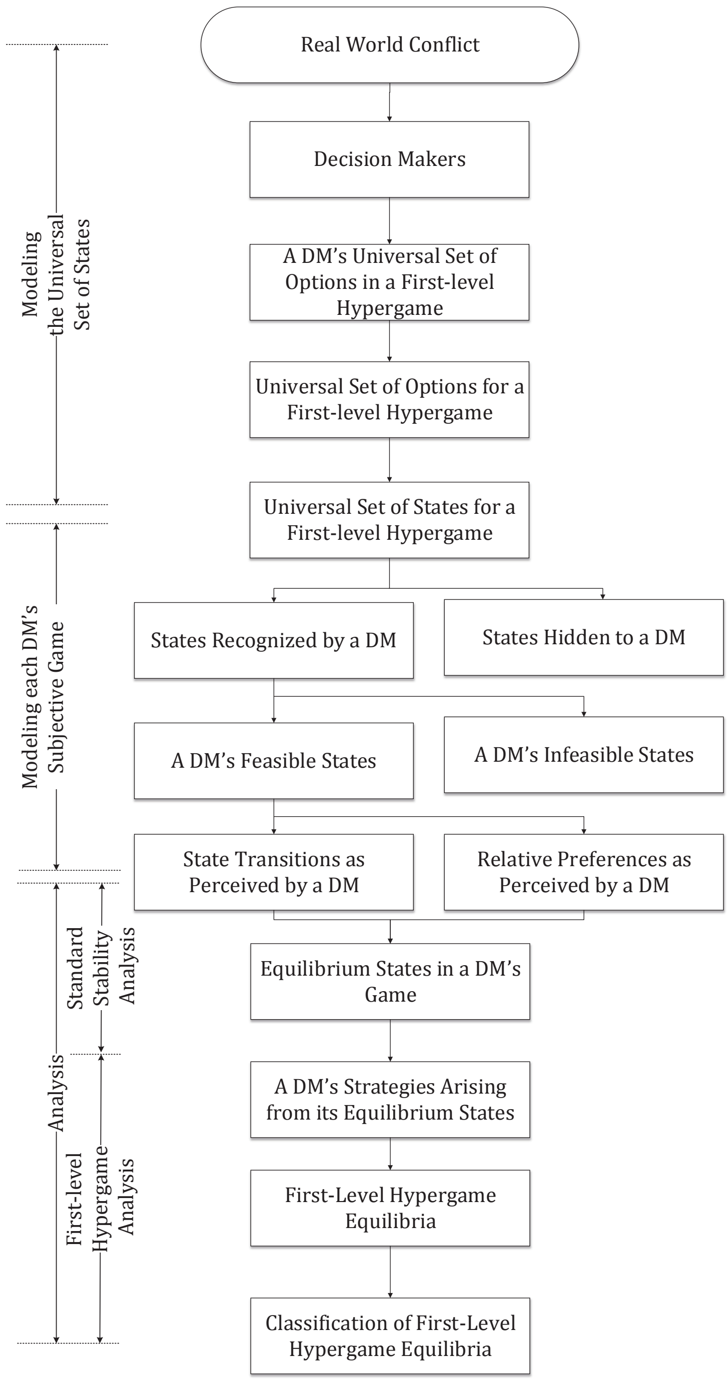


Figure 3.2: Architecture of a First-Level Hypergame Approach for Modeling and Analysis within the Graph Model

3.2.1 Universal Set of Options in a Two-Decision Maker First-Level Hypergame

As noted earlier, states in graph form are usually defined using option form (Howard, 1971). In GMCR, the available options for each DM are specified, and all DMs in the conflict are aware of them. Under misperception, however, a visualization of the real-life situation is established based on each DM's viewpoint (Jervis, 1968, 1976; Wang et al., 1988, 1989). That is, the set of options for a particular DM may be altered, not only by the DM itself (Levy, 1983; Betts, 2000; Trivers, 2000; Stoessinger, 2008), but also by its opponent. For instance, a DM can correctly perceive some of its courses of action but misperceive others. Similarly, a DM's opponent can misperceive some of its options while correctly perceiving others.

To take into account all possible perceptions (correct or incorrect) of options for a DM in a conflict model, the concept of a universal set of options for a DM in a 2-DM first-level hypergame is introduced. The universal set of options for a first-level hypergame consists of the universal sets of options of the participating DMs. An option in the universal set of options for a DM in a first-level hypergame can be explained using the following ideas: (1) correctly recognized by itself and possibly recognized by its opponent, (2) misperceived by the DM itself, and (3) misperceived by the DM's opponent. In fact, the DMs are not aware of their misperception, and, as a result, they cannot distinguish between the options that are correctly perceived and the courses of action that are misperceived. Hence, the grouping of options is performed by an external expert, who is aware of the asymmetry of perception among DMs. Since the varied perceptions are defined for a 2-DM graph model, until otherwise specified, assume that $N = \{i, j\}$.

A Decision Maker's Set of Options That Are Correctly Recognized by Itself and Possibly Recognized by Its Opponent

The set of these options represents a particular DM's correctly perceived options. Some of these courses of action may also be recognized by the DM's opponent (Wang et al., 1988, 1989). The sets of these options can be defined formally for a 2-DM model as follows, and are illustrated in Figure 3.3.

Definition 3.2.1.1 (**Set of a DM's Correctly Perceived Options**). *Let* O_{ii}^R *denote the set of DM i's options that are correctly identified by DM i itself. Also, let* O_{ij}^R *represent the set of DM i's options that are correctly identified by DM i itself and recognized by DM j. Then,* $O_i^R = O_{ii}^R \cap O_{ij}^R$ *represents the set of DM i's correctly perceived options that are recognized by both DMs.*

Note that $O_{ij}^R \subseteq O_{ii}^R$ by definition. Also note that $O_{ii}^R \setminus O_{ij}^R$ is the set of options that are correctly identified by *DM i* only. In a 2-DM model, $O_i^R = O_{ij}^R$.

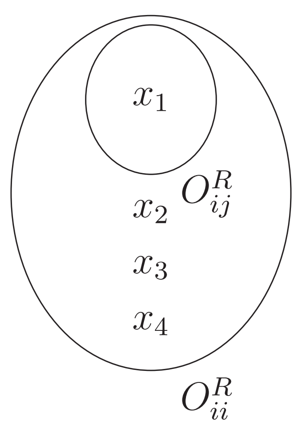


Figure 3.3: *DM i's* Correctly Perceived Options

As can be seen in Figure 3.3, *DM i* has four correctly perceived options: x_1, x_2, x_3, and x_4. Option x_1 is a commonly perceived course of action, and, hence, will be considered by both DMs i and j. Also, one can see that x_2, x_3, and x_4 are recognized by only *DM i* and will remain under its control. In other words, these options will only be considered by *DM i*. *DM j* does not recognize options x_2, x_3, and x_4 because of many reasons, such as lack of information or underestimating *DM i's* capabilities with regard to exercising these courses of action (Jervis, 1968, 1976; Wang et al., 1988; Ben-Zvi, 1995).

The Yom Kippur War, also called the Arab-Israeli war, constitutes a good example for the circumstances mentioned above (Ben-Zvi, 1995). In summary, the Arab coalition started with a stealth attack on Sinai and the Golan Heights for the purpose of expelling the Israeli forces who were controlling them. The war ended with the signing of the Camp David Accord in 1978, according to which Israel agreed to return Sinai to Egypt and part of the Golan Heights to Syria. In fact, Israel was aware of the Arab coalition's intention to launching an attack on Sinai and the Golan Heights, but it chose not to mitigate this risk because it underestimated its opponents' capability in regard to launching an attack. More specifically, Israel did not predict that Egypt would launch an attack on Sinai without obtaining bombers capable of offsetting Israeli air capability. Furthermore, because Israel did not predict an attack by Egypt on Sinai, it also did not expect an attack by Syria on the Golan Heights. That is, Israel had a mistaken belief and encountered a strategic surprise in the conflict (Ben-Zvi, 1995).

A Decision Maker's Set of Options That Are Misperceived by Itself

The DM's perception of itself is based on a combination of factors, such as emotion, race, ethnicity and culture, which may ultimately lead to a misperception (Stoessinger, 2008). Two classes of options can illustrate the DM's misperception: (a) options that are imagined by the DM itself and (b) options that are misunderstood by the DM itself. These two groups are formally defined below, and are illustrated in Figures 3.4 and 3.5, respectively.

Definition 3.2.1.2 (**Set of a DM's Imagined Options**). *Options that are assumed by a DM itself because of its misperception, such as overestimating its capabilities (Levy, 1983), mistaken beliefs (Stoessinger, 2008) or the impression of greatness (Betts, 2000; Trivers,* *2000) are called the DM's imagined options. The set of these options for DM i is denoted by O_{ii}^{I}.*

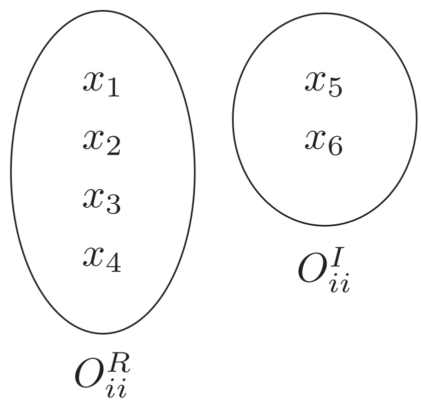


Figure 3.4: *DM i's* Options that are Imagined by Itself in Comparison with its Correctly Perceived Options

As can be seen in Figure 3.4, the options in O_{ii}^{R} are correctly perceived by *DM i* and are under its control. The options included in O_{ii}^{I} are the courses of action imagined by *DM i*, and, hence, do not have any connection with the options in O_{ii}^{R}. Although the options in O_{ii}^{I} are fictitious, *DM i*, because of its perception, will consider them as being viable courses of action for itself (Bluth, 2004; Kaufmann, 2004; Pollack, 2004).

The conflict between North and South Vietnam during the time period from 1957 to 1975 can be used as a good illustration for the aforesaid situation. In the war, North Vietnam was supported by both China and the Soviet Union, whereas South Vietnam was supported by the US and other anti-communist allies. The US participated in the war to prevent the communist expansionism led by the Soviet Union. The Vietnamese conflict was the first major defeat for the US in modern times. The US believed it had an upper hand in the dispute as its economy was in excellent condition, and it had the most advanced military technology (Levy, 1983; Tuchman, 1985; Trivers, 2000). The illusion of superiority caused the US to act aggressively and go to war. However, other options, such as economic sanctioning could have been used instead of going to actual war. In this case, the US's imagined option was to act aggressively and to go to war against North Vietnam.

Definition 3.2.1.3 (**Set of a DM's Misunderstood Options**). *Let* $O_i^{\bar{R}}$ *denote the set of DM i's options that exist in reality, but are misinterpreted by DM i. Also, let* O_{ii}^M *represent the set of DM i's options that are misunderstood in meaning by itself. Then, for every element* $\bar{t} \in O_i^{\bar{R}}$ *there is a misunderstood option* $t \in O_{ii}^M$.

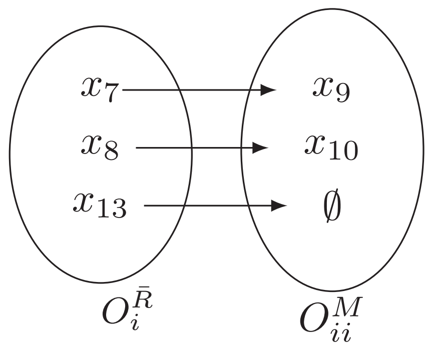


Figure 3.5: Options Misinterpreted by *DM i*

As can be seen in Figure 3.5, $O_i^{\bar{R}}$ contains three options: x_7, x_8, and x_{13}. *DM i* misunderstands the meaning of x_7 and x_8, mistaking them as x_9 and x_{10}, respectively. Hence, x_9 and x_{10} belong to O_{ii}^M. Also, x_{13} is interpreted by *DM i* as $\emptyset$, which means that *DM i* is not aware of the existence of x_{13}. As *DM i* cannot perceive x_7, x_8, and x_{13} as its options, they will be hidden to it, and, therefore, are not listed in O_{ii}^R. Moreover, because of DMs' varied perceptions, *DM j* may consider x_7, x_8, and x_{13} in its game or misunderstand their meaning (Jervis, 1968, 1976). Because of *DM i's* self-misperception, its set of options O_i for the standard GMCR defined in Section 2.1.2 is partitioned into two sets, O_{ii}^R and $O_i^{\bar{R}}$. As a hypothetical example to the situation as mentioned above, consider a possible conflict between a criminal and a police officer. Assume that the criminal attacks the officer and the officer acts in self-defense. The officer, intending to pull out a gun, realizes that the weapon is not a gun but a baton. In this case, the actual option "baton" is misunderstood by the officer as "gun".

Remark 3.2.1.1. *A misunderstood option is the course of action that is considered by a DM because of exaggerating the ability and capacity of the option than it really is. In comparison, an imagined option is the course of action that cannot be exercised in reality because a DM does not actually possess it. Also,* O_{ii}^{R} *includes DM i's options that are correctly perceived by the DM itself. That is,* O_{ii}^{R} *is free from any misperception, and hence, has no connection with* O_{ii}^{I} *and* O_{ii}^{M}*. Therefore, in this research,* O_{ii}^{R}*,* $O_{i}^{\bar{R}}$*,* O_{ii}^{I}*, and* O_{ii}^{M} *are assumed to be pairwise disjoint sets. Therefore,* $O_{ii}^{R} \cap O_{i}^{\bar{R}} = O_{ii}^{R} \cap O_{ii}^{I} = O_{ii}^{R} \cap O_{ii}^{M} = O_{i}^{\bar{R}} \cap O_{ii}^{I} = O_{i}^{\bar{R}} \cap O_{ii}^{M} = O_{ii}^{I} \cap O_{ii}^{M} = \emptyset$*. These relationships are illustrated in Figure 3.8.*

A Decision Maker's Set of Options That Are Misperceived by Its Opponent

An option that is not correctly perceived by the DM itself but is considered by its opponent in its subjective game can either be an imagined or misunderstood course of action as perceived by the opponent (Jervis, 1968, 1976; Wang et al., 1988, 1989). These two groups of options are defined below in Definitions 3.2.1.4 and 3.2.1.5, and are illustrated in Figures 3.6 and 3.7.

Definition 3.2.1.4 (Set of a DM's Options Imagined by its Opponent). *DM i's options that are imagined by DM j, denoted as* O_{ij}^{I}*, are composed of two types:*

- *Options in* $O_{i}^{\bar{R}}$ *that are still considered by DM j as courses of action for DM i, and*
- *Options for DM i that are completely unknown to itself because it has no idea about them but they are still assumed by DM j as possible courses of action for DM i.*

As can be seen in Figure 3.6, x_5, x_7, and x_{11} belong to O_{ij}^{I}, and, hence, remain in *DM j's* imagination. *DM j* considers them as they represent its viewpoint of the actual circumstances of the conflict. *DM j* may wrongly assume one or more courses of action for a given DM if it overestimates the focal DM's capabilities (Jervis, 1968, 1976; Levy,

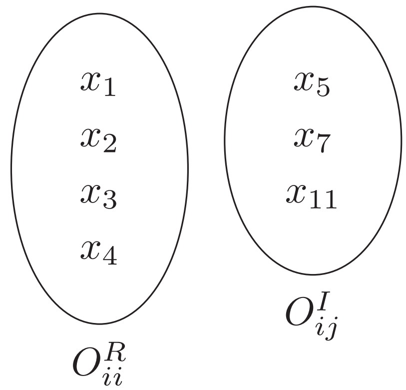


Figure 3.6: *DM i's* Options That Are Imagined by *DM j* in Comparison with *DM i's* Correctly Perceived Options

1983). Also, its experience from other situations may influence it to assume that the current conflict resembles earlier disputes, when no similarity, in fact, exists. Moreover, by investigating the information in Figures 3.4 and 3.6, one can see that x_5 is considered by both DMs i and j as an imagined option. In this case, one can conclude that both DMs share the same misperception. Hence, in this research, it is assumed that the expression $O_{ii}^I \cap O_{ij}^I \neq \emptyset$ may or may not hold. Also, by comparing the information in Figures 3.5 and 3.6 one can see that *DM j* captures x_7. This option is not recognized by *DM i* because he or she misunderstood its meaning. Because x_7 will not be considered by *DM i*, according to Definition 3.2.1.4, x_7 will be classified as an imagined option by *DM j*. Hence, in this research, it is assumed that the expression $O_i^{\bar{R}} \cap O_{ij}^I \neq \emptyset$ may or may not hold. This relationship is shown in Figure 3.8.

The decision made by the US and Britain leaders to attack Iraq in 2003 can serve as a good example of the situation described above. The US and Britain firmly believed that Iraq possessed weapons of mass destruction (WMD) and considered this to be a significant threat to international security. The report made by the UN International Atomic Energy Agency, however, stated that Iraq was far from obtaining WMD. As a result, the agency concluded that an attack on Iraq was unwarranted. According to this finding, many researchers questioned the justification of the US and Britain's decision to attack Iraq (Bluth, 2004; Kaufmann, 2004). However, the US and Britain did not believe the report by the UN International Atomic Energy Agency and still assumed that Iraq possessed WMD. This mistaken belief motivated these two allies to go to war with Iraq. In this circumstance, the assumption that Iraq had WMD at its disposal was an imagined option for Iraq by the US and Britain.

Definition 3.2.1.5 (Set of a DM's Options Misunderstood by Its Opponent). *Options in O_{ii}^R and $O_i^{\bar{R}}$ that are misperceived by DM j are referred to as DM i's options misunderstood by DM j, denoted by O_{ij}^M.*

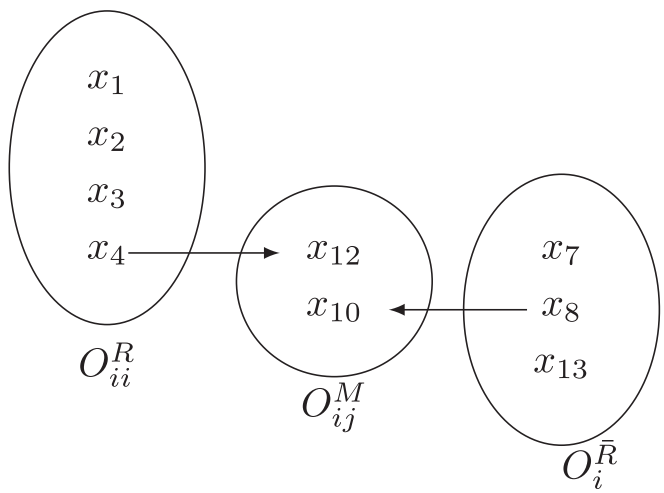


Figure 3.7: *DM i's* Options Misunderstood by *DM j* in Comparison with O_{ii}^R and $O_i^{\bar{R}}$

As can be seen in Figure 3.7, *DM j* misunderstands the meaning of x_4 and assumes it to be x_{12}. That is, *DM j* recognizes *DM i* has x_{12}. By investigating the data in Figures 3.5 and 3.7, one can identify that x_8, which is a member of $O_i^{\bar{R}}$, is misunderstood in meaning by both DMs i and j and is assumed to be x_{10} . In this situation, one can conclude that both DMs have the same misperception. It is important to note that *DM j* can, in fact, misunderstand the meaning of option x_8 differently than *DM i*. However, for this case, both DMs share the same misperception to show that this kind of perception is allowed in the proposed methodology. Therefore, in this research, it is assumed that the expression $O_{ii}^{M} \cap O_{ij}^{M} \neq \emptyset$ may or may not hold. This relationship is depicted in Figure 3.8.

The Japanese attack against the US at Pearl Harbor can be used as an insightful example to illustrate the aforesaid circumstance. The US had broken the Japanese code prior to its attack on Pearl Harbor on December 7, 1941 (Wohlstetter, 1962). Washington advised General Short in Pearl Harbor to anticipate hostile actions from Japan, by which it meant a surprise attack. General Short misunderstood this phrase as sabotage. As a result, General Short did not expect a surprise attack from Japan (Levy, 1983). In this case, Japan's actual option was launching a surprise air attack on Pearl Harbor and the misunderstood option by the US was to expect sabotage from the Japanese.

Remark 3.2.1.2. *Similar to Remark 1, O_{ij}^{R}, O_{ij}^{I}, and O_{ij}^{M} are assumed, in this research, to be pairwise disjoint sets. Therefore, $O_{ij}^{R} \cap O_{ij}^{I} = O_{ij}^{R} \cap O_{ij}^{M} = O_{ij}^{I} \cap O_{ij}^{M} = \emptyset$. These relationships are illustrated in Figure 3.8.*

Remark 3.2.1.3. *For a given situation a DM can only misperceive an option in one way. Hence, an option in O_{ij}^{I} cannot be part of O_{ii}^{R} and O_{ii}^{M}. Also, an element in O_{ij}^{M} cannot exist in O_{ii}^{R}, $O_{i}^{\bar{R}}$, and O_{ii}^{I}. Therefore, $O_{ij}^{I} \cap O_{ii}^{R} = O_{ij}^{I} \cap O_{ii}^{M} = \emptyset$ and $O_{ij}^{M} \cap O_{ii}^{R} = O_{ij}^{M} \cap O_{i}^{\bar{R}} = O_{ij}^{M} \cap O_{ii}^{I} = \emptyset$. These relationships are illustrated in Figure 3.8.*

As the definitions of various types of options have now been introduced, one can characterize the universal set of options for a DM in a first-level hypergame. *DM i's* universal set of options is defined below, and is illustrated in Figure 3.8.

Definition 3.2.1.6 (*DM i's* Universal Set of Options in a First-Level Hypergame). *Let $\ddot{O}_{i}^{1}$ denote DM i's universal set of options for a first-level hypergame. Then, $\ddot{O}_{i}^{1} = O_{ii}^{R} \cup O_{i}^{\bar{R}} \cup O_{ii}^{I} \cup O_{ij}^{I} \cup O_{ii}^{M} \cup O_{ij}^{M}$.*

Note that all the elements in $\ddot{O}_{i}^{1}$ are known to the analyst. However, because of DMs' different perceptions, some courses of action in $\ddot{O}_{i}^{1}$ may be unknown to either *DM i*, *DM j*,

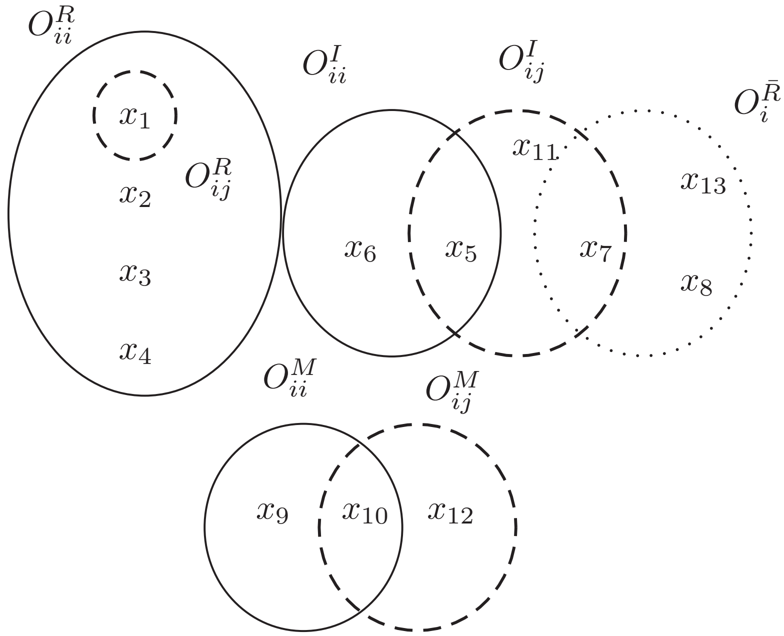


Figure 3.8: *DM i's* Universal Set of Options

or both. *DM i's* options that are unknown to itself are denoted by O_{ii}^U and can be expressed as $O_{ii}^U = \ddot{O}_i^1 \setminus (O_{ii}^R \cup O_{ii}^I \cup O_{ii}^M)$. Analogously, *DM i's* options that are unknown to *DM j*, O_{ij}^U, can be expressed as $O_{ij}^U = \ddot{O}_i^1 \setminus (O_{ij}^R \cup O_{ij}^I \cup O_{ij}^M)$.

DM j's universal set of options for a first-level hypergame, $\ddot{O}_j^1$, can be analogously defined. The universal set of options for the entire first-level hypergame, $\hat{O}^1$, can be formally defined as follows.

Definition 3.2.1.7 (**Universal Set of Options for the First-level Hypergame**). *The union of the universal set of options of DMs i and j is referred to as the universal set of options for the first-level hypergame, and is denoted by* $\hat{O}^1$. *Mathematically,* $\hat{O}^1 = \ddot{O}_i^1 \cup \ddot{O}_j^1$.

3.2.2 Universal Set of States in a Two-Decision Maker First-Level Hypergame

After defining $\hat{O}^1$, one can now formally define the universal set of states for a first-level hypergame. The universal set of states is the collection of all possible scenarios of all DMs' perceptions of a conflict under study. Generally, a state is formed after each DM selects its own strategy. A specific strategy for a DM, on the other hand, is a choice of its own options which are within its control: 1 if the option is chosen, and 0 if the option is not chosen (Howard, 1971).

Based on the mathematical definition of option form for which there are no misperceptions (Fang et al., 2003a,b; Xu et al., 2017), expressing states in option form for a first-level hypergame can be developed. Formally, let the universal set of options for a first-level hypergame be expressed by $\hat{O}^1 = \ddot{O}^1_i \cup \ddot{O}^1_j$, where $\ddot{O}^1_i = \{o^i_{\bar{k}} : \bar{k} = 1, 2, ..., m_i\}$ is *DM i's* universal set of options in a first-level hypergame, in which $o^i_{\bar{k}}$ is *DM i's* $\bar{k}^{th}$ option and m_i represents the total number of options for *DM i*. An option can either be chosen or not by the DM who possesses it. A strategy for a given DM is formed when it decides which of its options to choose or not. Mathematically, a strategy for *DM i* can be represented by a function $g_i : \ddot{O}^1_i \longrightarrow \{0, 1\}$, such that for $\bar{k} = 1, 2, ..., m_i$,

$$g_i(o^i_{\bar{k}}) = \begin{cases} 1, & \text{if } DM\ i \text{ selects option } o^i_{\bar{k}} \\ 0, & \text{otherwise} \end{cases}$$

A state, in option form, can be constructed by a λ-dimensional column vector, where λ is the number of options in $\hat{O}^1$. Such a vector represents each DM's strategy selection. Formally, a state can be defined by a mapping $f : \hat{O}^1 \longrightarrow \{0, 1\}$, such that

$$f(o^l_{\bar{k}}) = \begin{cases} 1, & \text{if } DM\ l \text{ selects option } o^l_{\bar{k}}, \forall\ l = \{i, j\} \\ 0, & \text{otherwise} \end{cases}$$

Note that a typical state defined above is a vector of the form: $(f(o_1^i), f(o_2^i), ..., f(o_{m_i}^i),$ $f(o_1^j), f(o_2^j), ..., f(o_{m_j}^j))^T$. The collection of all such states is the universal set of states for a first-level hypergame and is denoted by $\hat{S}^1$.

A state in $\hat{S}^1$ can be represented by s. The total number of states in $\hat{S}^1$ is 2^λ, where $\lambda = m_i + m_j$. Given a state $s \in \hat{S}^1$, let *DM i's* strategy associated with it be denoted by an m_i-dimensional column vector, g_i^s. Thus, $s = ((g_i^s)^T, (g_j^s)^T)^T$.

States in a Decision Maker's Subjective Game

In a first-level hypergame situation (Wang et al., 1988, 1989), a dispute is expressed by a number of games, where each game represents a specific DM's perception of the conflict. In fact, each DM's game must be constructed using its recognizable set of states. Some states in $\hat{S}^1$ may not be considered by a focal DM as they are based on the opponent's viewpoint of the dispute, which is different from the focal DM's perspective. Let the recognizable set of states for *DM i* be denoted by S_i. To formally define S_i, the universal set of options, $\hat{O}^1$, must be partitioned into four disjoint sets based on *DM i's* perception. In particular, *DM i's* universal set of options for a first-level hypergame, $\ddot{O}_i^1$, is partitioned into two disjoint sets $(O_{ii}^R \cup O_{ii}^I \cup O_{ii}^M)$ and O_{ii}^U. Likewise, based on *DM i's* perspective, *DM j's* universal set of options for a first-level hypergame, $\ddot{O}_j^1$, must be partitioned into two disjoint sets $(O_{ji}^R \cup O_{ji}^I \cup O_{ji}^M)$ and O_{ji}^U.

Based on *DM i's* perspective, the aforementioned disjoint sets of options can be used to partition $\hat{S}^1$ into two disjoint sets: (1) the set of states that are recognized by *DM i* in its game, denoted by S_i, and (2) the set of states that are hidden to *DM i* in its game, to be denoted by S_i^H. These disjoint sets of states are formally defined below.

Definition 3.2.2.1 (**Set of Recognizable States in a DM's Game**). *Let* $S_i \subseteq \hat{S}^1$ *denote the set of states considered in DM i's game. Then, a state* $s \in S_i \iff$ *there is a mapping* $f : \hat{O}^1 \rightarrow \{0, 1\}$ *such that* $s = (f(o_1^i), f(o_2^i), ..., f(o_{m_i}^i),\ f(o_1^j), f(o_2^j), ..., f(o_{m_j}^j))^T$ *satisfying* $f(o) = 0\ , \forall\ o \in O_{ii}^U \cup O_{ji}^U$.

Definition 3.2.2.2 (Set of Hidden States in a DM's Game). *Let* $S_i^H \subseteq \hat{S}^1$ *denote the set of states that are hidden to DM i in its game. Then, a state* $s \in S_i^H \iff$ *there is a mapping* $f : \hat{O}^1 \to \{0,1\}$ *such that* $s = (f(o_1^i), f(o_2^i), ..., f(o_{m_i}^i),\ f(o_1^j), f(o_2^j), ..., f(o_{m_j}^j))^T$ *satisfying* $\exists\ o \in O_{ii}^U \cup O_{ji}^U,\ f(o) = 1$.

Now S_i is partitioned into five subsets representing *DM i's* perception of the conflict situation. *DM i* is not aware of its misperception and, as a result, cannot distinguish between the group of states that are correctly perceived and the class of states that are misperceived. For this reason, the classification of states is performed by an external expert who is aware of the asymmetry of perceptions among DMs. The analyst can, in fact, distinguish the states that are correctly perceived from those that are misperceived by *DM i*. Furthermore, the analyst can find the group of states that are accurately perceived or misperceived by all DMs.

Classification of States in a Decision Maker's Subjective Game

To deal with *DM i's* perception in a conflict situation, S_i is divided into five disjoint sets, which are the groups of states that are: (1) correctly perceived by both *DM i* and *DM j*, denoted as S^R, (2) correctly perceived by only *DM i*, denoted as S_i^P, (3) imagined by *DM i*, denoted as S_i^I, (4) misunderstood by *DM i*, symbolized by S_i^M, and (5) imagined and misunderstood by *DM i*, symbolized by $S_i^{I,M}$. These subsets of states are formally defined below, and are displayed in Figure 3.9.

Definition 3.2.2.3 (Set of Correctly Perceived States by Both DMs). *Choose DM* $i \in N$. *A state* $s \in S_i$ *is correctly perceived by both DMs, that is,* $s \in S^R \subseteq S_i \iff$ *there is a mapping* $f : \hat{O}^1 \to \{0,1\}$ *such that* $s = (f(o_1^i), f(o_2^i), ..., f(o_{m_i}^i), f(o_1^j), f(o_2^j), ..., f(o_{m_j}^j))^T$ *satisfying* $f(o) = 0,\ \forall\ o \in \hat{O}^1 \setminus (O_i^R \cup O_j^R)$.

Definition 3.2.2.4 (Set of a DM's States Correctly Perceived by Itself Only). *A state* $s \in S_i$ *is correctly perceived by DM i only, that is,* $s \in S_i^P \subseteq S_i \iff$ *there is a*

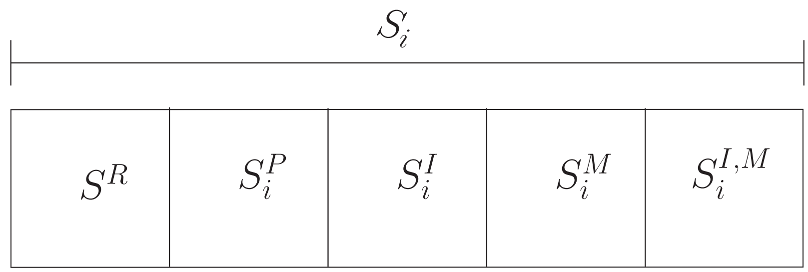


Figure 3.9: Classification of *DM i's* Set of Recognizable States

mapping $f : \hat{O}^1 \to \{0,1\}$ *such that* $s = (f(o_1^i), f(o_2^i), ..., f(o_{m_i}^i),\ f(o_1^j), f(o_2^j), ..., f(o_{m_j}^j))^T$ *satisfying* $\exists\ o \in O_{ii}^R \setminus O_i^R,\ f(o) = 1$ *and* $f(o^{'}) = 0, \forall\ o^{'} \in \hat{O}^1 \setminus (O_{ii}^R \cup O_j^R)$.

Definition 3.2.2.5 (**Set of a DM's States Imagined Only**). *Choose DM i* $\in N$. *A state* $s \in S_i$ *is imagined by DM i, such that* $s \in S_i^I \subseteq S_i \iff$ *there is a mapping* $f : \hat{O}^1 \to \{0,1\}$ *such that* $s = (f(o_1^i), f(o_2^i), ..., f(o_{m_i}^i),\ f(o_1^j), f(o_2^j), ..., f(o_{m_j}^j))^T$ *satisfying* $\exists\ o \in O_{ii}^I \cup O_{ji}^I,\ f(o) = 1$ *and* $f(o^{'}) = 0, \forall\ o^{'} \in \hat{O}^1 \setminus (O_{ii}^I \cup O_{ji}^I \cup O_{ii}^R \cup O_j^R)$.

Definition 3.2.2.6 (**Set of a DM's States Misunderstood Only**). *Choose DM i* $\in N$. *A state* $s \in S_i$ *is recorded in DM i's misunderstood scenarios, such that* $s \in S_i^M \subseteq S_i \iff$ *there is a mapping* $f : \hat{O}^1 \to \{0,1\}$ *such that* $s = (f(o_1^i), f(o_2^i), ..., f(o_{m_i}^i),$ $f(o_1^j), f(o_2^j), ..., f(o_{m_j}^j))^T$ *satisfying* $\exists\ o \in O_{ii}^M \cup O_{ji}^M,\ f(o) = 1$ *and* $f(o^{'}) = 0, \forall\ o^{'} \in \hat{O}^1 \setminus (O_{ii}^M \cup O_{ji}^M \cup O_{ii}^R \cup O_j^R)$.

Definition 3.2.2.7 (**Set of a DM's States Imagined and Misunderstood**). *Choose DM i* $\in N$. *A state* $s \in S_i$ *is recorded in DM i's imagined and misunderstood scenarios, such that* $s \in S_i^{I,M} \subseteq S_i \iff$ *there is a mapping* $f : \hat{O}^1 \to \{0,1\}$ *such that* $s = (f(o_1^i), f(o_2^i), ..., f(o_{m_i}^i),\ f(o_1^j), f(o_2^j), ..., f(o_{m_j}^j))^T$ *satisfying* $\exists\ o \in O_{ii}^I \cup O_{ji}^I,\ f(o) = 1,$ $\exists\ o^{'} \in O_{ii}^M \cup O_{ji}^M,\ f(o^{'}) = 1,$ *and* $f(o^{''}) = 0, \forall\ o^{''} \in \hat{O}^1 \setminus (O_{ii}^I \cup O_{ji}^I \cup O_{ii}^M \cup O_{ji}^M \cup O_{ii}^R \cup O_j^R)$.

3.2.3 Modeling and Analysis of a First-Level Hypergame with Two Decision Makers in Graph Form

In this subsection, the modeling and analysis procedures of a first-level hypergame with two DMs in graph form are put forward.

Mathematical Modeling of a First-level Hypergame with Two Decision Makers

After defining and classifying S_i, one can now define *DM i's* subjective game in a first-level hypergame as shown below.

$$G_i = \langle N, S_i, \{A_{ii}, A_{ji}\}, \{\succsim_{ii}, \succsim_{ji}\}\rangle \tag{3.2}$$

where $N = \{i, j\}$, G_i is the conflict situation as seen by *DM i*, and S_i denotes *DM i's* set of possible states, representing allowable distinct circumstances of the dispute. These scenarios are based on *DM i's* perception, and are thought of as vertices in *DM i's* graph. The subsets $A_{ii},\ A_{ji} \subseteq\ S_i\ \times\ S_i$ represent the sets of state transitions for DMs i and j, respectively, as perceived by *DM i*. More specifically, $(s_1,\ s_2)\ \in A_{ii}$ iff *DM i* perceives that it can cause the dispute to move from state s_1 to s_2. Finally, the preference relations $\succsim_{ii}$ and $\succsim_{ji}$ denote *DM i's* perceived preferences for itself and *DM j*, respectively. *DM j's* subjective game, G_j, can by defined analogously. Then, the collection of G_i and G_j defines a first-level hypergame, H^1, as follows.

$$H^1 = \langle G_i, G_j\rangle \tag{3.3}$$

Stability Analysis of a First-Level Hypergame with Two Decision Makers

A number of methods are available to analyze a first-level hypergame (Bennett, 1977, 1980; Takahashi et al., 1984; Wang et al., 1988, 1989). In the present research, the authors choose to advance the concepts developed by Wang et al. (1988, 1989) as they provide a better understanding and enhanced results. Three steps are needed to determine a first-level hypergame equilibrium.

- First, analyze *DM i's* and *DM j's* games separately by using GMCR stability definitions (Fraser and Hipel, 1984; Fang et al., 1993; Howard, 1971; Fraser and Hipel, 1979; Nash, 1950, 1951) to obtain the sets of *DM i's* and *DM j's* equilibrium states, respectively.

- Second, identify *DM i's* strategies from its equilibrium states obtained in its perceived game. Likewise, find *DM j's* strategies from its equilibrium states identified from its perceived game. Then, the first-level hypergame equilibria are determined by taking the Cartesian product of the sets of *DM i's* and *DM j's* individual equilibrium strategies.

- Third, the first-level hypergame equilibria are categorized into eight distinct classes to provide better insights into the dispute. The classification of the first-level hypergame equilibria is provided in Section 3.4 for the case of an n-DM ($n \geqslant 2$) conflict.

Step 1: Equilibrium States in a Decision Maker's Subjective Game

In order to determine the equilibria in each subjective game, the standard GMCR solution concepts, namely Nash (Nash) stability (Nash, 1950, 1951), sequential (SEQ) stability (Fraser and Hipel, 1984, 1979), general metarationality (GMR) (Howard, 1971), and symmetric metarationality (SMR) (Howard, 1971), are now defined within the structure of a first-level hypergame to identify the equilibrium states in *DM i's* game. Each state in *DM i's* game is checked for stability under various stability definitions, mentioned above, for *DM i* and *DM j*. When a state is stable for both DMs i and j under the same solution concept in G_i, it constitutes a possible resolution in G_i. The equilibrium results in G_i represent *DM i's* perception of the conflict situation. To perform the stability analysis for the states in G_i, the concepts of reachable list and unilateral improvement list for the case of 2-DM conflict must be defined first based on *DM i's* perception.

Definition 3.2.3.1 (**Reachable List by a DM**). *Let $q \in N = \{i, j\}$. The reachable list from $s_1 \in S_i$ by DM q in G_i is defined as $R_{qi}(s_1) = \{s_2 \in S_i : (s_1,\ s_2) \in A_{qi}\}$. $R_{qi}(s_1)$ represents the set of DM q's unilateral moves (UMs) starting from s_1.*

Definition 3.2.3.2 (**Unilateral Improvement List by a DM**). *A state $s_2 \in S_i$ is a unilateral improvement (UI) from $s_1 \in S_i$ for DM $q \in N$ in $G_i \iff s_2 \in R_{qi}(s_1)$ and $s_2 \succ_{qi} s_1$. The set of all UIs from $s_1 \in S_i$ by DM q is regarded as DM q's unilateral improvement list (UIL), and is denoted by $R^+_{qi}(s_1)$.*

Definition 3.2.3.3 (**Nash Stability**). *A state $s_1 \in S_i$ is Nash stable (Nash) for DM q $\in N$ in $G_i \iff R^+_{qi}(s_1) = \emptyset$. The set of all Nash stable states for DM q in G_i is denoted by $S_i^{Nash_{qi}}$.*

Definition 3.2.3.4 (**Sequential Stability**). *A state $s_1 \in S_i$ is sequentially stable (SEQ) for DM $q \in N$ in $G_i \iff$ for each $s_2 \in R^+_{qi}(s_1)$, $\exists\ s_3 \in R^+_{q'i}(s_2)$ such that $s_3 \precsim_{qi} s_1$, where q' denotes the opponent of q in N. The set of all SEQ stable states for DM q in G_i is denoted by $S_i^{SEQ_{qi}}$.*

Definition 3.2.3.5 (**General Metarationality**). *A state $s_1 \in S_i$ is general metarational stable (GMR) for DM $q \in N$ in $G_i \iff$ for each $s_2 \in R^+_{qi}(s_1)$, $\exists\ s_3 \in R_{q'i}(s_2)$ such that $s_3 \precsim_{qi} s_1$, where q' symbolizes the opponent of q in N. The set of all GMR stable states for DM q in G_i is denoted by $S_i^{GMR_{qi}}$.*

Definition 3.2.3.6 (**Symmetric Metarationality**). *A state $s_1 \in S_i$ is symmetric metarational stable (SMR) for DM $q \in N$ in $G_i \iff$ for each $s_2 \in R^+_{qi}(s_1)$, $\exists\ s_3 \in R_{q'i}(s_2)$ such that $s_3 \precsim_{qi} s_1$, and $s_4 \precsim_{qi} s_1$, $\forall\ s_4 \in R_{qi}(s_3)$, where q' denotes the opponent of q in N. The set of all SMR stable states for DM q in G_i is denoted by $S_i^{SMR_{qi}}$.*

After defining the aforementioned solution concepts, one can now determine the equilibrium states in G_i. A state is considered as an equilibrium in G_i under a specific stability concept if it is stable for every DM in G_i under that stability definition. Formally, the set of equilibria in G_i can be defined as follows.

Definition 3.2.3.7 (Equilibria). *A state $s_1 \in S_i$ that is stable for both DMs i and j in G_i according to a particular stability concept is an equilibrium in G_i under that stability definition. The set of all equilibrium states in G_i is denoted by E_i. The set of equilibrium states in DM j's subjective game, G_j, is constructed analogously, and is denoted by E_j.*

Step 2: First-Level Hypergame Equilibria for Two Decision Makers in Graph Form

Once the equilibrium states in each DM's subjective game are identified, then the possible compromise resolutions of the first-level hypergame can be determined by combining both DMs' strategies obtained from the equilibrium states in their perceived games. First, a hyper Nash equilibrium state for a 2-DM model is defined according to each DM's strategies, to be called Nash strategies, which are derived from the DM's Nash equilibrium states in its subjective game.

Definition 3.2.3.8 (Hyper Nash Equilibrium). *Let E_i^{Nash} denote the set of Nash equilibria in G_i. Assume $E_i^{Nash} = \{e_1^{Nash_i}, e_2^{Nash_i}, ..., e_{\varepsilon_i}^{Nash_i}\}$, where ε_i is the total number of Nash equilibrium states in G_i. Similarly, let $E_j^{Nash} = \{e_1^{Nash_j}, e_2^{Nash_j}, ..., e_{\varepsilon_j}^{Nash_j}\}$ represent the set of Nash equilibria in G_j, where ε_j is the total number of Nash equilibrium states in G_j. Let $g_i^{*Nash_i} = \{g_i^{e_1^{Nash_i}}, g_i^{e_2^{Nash_i}}, ..., g_i^{e_{\eta_i}^{Nash_i}}\}$, where $\eta_i \leqslant \varepsilon_i$, denote the set of DM i's distinct Nash strategies, where $g_i^{e_1^{Nash_i}}$ is DM i's strategy obtained from the equilibrium state $e_1^{Nash_i}$. Likewise, let $g_j^{*Nash_j} = \{g_j^{e_1^{Nash_j}}, g_j^{e_2^{Nash_j}}, ..., g_j^{e_{\eta_j}^{Nash_j}}\}$, where $\eta_j \leqslant \varepsilon_j$, be the set of DM j's distinct Nash strategies obtained from E_j^{Nash}. Then, the set of hyper Nash equilibrium states for the first-level hypergame can be defined as follows:*

$$HE^{1Nash} = g_i^{*Nash_i} \times g_j^{*Nash_j} \tag{3.4}$$

Any member of HE^{1Nash} is a hyper Nash equilibrium state of the dispute.

In fact, there are up to a total of $\varepsilon_i\varepsilon_j$ hyper Nash equilibrium states. As explained earlier, there are two steps to calculate the elements in HE^{1Nash} for a 2-DM first-level hypergame.

In the first step, one uses Definition 3.2.3.3 to compute E_i^{Nash} and E_j^{Nash} in G_i and G_j, respectively. A state is a Nash equilibrium in G_i if it is stable for both DMs i and j under the Nash solution concept. In the second step, one uses Definition 6.1.5.9 to compute HE^{1Nash} for a 2-DM first-level hypergame. This can be done by first isolating *DM i's* and *DM j's* Nash strategies from E_i^{Nash} and E_j^{Nash}, respectively, and then by taking the Cartesian product of *DM i's* and *DM j's* distinct Nash strategies. The sets of hyper SEQ equilibrium states (HE^{1SEQ}), hyper GMR equilibrium states (HE^{1GMR}), and hyper SMR equilibrium states (HE^{1SMR}) can be defined analogously.

3.3 First-Level Hypergame with n Decision Makers ($n \geqslant 2$) in Graph Form

To strategically model and analyze a conflict with more than two DMs, at least one of whom misperceives others' sets of options or preferences, a general structure for a first-level hypergame in graph form is put forward in this section. In particular, the ideas of the universal set of options and states for a 2-DM first-level hypergame in graph model, as well as the corresponding first-level hypergame stability analysis, are extended to an n-DM ($n \geqslant 2$) graph model in Sections 3.3.1, 3.3.2, and 3.3.3, respectively.

3.3.1 Universal Set of Options in an n-Decision Maker First-Level Hypergame

In Section 3.2.1, the definition for a given DM's universal set of options for a first-level hypergame is provided only for a 2-DM graph model. In this subsection, a given DM's universal set of options for an n-DM first-level hypergame in graph form is introduced to handle more than two DMs' perceptions in a conflict situation. The options put forward here to define a given DM's universal set of options are: (1) correctly identified by the DM itself and possibly recognized by some of its opponents, (2) misperceived by the DM itself, and (3) misperceived by some of the DM's opponents.

The set of a given DM's options that are correctly identified by the DM itself and possibly recognized by some of its opponents, formalized for a two-DM graph model in Definition 6.1.1.3, is defined first. Let the set of DMs in the hypergame be $N = \{1, 2, ..., i, ..., n\}$, $n \geqslant 2$. For $i \in N$, let O_{ii}^R denote the set of *DM i's* options that are correctly identified by *DM i* itself. For $j \in N - \{i\}$, let O_{ij}^R be the set of *DM i's* correctly perceived options that are recognized by *DM j*. Then, the set of *DM i's* options that are correctly perceived by all the DMs can be expressed as $O_i^R = \cap_{j=1}^n O_{ij}^R$.

Note that $O_i^R \subseteq O_{ii}^R$ and $O_{ii}^R \setminus O_i^R$ is the set of *DM i's* correctly perceived options that may be recognized by some of its opponents but not all. If O_{ij}^R is the same for each $j \in N - \{i\}$, then, $O_i^R = O_{ij}^R$. In this circumstance, $O_{ii}^R \setminus O_i^R$ contains *DM i's* correctly perceived options that are recognized by itself only.

The second collection of options is the set of a given DM's options misperceived by itself. As explained in Section 3.2.1, three classes of options can illustrate *DM i's* misperception: (a) options that are imagined by *DM i* itself, denoted as O_{ii}^I, (b) options that exist in reality, but are misinterpreted by *DM i*, symbolized as $O_i^{\bar{R}}$, and (c) options that are misunderstood by *DM i* itself, denoted as O_{ii}^M. Since the two classes of imagined and misunderstood options describe only *DM i's* misperception of itself and are not affected by any of its opponents' perception, they are not restrained by the number of DMs in the dispute. Thus, Definitions 3.2.1.2 and 3.2.1.3 remain the same for an n-DM model.

Like in Section 3.2.1, one or more of a DM's options may be misperceived by some of its opponents. This misperception can be because of opponents' imagination or misunderstanding. The two types of misperception are formally defined below.

Let $i \in N$. For $j \in N - \{i\}$, let O_{ij}^I and O_{ij}^M represent the sets of *DM i's* options that are imagined and misunderstood, respectively, by *DM j*. Then, $\cup_{j \in N-\{i\}} O_{ij}^I$ and $\cup_{j \in N-\{i\}} O_{ij}^M$ represent *DM i's* options that are imagined and misunderstood, respectively, by its opponents. In fact, $\cup_{j\in N-\{i\}}O_{ij}^{I}$ contains *DM i's* options that are imagined by at least one of its opponents. Similarly, $\cup_{j\in N-\{i\}}O_{ij}^{M}$ contains *DM i's* options that are misunderstood by at least one of its opponents.

Now that the definitions of different types of option perception have been introduced for the case of n-DM graph model, one can define *DM i's* universal set of options for a first-level hypergame as follows.

$$\ddot{O}_i^1 = O_{ii}^R \cup O_i^{\bar{R}} \cup (\cup_{j\in N}O_{ij}^I) \cup (\cup_{j\in N}O_{ij}^M). \tag{3.5}$$

As discussed in Section 3.2.1, all courses of action in $\ddot{O}_i^1$ are known to the analyst. Nonetheless, because of DMs' varied perceptions, some options in $\ddot{O}_i^1$ may be unknown to some DMs or all. For $j \in N$, let $O_{ij}^U = \ddot{O}_i^1 \setminus (O_{ij}^R \cup O_{ij}^I \cup O_{ij}^M)$ represent the set of *DM i's* options that are unknown to *DM j*.

Remark 3.3.1.1. *Similar to Section 3.2.1, the following relationships are assumed in this research:*

- $(\cup_{j\in N-\{i\}}O_{ij}^R) \subseteq O_{ii}^R$.
- $O_{ii}^I \cap O_i^{\bar{R}} = (\cup_{j\in N}O_{ij}^R) \cap O_i^{\bar{R}} = (\cup_{j\in N}O_{ij}^R) \cap (\cup_{j\in N}O_{ij}^I) = (\cup_{j\in N}O_{ij}^R) \cap (\cup_{j\in N}O_{ij}^M) = (\cup_{j\in N}O_{ij}^I) \cap (\cup_{j\in N}O_{ij}^M) = (\cup_{j\in N}O_{ij}^M) \cap O_i^{\bar{R}} = \emptyset$.
- $\cap_{j\in N}O_{ij}^I \neq \emptyset$ *may or may not hold.*
- $(\cup_{j\in N-\{i\}}O_{ij}^I) \cap O_i^{\bar{R}} \neq \emptyset$ *may or may not hold.*
- $\cap_{j\in N}O_{ij}^M \neq \emptyset$ *may or may not hold.*

The union of all DMs' universal sets of options generates the universal set of options for a first-level hypergame. As in the case of a 2-DM graph model having varied perception among DMs, let $\hat{O}^1$ also denote the universal set of options for an n-DM first-level hypergame. Then, $\hat{O}^1 = \cup_{i\in N}\ddot{O}_i^1$.

3.3.2 Universal Set of States in an n-Decision Maker First-Level Hypergame

Now that the universal set of options for a first-level hypergame with n DMs is defined, a procedure similar to the one defined in Section 3.2.2 is applied to formalize the universal set of states for a first-level hypergame with n-DM in graph form. Formally: let the universal set of options for a first-level hypergame be expressed by $\hat{O}^1 = \cup_{i \in N} \ddot{O}_i^1$, where $\ddot{O}_i^1 = \{o_{\bar{k}}^i : \bar{k} = 1, 2, ..., m_i\}$ is *DM i's* universal set of options in a first-level hypergame. Note that m_i represents the total number of options for *DM i*. As explained in Section 3.2.2, a strategy for *DM i* is a mapping $g_i : \ \ddot{O}_i^1 \longrightarrow \{0, 1\}$, such that for $\bar{k} = 1, 2, ..., m_i$,

$$g_i(o_{\bar{k}}^i) = \begin{cases} 1, & \text{if } DM\ i \text{ selects option } o_{\bar{k}}^i \\ 0, & \text{otherwise} \end{cases}$$

A state, in option form, can be represented by a λ-dimensional column vector, where $\lambda = m_1 + m_2 + ... + m_n$. States in a first-level hypergame can be defined as follows.

Definition 3.3.2.1 (**Universal Set of States for a First-Level Hypergame**). *A state can be defined by a mapping $f : \ \hat{O}^1 \longrightarrow \{0, 1\}$, such that,*

$$f(o_{\bar{k}}^i) = \begin{cases} 1, & \textit{if DM i selects option } o_{\bar{k}}^i, \ \textit{for } i = 1, 2, ..., n, \\ 0, & \textit{otherwise} \end{cases}$$

Note that a typical state, defined above, is a vector of the form: $(f(o_1^1), f(o_2^1), ..., f(o_{m_1}^1), ..., f(o_1^n), f(o_2^n), ..., f(o_{m_n}^n))^T$. The set of states defined above constitutes the universal set of states for a first-level hypergame and is denoted as $\hat{S}^1$.

A state in $\hat{S}^1$ can be represented by s. The total number of states in $\hat{S}^1$ is 2^λ, where $\lambda = \sum_{i}^{n} m_i$. Given a state $s \in \hat{S}^1$, let *DM i's* strategy associated with it be denoted by an m_i-dimensional column vector, g_i^s. Thus, for $N = \{1, 2, ..., i, ..., n\}$, $s = ((g_1^s)^T, (g_2^s)^T, ..., (g_i^s)^T, ..., (g_n^s)^T)^T$.

States in a Decision Maker's Subjective Game

States in a given DM's subjective game for an n-DM first-level hypergame can be defined in a similar fashion as formalized in a 2-DM first-level hypergame. The universal set of options for a first-level hypergame, $\hat{O}^1$, is partitioned into two disjoint sets based on *DM i's* perception: the collections of options that are (1) recognized by *DM i* in its game, expressed as $(\cup_{j\in N}O_{ji}^R)\cup(\cup_{j\in N}O_{ji}^I)\cup(\cup_{j\in N}O_{ji}^M)$, and (2) unknown to *DM i* in its game, expressed as $\cup_{j\in N}O_{ji}^U$. Now, one formally defines *DM i's* sets of recognizable and hidden states in its game as follows.

Definition 3.3.2.2 (**Set of Recognizable States in a DM's Game with** n **DMs**). *Let* $S_i \subseteq \hat{S}^1$ *denote the set of states considered in DM i's subjective game. Then, a state* $s \in S_i \iff$ *there is a mapping* $f : \hat{O}^1 \rightarrow \{0,1\}$ *such that* $s = (f(o_1^1), f(o_2^1), ..., f(o_{m_1}^1), f(o_1^2), f(o_2^2), ..., f(o_{m_2}^2), ..., f(o_1^n), f(o_2^n), ..., f(o_{m_n}^n))^T$ *satisfying* $f(o) = 0$ *,* $\forall\ o \in \cup_{j\in N}O_{ji}^U$.

Definition 3.3.2.3 (**Set of Hidden States in a DM's Game with** n **DMs**). *Let* $S_i^H \subseteq \hat{S}^1$ *denote the set of states that are hidden to DM i in its game. Then, a state* $s \in S_i^H \iff$ *there is a mapping* $f : \hat{O}^1 \rightarrow \{0,1\}$ *such that* $s = (f(o_1^1), f(o_2^1), ..., f(o_{m_1}^1), f(o_1^2), f(o_2^2), ..., f(o_{m_2}^2), ..., f(o_1^n), f(o_2^n), ..., f(o_{m_n}^n))^T$ *satisfying* $\exists\ o \in \cup_{j\in N}O_{ji}^U,\ f(o) = 1$.

Classification of States in a Decision Maker's Subjective Game

Section 3.2.2 provides criteria to classify a given DM's set of recognizable states into five disjoint sets of states for a 2-DM graph model. In this section, a given DM's set of recognizable states is divided into five categories to accommodate more than two DMs' perceptions in the model. For *DM* $i \in N$, its set of recognizable states, S_i, is categorized as follows: (1) correctly perceived by all DMs, denoted as S^R, (2) correctly perceived by *DM i* itself and possibly by some of its opponents but not all of them, symbolized as S_i^P, (3) imagined by *DM i* itself, denoted as S_i^I, (4) misunderstood by *DM i* itself, denoted as S_i^M, and (5) imagined and misunderstood by *DM i* itself, symbolized as $S_i^{I,M}$.

Definition 3.3.2.4 (**Set of Correctly Perceived States by all DMs**). *Choose DM* $i \in N$. *A state* $s \in S_i$ *is correctly perceived by all DMs, that is,* $s \in S^R \subseteq S_i \iff$ *there is a mapping* $f : \hat{O}^1 \to \{0,1\}$ *such that* $s = (f(o_1^1), f(o_2^1), ..., f(o_{m_1}^1),\ f(o_1^2), f(o_2^2),\ ..., f(o_{m_2}^2),\ ..., f(o_1^n), f(o_2^n), ..., f(o_{m_n}^n))^T$ *satisfying* $f(o) = 0,\ \forall\ o \in \hat{O}^1 \setminus (\cup_{i \in N} O_i^R)$.

Definition 3.3.2.5 (**Set of a DM's States Correctly Perceived by Itself and Possibly by Some of Its Opponents but Not by All**). *Choose DM* $i \in N$. *A state* $s \in S_i$ *is correctly perceived by DM* i *itself and possibly by some of its opponents but not by all, that is,* $s \in S_i^P \iff$ *there is a mapping* $f : \hat{O}^1 \to \{0,1\}$ *such that* $s = (f(o_1^1), f(o_2^1), ..., f(o_{m_1}^1),\ f(o_1^2), f(o_2^2),\ ..., f(o_{m_2}^2),\ ..., f(o_1^n), f(o_2^n), ..., f(o_{m_n}^n))^T$ *satisfying* $\exists\ o \in \cup_{j \in N}(O_{ji}^R \setminus O_j^R),\ f(o) = 1$ *and* $f(o') = 0, \forall\ o' \in \hat{O}^1 \setminus (\cup_{j\ \in\ N} O_{ji}^R)$.

Definition 3.3.2.6 (**Set of a DM's States Imagined Only**). *For* $i \in N$*, a state* $s \in S_i$ *is imagined by DM* i *in its game, that is,* $s \in S_i^I \iff$ *there is a mapping* $f : \hat{O}^1 \to \{0,1\}$ *such that* $s = (f(o_1^1), f(o_2^1),\ ..., f(o_{m_1}^1),\ f(o_1^2), f(o_2^2),\ ..., f(o_{m_2}^2),\ ..., f(o_1^n), f(o_2^n),\ ..., f(o_{m_n}^n))^T$ *satisfying* $\exists\ o \in \cup_{j \in N} O_{ji}^I,\ f(o) = 1$ *and* $f(o') = 0, \forall\ o' \in \hat{O}^1 \setminus [(\cup_{j \in N} O_{ji}^I) \cup (\cup_{j \in N} O_{ji}^R)]$.

Definition 3.3.2.7 (**Set of a DM's States Misunderstood Only**). *For* $i \in N$*, a state* $s \in S_i$ *is misunderstood by DM* i *in its game, that is,* $s \in S_i^M \iff$ *there is a mapping* $f : \hat{O}^1 \to \{0,1\}$ *such that* $s = (f(o_1^1), f(o_2^1), ..., f(o_{m_1}^1),\ f(o_1^2), f(o_2^2),\ ..., f(o_{m_2}^2),\ ..., f(o_1^n), f(o_2^n), ..., f(o_{m_n}^n))^T$ *satisfying* $\exists\ o \in \cup_{j \in N} O_{ji}^M,\ f(o) = 1$ *and* $f(o') = 0, \forall\ o' \in \hat{O}^1 \setminus [(\cup_{j \in N} O_{ji}^M) \cup (\cup_{j \in N} O_{ji}^R)]$.

Definition 3.3.2.8 (**Set of a DM's States Imagined and Misunderstood**). *For* $i \in N$*, a state* $s \in S_i$ *is included in DM* i*'s imagined and misunderstood scenarios, that is,* $s \in S_i^{I,M} \iff$ *there is a mapping* $f : \hat{O}^1 \to \{0,1\}$ *such that* $s = (f(o_1^1), f(o_2^1), ..., f(o_{m_1}^1),\ f(o_1^2), f(o_2^2),\ ..., f(o_{m_2}^2),\ ..., f(o_1^n), f(o_2^n), ..., f(o_{m_n}^n))^T$ *satisfying* $\exists\ o \in \cup_{j \in N} O_{ji}^I,\ f(o) = 1$, $\exists\ o' \in \cup_{j \in N} O_{ji}^M,\ f(o') = 1$, *and* $f(o'') = 0, \forall\ o'' \in \hat{O}^1 \setminus [(\cup_{j \in N} O_{ji}^I) \cup (\cup_{j \in N} O_{ji}^M) \cup (\cup_{j \in N} O_{ji}^R)]$.

In summary, the states formalized in Definition 3.3.2.4 are free from any misperception and recognized by all DMs in their subjective games. But the states in Definition 3.3.2.5

are unknown to at least one DM. Keep in mind that the condition $\exists\ \ o \in \cup_{j\in N}(O_{ji}^R \setminus O_j^R)$ in Definition 3.3.2.5 implies that there exists at least one option which is unknown to at least one DM. That is, the states related to the option being selected by the DM who possesses it are unknown to at least one DM. Moreover, according to Remark 4, a common misperception across all DMs is possible in this framework. Therefore, if $s \in S_i^I$, $s \in S_i^M$, or $s \in S_i^{I,M}$, then either (1) it is known to *DM i* only, (2) it is known to *DM i* and some other DMs but not to all, or (3) it is known to all DMs depending on the hypergame situation under investigation.

Theorem 3.3.2.1. *S^R, S_i^P, S_i^I, S_i^M, and $S_i^{I,M}$ are pairwise disjoint. In other words,* $S^R \bigcap S_i^P = S^R \bigcap S_i^I = S^R \bigcap S_i^M = S^R \bigcap S_i^{I,M} = S_i^P \bigcap S_i^I = S_i^P \bigcap S_i^M = S_i^P \bigcap S_i^{I,M} = S_i^I \bigcap S_i^M = S_i^I \bigcap S_i^{I,M} = S_i^M \bigcap S_i^{I,M} = \emptyset$.

Proof: Assume that $S^R \cap S_i^P \neq \emptyset$. Let $s \in S^R \cap S_i^P$. This implies that $s \in S^R$ and $s \in S_i^P$. As characterized in Definition 3.3.2.4, $s \in S^R \subseteq S_i \iff$ there is a mapping $f : \hat{O}^1 \rightarrow \{0,1\}$ such that $s = (f(o_1^1), f(o_2^1), ..., f(o_{m_1}^1),\ f(o_1^2), f(o_2^2),\ ..., f(o_{m_2}^2), ..., f(o_1^n), f(o_2^n), ..., f(o_{m_n}^n))^T$ satisfying $f(o) = 0,\ \forall\, o \in \hat{O}^1 \backslash (\cup_{i\in N} O_i^R)$. But since $(\cup_{j\in N} O_j^R) \subseteq (\cup_{j\in N} O_{ji}^R)$, the condition $f(o) = 0,\ \forall\, o \in \hat{O}^1 \backslash (\cup_{i\in N} O_i^R)$ can be expressed as $f(o) = 0,\ \forall\, o \in [\hat{O}^1 \setminus (\cup_{j\in N} O_{ji}^R)] \cup [\cup_{j\in N}(O_{ji}^R \setminus O_j^R)]$. Hence, a state $s \in S^R$ implies that $f(o) = 0,\ \forall\ o \in \cup_{j\in N}(O_{ji}^R \setminus O_j^R)$. This contradicts with the fact that $s \in S_i^P$ in which $\exists\, o \in \cup_{j\in N}(O_{ji}^R \setminus O_j^R)$, $f(o) = 1$, as can be found in Definition 3.3.2.5. Hence, the assumption that $S^R \bigcap S_i^P \neq \emptyset$ is not true. This proves by contradiction that $S^R \bigcap S_i^P = \emptyset$. In a similar way, one can prove that $S^R \bigcap S_i^I = S^R \bigcap S_i^M = S^R \bigcap S_i^{I,M} = S_i^P \bigcap S_i^I = S_i^P \bigcap S_i^M = S_i^P \bigcap S_i^{I,M} = S_i^I \bigcap S_i^M = S_i^I \bigcap S_i^{I,M} = S_i^M \bigcap S_i^{I,M} = \emptyset$. □

3.3.3 Modeling and Analysis of a First-Level Hypergame with n Decision Makers in Graph Form

As in Section 3.2.3, the modeling and analysis techniques of a first-level hypergame with n DMs in graph form are addressed here.

Mathematical Modeling of a First-Level Hypergame with n Decision Makers

As in Section 3.2.3, a first-level hypergame with more than two DMs can be represented by a number of subjective games, each of which takes into account a particular DM's view point of the conflict situation. For $i \in N$, *DM i's* subjective game in graph form, G_i, can be formalized as follows.

$$G_i = \langle N_i, S_i, \{A_{ji} : j \in N_i\}, \{\succsim_{ji} : j \in N_i\}\rangle \tag{3.6}$$

where $N_i \subseteq N$ is the set of DMs as perceived by *DM i* in G_i, S_i is the set of feasible states, A_{ji} represents the set of state transitions controlled by *DM j* as perceived by *DM i*, and $\succsim_{ji}$ represents *DM j's* preferences over S_i as perceived by *DM i*. Then, a first-level hypergame with more than two DMs in graph form can be represented as:

$$H^1 = \langle G_i : i \in N\rangle \tag{3.7}$$

As can be seen in Eq 3.6, a particular DM may not be aware that one or more DMs are involved in a conflict. If *DM i* is not aware of the presence of *DM j*, for example, then all of *DM j's* options are hidden to *DM i*. Consequently, the states related to these options being selected by *DM j* are also unknown to *DM i*.

Stability Analysis of a First-Level Hypergame with n Decision Makers

The stability analysis procedure for a first-level hypergame with more than two DMs in graph form can be formalized in a similar fashion as was carried out for a first-level hypergame with two DMs in Section 3.2.3. One can recall that there are three steps to determine the first-level hypergame equilibria. In the first step, one identifies the equilibrium states in each DM's subjective game. This can be accomplished by checking each state for stability using a range of solution concepts defined within the original GMCR structure. In the next step, one ascertains the first-level hypergame equilibria. This can be done by first isolating each DM's strategy from each of its equilibriuam states in its subjective game and then by taking the Cartesian product of all the DMs' strategy sets, each of which is constructed from a particular DM's equilibrium states in its subjective game. The resulting Cartesian product constitutes the set of first-level hypergame equilibria. In the third step, the first-level hypergame equilibria are classified into eight categories, as explained in Section 3.4, to provide better insights into the conflict.

Step 1: Equilibrium States in a Decision Maker's Subjective Game

The solution concepts presented in Section 3.2.3 are precisely defined for a first-level hypergame with two DMs in graph form in which a given DM has only one opponent. However, in a first-level haypergame with more than two DMs, the focal DM has more than one opponent. Thus, the stability definitions provided in Section 3.2.3 are not adequate for checking each state for stability in a first-level hypergame with more than two DMs. Accordingly, stability definitions for a general n-DM first-level hypergame in graph form are put forward to identify the equilibrium states in each DM's subjective game. For $i \in N$, the stability definitions for *DM i's* subjective game, G_i, require the extension of the concepts of reachable list and unilateral improvement list (UIL) by a single DM, formalized in Definitions 3.2.3.1 and 3.2.3.2, respectively, to those by a group of DMs.

Assume that the set of DMs in G_i is $N_i = \{1, 2, ..., i, ..., n_i\}$. Let $H \subseteq N_i$, $H \neq \emptyset$, be any subset of DMs in N_i. For $s_1 \in S_i$, recall that $R_{qi}(s_1)$ represents the set of *DM q's* unilateral moves (UMs) starting from s_1. Let $R_{Hi}(s_1)$ denote the set of all UMs from s_1 by one or more DMs in H through a valid sequence of moves starting from s_1. A sequence of moves by DMs in H is considered valid if no DM makes two consecutive moves. For $s_2 \in R_{Hi}(s_1)$, let $\Omega_{Hi}(s_1,\ s_2)$ denote the set of all last DMs in H in the valid sequences of moves from s_1 to s_2. The reachable list by $H \subseteq N_i$ can now be formalized.

Definition 3.3.3.1 (**Reachable List by** $H \subseteq N_i$)**.** *Let $s_1 \in S_i$. Then, $R_{Hi}(s_1)$ can be defined as follows:*

- *If $q \in H$ and $s_2 \in R_{qi}(s_1)$, then $s_2 \in R_{Hi}(s_1)$ and $q \in \Omega_{Hi}(s_1,\ s_2)$;*
- *If $s_2 \in R_{Hi}(s_1)$, $q \in H$, and $s_3 \in R_{qi}(s_2)$, then*

  (a) if $|\Omega_{Hi}(s_1,\ s_2)| = 1$ and $q \notin \Omega_{Hi}(s_1,\ s_2)$, then $s_3 \in R_{Hi}(s_1)$ and $q \in \Omega_{Hi}(s_1,\ s_3)$.

  (b) if $|\Omega_{Hi}(s_1,\ s_2)| > 1$, then $s_3 \in R_{Hi}(s_1)$ and $q \in \Omega_{Hi}(s_1,\ s_3)$.

  The induction stops when there is no new state s_3 that can be added to $R_{Hi}(s_1)$ and no change from $|\ \Omega_{Hi}(s_1,\ s_2)\ | = 1$ to $|\ \Omega_{Hi}(s_1,\ s_2)\ | > 1$ for any $s_2 \in R_{Hi}(s_1)$. Any state in $R_{Hi}(s_1)$ is a UM from s_1 by H.

Recall from Definition 3.2.3.2 that a state is considered as a unilateral improvement (UI) from a prespecified state by a particular DM if the state is reachable by the DM and is preferred to the initial state. As the concept of a reachable list by a set of DMs for the case of n DMs has now been introduced, one can define the idea of UIL by a group of DMs. For $s_1 \in S_i$, recall that $R^+_{qi}(s_1)$ denotes the set of *DM q's* UIL from s_1 in G_i. Let $R^+_{Hi}(s_1)$ denote the set of UIs from s_1 by a group of DMs, $H \subseteq N_i$ and $H \neq \emptyset$, and $\Omega^+_{Hi}(s_1,\ s_2)$ represent the set of all last DMs in valid sequences of unilateral improvements from s_1 to s_2;

Definition 3.3.3.2 (**Unilateral Improvement List by** $H \subseteq N_i$)**.** *Let $s_1 \in S_i$. The UIL $R^+_{Hi}(s_1)$ is constructed inductively as follows:*

- *If* $q \in H$ *and* $s_2 \in R^+_{qi}(s_1)$*, then* $s_2 \in R^+_{Hi}(s_1)$ *and* $q \in \Omega^+_{Hi}(s_1,\ s_2)$*;*
- *If* $s_2 \in R^+_{Hi}(s_1)$*,* $q \in H$*, and* $s_3 \in R^+_{qi}(s_2)$*, then*

  (a) if $|\ \Omega^+_{Hi}(s_1,\ s_2)\ |= 1$ *and* $q \notin \Omega^+_{Hi}(s_1,\ s_2)$*, then* $s_3 \in R^+_{Hi}(s_1)$ *and* $q \in \Omega^+_{Hi}(s_1,\ s_3)$*,*

  (b) if $|\ \Omega^+_{Hi}(s_1,\ s_2)\ |> 1$*, then* $s_3 \in R^+_{Hi}(s_1)$ *and* $q \in \Omega^+_{Hi}(s_1,\ s_3)$*.*

  The induction stops when there is no new state s_3 *that can be added to* $R^+_{Hi}(s_1)$ *and no change from* $|\ \Omega^+_{Hi}(s_1,\ s_2)\ |= 1$ *to* $|\ \Omega^+_{Hi}(s_1,\ s_2)\ |> 1$ *for any* $s_2 \in R^+_{Hi}(s_1)$*. Any state in* $R^+_{Hi}(s_1)$ *is a UI from* s_1 *by a group of DMs* H*.*

Now that the concepts of reachable list and UIL by a set of DMs $H \subseteq N_i$ have been introduced, one can formally define stability concepts in G_i with more than two DMs. The stability definitions put forward here are Nash stability, SEQ stability, GMR stability, and SMR stability. Note that a state is considered Nash stable for *DM q* in G_i if *DM q* has no UI from that particular state. That is, Nash stability does not take into account the responses by the opponents of a focal DM. Therefore, Definition 3.2.3.3 in Section 3.2.3 remains the same for the n-DM case. However, SEQ, GMR, and SMR stability definitions depend on the responses by the opponents of the focal DM. Hence, the definitions of these stabilities are generalized below:

Definition 3.3.3.3 (**SEQ Stability**)**.** *A state* $s_1 \in S_i$ *is sequentially stable (SEQ) for DM* $q \in N_i$ *in* $G_i \iff$ *for each* $s_2 \in R^+_{qi}(s_1)$*,* $\exists\ s_3 \in R^+_{(N_i-\{q\})i}(s_2)$ *such that* $s_3 \precsim_{qi} s_1$*. The set of all SEQ stable states for DM q in* G_i *is denoted by* $S_i^{SEQ_{qi}}$*.*

Definition 3.3.3.4 (**GMR Stability**)**.** *A state* $s_1 \in S_i$ *is general metarational stable (GMR) for DM* $q \in N_i$ *in* $G_i \iff$ *for each* $s_2 \in R^+_{qi}(s_1)$*,* $\exists\ s_3 \in R_{(N_i-\{q\})i}(s_2)$ *such that* $s_3 \precsim_{qi} s_1$*. The set of all GMR stable states for DM q in* G_i *is denoted by* $S_i^{GMR_{qi}}$*.*

Definition 3.3.3.5 (**SMR Stability**)**.** *A state* $s_1 \in S_i$ *is symmetric metarational stable (SMR) for DM* $q \in N_i$ *in* $G_i \iff$ *for each* $s_2 \in R^+_{qi}(s_1)$*,* $\exists\ s_3 \in R_{(N_i-\{q\})i}(s_2)$ *such that*

$s_3 \precsim_{qi} s_1$, *and* $s_4 \precsim_{qi} s_1$, $\forall\ s_4 \in R_{qi}(s_3)$. *The set of all SMR stable states for DM q in* G_i *is denoted by* $S_i^{SMR_{qi}}$.

Theorem 3.3.3.1. *In a first-level hypergame in graph form, the stability of states for DM q* $\in N_i$ *in* G_i *satisfies the following relationships:*

$$S_i^{Nash_{qi}} \subseteq S_i^{SMR_{qi}} \subseteq S_i^{GMR_{qi}} \tag{3.8}$$

$$S_i^{Nash_{qi}} \subseteq S_i^{SEQ_{qi}} \subseteq S_i^{GMR_{qi}} \tag{3.9}$$

Proof. The relationships in (3.8) and (3.9) are proven below.

- For the inclusion relationships in (3.8), if $s_1 \in S_i^{Nash_{qi}}$, then $R_{qi}^{+}(s_1) = \emptyset$. This mean that there are no UIs from s_1 by *DM q*. Therefore, Definition 3.3.3.5 satisfied and $s_1 \in S_i^{SMR_{qi}}$. As a result, $S_i^{Nash_{qi}} \subseteq S_i^{SMR_{qi}}$. Next, in light of Definition 3.3.3.5, if $s_1 \in S_i^{SMR_{qi}}$, it implies that all UIs for *DM q* are sanctioned by its opponents and *DM q* itself cannot escape from the sanctions. Any sanction in Definition 3.3.3.5 meets the conditions of Definition 3.3.3.4. Therefore, $s_1 \in S_i^{GMR_{qi}}$ and $S_i^{SMR_{qi}} \subseteq S_i^{GMR_{qi}}$. Thus, the inclusion relationships $S_i^{Nash_{qi}} \subseteq S_i^{SMR_{qi}} \subseteq S^{GMR_{qi}}$ are true.

- For the inclusion relationships in (3.9), if $s_1 \in S_i^{Nash_{qi}}$, then $R_{qi}^{+}(s_1) = \emptyset$, which implies by Definition 3.3.3.3 and as explained above that $s_1 \in S_i^{SEQ_{qi}}$. Thus, $S_i^{Nash_{qi}} \subseteq S_i^{SEQ_{qi}}$. Next, from Definition 3.3.3.3, one can observe that $R_{(N_i-\{q\})i}^{+}(s_2) \subseteq R_{(N_i-\{q\})i}(s_2)$. That is, $s_1 \in S_i^{SEQ_{qi}}$ for *DM q* must also be GMR stable for *q*. Hence, $S_i^{SEQ_{qi}} \subseteq S_i^{GMR_{qi}}$. Therefore, the inclusion relationships $S_i^{Nash_{qi}} \subseteq S_i^{SEQ_{qi}} \subseteq S_i^{GMR_{qi}}$ are true.

□

As various stability definitions for the n-DM case have now been introduced, one can determine the equilibrium states for *DM i's* subjective game. A state is considered as an equilibrium in G_i under a specific solution concept if it is stable for every DM in N_i under that stability concept. Hence, Definition 3.2.3.7 remains the same for the n-DM case.

Step 2: First-Level Hypergame Equilibria for n Decision Makers in Graph Form

In this step, first-level hypergame equilibria are identified. This can be done by determining each DM's strategy from each of its Nash, SEQ, GMR, and SMR equilibrium states. Next, one takes the Cartesian product of all the DMs' strategy sets for each of the above mentioned solution concepts. Note that the Cartesian product of all DMs' Nash strategy sets, each of which is obtained from a given DM's Nash equilibrium states within its subjective game, is referred to as the set of hyper Nash equilibrium states for a first-level hypergame.

Definition 3.3.3.6 (**Hyper Nash Equilibrium**). *For $i \in N$, recall that E_i^{Nash} and $g_i^{*Nash_i}$ denote the sets of DM i's Nash equilibrium states and distinct Nash strategies, respectively, as perceived by DM i in its subjective game G_i. Then, the set of hyper Nash equilibria for the first-level hypergame is defined as follows:*

$$HE^{1Nash} = \prod_{i \in N} g_i^{*Nash_i} \tag{3.10}$$

Any member of HE^{1Nash} is a hyper Nash equilibrium state of the dispute. In fact, there are up to a total of $\varepsilon_i \varepsilon_j ... \varepsilon_n$ hyper Nash equilibrium states.

HE^{1SEQ}, HE^{1GMR}, and HE^{1SMR} can be constructed analogously.

3.4 Classification of First-Level Hypergame Equilibria

The classification of first-level hypergame equilibria is the final step of the analysis of a conflict with misperception. The classification of the hypergame equilibria is not accomplished by the participants of the conflict, but instead by an insightful external analyst who is aware of the asymmetry of viewpoints of the DMs. As a result, the analyst can, in fact, distinguish between the states that are correctly perceived and misperceived by the DMs. These equilibrium classes provide information with regards to the types of DMs' misperception and the consequences of misperception on resolutions of the dispute. The equilibrium classes put forward here are steady hyper equilibrium, unsteady hyper equilibrium, stealthy hyper equilibrium, unsteady stealthy hyper equilibrium, contingent hyper equilibrium, unsteady contingent hyper equilibrium, self-contingent hyper equilibrium, and emergent hyper equilibrium.

For an n-DM hypergame ($n \geqslant 2$), a hypergame equilibrium state that is recognized by all DMs as a correctly perceived state in their subjective games and is also an equilibrium scenario under a specific solution concept (Nash, 1950, 1951; Howard, 1971; Fraser and Hipel, 1984; Fang et al., 1993) in their subjective games, is considered as a steady hyper equilibrium state for the dispute. Formally, a steady hyper Nash ($SHNash^1$) equilibrium state for a first-level hypergame can be defined as follows:

Definition 3.4.0.7 ($SHNash^1$ **Equilibrium**). *A hyper Nash equilibrium state,* $s \in HE^{1Nash}$*, is called* $SHNash^1$ *equilibrium iff* $s \in S^R$ *and* $s \in \cap_{i \in N} E_i^{Nash}$.

Please notice that the superscript 1 in $SHNash^1$ indicates that it is an equilibrium for a first-level hypergame. In this situation, even though the hyper Nash equilibrium state is obtained from subjective games, DMs' misperception does not affect the equilibrium state as no information about misperception is included in the state. In such a case, DMs' misperceptions are found to be preserved and all DMs may not be motivated to move the dispute into future rounds and, as a result, the equilibrium state can be considered as a final resolution of the dispute (Sasaki and Kijima, 2008; Aljefri et al., 2014a,b).

A steady hyper SEQ ($SHSEQ^1$) equilibrium, a steady hyper GMR ($SHGMR^1$) equilibrium, and a steady hyper SMR ($SHSMR^1$) equilibrium for a first-level hypergame can be defined analogously.

In contrast with the idea of an $SHNash^1$ equilibrium state, if a hyper Nash equilibrium state is recognized by all DMs as a feasible scenario in their subjective games and not predicted by at least one DM as a Nash equilibrium in its subjective game, the state is an unsteady hyper Nash ($UHNash^1$) equilibrium. Formally:

Definition 3.4.0.8 ($UHNash^1$ **Equilibrium**). *A hyper Nash equilibrium state,* $s \in HE^{1Nash}$*, is called a* $UHNash^1$ *equilibrium iff* $s \in S^R$ *and* $s \notin \cap_{i \in N} E_i^{Nash}$*.*

In this circumstance, the conflict may evolve into future rounds, as some information concerning DMs' preference misperception may be revealed to some DMs. For instance, if a hyper Nash equilibrium state is not a Nash equilibrium in *DM i's* game, then *DM i* believes that it has a UI from the hyper Nash equilibrium state to another state, or it assumes that some DMs can move to a more preferred state.

An unsteady hyper SEQ ($UHSEQ^1$) equilibrium, unsteady hyper GMR ($UHGMR^1$) equilibrium, and unsteady hyper SMR ($UHSMR^1$) equilibrium for a first-level hypergame can be defined in a similar way.

If a hypergame equilibrium state under a specific solution concept is correctly recognized by a given DM and possibly by some of its opponents but not by all, then the state is a stealthy hyper equilibrium under that specific stability definition for those DMs who are unaware of the existence of this particular state in reality. Formally:

Definition 3.4.0.9 (Stealthy Hyper Nash ($STHNash^1$) **Equilibrium).** *A hyper Nash equilibrium state,* $s \in HE^{1Nash}$*, is called an* $STHNash^1$ *iff* $\exists\, i \in N$*,* $s \in S_i^P \cap E_i^{Nash}$*.*

A hyper Nash equilibrium state is called an $STHNash^1$ equilibrium state for a first-level hypergame (Aljefri et al., 2014a,b) whenever it is a surprise for at least one DM and constitutes a Nash equilibrium by the other DMs who correctly perceived the state in their subjective games. In fact, a DM can fail to correctly perceive one or more real scenarios in its opponents' subjective games because of underestimating opponents' capability in exercising certain courses of action in reality (Jervis, 1968, 1976; Levy, 1983; Ben-Zvi, 1995). These courses of action are strategic surprises for the DMs who are unaware of the existence of this particular state in reality. Once the $STHNash^1$ equilibrium state occurs in a dispute, then at least one DM will acquire some information about its misperception, motivating it to evolve the dispute into a future round.

A stealthy hyper SEQ ($STHSEQ^1$) equilibrium, a stealthy hyper GMR ($STHGMR^1$) equilibrium, and a stealthy hyper SMR ($STHSMR^1$) equilibrium for a first-level hypergame can be defined in a similar way.

If the hypergame equilibrium state under a specific solution concept is unknown to at least one DM and does not constitute an equilibrium for at least one of the DMs who correctly perceive the state in their subjective games under that specific solution concept, then the state is an unsteady stealthy hyper equilibrium for the dispute. Formally:

Definition 3.4.0.10 (Unsteady $STHNash^1$ ($USTHNash^1$) Equilibrium). *A hyper Nash equilibrium state, $s \in HE^{1Nash}$, is called a $USTHNash^1$ equilibrium for a first-level hypergame iff $s \in S_i^P \setminus E_i^{Nash}$ for at least one $i \in N$.*

This equilibrium is unsustainable for DMs. For example, DMs who fail to perceive this state in their subjective games will face a strategic surprise, and the conflict may evolve after they obtain updated information about their misperception. Other DMs, on the other hand, who are aware of the state in their subjective games, but fail to predict it as a Nash equilibrium in their subjective games, may believe that either they or the other DMs may have a UI from this state to a more preferred state. After DMs filter their misperception, a new round of the conflict may occur.

An unsteady $STHSEQ^1$ ($USTHSEQ^1$) equilibrium, unsteady $STHGMR^1$ ($USTHGMR^1$) equilibrium, and unsteady $STHSMR^1$ ($USTHSMR^1$) equilibrium for a first-level hypergame can be defined analogously.

If the hypergame equilibrium state under a specific solution concept is recognized by DMs as a misperceived state and it constitutes an equilibrium in their individual games under that specific solution concept, then the state is called a contingent hyper equilibrium under that stability definition for the dispute. Formally:

Definition 3.4.0.11 (Contingent Hyper Nash ($CHNash^1$) Equilibrium). *A hyper Nash equilibrium state, $s \in HE^{1Nash}$, is called a $CHNash^1$ equilibrium $\iff$ either $s \in \cap_{i \in N} S_i^I$, $s \in \cap_{i \in N} S_i^M$, or $s \in \cap_{i \in N} S_i^{I,M}$ and $s \in \cap_{i \in N} E_i^{Nash}$.*

Despite the fact that all DMs consider a $CHNash^1$ equilibrium as an possible resolution in their subjective games, this state is considered as an illusionary equilibrium for the dispute (Jervis, 1976; Betts, 2000). As a hypothetical example to the situation mentioned above, consider a possible military confrontation between countries "A" and "B". Suppose that A's actual option is to exercise a weak military attack against B. However, because A overestimated its capabilities and B underestimated its strength, both countries believe that A can launch a possible massive attack against B. In this circumstance, one can conclude that both DMs share the same misperception. If all DMs find this state as an acceptable outcome, the state is considered as a final resolution of the dispute. However, if at least one DM is not satisfied with this equilibrium, then the conflict may evolve into a future round. In this case, a new analysis is required.

A contingent hyper SEQ ($CHSEQ^1$) equilibrium, a contingent hyper GMR ($CHGMR^1$) equilibrium, and a contingent hyper SMR ($CHSMR^1$) equilibrium for a first-level hypergame can be defined in a similar fashion.

If a hypergame equilibrium state under a specific solution concept is recognized by all DMs as a misperceived state and it is not an equilibrium in at least one DM's game, then it is called an unsteady contingent hyper equilibrium for the dispute under that specific solution concept. Formally:

Definition 3.4.0.12 (**Unsteady** $CHNash^1$ ($UCHNash^1$) **Equilibrium**). *A hyper Nash equilibrium state, $s \in HE^{1Nash}$, is called an $UCHNash^1$ equilibrium $\Longleftrightarrow$ either $s \in \cap_{i \in N} S_i^I$, $s \in \cap_{i \in N} S_i^M$, or $s \in \cap_{i \in N} S_i^{I,M}$ and $s \notin \cap_{i \in N} E_i^{Nash}$.*

An unsteady $CHSEQ^1$ ($UCHSEQ^1$) equilibrium, unsteady CHGMR ($UCHGMR^1$) equilibrium, and unsteady $CHSMR^1$ ($UCHSMR^1$) equilibrium for a first-level hypergame can be defined in a similar way.

If a hypergame equilibrium state under a specific solution concept is recognized as (1) a misperceived state by at least one DM but not by all of them, and (2) an unknown state by the remaining DMs, then it is called a self-contingent hyper equilibrium for the dispute under that solution concept. Formally:

Definition 3.4.0.13 (**Self-**$CHNash^1$ ($SCHNash^1$) **Equilibrium**). *A hyper Nash equilibrium state, $s \in HE^{1Nash}$, is called an $SCHNash^1$ equilibrium for the dispute iff there is a DM $i \in N$ such that $s \in (S_i^I \cup S_i^M \cup S_i^{I,M}) \setminus \cap_{j \in N}(S_j^I \cup S_j^M \cup S_j^{I,M})$.*

An $SCHNash^1$ equilibrium is an illusionary and unrealistic outcome for the DM who perceives it (Betts, 2000; Trivers, 2000). Because of uncertainty of the impact of exercising imagined or misunderstood, or imagined and misunderstood options by a DM, the DM may either improve its position relative to that of its opponents or put itself into a worse position (Betts, 2000). If this possible resolution is exercised, any DM may obtain new information, motivating it to evolve the dispute into a future round, thereby making an $SCHNash^1$ equilibrium state unstable, and further analysis may be needed.

A self-$CHSEQ^1$ ($SCHSEQ^1$) equilibrium, self-$CHGMR^1$ ($SCHGMR^1$) equilibrium, and self-$CHSMR^1$ ($SCHSMR^1$) equilibrium for a first-level hypergame can be defined in a similar fashion.

If a hypergame equilibrium state under a specific solution concept is hidden to all DMs, then it is called an emergent hyper equilibrium state under that solution concept for the conflict. Formally:

Definition 3.4.0.14 (**Emergent Hyper Nash** ($EHNash^1$) **Equilibrium**). *A hyper Nash equilibrium state, $s \in HE^{1Nash}$, is called an $EHNash^1$ equilibrium iff $s \notin \cup_{i \in N} S_i$.*

An $EHNash^1$ equilibrium constitutes a possibility of surprise to all DMs. In particular, a state possesses a possibility of shock if each DM exercises a strategy that is only recognized by itself. The combination of these strategies forms a state that is hidden to all DMs. This circumstance may occur in a real-life conflict under either asymmetry of information between the DMs or their misperception of the dispute. From the equilibrium state, DMs are expected to obtain additional information about each other's options, strategies, and preferences. This new knowledge may motivate DMs to cause the dispute to change by invoking possible moves.

An emergent hyper SEQ ($EHSEQ^1$) equilibrium, emergent hyper GMR ($EHGMR^1$) equilibrium, and emergent hyper SMR ($EHSMR^1$) equilibrium for a first-level hypergame can be defined in a similar fashion.

In summary, the eight types of first-level hypergame Nash equilibria as defined in Definitions 37 to 44 are mutually exclusive and cover every possible first-level hypergame equilibrium. If a hypergame equilibrium state is recognized by all DMs in their subjective games, then the state is either a steady hyper, unsteady hyper, contingent hyper, or unsteady contingent hyper equilibrium for the first-level hypergame. However, if a hypergame equilibrium state is limited to at least one DM's perception, then it is either a stealthy, unsteady stealthy, or self-contingent hyper equilibrium for the hypergame. If, on the other hand, a hypergame equilibrium is hidden to all DMs, then it is an emergent hyper equilibrium for the dispute.

Theorem 3.4.0.2. *$SHNash^1$, $UHNash^1$, $STHNash^1$, $USTHNash^1$, $CHNash^1$, $UCHNash^1$, $SCHNash^1$, and $EHNash^1$ are pairwise disjoint.*

Proof: Let $s \in HE^{1Nash}$ be a hyper Nash equilibrium state for a first-level hypergame. Assume that $SHNash^1 \cap UHNash^1 \neq \emptyset$. Let $s \in SHNash^1 \cap UHNash^1$. This implies that $s \in SHNash^1$ and $s \in UHNash^1$. As formalized in Definition 3.4.0.7, $s \in HE^{1Nash}$ is called a $SHNash^1$ if $s \in S^R$ and $s \in \cap_{i \in N} E_i^{Nash}$. Hence, a state $s \in SHNash^1$ implies that $s \in \cap_{i \in N} E_i^{Nash}$. This contradicts with the fact that $s \in UHNash^1$ in which $s \notin \cap_{i \in N} E_i^{Nash}$, as can be found in Definition 3.4.0.8. Therefore, the assumption that $SHNash^1 \cap UHNash^1 \neq \emptyset$ is not true. This proves by contradiction that $SHNash^1 \cap UHNash^1 = \emptyset$. In a similar way, the other relationships can be proven. □

3.5 Chapter Summary

The first-level hypergame in graph form is developed in this research for 2-DM and n-DM conflicts. The main objectives of this new hypergame approach are to predict the possible compromise resolutions for real-life conflicts that have misperceptions by participating DMs, and to investigate the strategic consequences of various types of equilibria that could occur. More specifically, the new method of a first-level hypergame in graph form provides better insights about a dispute as it categorizes the first-level hypergame equilibria into eight classes, each of which provides different understandings about the type of misperception involved in the dispute and possible reactions by DMs after learning about their misperceptions.

Chapter 4

Application of the First-Level Hypergame Methodology to Two Real-Life Conflicts

To test the applicability and efficacy of the first-level hypergame in graph form defined in Chapter 3 as well as to show how to apply it in practice, two real-life case studies are investigated. In particular, the 2011 conflict between North and South Sudan over South Sudanese oil exports and the dispute over the unexpected nationalization of the Suez Canal in 1956 by Egyptian President Gamal Abdul Nasser are studied within the structure of a 2-DM first-level hypergame in graph form. The contents of this chapter are based on the hypergame analyses of the South Sudan conflict by Aljefri et al. (2013, 2014b) and the Suez Canal crisis by Aljefri et al. (2016a).

4.1 Case Study I: The Oil Export Pipeline Conflict between North and South Sudan

4.1.1 Historical Background

South Sudan gained its independence on July 9, 2011 from the Republic of North Sudan. The Comprehensive Peace Agreement (CPA) signed in 2005 (Brosché, 2008) stipulated important settlements that facilitated South Sudanese independence (Oil and Energy Trends, 2011). Within the agreement, desperate to stop the long civil war, South Sudan agreed to give more than 48% of the southern region's oil revenue to the government of North Sudan. The situation was very different, however, after South Sudan gained its independence (Oil and Energy Trends, 2011). South Sudan attempted to negotiate reducing North Sudan's revenue to about 25%. North Sudan rejected the proposal and demanded more than 50% of the income from the southern region's oil revenue. In August 2011, North Sudan imposed an exorbitant fee for the use of its export pipeline to Port Sudan, which is the only available route for South Sudanese oil exports. South Sudan's response was to reject the overpriced fee but keep using the North Sudanese export pipeline and begin looking for an alternative export route (Oil and Energy Trends, 2011; Aljefri et al., 2013).

According to (Oil and Energy Trends, 2011), South Sudan is examining the possibility of constructing a new route for oil exports. The plan is to build a new pipeline with a capacity of 450,000 barrels per day (bpd) to Kenya, which can then be connected with the existing Kenyan pipeline to the port of Mombasa (Oil and Energy Trends, 2011). Under the current economic conditions, however, it is doubtful that South Sudan can finance the project.

On December 1, 2011, North Sudan escalated the situation by taking 23% of South Sudan's oil as compensation for South Sudan's failure to pay the stipulated fee for using its pipeline, without obtaining South Sudan's approval (Sudan Tribune, 2011b,a). The conflict between North and South Sudan is studied using a first-level hypergame in a graph model to explain the effect of DMs' perceptions on the outcomes of the dispute.

4.1.2 Modelling the Dispute

The conflict between North and South Sudan encountered a type of hypergame, which is a game in which one or more of the DMs fails to perceive his or her opponent's true set of options and preferences. As a result, an individual game and an integrated graph model must be developed for each DM to represent his or her viewpoint of the dispute. The conflict between North and South Sudan is modeled as a first-level hypergame, as none of the DMs are aware of the occurrence of any misperception. The architecture of the first-level hypergame in a graph form is displayed in Figure 3.2. The modeling of the universal set of states for the dispute is put forward in the next subsection.

Modelling the Universal Set of States for the Dispute

The DMs and options for the conflict are given in Table 4.1. Note that two DMs are identified for the oil export conflict (Oil and Energy Trends, 2011; Aljefri et al., 2013; Sudan Tribune, 2011b,a): North Sudan (denoted by C) and South Sudan (denoted by D). As of December 1, 2011, North Sudan had three options: request exorbitant fee, shut down the oil export pipeline, or self-reimburse by taking 23% of South Sudan's oil without South Sudan's approval. South Sudan also had three options: accept the exorbitant fee, explore alternative independent oil export route, or stop oil production. As can be found in Table 4.1, South Sudan misperceives North Sudan's options. As explained in Oil and Energy Trends (2011); Sudan Tribune (2011a,b); Aljefri et al. (2013), South Sudan did not expect North Sudan to self-reimburse for the unpaid exorbitant fees without first reaching an agreement with South Sudan. That is, the option self-reimburse is unknown to South Sudan and will not be considered in its game. Each option in Table 4.1 is labeled with a number and can either be chosen (Y for yes) or not (N for no). For example, option 1 (request exorbitant fee) is the circumstance in which North Sudan imposes a high export fee for the use of its export pipeline. Choosing this option, as indicated by Y for yes, means North Sudan imposes a high fee, while not selecting this course of action, N for no, implies the situation in which North Sudan decides to charge South Sudan a reasonable fee for the use of its export pipeline.

Table 4.1: Decision Makers and Options for the Dispute

DM	Option	Choice	Description	Recognized by
North Sudan (C)	1. Request Exorbitant Fee.	Y	North Sudan imposes an exorbitant fee for the use of its export pipeline to Port Sudan, which is the only available route for South Sudanese oil exports.	North and South Sudan
		N	North Sudan charges a reasonable fee for the use if its export pipeline.	
	2. Shut Down.	Y	Prevent South Sudan from using North Sudan's Export pipeline.	North and South Sudan
		N	Allow South Sudan to use North Sudan's export pipeline.	
	3. Self-Reimburse.	Y	North Sudan takes 23% of South Sudan's oil as compensation for South Sudan's failure to pay the stipulated fee for using its pipeline.	North Sudan
		N	The option is not exercise.	
South Sudan (D)	4. Accept the Exorbitant Fee.	Y	South Sudan accepts paying the exorbitant fee.	North and South Sudan
		N	South Sudan rejects paying the exorbitant fee.	
	5. Alternative Export Route.	Y	South Sudan explores an alternative export route.	North and South Sudan
		N	South Sudan uses North Sudan's pipeline.	
	6. Stop Oil Production.	Y	South Sudan stop producing oil.	North and South Sudan
		N	South Sudan continues producing oil.	

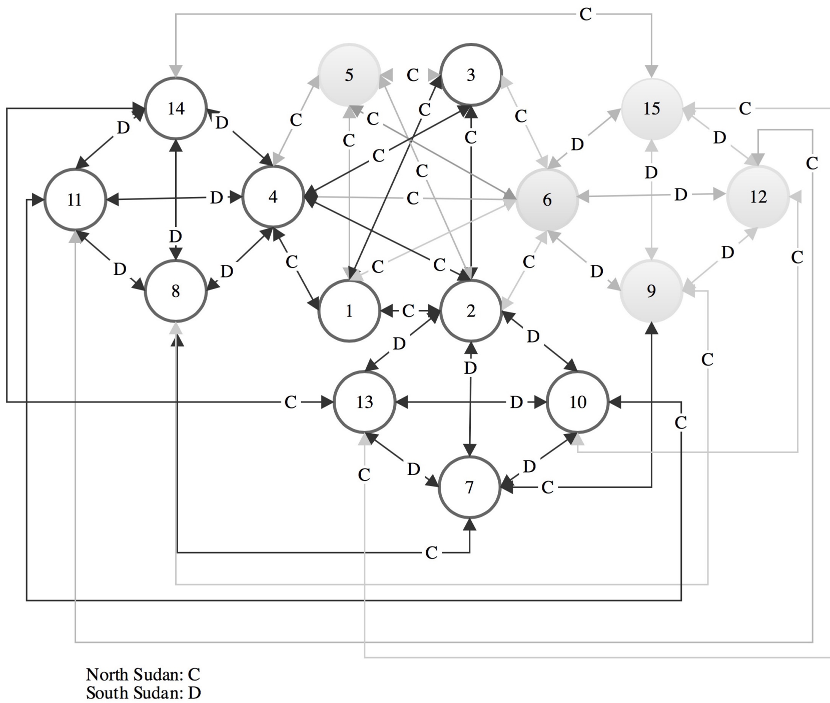
The courses of action in Table 4.1 are considered in defining the universal set of states. In option form, an option can be taken or not by the DM who controls it; therefore, the total number of possible states in a conflict can be mathematically computed as 2^{λ}. In this conflict, the total number of possible states comes to $2^6 = 64$. These states represent the total number of mathematical states for the universal set of states, $\mid \hat{S}^1 \mid = 64$. However, some states are removed because they are categorized as being infeasible (Fang et al., 2003a,b). Note that there are two infeasible circumstances: mutually exclusive states are not considered (e.g., South Sudan cannot accept the exorbitant fees and then stop oil production), and logically infeasible states are not considered (e.g., the states in which North Sudan does not request exorbitant fees and South Sudan accepts the charges are removed). As a result, 49 states are removed from $\hat{S}^1$. Hence, 15 states are found to be feasible for the dispute. The set of feasible states for a first-level hypergame is furnished in Table 4.2.

Table 4.2: Universal Set of States for a First-level Hypergame

	States														
Decision Makers	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
North Sudan															
1. Request Exorbitant Fees	N	Y	N	Y	N	Y	Y	Y	Y	Y	Y	Y	Y	Y	Y
2. Shut Down the Export Pipeline	N	N	Y	Y	N	N	N	Y	N	N	Y	N	N	Y	N
3. Self-Reimburse	N	N	N	N	Y	Y	N	N	Y	N	N	Y	N	N	Y
South Sudan															
4. Accept the Exorbitant Fees	N	N	N	N	N	N	Y	Y	Y	N	N	N	N	N	N
5. Explore Alternative Export Route	N	N	N	N	N	N	N	N	N	Y	Y	Y	N	N	N
6. Stop Oil Production	N	N	N	N	N	N	N	N	N	N	N	N	Y	Y	Y

North Sudan's Graph Model

North Sudan correctly capture the conflict situation and is aware of all the states in $\hat{S}^1$. That is, the set of recognizable states in North Sudan subjective game S_C is identical to $\hat{S}^1$. S_C is partitioned into two disjoint subsets: the class of states that are correctly perceived by both North and South Sudan, $S^R = \{1, 2, 3, 4, 7, 8, 10, 11, 13, 14\}$, and the states that are correctly perceived by only North Sudan and hidden to South Sudan, $S_C^P = \{5, 6, 9, 12, 15\}$. State 2 is the status quo, at which the conflict started on December 1, 2011. Figure 4.1 shows the integrated graph model for the North Sudan game. The number in the nodes refers to the state number as depicted in Table 4.2. The arcs between the nodes are the possible moves that can be performed by the specified DM. The nodes and arcs that are shaded in Figure 4.1 indicate the states and their related state transitions that are only perceived by North Sudan.



North Sudan's Game

South Sudan's Preferences as Perceived by North Sudan:	<2, 10, 12, 6, 11, 4, 14, 13, 8, 15, 9, 7, 3, 1, 5>
North Sudan's Own Preferences:	<7, 6, 12, 9, 2, 10, 8, 11, 14, 4, 1, 3, 5, 13, 15>

Figure 4.1: North Sudan's Graph Model

North Sudan's preference relationships among the set of feasible states S_C is based on the overwhelming desire to continue imposing exorbitant fees and for South Sudan to accept the fees. Afterward, North Sudan is expected to rapidly escalate the dispute by self-reimbursing if South Sudan rejects paying the overpriced fees. As a result, the preference relationships for North Sudan is expressed by ordinal preferences as $\langle 7, 6, 12, 9, 2, 10, 8, 11, 14, 4, 1, 3, 5, 13, 15 \rangle$ (Oil and Energy Trends, 2011; Sudan Tribune, 2011a,b). Based on North Sudan's perception, South Sudan wanted to keep producing the oil and dispatch it to Port Sudan while ignoring paying the exorbitant fees. That is, the preference relationships for South Sudan can be expressed by ordinal preferences as $\langle 2, 10, 12, 6, 11, 4, 14, 13, 8, 15, 9, 7, 3, 1, 5\rangle$.

South Sudan's Graph Model

South Sudan misperceives North Sudan' options and preferences in the dispute. As a result, some states in $\hat{S}^1$ are hidden to South Sudan. The states in which North Sudan chooses to select the option of self-reimbursement are hidden to South Sudan. That is, South Sudan set of recognizable states S_D is a sub set of $\hat{S}^1$, $S_D \subseteq \hat{S}^1$. S_D is depicted in Table 4.3.

Table 4.3: Feasible States for South Sudan's Game

	States									
Decision Makers	1	2	3	4	7	8	10	11	13	14
North Sudan										
1. Request Exorbitant Fees	N	Y	N	Y	Y	Y	Y	Y	Y	Y
2. Shut Down the Export Pipeline	N	N	Y	Y	N	Y	N	Y	N	Y
3. Self-Reimburse	N	N	N	N	N	N	N	N	N	N
South Sudan										
4. Accept the Exorbitant Fees	N	N	N	N	Y	Y	N	N	N	N
5. Explore Alternative Export Route	N	N	N	N	N	N	Y	Y	N	N
6. Stop Oil Production	N	N	N	N	N	N	N	N	Y	Y

Because all the states in S_D are recognized by North Sudan, $S_D = S^R$. Figure 4.2 shows the integrated graph model for South Sudan's game. As can be seen, the nodes with the numbers $5, 6, 15, 9$, and 12 in Figure 4.1 are not considered by South Sudan in its graph model because of its failure to perceive them.

South Sudan's preference relationships among the set of feasible states S_D is based on the overwhelming desire to continue producing oil and transfer it to Port Sudan by using North Sudan's export pipeline without paying the overpriced fees. South Sudan's preferences over S_D can be expressed by ordinal ranking (most to least preferred) as

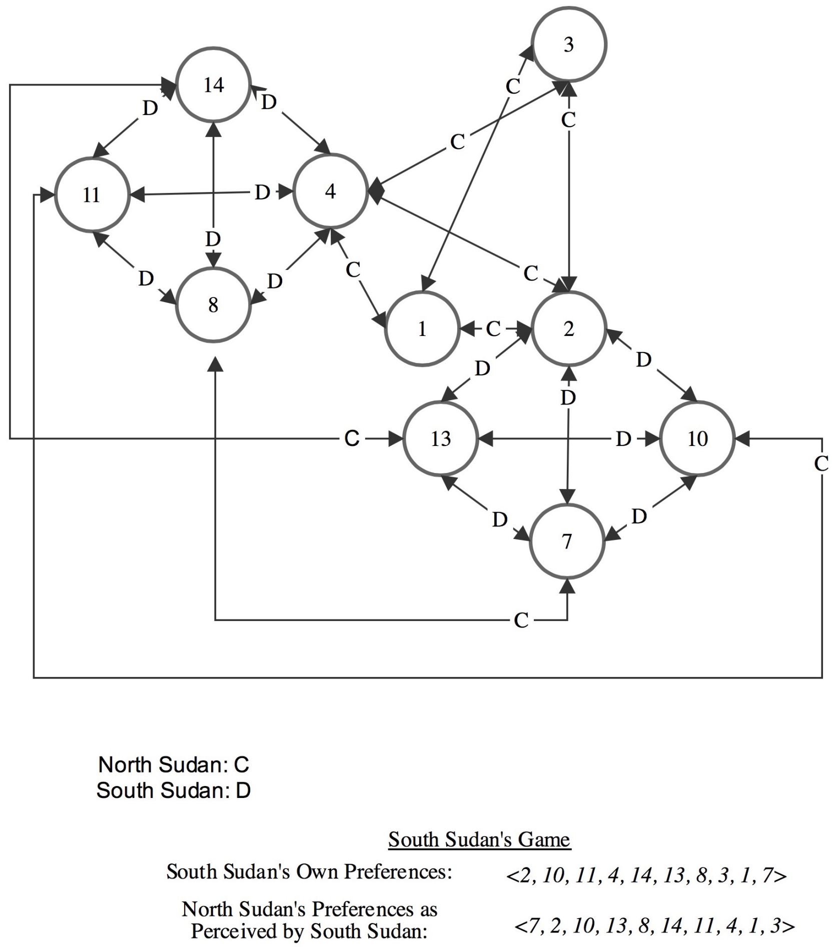


Figure 4.2: South Sudan's Graph Model

$\langle 2, 10, 11, 4, 14, 13, 8, 3, 1, 7\rangle$. Furthermore, North Sudan's preference relationships among the set of feasible states S_D is represented based on South Sudan's viewpoint as $\langle 7, 2, 10, 13, 8, 14, 11, 4, 1, 3\rangle$. South Sudan expects a weak response from North Sudan regarding its refusal to pay the overpriced export fees (Oil and Energy Trends, 2011; Sudan Tribune, 2011a,b). In addition, South Sudan believes that the only way North Sudan can escalate the dispute is by shutting down the export pipeline to prevent South Sudan from using it.

Stability Analysis and Equilibria

The objective of this section is determine the equilibrium states for a first-level hypergame. The procedures explained in Subsection 3.2.3 can be used to ascertain a first-level hypergame equilibria. Three steps are needed to calculate the first-level hypergame equilibria. In the first step, one need to analyze both North and South Sudan's subjective games utilizing GMCR's solution concepts to identify the equilibria in each DM's game. Next, one isolate North Sudan's strategies out of the equilibrium states obtained from North Sudan's game. The same procedure applied for South Sudan's game. Finally, the Cartesian product of both North and South Sudan's strategies, each of which is obtained from the set of equilibrium states in a DM's game, generate the equilibria for a first-level hypergame. A computerized software called GMCR II (Fang et al., 2003a,b) can be used to obtain the equilibrium states for North Sudan's game and South Sudan's game.

Table 4.4 shows the stability and equilibrium results for North Sudan's game. As can be seen, states 2, 6, 10, and 11 are found to be weak equilibria for the dispute; that is, they are equilibria under GMR and SMR solution concepts. On the other hand, state 12 is found to be a strong equilibrium for the dispute; that is, it is an equilibrium under Nash, SEQ, GMR, and SMR solution concepts. North Sudan's strategy related to state 2 is $g_C^2 = (YNN)^T$, its strategies for states 6 and 12 are $g_C^6 = g_C^{12} = (YNY)^T$, and its strategies for states 10 and 11 are $g_C^{10} = g_C^{11} = (YYN)^T$. Note that the strategies $g_C^2, g_C^6, g_C^{10},$, g_C^{11}, and g_C^{12} are found to be equilibria in North Sudan's game under GMR and SMR solution concepts. Hence, g_C^2, g_C^6, g_C^{10}, g_C^{11}, $g_C^{12} \in g_C^{*GMR} \cup g_C^{*SMR}$, in which g_C^{*GMR} and g_C^{*SMR} denote the sets of North Sudan's strategies that are equilibria under GMR and SMR, respectively. Also, notice that the strategy g_C^{12} is found to be a resolution in North Sudan's game under Nash stability definition. Hence, $g_C^{12} \in g_C^{*Nash}$, in which g_C^{*Nash} represents the set of North Sudan's strategies that are equilibria under the Nash solution concept.

Table 4.5 shows the individual stability and equilibria results for South Sudan game. State 2 is found to be a strong equilibrium; that is, it is an equilibrium under Nash, SEQ,

Table 4.4: Stability Analysis and Equilibria Results for North Sudan's Game

		States														
	Solution Concepts	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
North Sudan	Nash	/	/	/	/	/	YES	YES	/	/	/	/	YES	/	YES	/
	SEQ	/	/	/	/	/	YES	YES	YES	YES	/	/	YES	/	YES	/
	GMR	/	YES	/	YES	/	YES	YES	YES	YES	YES	YES	YES	/	YES	/
	SMR	/	YES	/	/	/	YES	YES	YES	YES	YES	YES	YES	/	YES	/
South Sudan	Nash	YES	YES	YES	/	YES	/	/	/	/	/	YES	YES	/	/	/
	SEQ	YES	YES	YES	/	YES	/	/	/	/	YES	YES	YES	/	/	/
	GMR	YES	YES	YES	/	YES	YES	/	/	/	YES	YES	YES	/	/	/
	SMR	YES	YES	YES	/	YES	YES	/	/	/	YES	YES	YES	/	/	/
Equilibrium	Nash	/	/	/	/	/	/	/	/	/	/	/	E	/	/	/
	SEQ	/	/	/	/	/	/	/	/	/	/	/	E	/	/	/
	GMR	/	E	/	/	/	E	/	/	/	E	E	E	/	/	/
	SMR	/	E	/	/	/	E	/	/	/	E	E	E	/	/	/

GMR, and SMR solution concepts, whereas state 10 is a weak equilibrium, because it is a resolution under GMR and SMR stability definitions. South Sudan's strategy related to state 2 is $g_D^2 = (NNN)^T$, and for state 10 is $g_D^{10} = (NYN)^T$. The strategy g_D^2 is a Nash strategy for South Sudan, $g_D^2 \in g_D^{*Nash}$. The strategies g_D^2 and g_D^{10} are GMR and SMR strategies for South Sudan, such that $g_D^2, g_D^{10} \in g_D^{*GMR} \cup g_D^{*SMR}$.

Table 4.5: Stability Analysis and Equilibria Results for South Sudan Game

		States									
	Solution Concepts	1	2	3	4	7	8	10	11	13	14
North Sudan	Nash	/	YES	/	/	YES	/	YES	/	YES	/
	SEQ	/	YES	/	/	YES	/	YES	/	YES	/
	GMR	/	YES	/	/	YES	/	YES	/	YES	/
	SMR	/	YES	/	/	YES	/	YES	/	YES	/
South Sudan	Nash	YES	YES	YES	/	/	/	/	YES	/	/
	SEQ	YES	YES	YES	/	/	/	/	YES	/	/
	GMR	YES	YES	YES	/	/	/	YES	YES	/	/
	SMR	YES	YES	YES	/	/	/	YES	YES	/	/
Equilibrium	Nash	/	E	/	/	/	/	/	/	/	/
	SEQ	/	E	/	/	/	/	/	/	/	/
	GMR	/	E	/	/	/	/	E	/	/	/
	SMR	/	E	/	/	/	/	E	/	/	/

After determining the equilibrium results for North Sudan in its game in Table 4.4, and for South Sudan in its game in Table 4.5, a first-level hypergame equilibria can be ascertained by taking the Cartesian product of North Sudan's strategies that are identified from its equilibrium states and South Sudan's strategies that are obtained from its equilibrium states. Table 4.6 shows the equilibrium results for a first-level hypergame.

Table 4.6: Cartesian Product of Decision Makers' Stable Strategies

		***DM D's* Wining Strategies**	
		(NNN)	**(NYN)**
***DM C's* Wining Strategies**	**(YNN)**	State 2	State 10
	(YNY)	State 6	State 12
	(YYN)	State 4	State 11

In Table 4.6, if North Sudan's and South Sudan's strategies related to a state are found to be either Nash, SEQ, GMR, or SMR, then the state is either called hyper Nash equilibrium (HE^{1Nash}), hyper SEQ equilibrium (HE^{1SEQ}), hyper GMR equilibrium (HE^{1GMR}) or hyper SMR equilibrium (HE^{1SMR}), respectively. Table 4.7 shows the equilibrium results under a specific stability definition for a first-level hypergame. For example, because North Sudan's and South Sudan's strategies related to state 2 are stable under GMR and SMR solution concepts, state 2, which is identified as a resolution for a first-level hypergame in Table 4.6, is considered as HE^{1GMR} and HE^{1SMR} equilibrium for a first-level hypergame.

A first-level hypergame equilibria are categorized into useful classes to address variation in awareness among DMs. In Table 4.7, the results reveal that states 2 and 10, which are resolutions under North Sudan's and South Sudan's subjective games, are considered as steady hyper equilibria for a first-level hypergame. Thus, if either state 2 or 10 is achieved, then the equilibrium states most likely constitute sustainable compromise resolutions for the dispute because no information concerning misperception can be extracted from these states.

In addition, states 4 and 11 are classified as unsteady hyper equilibria for a first-level hypergame under GMR and SMR solution concepts. State 4 is recognized but does not constitute a resolution in both North Sudan's and South Sudan's subjective games.

Table 4.7: Equilibrium Results for a First-Level Hypergame

		Equilibrium States					
Decision Maker	**Stability of a Strategy for a DM**	2	4	6	10	11	12
North Sudan	$g^{*Nash}{}_{C}$	NO	NO	YES	NO	NO	YES
	$g^{*SEQ}{}_{C}$	NO	NO	YES	NO	NO	YES
	$g^{*GMR}{}_{C}$	YES	YES	YES	YES	YES	YES
	$g^{*SMR}{}_{C}$	YES	YES	YES	YES	YES	YES
South Sudan's Strategy	$g^{*Nash}{}_{D}$	YES	YES	YES	NO	NO	NO
	$g^{*SEQ}{}_{D}$	YES	YES	YES	NO	NO	NO
	$g^{*GMR}{}_{D}$	YES	YES	YES	YES	YES	YES
	$g^{*SMR}{}_{D}$	YES	YES	YES	YES	YES	YES
First-level Hypergame Equilibrium	HE^{Nash}	/	/	E	/	/	/
	HE^{SEQ}	/	/	E	/	/	/
	HE^{GMR}	E	E	E	E	E	E
	HE^{SMR}	E	E	E	E	E	E

Hence, if state 4 is attained, then both DMs will obtain new information about their misperception, which may motivate them to escalate the dispute. State 11 is identified by both DMs but does not constitute a resolution in South Sudan's game. South Sudan will obtain some information about North Sudan's preferences, which will ultimately ameliorate South Sudan's misperception and may motivate it to escalate the dispute.

Furthermore, states 6 and 12 are classified as stealthily equilibria for South Sudan. These states constitute resolutions for North Sudan in its game. State 6 is a stealthy hyper equilibrium under Nash solution concept, whereas state 12 is a stealthy hyper equilibrium under GMR and SMR stability definitions. Thus, if either state 6 or 12 is attained, South Sudan would be given some information about North Sudan's option that is unknown to itself in its game. This observed evidence would motivate South Sudan to escalate the dispute once it learns about the new option.

Historically, state 12, the stealthy equilibrium, was the resolution for the dispute in December 2011. State 12 represents the scenario in which North Sudan requested a higher charge for the use of it export pipeline, and self-reimbursed by taking 23% of South Sudan's oil, while South Sudan kept producing the oil and exploring the possibility of having an independent oil export route. After South Sudan become aware of its misperception, it escalated the conflict by stop oil production.

4.2 Case Study II: The Suez Canal Conflict

In the 1956 Suez Canal conflict (Shupe et al., 1980; Fraser and Hipel, 1984), the Egyptian President Gamal Abdul Nasser surprised both Britain and the US by nationalizing the Suez Canal. President Nasser chose this course of action after Britain and the US had withdrawn their offer to provide a series of loans and grants to Egypt to help build the High Aswan Dam on the Nile River. This action caused the termination of the trade relationship between Egypt and Britain with regard to operating the Suez Canal. The British wanted to control the Suez Canal as it handled a significant amount of British marine traffic and was a main gateway of the oil supply from the Middle East to European countries. Further information about the dispute can be found in Shupe et al. (1980); Wright et al. (1980); Fraser and Hipel (1984).

4.2.1 Modeling the Universal Set of States of the Suez Canel Conflict

The conflict between the Britain/ US partnership, and Egypt can be represented as a specific type of hypergame. In particular, Britain and the US misperceived Egypt's actual set of options and preferences. Hence, a separate game must be constructed for each DM to represent his or her understanding of the dispute. Because none of the DMs are aware of the misunderstanding, the Suez Canal dispute is modeled as a first-level hypergame.

As of February 9, 1956, the DMs and options for the conflict are given in Table 4.8. Note that two DMs are identified for the Suez Canal conflict (Shupe et al., 1980; Fraser and Hipel, 1984): Britain and the US (denoted by BS) which are represented as one DM because of shared interests and goals, and Egypt (denoted by EG). As explained in Shupe et al. (1980); Fraser and Hipel (1984), Britain and the US were not aware of President Gamal Abdul Nasser's course of action to nationalize the Suez Canal. Thus, this option is hidden to both Britain and the US and will not be considered in their subjective game.

Table 4.8: DMs and Options for the Dispute

DM	Option	Recognized by
Britain and US (BS)	1. Grant loan with original terms	Britain and US and Egypt
	2. Grant loan on Nasser's conditions	Britain and US and Egypt
Egypt (EG)	3. Negotiate loan with original terms	Britain and US and Egypt
	4. Negotiate loan on Nasser's conditions	Britain and US and Egypt
	5. Mitigate Britain and US	Britain and US and Egypt
	6. Seek loan with Russians	Britain and US and Egypt
	7. Pursue loan from Russians and, if this does not succeed, nationalize the Suez Canal	Egypt

The courses of action in Table 4.8 are considered in defining the universal set of states for a first-level hypergame, $\hat{S}^1$. In option form, an option can be selected or not by the focal DM who possesses it; as a result, the mathematical number of states in $\hat{S}^1$ is equal to 2^λ where λ is the number of options in the conflict. Some of the mathematically possible states are infeasible and need to be removed from $\hat{S}^1$. For instance, the circumstance in which Egypt decided to negotiate both loan conditions are infeasible (mutually exclusive). A complete list of infeasible states can be found in Shupe et al. (1980); Fraser and Hipel (1984). Table 4.9 shows the feasible states for the Suez Canal conflict. Each option in

Table 4.9 is labeled with a number and can either be chosen (Y for yes) or not (N for no).

Table 4.9: Universal Set of States, $\hat{S}^1$

Decision Makers and Options	***Possible Scenarios (States)***															
	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16
Britain and US																
1. Grant original loan	N	Y	N	Y	N	Y	N	N	Y	N	Y	N	N	Y	N	Y
2. Agree to Nassr's conditions	N	N	N	N	N	N	Y	N	N	N	N	Y	N	N	N	N
Egypt																
3. Negotiate original loan	N	N	Y	Y	N	N	N	Y	Y	N	N	N	N	N	N	N
4. Negotiate Nasser's terms	N	N	N	N	Y	Y	Y	N	N	Y	Y	Y	N	N	N	N
5. Mitigate West	N	N	N	N	N	N	N	Y	Y	Y	Y	Y	N	N	N	N
6. Russian loan	N	N	N	N	N	N	N	N	N	N	N	N	Y	Y	N	N
7. Russian loan/ Nationalization	N	N	N	N	N	N	N	N	N	N	N	N	N	N	Y	Y

4.2.2 Stability Analysis of the Suez Canel Conflict

Equilibrium Results for Britain and US's Game

The set of possible states in Britain and the US's game is $S_{BS} = \{1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14\}$. Note that some states in the universal set of states $\hat{S}^1$ are hidden to Britain and the US. The states in which Egypt chooses to select the option of nationalizing the Suez Canal are hidden to Britain and the US. As a result, states 15 and 16 in Table 4.9 are not considered by Britain and the US in their game.

To perform the stability analysis for Britain and the US's game, states are put in sequence of preference for both DMs as perceived by Britain and the US. To begin with, Britain and the US's preference relationships are expressed by ordinal ranking as $\{9 \succ 8 \sim 10 \sim 11 \succ 1 \sim 2 \sim 3 \sim 5 \sim 6 \succ 4 \succ 12 \succ 13 \sim 14 \succ 7\}$ (Shupe et al., 1980; Fraser and Hipel, 1984). As can be seen, the most preferred state for Britain and the US is state 9. This scenario represents the circumstance in which Britain and the US along with the World Bank proposed to grant Egypt a series of loans to build the High Aswan Dam in the Nile River, and Egypt agreed to negotiate the proposed loans under Britain and the US's terms. The least preferred scenario for Britain and the US is state 7. This state represents the situation in which Britain and the US offered a loan to Egypt under President Nasser's conditions, and Egypt decided to negotiate the offer.

Egypt's preference relationships as perceived by Britain and the US are expressed by an ordinal ranking as $\{7 \succ 4 \succ 13 \sim 14 \succ 1 \sim 2 \sim 3 \sim 5 \sim 6 \succ 8 \sim 10 \sim 11 \succ 12 \succ 9\}$ (Shupe et al., 1980; Fraser and Hipel, 1984). The most desirable scenario for Egypt is state 7. This state, as mentioned before, is the least preferred state for Britain and the US. The worst possible scenario for Egypt is state 9, which represents the situation in which Britain and the US grant a loan to Egypt with their original conditions and President Gamal Abdul Nassar negotiates in their terms.

Table 4.10 shows the stability and equilibrium results for Britain and the US's game. As can be seen, states 4 and 14 are found to be weak equilibria for the conflict because they are equilibria under the SEQ, GMR and SMR solution concepts but not Nash. On the other hand, state 13 is determined to be a strong equilibrium for the conflict because it is an equilibrium under the Nash, SEQ, GMR, and SMR solution concepts. Britain and the US's strategies related to states 4 and 14 are $g^4_{BS} = g^{14}_{BS} = (YN)^T$, and their strategy for state 13 is $g^{13}_{BS} = (NN)^T$. Hence, one can obtain g^{*Nash}_{BS} as follows:

- $g^{*Nash}_{BS} = \{g^{13}_{BS}\} = \{(NN)^T\}$.

The sets of strategies, g^{*SEQ}_{BS}, g^{*GMR}_{BS}, and g^{*SMR}_{BS}, can be obtained analogously as follows:

- $g^{*SEQ}_{BS} = \{g^4_{BS}, g^{13}_{BS}, g^{14}_{BS}\} = \{(NN)^T, (YN)^T\}$,
- $g^{*GMR}_{BS} = \{g^4_{BS}, g^{13}_{BS}, g^{14}_{BS}\} = \{(NN)^T, (YN)^T\}$, and
- $g^{*SMR}_{BS} = \{g^4_{BS}, g^{13}_{BS}, g^{14}_{BS}\} = \{(NN)^T, (YN)^T\}$.

These strategies are displayed in the top part of Table 4.12.

Table 4.10: Stability Analysis and Equilibrium Results for Britain and US's Game

States		1	2	3	4	5	6	7	8	9	10	11	12	13	14
Britain and US	Nash	YES	YES	YES	NO	YES	YES	NO	NO	YES	YES	YES	NO	YES	YES
	SEQ	YES	YES	YES	YES	YES	YES	NO	YES	YES	YES	YES	YES	YES	YES
	GMR	YES	YES	YES	YES	YES	YES	NO	YES	YES	YES	YES	YES	YES	YES
	SMR	YES	YES	YES	YES	YES	YES	NO	YES	YES	YES	YES	YES	YES	YES
Egypt	Nash	NO	NO	NO	YES	NO	NO	YES	NO	NO	NO	NO	NO	YES	NO
	SEQ	NO	NO	NO	YES	NO	NO	YES	NO	NO	NO	NO	NO	YES	YES
	GMR	NO	NO	NO	YES	NO	NO	YES	NO	NO	NO	NO	NO	YES	YES
	SMR	NO	NO	NO	YES	NO	NO	YES	NO	NO	NO	NO	NO	YES	YES
Equilibrium	Nash	/	/	/	/	/	/	/	/	/	/	/	/	**E**	/
	SEQ	/	/	/	**E**	/	/	/	/	/	/	/	/	**E**	**E**
	GMR	/	/	/	**E**	/	/	/	/	/	/	/	/	**E**	**E**
	SMR	/	/	/	**E**	/	/	/	/	/	/	/	/	**E**	**E**

Equilibrium Results for Egypt's Game

President Nasser correctly perceived Britain and the US's available courses of actions and preferences; as a result, he correctly evaluated the consequences of the dispute and avoided facing any strategic surprise (Shupe et al., 1980; Fraser and Hipel, 1984). The set of possible states in Egypt's game, $S_{EG} = \{1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16\}$, is identical to $\hat{S}^1$. However, based on Egypt's perception, S_{EG} is partitioned into two disjoint sets: the class of states that are correctly perceived by Egypt as well as Britain and the US, denoted as $S^R = \{1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14\}$; and the group of states that are correctly perceived by only Egypt and hidden to Britain and the US, expressed as $S^P_{EG} \cap S^H_{BS} = \{15, 16\}$.

Egypt's preference relationships among the set of feasible states S_{EG} are based on its ambition to obtain a series of loans from the West without forfeiting its national independence. However, if President Nasser fails to obtain a loan from the West, then he will approach Russia, and if all of his efforts in securing a loan for constructing the High Aswan Dam fail, then he will nationalize the Suez Canal. That is, Egypt's preference relationships are expressed by an ordinal ranking as $7 \succ 4 \succ 15 \sim 16 \succ 13 \sim 14 \succ 1 \sim 2 \sim 3 \sim 5 \sim 6 \succ 8 \sim 10 \sim 11 \succ 12 \succ 9\}$ (Shupe et al., 1980; Fraser and Hipel, 1984).

Based on Egypt's perspective, Britain and the US want to prevent Nasser from securing a loan from Russia by offering him a loan under the condition that Egypt shares some of its sovereignty with the West with respect to managing the Suez Canal. The least preferred scenario for Britain and the US is the state in which Egypt chooses to nationalize the Suez Canal to secure funds for the High Aswan Dam. That is, the preference relationships for Britain and the US as perceived by Egypt can be expressed by an ordinal ranking as $\{9 \succ 8 \sim 10 \sim 11 \succ 1 \sim 2 \sim 3 \sim 5 \sim 6 \succ 4 \succ 12 \succ 13 \sim 14 \succ 7 \succ 15 \sim 16\}$ (Shupe et al., 1980; Fraser and Hipel, 1984).

Table 4.11 shows the stability and equilibrium results for Egypt's game. As can be seen, states 4, 7, and 16 are determined to be weak equilibria for the conflict because they are equilibria under the SEQ, GMR and SMR solution concepts but not Nash. On the other hand, state 15 is found to be a strong equilibrium for the conflict since it is an equilibrium under the Nash, SEQ, GMR, and SMR solution concepts. Egypt's strategies related to states 4 and 7 are $g_{EG}^{4} = (YNNNN)^T$, and $g_{EG}^{7} = (NYNNN)^T$, respectively. Moreover, Egypt's strategies for states 15 and 16 are $g_{EG}^{15} = g_{EG}^{16} = (NNNNY)^T$. Hence, one can obtain g_{EG}^{*Nash} as follows:

- $g_{EG}^{*Nash} = \{g_{EG}^{15}\} = \{(NNNNY)^T\}$.

In a similar fashion, one can obtain the sets of strategies, $g_{EG}^{*SEQ}, g_{EG}^{*GMR}$, and g_{EG}^{*SMR}, as follows:

- $g_{EG}^{*SEQ} = \{g_{EG}^{4}, g_{EG}^{7}, g_{EG}^{15}, g_{EG}^{16}\} =$
  $\{(YNNNN)^T, (NYNNN)^T, (NNNNY)^T\}$,
- $g_{EG}^{*GMR} = \{g_{EG}^{4}, g_{EG}^{7}, g_{EG}^{15}, g_{EG}^{16}\} =$
  $\{(YNNNN)^T, (NYNNN)^T, (NNNNY)^T\}$, and
- $g_{EG}^{*SMR} = \{g_{EG}^{4}, g_{EG}^{7}, g_{EG}^{15}, g_{EG}^{16}\} =$
  $\{(YNNNN)^T, (NYNNN)^T, (NNNNY)^T\}$.

These strategies are shown in Table 4.12.

Table 4.11: Stability Analysis and Equilibrium Results for Egypt's Game

States		1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16
Britain and US	Nash	YES	YES	YES	/	YES	YES	/	/	YES	YES	YES	/	YES	YES	YES	YES
	SEQ	YES	YES	YES	YES	YES	YES	YES	YES	YES	YES	YES	YES	YES	YES	YES	YES
	GMR	YES	YES	YES	YES	YES	YES	YES	YES	YES	YES	YES	YES	YES	YES	YES	YES
	SMR	YES	YES	YES	YES	YES	YES	YES	YES	YES	YES	YES	YES	YES	YES	YES	YES
Egypt	Nash	/	/	/	YES	/	/	YES	/	/	/	/	/	/	/	YES	/
	SEQ	/	/	/	YES	/	/	YES	/	/	/	/	/	/	/	YES	YES
	GMR	/	/	/	YES	/	/	YES	/	/	/	/	/	/	/	YES	YES
	SMR	/	/	/	YES	/	/	YES	/	/	/	/	/	/	/	YES	YES
Equilibrium	Nash	/	/	/	/	/	/	/	/	/	/	/	/	/	/	**E**	/
	SEQ	/	/	/	**E**	/	/	**E**	/	/	/	/	/	/	/	**E**	**E**
	GMR	/	/	/	**E**	/	/	**E**	/	/	/	/	/	/	/	**E**	**E**
	SMR	/	/	/	**E**	/	/	**E**	/	/	/	/	/	/	/	**E**	**E**

Equilibrium Results for a First-level Hypergame

After determining the equilibrium results for Britain and the US in their game in Table 4.10, and for Egypt in its game in Table 4.11, the first-level hypergame equilibria can be ascertained by taking the Cartesian product of Britain and the US's strategies that are identified from its equilibrium states and Egypt's strategies that are obtained from its equilibrium states. Table 4.12 shows the results of equilibrium analysis for the first-level hypergame. As can be seen in Table 4.12, states 3, 5, 4, 6, and 16 are found to be weak hypergame equilibria (HE) for the first-level hypergame since they are equilibria under SEQ, GMR, and SMR. However, state 15 is a strong hypergame equilibrium for the first-level hypergame because it is a resolution under the Nash stability definition.

The first-level hypergame equilibria are categorized into useful classes to address variation in awareness among DMs. In Table 4.12, the results reveal that states 3, 5, and 6 $\in S^R$ are not a resolution in either Britain and the US's game or Egypt's game. That is, states 3, 5, and 6 are considered as unsteady hyper equilibrium states under the SEQ, GMR, and SMR for the first-level hypergame. States 3, 5, and 6 are recognized but do not constitute resolutions in both DMs' respective games. Hence, if either state 3, 5, or 6 is attained, then both DMs will obtain new information about their misperception, which may motivate them to escalate the dispute into future rounds.

Moreover, state 4, which is a hypergame equilibrium and also constitutes a resolution under Britain and the US's as well as Egypt's subjective games, is considered to be a steady hyper SEQ, GMR, and SMR equilibrium for the first-level hypergame. Thus, if state 4 is reached, then this equilibrium most likely constitutes a sustainable compromise resolution for the dispute because no information concerning misperception can be extracted from state 4.

Furthermore, states 15 and 16 are classified as stealthy hypergame equilibria for the first-level hypergame. These states are only recognized by Egypt in its game and also constitute resolutions in its subjective game. State 15 is a stealthy hyper Nash equilibrium state for the first-level hypergame because it is a resolution under the Nash stability concept, whereas state 16 is a stealthy hyper SEQ, SMR, and GMR equilibrium because it is a resolution under the SEQ, SMR and GMR solution concepts. If either state 15 or 16 is attained, Britain and the US would be given some information about Egypt's option that is unknown to them in their game and would be facing a strategic surprise. This observed evidence would motivate Britain and the US to escalate the conflict once they learn about the new option. Britain and the US misperceived Egypt's true set of options and miscalculated the consequences of the conflict.

Historically, state 15, the stealthy hyper Nash equilibrium state, was the resolution of the dispute. Table 4.13 outlines the evolution of the Suez Canal dispute. The conflict evolves from the status quo (state 2) to the first transition state (state 6), in which Nasser rejects the original proposal made by Britain and the US, and offers alternative conditions. This chain of events leads to another transitional state (state 5), in which Britain and the US officially withdraw their offer to provide Egypt with a series of loans to help construct the High Aswan Dam. Britain and the US withdraw this offer because of a change in US national politics, actions that Nasser took were hostile to the interests of Western countries, and Russias inability to support the construction of the High Aswan Dam. These events

Table 4.12: Equilibrium Results for a First-Level Hypergame

		Winning Strategy					
Britain and US (BS)	Stability	NN	NN	NN	YN	YN	YN
	Nash	YES	YES	YES	NO	NO	NO
	SEQ	YES	YES	YES	YES	YES	YES
	GMR	YES	YES	YES	YES	YES	YES
	SMR	YES	YES	YES	YES	YES	YES
		Winning Strategy					
Egypt (EG)	Stability	YNNNN	NYNNN	NNNNY	YNNNN	NYNNN	NNNNY
	Nash	NO	NO	YES	NO	NO	YES
	SEQ	YES	YES	YES	YES	YES	YES
	GMR	YES	YES	YES	YES	YES	YES
	SMR	YES	YES	YES	YES	YES	YES
States		3	5	15	4	6	16
First-Level Hypergame Equilibrium	Nash	/	/	*HE*	/	/	/
	SEQ	*HE*	*HE*	*HE*	*HE*	*HE*	*HE*
	GMR	*HE*	*HE*	*HE*	*HE*	*HE*	*HE*
	SMR	*HE*	*HE*	*HE*	*HE*	*HE*	*HE*
Classification of the First-Level Hypergame Equilibria	Nash	*UHNash*	*UHNash*	*STHNash*		*UHNash*	/
	SEQ	*UHSEQ*	*UHSEQ*	*STHSEQ*	*SHSEQ*	*UHSEQ*	*STHSEQ*
	GMR	*UHGMR*	*UHGMR*	*STHGMR*	*SHGMR*	*UHGMR*	*STHGMR*
	SMR	*UHSMR*	*UHSMR*	*STHSMR*	*SHSMR*	*UHSMR*	*STHSMR*

lead to a resolution in state 15, in which Nasser nationalizes the Suez Canal on July 26, 1956. As a result, Egypt gains full control over the canal, secures the funds needed to build the High Aswan Dam, and ends its association with the West.

State 15 represents the scenario in which Egypt decides to nationalize the Suez Canal after it failed to secure a series of loans from the West and Russia. This circumstance represents the occurrence of strategic surprise in a conflict situation. The resolution in 1956 (Shupe et al., 1980; Fraser and Hipel, 1984), as a matter of fact, was very risky as it caused the Suez Canal invasion later in 1957 (Wright et al., 1980). In the dispute, Egypt was invaded by Israel, Britain and France for the purpose of regaining control of the Suez Canal and to remove President Nasser from the power (Wright et al., 1980).

Table 4.13: Evolution of the Suez Canal Conflict

Decision Makers and Options	Status Quo	Transition State I	Transition state II	Equilibrium State
Britain and US				
1. Grant original loan	Y	Y →	N	N
2. Agree to Nassr's conditions	N	N	N	N
Egypt				
3. Negotiate original loan	N	N	N	N
4. Negotiate Nasser's terms	N →	Y	Y →	N
5. Mitigate West	N	N	N	N
6. Russian loan	N	N	N	N
7. Russian loan/ Nationalization	N	N	N →	Y
State	2	6	5	15

4.3 Chapter Summary

The first-level hypergame in graph form has been utilized to model and analyze the 2011 conflict between North and South Sudan over South Sudanese oil exports as well as the 1956 nationalization of the Suez Canal dispute to predict the possible compromise resolutions, and to investigate the resilience of these equilibrium states after DMs learn about their misperception in reality. In both conflicts, the historical equilibrium state were under the definition of the stealthy hyper equilibrium concept of a first-level hypergame in graph form. This resolution is considered as being unstable because DMs may decide to escalate the situation into future rounds to improve their position.

After North Sudan self-reimbursed by taking 23% of South Sudan's oil, South Sudan escalated the situation by stopping bumping the oil to Port Sudan using North Sudan pipelines. Also, after Egypt nationalized the canal, an alliance of Britain, France, and Israel undertook a surprise invasion of the Suez Canal to gain control (Shupe et al., 1980; Wright et al., 1980; Fraser and Hipel, 1984). This outcome confirms the applicability of the modeling and analysis techniques of a first-level hypergame in graph form

Chapter 5

Second-Level Hypergame in Graph Form

In a second-level hypergame, DMs are playing a different games and at least one DM think that he or she is aware of the misperceptions of other DMs. The objectives of this chapter are fourfold. Firstly, a key goal is to incorporate a second-level hypergame situation into the framework of GMCR. Consequently, not only conflicts with complete information can be modeled within GMCR, but also disputes caused by misperceptions among the DMs. A second objective is to allow a DM to have a misperception about its options, strategies, possible states, and preferences pertaining to the dispute. A third goal is to design and analyze starting from the option stage rather than the traditional state level by identifying different options for the misperceptions, and to investigate their consequences on the possible states of the dispute. Finally, stability analysis procedures are designed and implemented within the second-level hypergame in graph form, thereby, supporting the calculation of the second-level hypergame equilibria. Chapter 5 is partially based on the published extended abstract by Aljefri et al. (2016b)

5.1 Second-Level Hypergame With n Decision Makers in Graph Form

In a first-level hypergame, stakeholders are not entirely aware about the circumstances of the dispute. All the information about the dispute such as the set of DMs, set of DMs' options, the set of states, and each DM's preferences over the set of possible states, are perceived by each DM in a subjective fashion (Aljefri et al., 2014a,b, 2015, 2017a). In a second-level hypergame, however, at least one DM comprehend that one or more DMs are playing different subjective games, perhaps because of acquiring some extra information about the situation or because of misleading the other DMs into perceiving what he or she wants them to believe, whether it is true or not (Bennett, 1977, 1980; Takahashi et al., 1984; Hipel et al., 1988; Wang et al., 1988, 1989; Leary, 1995). Therefore, this particular DM attempts to picture what constitutes the subjective games of its adversaries. For example, in a two-DM conflict, $N = \{i, j\}$, the structure of second-level hypergame reflects how *DM i* sees the viewpoint of *DM j* with respect to the options and preferences of *DM i* in the dispute.

The Suez Canal invasion in October 1956 is an example of a second-level hypergame conflict (Shupe et al., 1980; Wright et al., 1980). After President Nasser of Egypt nationalized the Suez Canal on July 26, 1956, western countries started a three-month negotiation with Egypt to regain some control over the Suez Canal. However, after negotiations with Egypt reached a dead end, a coalition composed of Britain, France, and Israel launched an unexpected attack on Egypt on October 29, 1956 to take back the canal. Egypt did not anticipate a military intervention from the allied western countries on the Sinai peninsula and the Suez Canal after the negotiations failed. Consequently, Egypt did not take any mitigated action to defend itself against the allied attack of the western countries, and Egypt was surprised when the attack occurred. The allied western countries were aware of the misperception of Egypt and utilized this information to launch a successful stealthy attack on Egypt. This situation is an example of the use of strategic surprise in a conflict situation.

To permit GMCR to model conflict situations having misperceptions among the participating DMs in which at least one DM has knowledge of the subjective games of other DMs, a new method for a second-level hypergame in graph form is proposed in this chapter. The overall layout of a second-level hypergame with n-DM ($n \geqslant 2$) in graph form is outlined in Figure 5.1. Similar to a first-level hypergame in graph form (Aljefri et al., 2017a), the suggested graph model for a second-level hypergame analysis is composed of three components: one to develop the universal set of states for a second-level hypergame, another to model the subjective first-level hypergame of each DM, and a third to perform stability analyses and calculate the equilibria for a second-level hypergame.

Beginning from the top with Step 1 in Figure 5.1, the universal set of states for a second-level hypergame is designed in a procedure resembling the one conducted to formalize the universal set of states for a first-level hypergame (Aljefri et al., 2017a). In particular, in graph form, states are defined in option form by using the universal sets of options of the DMs for a second-level hypergame. The universal set of options of a DM for a second-level hypergame is defined by extending the universal set of options of a given DM for a first-level hypergame to include not only options that are considered by the DM itself and its adversaries, but also the courses of action of a DM that are considered by its adversaries as assumed by any DM who assume knowledge of the different of perception among the DMs. The union of the universal sets of options of all the DMs for a second-level hypergame generates the universal set of options for the unified second-level hypergame. This unified set of options is then utilized to produce the universal set of states for a second-level hypergame. This set is used to lay out states in the subjective first-level hypergame of each DM within a second-level hypergame situation.

In Step 2 of Figure 5.1, the modeling of a second-level hypergame begins by modeling the subjective first-level hypergame of each DM, which can be formed in a similar way to a first-level hypergame in graph form (Aljefri et al., 2017a). Because in a second-level hypergame a DM may attempt to perceive the subjective games of its adversaries, its subjective

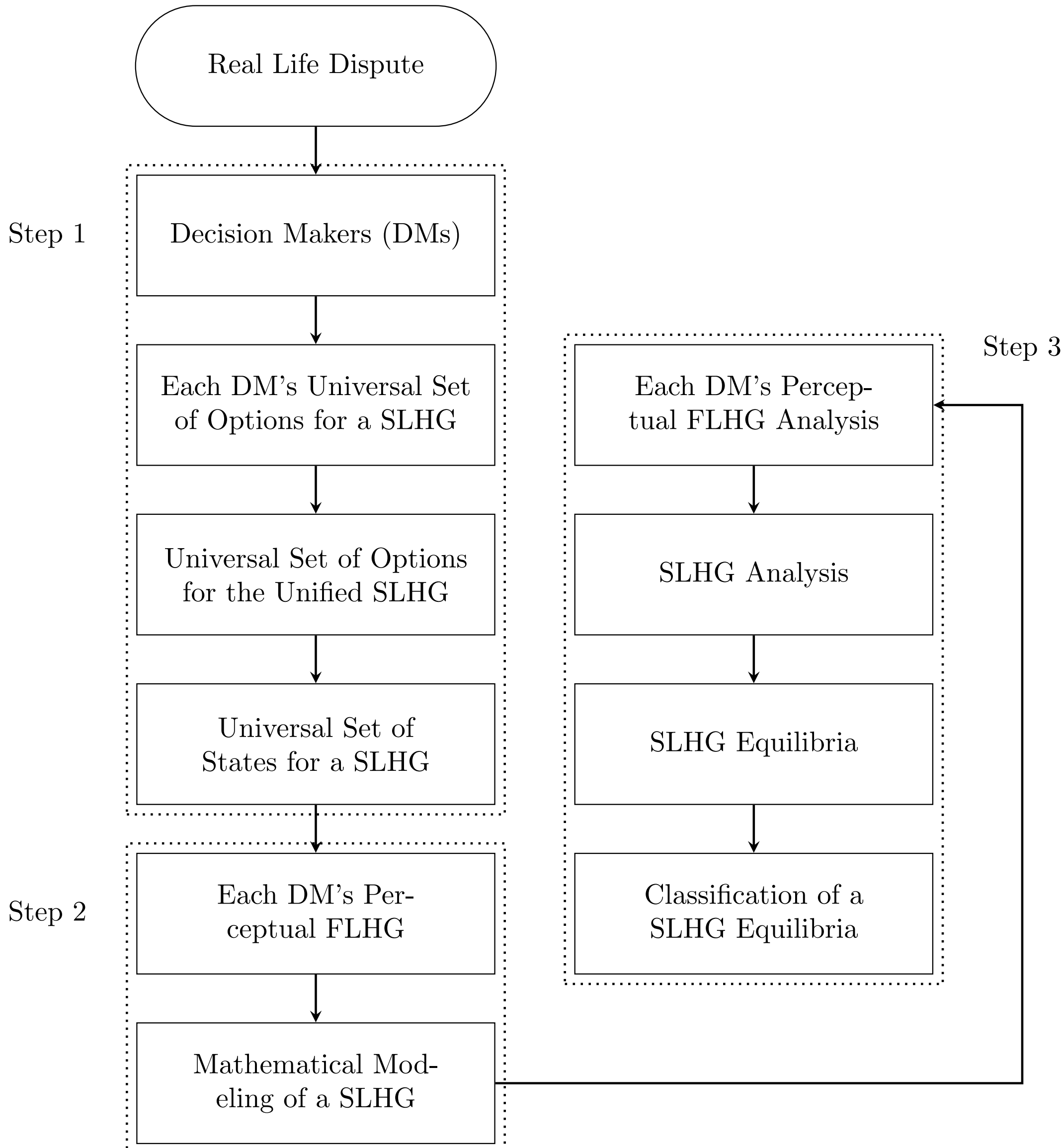


Figure 5.1: Design of a Second-level Hypergame in the Graph Form

first-level hypergame is formed by a system of subjective games. The combination of the subjective first-level hypergame of all the DMs generates the mathematical model of a second-level hypergame.

In Step 3 of Figure 5.1, the analysis module is further composed of each DM's subjective first-level hypergame analyses and the overall second-level hypergame analyses. The analysis of each DM's subjective first-level hypergame is always initiated by analyzing the subjective games of the adversaries by employing any of the standard GMCR stability concepts. Consequently, the possible compromise resolutions in the subjective games of the adversaries as seen by a given DM are identified. Next, the strategies of the adversaries are isolated from their equilibrium states within their subjective games. These strategies are referred to as a DM's winning strategies. After that, one calculate the Cartesian product of all of the opponents' winning strategies. Finally, the stability of the states for a given DM in its subjective game within its subjective first-level hypergame that are related to the winning strategies of its adversaries are investigated. If the state is stable for a DM in its subjective game, it is considered as an equilibrium in its subjective game and in its subjective first-level hypergame. The analysis of the overall second-level hypergame starts by first isolating each DM's strategies out of the equilibrium states in its subjective game within its subjective first-level hypergame. Next, one take the Cartesian product of all DMs' winning strategies to produce the possible equilibria for a second-level hypergame.

The outline of this section is as follows. First, the mathematical modeling of a second-level hypergame with n-DM in graph form is presented in Section 5.1.1. In Section 5.1.2 the structures of the universal set of options of a DM for a second-level hypergame and the design of the universal set of options for the entire second-level hypergame are proposed. The modeling of the universal set of states for a second-level hypergame with more than two DMs in graph form is discussed in Section 5.1.3. Finally, the analysis of a second-level hypergame with n-DM in graph form is described in Section 5.1.4.

5.1.1 Mathematical Modeling of a Second-Level Hypergame with n Decision Makers

A second-level hypergame is a system which consists of subjective first-level hypergames, each of which depicts the perception of a particular DM of the conflict circumstance. By utilizing Eq. 3.1, a second-level hypergame H^2 can be furnished as given below:

$$H^2 = \langle H_1^1, H_2^1, ..., H_i^1, ..., H_n^1 \rangle \tag{5.1}$$

For $i \in N$, the subjective first-level hypergame of *DM* i, H_i^1, is expressed as shown below:

$$H_i^1 = \langle G_{ji} : j \in N_i \rangle \tag{5.2}$$

where G_{ji} is the subjective game of *DM* j as seen by *DM* i within H_i^1, and $N_i \subseteq N$ denotes the set of DMs as perceived by *DM* i in H_i^1. Mathematically, G_{ji} is defined as follows:

$$G_{ji} = \langle N_{ji}, S_{ji}, \{A_{kji} : k \in N_{ji}, j \in N_i\}, \{\succsim_{kji}: k \in N_{ji}, j \in N_i\} \rangle \tag{5.3}$$

where $N_{ji} \subseteq N_i$ is the set of DMs as perceived by first *DM* j and then by *DM* i. S_{ji} is the set of feasible states in G_{ji}. A_{kji} represents the set of state transitions available for *DM* k from one state to another in S_{ji} as perceived by *DM* j and then by *DM* i. $\succsim_{kji}$ denotes the preference relations of *DM* k over the states in S_{ji} as perceived by *DM* j and then by *DM* i. Note that if $k = j$, then A_{jji} and $\succsim_{jji}$ are identical to A_{ji} and $\succsim_{ji}$, respectively. A_{ji} and $\succsim_{ji}$ represent the allowed state transitions of *DM* j and its relative preferences as perceived by *DM* i, respectively.

Please note that if $j = i$, then G_{ii} is identical to G_i, which as mentioned earlier represent *DM i's* actual subjective game.

5.1.2 Universal Set of Options in a Second-Level Hypergame with Two or More Decision Makers

As mentioned earlier, states in GMCR (Fang et al., 1993; Fang et al., 2003a,b) are defined in option form by using the courses of action of the competing DMs (Howard, 1971). Within this paradigm, complete information is always assumed among the DMs. Hence, the DMs are assumed to correctly perceive the conflict situation, and thereby accurately recognize the model parameters of other DMs in the conflict situation, such as options, strategies, and preferences. Therefore, the dispute is represented by a single graph model. Under a second-level hypergame (Bennett, 1977, 1980; Takahashi et al., 1984; Hipel et al., 1988; Wang et al., 1988, 1989; Leary, 1995), however, the dispute is perceived by each DM in a subjective fashion, and at least one DM possesses knowledge of the asymmetry of the viewpoint among the DMs. Thus, the set of options for a focal DM can be extended based on the perception (correct or incorrect) of the focal DM itself, its adversaries, and its adversaries as perceived by the other DMs in the dispute. For instance, because of misperception, a DM may consider some realistic and unrealistic options for itself in the dispute (Aljefri et al., 2014a,b, 2015, 2017a). Similarly, the adversaries of a given DM may correctly perceive, fail to perceive, and/or invent some options for a DM (Aljefri et al., 2014a,b, 2015, 2017a). Moreover, a DM may be cognizant of the misperception of its adversaries, and thereby try to perceive which options might be considered by its adversaries, and mistakes may occur during this process (Bennett, 1977, 1980; Takahashi et al., 1984; Hipel et al., 1988; Wang et al., 1988, 1989; Leary, 1995).

To consider all viable options for a DM in a second-level hypergame situation, the idea of a universal set of options for the entire second-level hypergame with n DMs in graph form is addressed in this subsection. In particular, the concept of a universal set of options for the entire first-level hypergame in graph form (Aljefri et al., 2015, 2017a) is extended to define options for a second-level hypergame in graph form. A course of action in the universal set of options for a given DM in a second-level hypergame can be illustrated using the following classes (Aljefri et al., 2015, 2017a): (1) correctly perceived by the DM itself, (2) misconceived by the DM itself, and (3) misconceived by its adversaries. Similar to a first-level hypergame in graph form (Aljefri et al., 2015, 2017a), the identification of the universal set of options for a given DM in a second-level hypergame is conducted by an external analyst who is cognizant of the differences in viewpoints among the competing DMs (Aljefri et al., 2017a).

The set of DMs in a second-level hypergame is denoted by $N = \{1, 2, ..., i, ..., n\}$. For $i \in N$, let $N_i \subseteq N$ stand for the set of DMs as perceived by *DM i* within H_i^1. If $N_i = N$, then one concludes that *DM i* correctly recognizes all the DMs in the dispute, whereas, if $N_i \neq N$, then one assumes that *DM i* misconceives the number of DMs participating in the dispute.

A DM's Set of Options Correctly Perceived by Itself

The set of options of a given DM that are correctly perceived by itself contains elements that are real, free from any misunderstanding, and recognized by a given DM, and represent its actual capability in the dispute (Aljefri et al., 2015, 2017a). However, because of misperception, the adversaries of a given DM may fail to consider some of the actual options of the DM (Aljefri et al., 2017a), perhaps because of incomplete information or the attempt of a given DM to hide one or more of its actual options in the dispute. In some real-life conflicts, a DM may be aware of the misperception of its adversaries, and as a result, try to visualize what its adversaries know, and mistakes may occur during this process.

Definition 5.1.2.1 (**Set of Correctly Perceived Courses of Action of a Focal DM**). *Choose $i \in N$. The set of courses of action of DM i that are correctly considered by itself is denoted by O_{ii}^R. For $j \in N - \{i\}$, let O_{ij}^R symbolizes the set of actual options of DM i that are recognized by DM j. Then, the actual real options of DM i that are considered by itself and other DMs in N can be expressed as $O_i^R = \cap_{j=1}^{n} O_{ij}^R$. For $p \in N - \{i\}$ and $q \in N$,* *let O^R_{ipq} denote the set of DM i's actual options that are considered by DM p as seen by DM q. Then, DM i's actual options that are considered by its adversaries as perceived by any DM q can be expressed as $(\cup_{p\in N-\{i\}} \cup_{q\in N} O^R_{ipq})$. It is assumed that $(\cup_{j\in N} O^R_{ij}) \subseteq O^R_{ii}$ and $(\cup_{p\in N-\{i\}} \cup_{q\in N} O^R_{ipq}) \subseteq O^R_{ii}$.*

To provide good illustration about all the possible types of a DM's options within the second-level hypergame, the hypothetical examples that are depicted in Figures 5.2, 5.3, 5.4, 5.5, 5.6, and 5.7 are given for the case of two DMs, $N = \{i, j\}$.

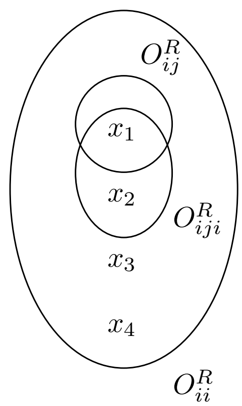


Figure 5.2: *DM i's* Correctly Recognized Courses of Action

The set of options of *DM i* that are correctly perceived by itself is illustrated in Figure 5.2. As can be seen, O^R_{ii} includes four real options: x_1, x_2, x_3, and x_4. These courses of action are within the control of *DM i*. However, because of the misperception of *DM j*, it does not see some of the correctly perceived options of *DM i*. *DM j* correctly recognizes x_1, but not x_2, x_3, and x_4. Therefore, in this research it is assumed that $O^R_{ij} \subseteq O^R_{ii}$. Moreover, *DM i* may assume knowledge of the misperception of *DM j* and, as a result, attempt to perceive which of its correctly perceived options are recognized by *DM j*. It can be seen that *DM i* assumes that *DM j* has recognized x_1 and x_2 in O^R_{iji}. *DM i* wrongly believes that *DM j* perceives the option x_2. Despite *DM i's* mistaken belief, it will still assume that *DM j* is aware of x_1 and x_2, but not options x_3 and x_4. Similar to O^R_{ij}, it is assumed that $O^R_{iji} \subseteq O^R_{ii}$.

A DM's Set of Options that are Misconceived by Itself

The second notion of options includes a DM's courses of action that are misconceived by itself. Similar to the case of a DM's universal set of options for a first-level hypergame (Aljefri et al., 2017a), two sets of options can represent *DM i's* self-misperception: the groups of options of *DM i* that are (1) imagined and (2) misunderstood by itself, symbolized as O_{ii}^{I} and O_{ii}^{M}, respectively. *DM i* cannot exercise any of the options in O_{ii}^{I} because it does not possess them in reality. O_{ii}^{M} is defined by the mapping function $\Psi_i : O_i^{\bar{R}} \longrightarrow O_{ii}^{M}$, such that for each option in $O_i^{\bar{R}}$ there is a misinterpreted option in O_{ii}^{M}. According to the definition in Aljefri et al. (2017a), $O_i^{\bar{R}}$ represents the set of courses of action of *DM i* that are valid in reality, but misunderstood by *DM i* because of its self-misperception.

Figure 5.3 shows an illustrative example of the set of options of *DM i* that are imagined by itself. As can be seen, *DM i* imagined options x_5 and x_6. These two options cannot be exercised in reality because *DM i* does not actually possess them. In a second-level hypergame, *DM i* may imagine a course of action for itself because of its misperception or its attempt to deceive its opponents by making them believe that it has some important options in the dispute where in fact it does not hold them.

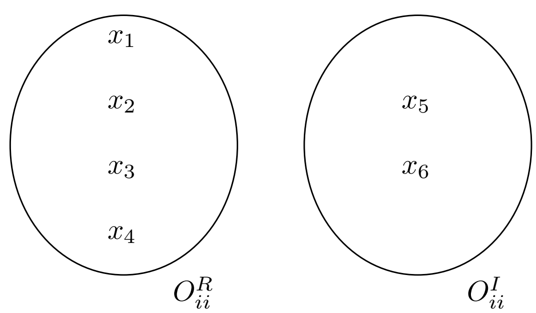


Figure 5.3: Options in O_{ii}^{I} in Comparison with O_{ii}^{R}

A situation in which *DM i* misunderstands the meaning of its options is depicted in Figure 5.4. It is clear that all elements in $O_i^{\bar{R}}$ are mapped to O_{ii}^{M}, but with a different interpretation. x_7 and x_8 are respectively comprehended by *DM i* as x_9 and x_{10}. However,

$DM\ i$ underperceives x_{13} as it cannot visualize it. Note that these sets of options pertain only to the self-misperception of $DM\ i$. Hence, they are not restrained by the awareness of $DM\ i$ of the misperception of its adversaries. Thus, their definitions and examples given in Aljefri et al. (2017a), remain the same for its universal set of options for a second-level hypergame.

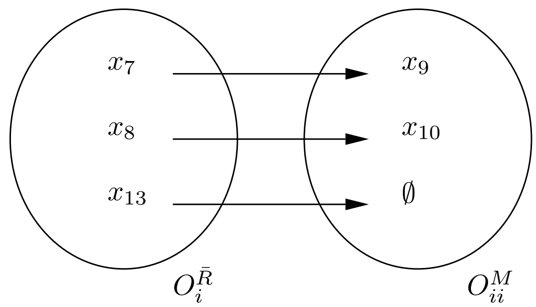


Figure 5.4: Options in $O_i^{\bar{R}}$ Misinterpreted by $DM\ i$ in O_{ii}^M

A DM's Set of Options that are Misconceived by Its Adversaries

Similar to the universal set of options of a given DM for first-level hypergame (Aljefri et al., 2017a), the courses of action of a particular DM that are not considered by itself in the dispute but are still assumed by its adversaries, or a particular DM thought that its adversaries contemplated them, can be regarded as being either imagined or misunderstood options. The sets of these options are formally defined below:

Definition 5.1.2.2 (**Set of Options of a DM that are Imagined by Its Adversaries**). *Select $i \in N$. For $j \in N - \{i\}$, let O_{ij}^I denote the set of options of DM i that are imagined by DM j. For $p \in N - \{i\}$ and $q \in N$, let O_{ipq}^I be the set of options of DM i that are imagined by DM p as perceived by DM q. Then, $(\cup_{j \in N-\{i\}} O_{ij}^I)$ and $(\cup_{p \in N-\{i\}} \cup_{q \in N} O_{ipq}^I)$ designate the options of DM i that are imagined by its adversaries and by its adversaries as perceived by any DM q, respectively.*

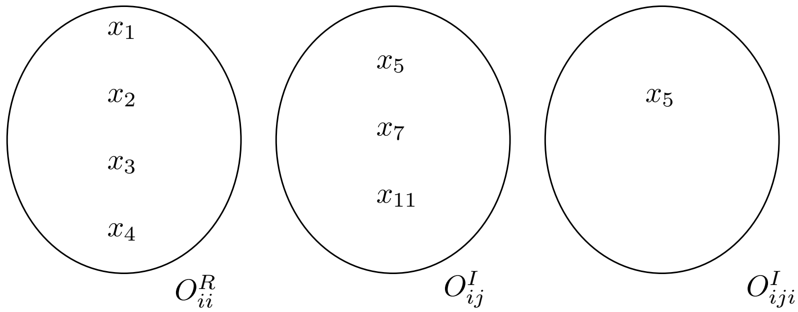
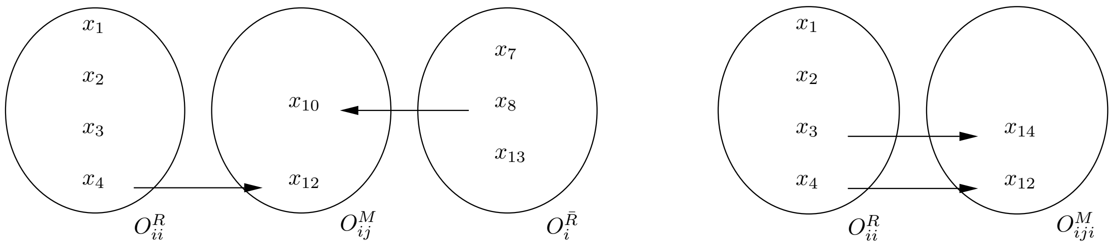


Figure 5.5: Courses of Action of *DM i* as Imagined by its Adversaries

Figure 5.5 displays the sets of courses of action of *DM i*, namely, O_{ii}^R, O_{ij}^I, and O_{iji}^I. The notation O_{iji}^I is used rather than the general form O_{ipq}^I to illustrate clearly the idea of misperception that may occur in a second-level hypergame. It can be seen that O_{ij}^I includes x_5, x_7, and x_{11}. These elements are considered by *DM j* in G_{jj} within H_j^1. Further, by comparing the options in O_{ij}^I with the option in O_{iji}^I, one can see that x_5 is common to both sets. In this situation, one can assume that *DM i* correctly captures the misperception of *DM j* with respect to x_5. Note that *DM i* can, in fact, incorrectly picture the misperception of *DM j*. *DM i* fails to perceive x_7 and x_{11} that are imagined by *DM j*. Hence, *DM i* within the second-level hypergame framework can observe (correctly or incorrectly) the misperceptions of its adversaries in a conflict situation. Moreover, by investigating the elements in Figures 5.3 and 5.5, one can observe that x_5 belongs to O_{ii}^I, O_{ij}^I, and O_{iji}^I. In this case, one can assume that *DM i* is successful in deceiving *DM j* by making it believe that *DM i* possesses x_5, when in reality i does not hold it.

Definition 5.1.2.3 (Set of Options of a DM that are Misunderstood by Its Adversaries). *Select $i \in N$. For $j \in N - \{i\}$, the courses of action in O_{ii}^R and $O_i^{\bar{R}}$ that are misinterpreted by DM j are designated as the courses of action of DM i misunderstood by DM j, symbolized as O_{ij}^M. Then, the set of options of DM i that are misunderstood by all of its adversaries is expressed as $(\cup_{j \in N-\{i\}} O_{ij}^M)$. For $p \in N - \{i\}$ and $q \in N$, options in O_{ii}^R that are misinterpreted by DM p as perceived by DM q are contained in the set O_{ipq}^M. Then, $(\cup_{p \in N-\{i\}} \cup_{q \in N} O_{ipq}^M)$ represents the options of DM i that are misunderstood by its*



(a) *DM i's* options Misunderstood by *DM j*

(b) Options in O_{ii}^R Misunderstood by *DM j* as Seen by *DM i*

Figure 5.6: Options of *DM i* that are Misunderstood by its Adversaries

adversaries as perceived by any DM q.

The sets of *DM i's* options that are misunderstood by its opponents are illustrated in Figure 5.6. Similar to the aforementioned cases, the notation O_{iji}^M in Figure 5.6b is used rather than the general notation O_{ipq}^M to clearly explain the concept. In Figure 5.6a, it can be seen that O_{ij}^M represents the options of *DM i* that are misunderstood in meaning by *DM j* in G_{jj} within H_j^1. Further, O_{iji}^M denotes the courses of action of *DM i* that are misunderstood by *DM j* as seen by *DM i* in G_{ji} within H_i^1. By observing the elements in the sets O_{ij}^M and O_{iji}^M, one can conclude that *DM i* recognizes the misperception of *DM j* in misunderstanding the meaning of x_4 and thought it to be x_{12}. This insightful information may assist *DM i* in achieving a better result in the dispute. Moreover, as can be seen in Figure 5.6b, *DM i* is not aware of the misperception of *DM j* in perceiving x_{10}, since x_{10} is unknown to i in G_{ii} within H_i^1. Furthermore, by investigating the entries in Figure 5.6, one can recognize that *DM i* is not correct in perceiving the misperception of *DM j*, because x_3 is unknown to *DM j*, but *DM i* thought that *DM j* interpreted x_3 as x_{14}. Hence, within the second-level hypergame framework in graph form, a DM can perceive the misperception of its opponents correctly, incorrectly, or may be unaware of its opponents' misperception.

The descriptions of different types of perceptions of options have now been formalized for the case of n-DM. Therefore, one can now formally define the universal set of options for a DM in a second-level hypergame. In particular, *DM i's* universal set of options for a second-level hypergame is defined as follows and depicted in Figure 5.7.

Definition 5.1.2.4 (*DM i's* Universal Set of Options in a Second-level Hypergame). *Select $i \in N$. Let $\ddot{O}_i^2$ represent the universal set of options of DM i for a second-level hypergame H^2. Then,* $\ddot{O}_i^2 = (\cup_{j\in N} O_{ij}^R) \cup O_i^{\bar{R}} \cup (\cup_{j\in N} O_{ij}^I) \cup (\cup_{j\in N} O_{ij}^M) \cup (\cup_{p\in N-\{i\}} \cup_{q\in N} O_{ipq}^I) \cup (\cup_{p\in N-\{i\}} \cup_{q\in N} O_{ipq}^M)$

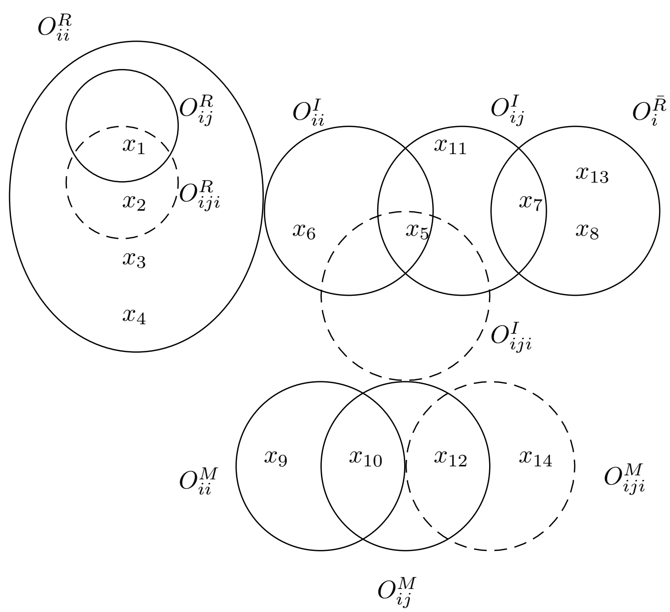


Figure 5.7: Universal Set of Options of *DM i* for Second-level Hypergame

Figure 5.7 shows the assumed relationships among the set of options of *DM i* that are perceived by itself, perceived by its opponent *DM j*, and perceived by *DM j* as contemplated by *DM i*. These relations is identical to the general case notation. These assumed relationships are listed below for the general case:

- $(\cup_{j\in N-\{i\}} O_{ij}^R) \subseteq O_{ii}^R$.
- $(\cup_{p\in N-\{i\}} \cup_{q\in N} O_{ipq}^R) \subseteq O_{ii}^R$.
- $(\cup_{j\in N-\{i\}} O_{ij}^R) \cap (\cup_{p\in N-\{i\}} \cup_{q\in N} O_{ipq}^R) \neq \emptyset$ may or may not exist.

- $\cap_{j\in N}O^I_{ij} \neq \emptyset$ may or may not hold.

- $(\cup_{j\in N}O^I_{ij}) \cap (\cup_{p\in N-\{i\}} \cup_{q\in N} O^I_{ipq}) \neq \emptyset$ may or may not exist.

- $(\cup_{j\in N-\{i\}}O^I_{ij}) \cap O^{\bar{R}}_i \neq \emptyset$ may or may not hold.

- $\cap_{j\in N}O^M_{ij} \neq \emptyset$ may or may not exist.

- $(\cup_{j\in N}O^M_{ij}) \cap (\cup_{p\in N-\{i\}} \cup_{q\in N} O^M_{ipq}) \neq \emptyset$ may or may not be true.

- $(\cup_{j\in N}O^R_{ij}) \cap (\cup_{j\in N}O^I_{ij}) = (\cup_{j\in N}O^R_{ij}) \cap (\cup_{p\in N-\{i\}} \cup_{q\in N} O^I_{ipq}) = (\cup_{j\in N}O^R_{ij}) \cap O^{\bar{R}}_i = O^I_{ii} \cap O^{\bar{R}}_i = (\cup_{j\in N}O^I_{ij}) \cap (\cup_{j\in N}O^M_{ij}) = (\cup_{p\in N-\{i\}} \cup_{q\in N} O^I_{ipq}) \cap (\cup_{p\in N-\{i\}} \cup_{q\in N} O^M_{ipq}) = (\cup_{p\in N-\{i\}} \cup_{q\in N} O^I_{ipq}) \cap O^{\bar{R}}_i = (\cup_{j\in N}O^M_{ij}) \cap O^{\bar{R}}_i = (\cup_{p\in N-\{i\}} \cup_{q\in N} O^M_{ipq}) \cap O^{\bar{R}}_i = (\cup_{p\in N-\{i\}} \cup_{q\in N} O^M_{ipq}) \cap O^M_{ii} = \emptyset$

Keep in mind that all courses of action in $\ddot{O}^2_i$ are recognized by an external expert. However, because the DMs in the second-level hypergame may have different perceptions, some options in $\ddot{O}^2_i$ may be known to them while others may not. For $j \in N$, the set of options of *DM i* that are unknown to *DM j* is expressed in the same way as performed in first-level hypergame in graph form by Aljefri et al. (2017a), as $O^U_{ij} = \ddot{O}^2_i \setminus (O^R_{ij} \cup O^I_{ij} \cup O^M_{ij})$. Further, for $p \in N - \{i\}$ and $q \in N$, the set of options of *DM i* that are unknown to *DM p* as perceived by *DM q* is defined as: $O^U_{ipq} = \ddot{O}^2_i \setminus (O^R_{ipq} \cup O^I_{ipq} \cup O^M_{ipq})$.

The universal set of options of other DMs for a second-level hypergame can be analogously defined. The union of the universal sets of options of all the DMs for a second-level hypergame mathematically defines the universal set of options for the entire second-level hypergame $\hat{O}^2$ as follows:

$$\hat{O}^2 = \cup_{i\in N}\ddot{O}^2_i. \tag{5.4}$$

5.1.3 Universal Set of States in an n-Decision Maker Second-Level Hypergame

Having now introduced the universal set of options for a second-level hypergame, a technique similar to that used for defining the universal set of states for first-level hypergame by Aljefri et al. (2017a) is employed to construct the states in the universal set of states for a second-level hypergame. This set of states includes the perception of a DM, the perception of its adversaries, and the perception of adversaries as contemplated by the other DMs about the conflict situation. According to the mathematical representation of option form for which complete information is assumed (Howard, 1971; Fraser and Hipel, 1979; Fang et al., 1993; Fang et al., 2003a,b), representative states in option form for an n-DM second-level hypergame can be formed as follows:

Definition 5.1.3.1 (**Universal Set of States for a Second-level Hypergame**). *Let $N = \{1, 2, ..., i, ..., n\}$. For $i \in N$, note that $\ddot{O}_i^2 = \{o_{\bar{k}}^i : \bar{k} = 1, 2, ..., m_i\}$ denotes the universal set of options of DM i for a second-level hypergame, where $o_{\bar{k}}^i$ is the $\bar{k}^{th}$ option of DM i and $m_i =| \ddot{O}_i^2 |$ is the total number of options in $\ddot{O}_i^2$. A course of action can either be chosen or not by the DM controlling it. Therefore, the strategy of DM i can be expressed by the mapping $g_i : \ \ddot{O}_i^2 \longrightarrow \{0, 1\}$. Recall that $\hat{O}^2 = \ddot{O}_1^2 \cup \ddot{O}_2^2 \cup ... \cup \ddot{O}_i^2 \cup ... \cup \ddot{O}_n^2$ denotes the universal set of options for the entire second-level hypergame, and let $\lambda = \sum_{i=1}^{n} m_i$ denote the number of elements in $\hat{O}^2$. Then, a state s in option form is a λ-dimensional column vector and is expressed by the mapping $f : \ \hat{O}^2 \longrightarrow \{0, 1\}$, such that $f(o_{\bar{k}}^i) =$ either 0 or 1, for $i = 1, 2, 3, ..., n$.*

Keep in mind that a state is a vector in the structure $(f(o_1^1), ..., f(o_{m_1}^1), ..., f(o_1^n), \ ...,$ $f(o_{m_n}^n))^T$. Then, the set of mathematically feasible states for a second-level hypergame is denoted by $\hat{S}^2 = \{s_1, s_2, ..., s_{2^\lambda}\}$, where 2^λ is the total number of the mathematically feasible states. Similar to the standard GMCR, some of the mathematically feasible states in $\hat{S}^2$ are removed from the model because they are infeasible within four option conditions

(Fang et al., 2003a,b). DMs' strategies corresponding to state s_1 is denoted by $g_1^{s_1}, g_2^{s_1}, g_3^{s_1}$, $..., g_n^{s_1}$. Therefore, $s_1 = ((g_1^{s_1})^T, (g_2^{s_1})^T, ..., (g_i^{s_1})^T, ..., (g_n^{s_1})^T)^T$.

States in a DM's Subjective First-Level Hypergame

It was noted earlier that second-level hypergame is described by a system of subjective first-level hypergames, each of which points out not only the perception of a particular DM of the dispute, but also the perception of its adversaries about the conflict situation as seen by a particular DM. For $i \in N$, one should keep in mind that $H_i^1 = \langle G_{ji} : j \in N_i \rangle$. To identify states in G_{ji}, $\hat{O}^2$ must be partitioned based on the perception of *DM j* as perceived by *DM i* into two collections: the groups of options that are (1) realized by *DM j* as assumed by *DM i*, expressed as $(\cup_{k \in N_{ji}} O_{kji}^R) \cup (\cup_{k \in N_{ji}} O_{kji}^I) \cup (\cup_{k \in N_{ji}} O_{kji}^M)$, where N_{ji} is the set of DMs as perceived by *DM j* and then by *DM i*, and (2) hidden to *DM j* in its subjective game as seen by *DM i*, represented as $\cup_{k \in N_{ji}} O_{kji}^U$. Note that, if $k = j$, then O_{jji}^R, O_{jji}^I, O_{jji}^M, O_{jji}^U are identical to O_{ji}^R, O_{ji}^I, O_{ji}^M, O_{ji}^U, respectively. With the collections of options defined above, one can now define the sets of recognizable and hidden states in G_{ji} as follows:

Definition 5.1.3.2 (**Set of Recognizable States in** G_{ji}). *Select* $i \in N$ *and* $j \in N_i$. *Let* $S_{ji} \subseteq \hat{S}^2$ *be the set of states perceived by DM j in* G_{ji} *as seen by DM i in* H_i^1. *A state* $s \in S_{ji} \iff$ *there is a mapping* $f : \hat{O}^2 \rightarrow \{0, 1\}$ *satisfying* $f(o) = 0$ $, \forall\ o \in \cup_{k \in N_{ji}} O_{kji}^U$.

Definition 5.1.3.3 (**Set of Hidden States in** G_{ji}). *Choose* $i \in N$ *and* $j \in N_i$. *Denote by* $S_{ji}^H \subseteq \hat{S}^2$ *the set of states that are hidden to DM j in* G_{ji} *as seen by DM i in* H_i^1. *A state* $s \in S_{ji}^H \iff$ *there is a mapping* $f : \hat{O}^2 \rightarrow \{0, 1\}$ *satisfying* $\exists\ o \in \cup_{k \in N_{ji}} O_{kji}^U, f(o) = 1$.

Similar to the case of a first-level hypergame with n DMs in graph form in Aljefri et al. (2017a), the set of recognizable states of a DM in its subjective game within its subjective first-level hypergame is further classified by an external expert or analyst into five distinct classes of states. Note that in a second-level hypergame, the analysis of a DM subjective first-level hypergame always starts by analyzing its opponents' games as seen by itself within its own subjective first-level hypergame. After that, a DM utilizes this insightful information to calculate the equilibrium states in its game within its subjective first-level hypergame. Finally, second-level hypergame equilibria are calculated by evaluating all DMs' subjective games out of their subjective first-level hypergames. Therefore, within second-level hypergame, the classification of states is limited to the subjective game of a DM within its subjective first-level hypergame.

For $i \in N$, the set of recognizable states of *DM i*, S_{ii}, in G_{ii} within H_i^1 is divided as follows: the group of states that are (1) correctly perceived by all DMs in the dispute, denoted by S^R; (2) correctly identified by *DM i* and perhaps by some of its adversaries but not by all, represented by S_{ii}^P; (3) imagined by *DM i*, symbolized as S_{ii}^I; (4) misunderstood in meaning by *DM i*, symbolized as S_{ii}^M; and (5) imagined and misunderstood by *DM i*, denoted by $S_{ii}^{I,M}$. The definitions of the five classes of states are summarized in Table 5.1 for the case of n-DM second-level hypergame. These distinct sets assist the analyst in classifying the second-level hypergame's equilibria into meaningful categories to provide better strategic insights about the hypergame situation. Also, these equilibria explore the possible moves of a DM after observing others' misperceptions in reality. For further discussion about the classification of S_{ii} in G_{ii}, see Aljefri et al. (2017a) and the references contained therein.

5.1.4 Analysis of a Second-Level Hypergame with n-DM in Graph Form

Several techniques are accessible to analyze a second-level hypergame (Bennett, 1977, 1980; Takahashi et al., 1984; Wang et al., 1988, 1989). In this research, the methods used in Wang et al. (1988, 1989) are improved and applied within the framework of a second-level hypergame in graph form to anticipate the equilibria of the situation under investigation. Figure 5.8 shows the overall structure of a second-level hypergame analysis in graph form.

Table 5.1: Partitioning of S_{ii} in G_{ii} within H_i^1

State Type	Definition
S^R	A state $s \in S_{ii}$ is correctly recognized by all DMs, that is, $s \in S^R \subseteq S_{ii} \iff$ there is a mapping $f : \hat{O}^2 \rightarrow \{0,1\}$ satisfying $f(o) = 0,\ \forall\ o \in \hat{O}^2 \setminus (\cup_{i \in N} O_i^R)$.
S_{ii}^P	A state $s \in S_{ii}$ is correctly perceived by *DM* i itself in G_{ii} and possibly by some of its adversaries but not by all, that is, $s \in S_{ii}^P \iff$ there is a mapping $f : \hat{O}^2 \rightarrow \{0,1\}$ satisfying $\exists\ o \in \cup_{j \in N}(O_{ji}^R \setminus O_j^R)$, $f(o) = 1$ and $f(o') = 0, \forall\ o' \in \hat{O}^2 \setminus (\cup_{j \in N} O_{ji}^R)$.
S_{ii}^I	A state $s \in S_{ii}$ is imagined by *DM* i in G_{ii}, that is, $s \in S_{ii}^I \iff$ there is a mapping $f : \hat{O}^2 \rightarrow \{0,1\}$ satisfying $\exists\ o \in \cup_{j \in N} O_{ji}^I$, $f(o) = 1$ and $f(o') = 0, \forall\ o' \in \hat{O}^2 \setminus [(\cup_{j \in N} O_{ji}^I) \cup (\cup_{j \in N} O_{ji}^R)]$.
S_{ii}^M	A state $s \in S_{ii}$ is misunderstood by *DM* i in G_{ii}, that is, $s \in S_{ii}^M \iff$ there is a mapping $f : \hat{O}^2 \rightarrow \{0,1\}$ satisfying $\exists\ o \in \cup_{j \in N} O_{ji}^M$, $f(o) = 1$ and $f(o') = 0, \forall\ o' \in \hat{O}^2 \setminus [(\cup_{j \in N} O_{ji}^M) \cup (\cup_{j \in N} O_{ji}^R)]$.
$S_{ii}^{I,M}$	A state $s \in S_{ii}$ is included in the imagined and misunderstood scenarios of *DM* i in G_{ii}, that is, $s \in S_{ii}^{I,M} \iff$ there is a mapping $f : \hat{O}^2 \rightarrow \{0,1\}$ satisfying $\exists\ o \in \cup_{j \in N} O_{ji}^I$, $f(o) = 1$, $\exists\ o' \in \cup_{j \in N} O_{ji}^M$, $f(o') = 1$, and $f(o'') = 0, \forall\ o'' \in \hat{O}^2 \setminus [(\cup_{j \in N} O_{ji}^I) \cup (\cup_{j \in N} O_{ji}^M) \cup (\cup_{j \in N} O_{ji}^R)]$.

The analysis is performed in two phases: (a) the analysis of the subjective first-level hypergame of each DM and (b) the overall second-level hypergame analysis.

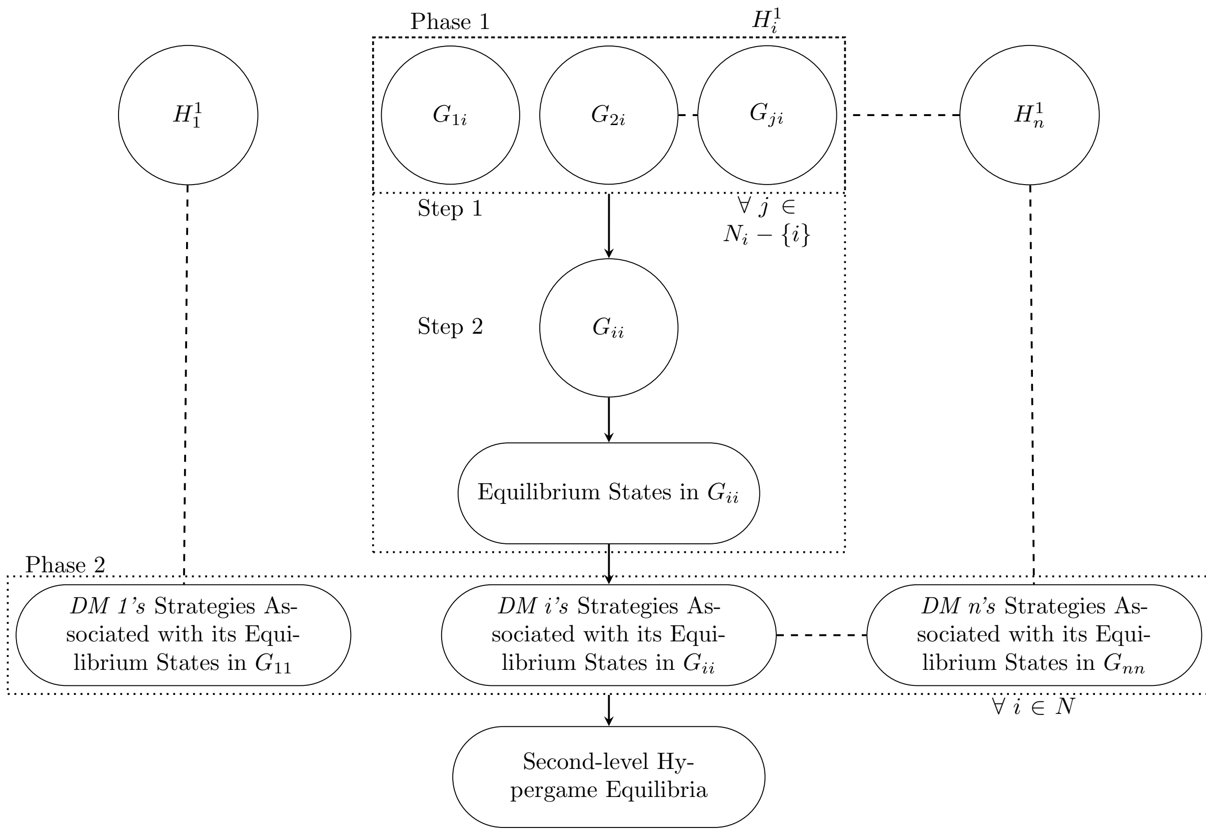


Figure 5.8: Stability Analysis Procedure for a Second-level Hypergame with n-DM in Graph Form

Starting at the top part of Phase 1 of Figure 5.8, Step 1 is executed by investigating the stability of states in G_{ji} using any of the standard GMCR solution concepts. Therefore, the equilibria in the subjective game of each DM other than *DM i* within H_i^1 are anticipated.

In Step 2 of Phase 1 of Figure 5.8, the equilibria in each DM's subjective game other than *DM i* within H_i^1 are utilized to identify the subjective equilibria in G_{ii}. within Step 2, three stages need to be performed as follows:

- For each $j \in N_i - \{i\}$, isolate *DM j's* strategies from its equilibrium states in G_{ji}. These strategies are to be called a DM's winning strategies.
- Calculate the Cartesian product for all DMs' winning strategies, for all $j \in N_i - \{i\}$.
- In G_{ii}, calculate the stability of states for *DM* i using GMCR solution concepts. If a state is stable for *DM* i and its opponents' strategies related to this state are members of the set of strategies identified in stage 2 under a solution concept, the state is an equilibrium in G_{ii} and in H_i^1 within this particular solution concept.

It can be seen from the bottom part of Figure 5.8, Phase 2 starts by isolating the strategies of *DM* i from the equilibria in G_{ii}. Then, second-level hypergame equilibria are obtained by taking the ordered collections (Cartesian product) of all the strategy sets of the DMs, each of which is obtained from the set of equilibria of a given DM in its subjective game within its subjective first-level hypergame. Note, that if the strategies of the DMs are obtained from their Nash equilibrium states, then the resulting Cartesian product is called hyper Nash equilibria for a second-level hypergame.

Analysis of a DM's Subjective First-Level Hypergame

For $i \in N$, recall that $H_i^1 = \langle G_{ji} : j \in N_i \rangle$. As noted earlier, the analysis of H_i^1 starts by analyzing G_{ji} based on a collection of stability definitions, each of which imitates different possible human behavior under conflict. These stability definitions, formalized within the paradigm of a first-level hypergame in graph form to ascertain the equilibrium states in the subjective game of each DM (Aljefri et al., 2017a), are used to calculate the equilibrium states in G_{ji}. To furnished the stability analysis definitions in G_{ji} for the case of n-DM, the concepts of reachable list and unilateral improvement list by a group of DMs are put forward below.

Let the set of DMs in G_{ji} be $N_{ji} = \{1, 2, ..., j, ..., n_{ji}\}$. Assume that $H \subseteq N_{ji}$, $H \neq \emptyset$, be any subgroup of players in N_{ji}. For $s_1 \in S_{ji}$ and $k \in N_{ji}$, let $R_{kji}(s_1)$ be the set of unilateral moves (UMs) of *DM* k beginning from s_1. Also, let $R_{Hji}(s_1)$ symbolize the set of all UMs from s_1 by any number of DMs $\in H$ over a legal sequence of moves beginning from s_1. A sequence of moves by players in H is legal if no player makes two successive moves. For $s_2 \in R_{Hji}(s_1)$, let $\Omega_{Hji}(s_1,\ s_2)$ symbolize the set of all last players in H in the legal sequences of moves from s_1 to s_2. Further, let $R^+_{Hji}(s_1)$ denote the set of all UIs from s_1 by any number of DMs $\in H$ over a legal sequence of moves beginning from s_1. For $s_2 \in R^+_{Hji}(s_1)$, let $\Omega^+_{Hji}(s_1,\ s_2)$ symbolize the set of all last players in H in the legal sequences of UI from s_1 to s_2. The unilateral moves and the unilateral improvement list by $H \subseteq N_{ji}$ can now be defined as follows.

Definition 5.1.4.1 (**Unilateral Moves by** $H \subseteq N_{ji}$)**.** *Let* $s_1 \in S_{ji}$*. Then,* $R_{Hji}(s_1)$ *can be defined as follows:*

- *If* $k \in H$ *and* $s_2 \in R_{kji}(s_1)$*, then* $s_2 \in R_{Hji}(s_1)$ *and* $k \in \Omega_{Hji}(s_1,\ s_2)$*.*
- *If* $s_2 \in R_{Hji}(s_1)$*,* $k \in H$*, and* $s_3 \in R_{kji}(s_2)$*, then*
    1. *if* $|\Omega_{Hji}(s_1,\ s_2)| = 1$ *and* $k \notin \Omega_{Hji}(s_1,\ s_2)$*, then* $s_3 \in R_{Hji}(s_1)$ *and* $k \in \Omega_{Hji}(s_1,\ s_3)$*.*
    2. *if* $|\Omega_{Hji}(s_1,\ s_2)| > 1$*, then* $s_3 \in R_{Hji}(s_1)$ *and* $k \in \Omega_{Hji}(s_1,\ s_3)$*.*

  The process ends when no new state s_3 *can be included in* $R_{Hji}(s_1)$ *and no differences occur from* $|\ \Omega_{Hji}(s_1,\ s_2)\ | = 1$ *to* $|\ \Omega_{Hji}(s_1,\ s_2)\ | > 1$ *for any* $s_2 \in R_{Hji}(s_1)$*. Every state in* $R_{Hji}(s_1)$ *is considered as a UM from* s_1 *by* H*.*

Definition 5.1.4.2 (**Unilateral Improvement List by the Subgroup of DMs** $H \subseteq N_{ji}$)**.** *Let* $s_1 \in S_{ji}$*. A Unilateral Improvement List (UIL)* $R^+_{Hji}(s_1)$ *is built as shown below:*

- *If* $k \in H$ *and* $s_2 \in R^+_{kji}(s_1)$*, then* $s_2 \in R^+_{Hji}(s_1)$ *and* $k \in \Omega^+_{Hji}(s_1,\ s_2)$*.*
- *If* $s_2 \in R^+_{Hji}(s_1)$*,* $k \in H$*, and* $s_3 \in R^+_{kji}(s_2)$*, then*

1. if $\mid \Omega^+_{Hji}(s_1,\ s_2) \mid = 1$ *and* $k \notin \Omega^+_{Hji}(s_1,\ s_2)$*, then* $s_3 \in R^+_{Hji}(s_1)$ *and* $k \in \Omega^+_{Hji}(s_1,\ s_3)$*,*

2. if $\mid \Omega^+_{Hji}(s_1,\ s_2) \mid > 1$*, then* $s_3 \in R^+_{Hji}(s_1)$ *and* $k \in \Omega^+_{Hji}(s_1,\ s_3)$*.*

The process ends when there is no new state s_3 *that can be included in* $R^+_{Hji}(s_1)$ *and the condition* $\mid \Omega^+_{Hji}(s_1,\ s_2) \mid = 1$ *to* $\mid \Omega^+_{Hji}(s_1,\ s_2) \mid > 1$ *is not altered for any state* $s_2 \in R^+_{Hji}(s_1)$*. Every state in* $R^+_{Hji}(s_1)$ *is a UI starting from* s_1 *by a subgroup of DMs* H*.*

If $k = j$, then remember that $R_{jji}(s_1)$ and $R^+_{jji}(s_1)$ are identical to $R_{ji}(s_1)$ and $R^+_{ji}(s_1)$, respectively. After the concepts of UM and UI for a group of DMs H are introduced, the solution concepts of Nash, SEQ, GMR, and SMR can be formally defined within the paradigm of a second-level hypergame as follows.

Definition 5.1.4.3 (**Nash Stability**). *A state* $s_1 \in S_{ji}$ *is Nash stable (Nash) for DM* $k \in N_{ji}$ *in* $G_{ji} \iff R^+_{kji}(s_1) = \emptyset$*. The group of all Nash stable states for DM k in* G_{ji} *is symbolized by* $S_{ji}^{Nash_{kji}}$*.*

Definition 5.1.4.4 (**SEQ Stability**). *A state* $s_1 \in S_{ji}$ *is sequentially stable (SEQ) for DM* $k \in N_{ji}$ *in* $G_{ji} \iff$ *for each* $s_2 \in R^+_{kji}(s_1)$*,* $\exists\ s_3 \in R^+_{(N_{ji}-\{k\})ji}(s_2)$ *such that* $s_3 \precsim_{kji} s_1$*. The group of all SEQ stable states for DM k in* G_{ji} *is denoted by* $S_{ji}^{SEQ_{kji}}$*.*

Definition 5.1.4.5 (**GMR Stability**). *A state* $s_1 \in S_{ji}$ *is general metarational stable (GMR) for DM* $k \in N_{ji}$ *in* $G_{ji} \iff$ *for each* $s_2 \in R^+_{kji}(s_1)$*,* $\exists\ s_3 \in R_{(N_{ji}-\{k\})ji}(s_2)$ *such that* $s_3 \precsim_{kji} s_1$*. The group of all GMR stable states for DM k in* G_{ji} *is symbolized by* $S_{ji}^{GMR_{kji}}$*.*

Definition 5.1.4.6 (**SMR Stability**). *A state* $s_1 \in S_{ji}$ *is SMR stable for DM* $k \in N_{ji}$ *in* $G_{ji} \iff$ *for each* $s_2 \in R^+_{kji}(s_1)$*,* $\exists\ s_3 \in R_{(N_{ji}-\{k\})ji}(s_2)$ *such that* $s_3 \precsim_{kji} s_1$*, and* $s_4 \precsim_{kji} s_1$*,* $\forall\ s_4 \in R_{kji}(s_3)$*. The group of all SMR stable states for DM k in* G_{ji} *is represented by* $S_{ji}^{SMR_{kji}}$*.*

The interrelationships among the aforementioned solution concepts were investigated within the structure of the GMCR for which there are no misperceptions (Fang et al., 1993). Here, the same properties of the solution concepts are studied within the structure of the second-level hypergame in graph form, for which the findings are summarized in the Venn diagram in Figure 5.9. The investigation reveals that the same relationships between the solution concepts found in GMCR hold in the second-level hypergame in graph form.

Theorem 5.1.4.1. *Let $k \in N_{ji}$ and $s_1 \in S_{ji}$. If $s_1 \in S_{ji}^{Nash_{kji}}$ for DM k in G_{ji}, then $s_1 \in S_{ji}^{SMR_{kji}}$ for DM k in G_{ji}; if $s_1 \in S_{ji}^{SMR_{kji}}$ for DM k in G_{ji}, then $s_1 \in S_{ji}^{GMR_{kji}}$ for DM k in G_{ji}.*

Proof. According to Definition 5.1.4.3, s_1 is Nash for *DM k* if $R_{kji}^{+}(s_1) = \emptyset$. This condition is trivially satisfied in Definition 5.1.4.6. Therefore, $S_{ji}^{Nash_{kji}} \subseteq S_{ji}^{SMR_{kji}}$. Next, from Definition 5.1.4.6, a state $s_1 \in S_{ji}^{SMR_{kji}} \iff$ for each $s_2 \in R_{kji}^{+}(s_1)$ $\exists$ $s_3 \in R_{(N_{ji}-\{k\})ji}(s_2)$ such that $s_3 \precsim_{kji} s_1$, and $s_4 \precsim_{kji} s_1$ $\forall$ $s_4 \in R_{kji}(s_3)$. The condition for each $s_2 \in R_{kji}^{+}(s_1)$ $\exists$ $s_3 \in R_{(N_{ji}-\{k\})ji}(s_2)$ such that $s_3 \precsim_{kji} s_1$ in Definition 5.1.4.5, implies that $s_1 \in S_{ji}^{GMR_{kji}}$. Hence, $S_{ji}^{SMR_{kji}} \subseteq S_{ji}^{GMR_{kji}}$. Thus, $S_{ji}^{SMR_{kji}} \subseteq S_{ji}^{GMR_{kji}}$. □

Theorem 5.1.4.2. *Let $k \in N_{ji}$ and $s_1 \in S_{ji}$. If $s_1 \in S_{ji}^{Nash_{kji}}$ for DM k in G_{ji}, then $s_1 \in S_{ji}^{SEQ_{kji}}$ for DM k in G_{ji}; if $s_1 \in S_{ji}^{SEQ_{kji}}$ for DM k in G_{ji}, then $s_1 \in S_{ji}^{GMR_{kji}}$ for DM k in G_{ji}.*

Proof. If $s_1 \in S_{ji}^{Nash_{kji}}$, then Definition 5.1.4.4 is obviously satisfied since $R_{kji}^{+}(s_1) = \emptyset$. Hence, $S_{ji}^{SEQ_{kji}} \subseteq S_{ji}^{Nash_{kji}}$. Also, since $R_{(N_{ji}-\{k\})ji}^{+}(s_2) \subseteq R_{(N_{ji}-\{k\})ji}(s_2)$, $S_{ji}^{SEQ_{kji}} \subseteq S_{ji}^{GMR_{kji}}$. □

Definition 5.1.4.7 (Equilibria in G_{ji}). *A state $s_1 \in S_{ji}$ that is stable for all the DMs in G_{ji} under the same stability concept is an equilibrium in G_{ji} according to that stability definition. The set of all equilibrium states in G_{ji} as seen by DM i is symbolized by E_{ji}.* (Note that if a state is either Nash, SEQ, GMR, or SMR stable across all the DMs in G_{ji},

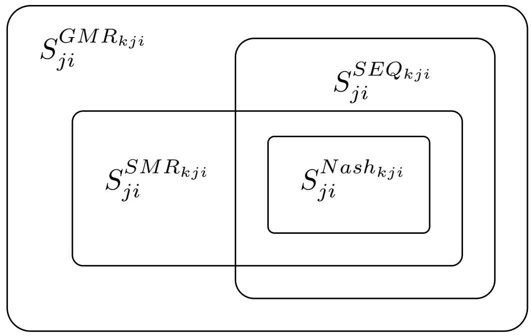


Figure 5.9: Characteristics of the Solution Concepts in G_{ji}

it is considered as either Nash, SEQ, SMR, or GMR equilibrium in G_{ji}. The sets of all Nash, SEQ, SMR, and GMR equilibrium states are denoted, respectively, by E_{ji}^{Nash}, E_{ji}^{SEQ}, E_{ji}^{GMR}, and E_{ji}^{SMR}).

Once the set of equilibria in the subjective game of *DM j* as envisioned by *DM i*, G_{ji}, is identified, the equilibrium states in G_{ii} within H_i^1 can be determined (See Step 2 in Figure 5.8). This can be achieved by investigating the stability of states for *DM i*, which are related to its opponents' winning strategies, obtained from G_{ji}, for all $j \in N_i - \{i\}$. The set of Nash equilibrium states of *DM i* in G_{ii} can be defined as follows:

Definition 5.1.4.8 (Nash Equilibria in the Actual Subjective Game of a DM). *For every $j \in N_i - \{i\}$, one should recall that E_{ji}^{Nash} denotes the set of Nash equilibria in G_{ji}. Assume that $E_{ji}^{Nash} = \{e_{1ji}^{Nash_{ji}}, e_{2ji}^{Nash_{ji}}, ..., e_{\varepsilon_{ji}}^{Nash_{ji}}\}$, where ε_{ji} is the total number of Nash equilibrium states in G_{ji}. Let $g_{ji}^{*Nash_{ji}} = \{g_{ji}^{e_1^{Nash_{ji}}}, g_{ji}^{e_2^{Nash_{ji}}}, ..., g_{ji}^{e_{\varepsilon_{ji}}^{Nash_{ji}}}\}$ constitute the set of Nash strategies of DM j in G_{ji} in H_i^1, where $g_{ji}^{e_{1ji}^{Nash_{ji}}}$ is the strategy of DM j as seen by DM i that is obtained from the equilibrium state $e_{1ji}^{Nash_{ji}}$ in G_{ji}. Then, the Cartesian product for all DMs' Nash strategies in $N_i - \{i\}$ is represented as ($\prod_{j \in N_i - \{i\}} g_{ji}^{*Nash_{ji}}$). Next, for $s_1 \in S_{ii}$ in G_{ii} within H_i^1, let $s_1 = \{(g_i^{s_1})^T, (g_{N_i-\{i\}}^{s_1})^T\}$, where $g_i^{s_1}$ is DM i's strategy related to s_1 and $g_{N_i-\{i\}}^{s_1}$ is the opponents' strategies associated with s_1. Then, s_1 is a Nash* *equilibrium for DM i in* $G_{ii} \Leftrightarrow s_1 \in S_{ii}^{Nash_{ii}}$ *and* $g_{N_i-\{i\}}^{s_1} \in (\prod_{j \in N_i - \{i\}} g_{ji}^{*Nash_{ji}})$. *The set of Nash equilibrium states of DM i in* G_{ii} *within* H_i^1 *is symbolized as* E_{ii}^{Nash}.

The SEQ, GMR, and SMR equilibrium states of *DM i* in G_{ii} can be analogously defined.

Analysis of the Overall Second-Level Hypergame Equilibria

In this phase, second-level hypergame equilibria are obtained. Similar to a first-level hypergame in graph form (Aljefri et al., 2017a), this can be achieved by first identifying the strategies of all the DMs obtained from the equilibrium states in their subjective games within their subjective first-level hypergame. Then, the ordered collections (Cartesian product) of the Nash, SEQ, GMR, and SMR strategies of all the DMs constitute the hyper Nash, hyper SEQ, hyper GMR, and hyper SMR equilibria, respectively, for a second-level hypergame. The hyper Nash equilibrium for a second-level hypergame in an n-DM model is defined first.

Definition 5.1.4.9 (**Hyper Nash Equilibrium for a Second-Level Hypergame**). *For* $i \in N$, *note that* E_{ii}^{Nash} *symbolizes the set of Nash equilibria in* G_{ii} *within* H_i^1. $E_{ii}^{Nash} = \{e_{1ii}^{Nash_{ii}}, e_{2ii}^{Nash_{ii}}, ..., e_{\varepsilon_{ii}}^{Nash_{ii}}\}$, *where* ε_{ii} *is the number of Nash equilibria in* G_{ii}. *Let* $g_{ii}^{*Nash_{ii}} = \{g_{ii}^{e_{1ii}^{Nash_{ii}}}, g_{ii}^{e_{2ii}^{Nash_{ii}}}, ..., g_{ii}^{e_{\eta_{ii}}^{Nash_{ii}}}\}$ *represent the set of different Nash strategies of DM i in* G_{ii}, *where* $g_{ii}^{e_{1ii}^{Nash_{ii}}}$ *is the strategy of DM i attained from the equilibrium* $e_{ii}^{Nash_{ii}}$ *in* G_{ii}. *The set of hyper Nash equilibria for the second-level hypergame is formalized as follows:*

$$HE^{2Nash} = \prod_{i \in N} g_{ii}^{*Nash_{ii}} \tag{5.5}$$

Keep in mind that the notation $\prod$ in Eq. 5.5 stands for the Cartesian product. Similar to the first-level hypergame equilibria in graph form Aljefri et al. (2017a), the total number of hyper Nash equilibria is $\varepsilon_{ii}\varepsilon_{jj}$. A hyper SEQ equilibrium state (HE^{2SEQ}), a hyper GMR equilibrium state (HE^{2GMR}), and a hyper SMR equilibrium state (HE^{2SMR}) for second-level hypergame are analogously defined. HE^{2SEQ}, HE^{2GMR}, and HE^{2SMR} can be defined in a similar fashion.

5.2 Classification of Second-Level Hypergame Equilibria

Based on the description of a first-level hypergame with n DMs in graph form in Aljefri et al. (2017a), the classification of the second-level hypergame equilibria is conducted by an insightful specialist or analyst who is cognizant of the difference in understanding among the DMs. Hence, one can identify the equilibrium states that are understood by all the DMs and those that are not. These equilibrium classes for a second-level hypergame provide information about the sources of misperception that underly the dispute and the possible reactions of the DMs after they become aware of their misperception in reality. Similar to a first-level hypergame with two or more DMs in graph form, the groups of equilibrium states addressed here are steady, unsteady, stealthy, unsteady stealthy, contingent, unsteady contingent, self-contingent, and emergent hyper-equilibrium states for second-level hypergame. The formal definitions of these classes of equilibria are summarized here for a second-level hypergame with n-DM. For further discussion about these distinct equilibrium classes, please refer to Aljefri et al. (2017a).

Definition 5.2.0.10 (Steady Hyper Nash ($SHNash^2$) Equilibrium). *A hyper Nash equilibrium state for a second-level hypergame, $s \in HE^{2\ Nash}$, is referred to as $(SHNash^2)$ equilibrium iff $s \in S^R$ and $s \in \cap_{i \in N} E_{ii}^{Nash}$.*

Please notice that the superscript 2 in $SHNash^2$ indicates that it is an equilibrium for a second-level hypergame. A steady hyper SEQ *($SHSEQ^2$)* equilibrium, a steady hyper GMR *($SHGMR^2$)* equilibrium, and a steady hyper SMR *($SHSMR^2$)* equilibrium for a second-level hypergame can be similarly formalized.

Definition 5.2.0.11 (**Unsteady Hyper Nash** ($UHNash^2$) **Equilibrium**). *A hyper Nash equilibrium state for a second-level hypergame,* $s \in HE^{2\ Nash}$, *is called a UHNash*^{2} *equilibrium iff* $s \in S^R$ *and* $s \notin \cap_{i \in N} E_{ii}^{Nash}$.

An unsteady hyper SEQ *(UHSEQ*^{2}*)* equilibrium, unsteady hyper GMR *(UHGMR*^{2}*)* equilibrium, and unsteady hyper SMR *(UHSMR*^{2}*)* equilibrium for a second-level hypergame can be defined in a similar manner.

Definition 5.2.0.12 (**Stealthy Hyper Nash** ($STHNash^2$) **Equilibrium**). *A hyper Nash equilibrium state for a second-level hypergame,* $s \in HE^{2\ Nash}$, *is called an STHNash*^{2} *equilibrium iff* $\exists\ i \in N,\ s \in S_{ii}^P \cap E_{ii}^{Nash}$.

A stealthy hyper SEQ *(STHSEQ*^{2}*)* equilibrium, a stealthy hyper GMR *(STHGMR*^{2}*)* equilibrium, and a stealthy hyper SMR *(STHSMR*^{2}*)* equilibrium for a second-level hypergame can be analogously formalized.

Definition 5.2.0.13 (**Unsteady STHNash**^{2} **(**$USTHNash^2$**) Equilibrium**). *A hyper Nash equilibrium state for a second-level hypergame,* $s \in HE^{2\ Nash}$, *is called a USTHNash*^{2} *iff* $s \in S_{ii}^P \setminus E_{ii}^{Nash}$ *for at least one* $i \in N$.

An unsteady STHSEQ^2 ($USTHSEQ^2$) equilibrium, unsteady STHGMR^2 ($USTHGMR^2$) equilibrium, and unsteady STHSMR^2 ($USTHSMR^2$) equilibrium for a second-level hypergame can be defined in a similar manner.

Definition 5.2.0.14 (**Contingent Hyper Nash *(CHNash*^{2}*)* Equilibrium**). *A hyper Nash equilibrium state for a second-level hypergame,* $s \in HE^{2\ Nash}$, *is called a CHNash*^{2} *equilibrium* $\iff$ *either* $s \in \cap_{i \in N} S_{ii}^I$, $s \in \cap_{i \in N} S_{ii}^M$, *or* $s \in \cap_{i \in N} S_{ii}^{I,M}$ *and* $s \in \cap_{i \in N} E_{ii}^{Nash}$.

A contingent hyper SEQ *(CHSEQ*^{2}*)* equilibrium, a contingent hyper GMR *(CHGMR*^{2}*)* equilibrium, and a contingent hyper SMR *(CHSMR*^{2}*)* equilibrium for a second-level hypergame can be defined in a similar way.

Definition 5.2.0.15 (**Unsteady CHNash^2 ($UCHNash^2$) Equilibrium**). *A hyper Nash equilibrium state for a second-level hypergame, $s \in HE^{2\ Nash}$, is called an UCHNash^2 equilibrium $\iff$ either $s \in \cap_{i\in N} S_{ii}^{I}$, $s \in \cap_{i\in N} S_{ii}^{M}$, or $s \in \cap_{i\in N} S_{ii}^{I,M}$ and $s \notin \cap_{i\in N} E_{ii}^{Nash}$.*

An unsteady contingent hyper SEQ ($UCHSEQ^2$) equilibrium, unsteady contingent hyper GMR *(UCHGMR^2)* equilibrium, and unsteady contingent hyper SMR *(UCHSMR^2)* equilibrium for a second-level hypergame can be defined in a similar manner.

Definition 5.2.0.16 (**Self-CHNash^2** ($SCHNash^2$) **Equilibrium**). *A hyper Nash equilibrium state for a second-level hypergame, $s \in HE^{2\ Nash}$, is called an $SCHNash^2$ equilibrium for the dispute iff there is a DM $i \in N$ such that $s \in (S_{ii}^{I} \cup S_{ii}^{M} \cup S_{ii}^{I,M}) \setminus \cap_{j\in N}(S_{jj}^{I} \cup S_{jj}^{M} \cup S_{jj}^{I,M})$.*

A self-CHSEQ^2 *(SCHSEQ^2)* equilibrium, self-CHGMR^2 *(SCHGMR^2)* equilibrium, and self-CHSMR^2 *(SCHSMR^2)* equilibrium for a second-level hypergame can be defined in a like-wise manner.

Definition 5.2.0.17 (**Emergent Hyper Nash** ($EHNash^2$) **Equilibrium**). *A hyper Nash equilibrium state for a second-level hypergame, $s \in HE^{2\ Nash}$, is called an $EHNash^2$ equilibrium for the conflict iff $s \notin \cup_{i\in N} S_{ii}$.*

An emergent hyper SEQ *(EHSEQ^2)* equilibrium, emergent hyper GMR *(EHGMR^2)* equilibrium, and emergent hyper SMR *(EHSMR^2)* equilibrium for a second-level hypergame are defined in a similar fashion.

5.3 Chapter Summary

A new methodology for SLHG with two or more DMs in graph form is put forward in this chapter. The aim of this novel approach is to investigate a conflict situation having misperceptions among the engaged DMs, with at least one DM being aware of the differences in understanding among the participating DMs. The foundations of this encompassing procedure is the development of the concept of a universal set of options, which is then extended for designing a universal set of states for SLHG. This universal set of states for SLHG is utilized to generate the states in a subjective FLHG for each DM. Consequently, an expert or analyst can recognize the collection of states that are viewed across the subjective FLHGs of all the DMs and those that are seen privately. This important feature allows the expert to classify the SLHG equilibria into eight classes, each of which contains information about the type of misperception that produced the conflict. For example, if a state were found to be an equilibrium for SLHG, and also an equilibrium in the actual subjective game of a given DM, but at least one of the adversaries of the DM is unaware of the existence of the state in reality within its actual subjective game, then this state is defined as a stealthy hyper equilibrium state for SLHG. This type of equilibrium constitutes an example of the use of strategic surprise by a DM in a conflict situation. Hence, the new approach of SLHG in graph form developed here is a truly general method for modeling and analyzing conflict situations having asymmetry of perception among the DMs.

Chapter 6

h-Level Hypergame ($h \geqslant 1$) in Graph Form

In this chapter, misperceptions are incorporated into GMCR to express misunderstanding for the most general situation. In particular, the idea of a first-level hypergame in graph form descried in Chapter 3 (Aljefri et al., 2017a) and the notion of a second-level hypergame in graph form provided in Chapter 5 are generalized to handle any h-level hypergames having n-DM, where $h \geqslant 1$ and $n \geqslant 2$. To do this, the collections of options and states for GMCR are expanded to include fictitious options and states for any level of hypergame. These two sets capture all DMs' perceptions about the conflict situation. Because of the asymmetry of perception among DMs, some of the states in the universal set of states are known to a particular DM while others may be hidden. Since the source of these states is the universal set of states, an external analyst can formally distinguish between the states that are considered by all the players and the states that are taken into account individually. Moreover, a practical procedure for implementing hypergame stability analysis for an n-DM hypergame at any level of perception is proposed to predict the resolutions of the hypergame and to obtain valuable strategic insights. To investigate how DMs may behave after they become aware of their misperceptions, a classification of hypergame equilibria is developed for any level of hypergame.

6.1 Methodology to Incorporate an h-Level Hypergame with More Than Two DMs into the Graph Model

Similar to a first- and a second-level hypergame in graph form (Aljefri et al., 2017a,b), the suggested graph model for an h-level hypergame description includes three components: one to produce the universal set of states for the h-level hypergame, another to construct each DM's $(h-1)$-level hypergame, and the last is to perform the hypergame analysis and to predict the possible equilibria for a hypergame.

According to the representation of the option form for which there is no misinterpretation (Fang et al., 2003a,b), displaying scenarios for the h-level hypergame can be produced. In particular, the set of all DMs' options in the standard GMCR is modified to the universal set of options for the entire h-level hypergame. This set is a collection of individual universal sets of options for the h-level hypergame, each of which describes a particular DM's options that are perceived (correct or fictitious) by either the focal DM itself or its opponents at all levels of perception beginning from $\bar{h} = 1$ to h.

In the second part, the modeling of each DM's subjective $(h-1)$-level hypergame is developed. Each DM's subjective hypergame consists of a group of games, each of which characterizes not only a given DM's viewpoint of the conflict situation but also the way that particular DM sees the other DMs' games. The universal set of states is used to lay out states in each of the individual games.

In the last part, the hypergame analysis is conducted by first analyzing each DM's subjective hypergame and then predicting the overall hypergame equilibria. Because of asymmetry of understanding among players, an equilibrium state for the h-level hypergame may be known by all DMs, unknown to some DMs, or unknown to all DMs.

A description of the universal set of options for the entire h-level hypergame for the case of n-DM is provided in Section 6.1.1. Section 6.1.2 details the construction of the universal set of the states for the h-level hypergame. Section 6.1.3 includes a description of a DM's subjective hypergame in graph form. Section 6.1.4 provides the mathematical definitions of a given DM's subjective hypergame. Finally, Section 6.1.5 discusses the technique used to analyze the h-level hypergame in graph form using a range of solution concepts.

6.1.1 Universal Set of Options in an n-Decision Maker h-Level Hypergame

To consider all perceived courses of action for a particular player in the h-level hypergame, the idea of a universal set of options for a DM in an n-DM h-level hypergame is put forward. The notions of the universal sets of options for the first- and second-level hypergames in graph form (Aljefri et al., 2017a,b) are extended to define options for the h-level hypergame in graph form. This set includes options for a particular DM that are considered by itself and its opponents at all levels of perception. Three categories of options can define a given DM's universal set of options in the h-level hypergame: a DM's set of courses of action that are (1) correctly perceived, (2) imagined, and (3) misunderstood. Similar to a DM's universal sets of options for a first- and second-level hypergame (Aljefri et al., 2017a,b), the identification and classification of a DM's set of options in the h-level hypergame are performed by an outside expert, also known as analyst, who knows the differences of perception among the players in a conflict setting.

Figure 6.1 portrays the logical structure of *DM i's* set of options that are perceived by itself and its opponents in the h-level hypergame. As can be seen, *DM i's* set of options, O_i is altered by the perceptions of three DMs, *"i"*, *"j"*, and *"k"*, at different levels of a hypergame ranging from level 0 to level h. O_i is *DM i's* set of options for a zero-level hypergame. This set has no misperception, and all DMs in the game correctly capture it.

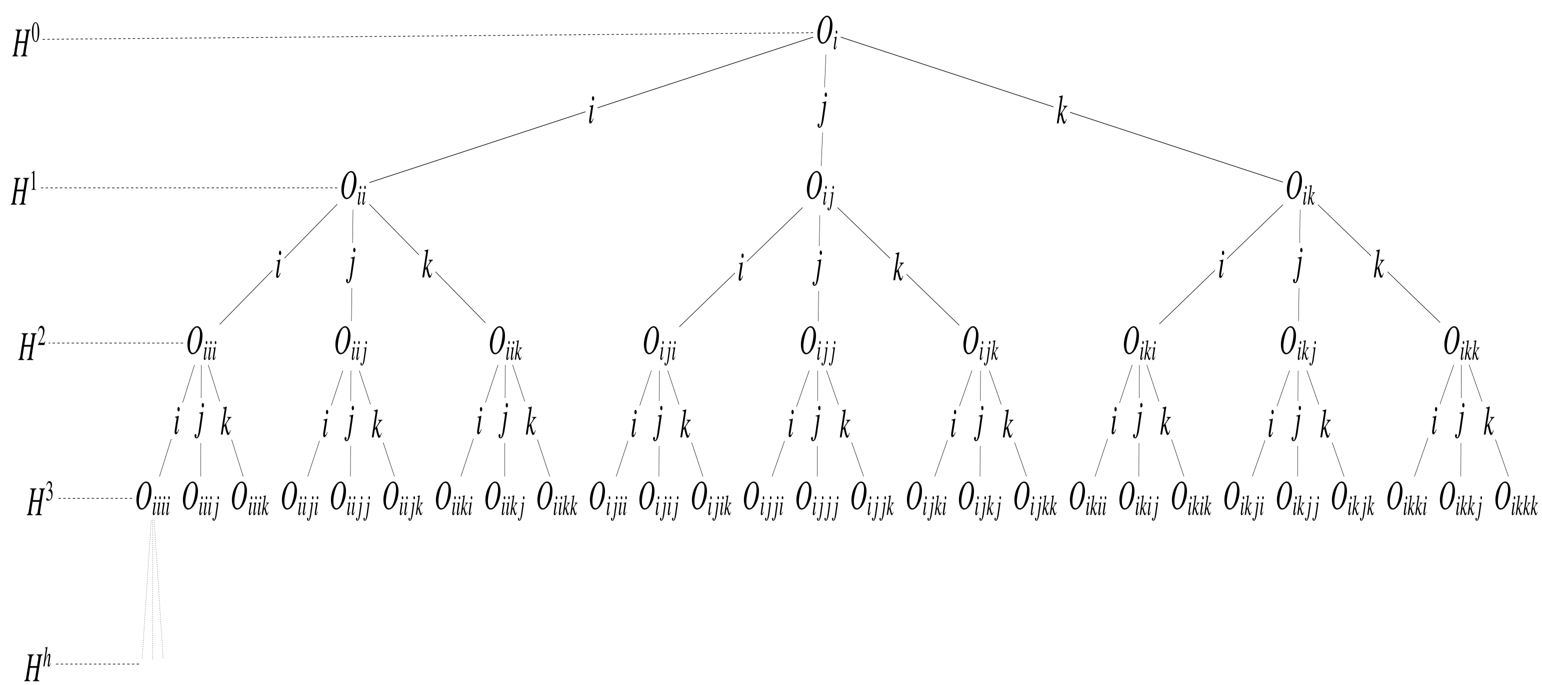


Figure 6.1: Logical Structure of *DM i's* Options that are Perceived by Itself and its Opponents at Different Levels of Hypergame

Thus, the first subscript, i, is used to indicate to whom the set of options belongs. DMs' interpretations of O_i can be different as a result of the asymmetry of perception among them. For instance, in a first-level hypergame, O_i is separately perceived by each DM, and misunderstandings may happen. Hence, O_{ii}, O_{ij}, and O_{ik} stand for *DM i's* sets of options as perceived by itself, as seen by *DM j*, and as perceived by *DM k*, respectively. The second subscript in O_{ik}, for example, stands for the particular DM who views the set O_i. In O_{ik}, the order of DMs' perception equals one. Recall that self-misperception is permitted within the current research. Therefore, $O_i \neq O_{ii}$ whenever *DM i* makes a mistake in perceiving its options in the dispute. For examples of the situations in which a DM misperceives its options, the reader is referred to the work by Aljefri et al. (2017a), and the references contained therein. In a second-level hypergame, at least one DM is assumed to understand different perceptions among the players. Therefore, a DM is trying to interpret how the other DMs view the conflict situation. For example, O_{iji} denotes the set of *DM i's* options that are perceived by *DM j* and then interpreted by *DM i*. O_{iji} has a subscript of length three. The first subscript stands for the ownership of the set, while the other two subscripts stand for the players who perceive O_i. Therefore, the length of the subscript includes h sequences of DMs' perception, and the total length of the subscript is equal to $h + 1$. Depending on the level of a hypergame, the length of the subscript is extended. It is worthwhile noting that the universal set of options for the h-level hypergame must contain all sets of options considered from $\bar{h} = 1$ to h. The logical structures of options in the hypergame of level h for DMs j and k as portrayed in Figure 6.1 can be constructed analogously. These options are utilized to mathematically define states for a hypergame, which can then be used to model each DM's subjective hypergame. To simplify the notation, the concept of a string can be used as a part of the subscript.

Let w be an ordered string of DMs in a hypergame. Let $w = i_1 i_2 i_3 ... i_h$, where i_1, i_2, i_3,..., $i_h \in N$. The length of w is equal to $h \geqslant 0$. Also, let Σ^1, Σ^2, Σ^3, and Σ^h stand for the sets of all order strings of DMs of lengths $1, 2, 3$, and h, respectively. If the total number of DMs in N is n, the total number of strings in Σ^h is n^h. Then, the set of all strings of

DMs from length 1 to length h is denoted by Σ^{h*} and is defined as follows.

$$\Sigma^{h*} = \Sigma^1 \cup \Sigma^2 \cup \Sigma^3 \cup ... \cup \Sigma^h \tag{6.1}$$

For example, the sets Σ^1, Σ^2, and Σ^3 for the case of 2-DM, $N = \{i, j\}$, are computed as follows:

- $\Sigma^1 = \{i, j\}$
- $\Sigma^2 = \{ii, ij, ji, jj\}$
- $\Sigma^3 = \{iii, iji, jii, jji, iij, ijj, jij, jjj\}$

With the help of string w, various types of *DM i's* option misperception can be generalized for the case of n-DM hypergame at any length of DMs' perception.

Remark 6.1.1.1. *Similar to Remark 3.1.0.1, self-misperception is permitted within the hypergame theory in graph form. That is, $O_i \neq O_{ii}$ whenever DM i misunderstands its possible courses of action in the dispute. Moreover, it is assumed that a DM is not aware of its misperception in a conflict situation. Hence, $O_{ii} = O_{iii...i}$. Lastly, a DM is assumed to hold its own perception about a given DM at any level of hypergame. That is, $O_{ji} = O_{jji}$*

A DM's Set of Correctly Perceived Options

This group of options includes a given DM's courses of action that are correctly considered by itself. These options are not fictitious and are sensible to be implemented by the DM who possesses them in a real-life situation. Here, one assumes that a DM is aware of theses options. However, because of the other DMs' misperception, some or all of these options may be unknown to them. Since hypergame theory allows DMs to be aware of each other's misperception, they are trying to predict which collection of options is comprehended by their opponents, but mistakes may occur.

Definition 6.1.1.1 (***DM i's* Set of Correctly Perceived Options**). *For $i \in N$, let the collection of options for DM i that are free from any misperception and known to itself be denoted by O_{ii}^R. For $j \in N - \{i\}$ recall that $O_{ij}^R \subseteq O_{ii}^R$ stands for the set of DM i's options that are assumed by DM j. Now, DM i's options that are assumed by all DMs in the dispute are expressed as $O_i^R = \cap_{j \in N} O_{ij}^R$ (Aljefri et al., 2017a). For w, a sequence of DMs, let O_{iw}^R stand for the set of DM i's correct options that are correctly considered by w.*

Note that $O_{iw}^R \subseteq O_{ii}^R$ by definition. Then, the set of *DM i's* correctly perceived options that are considered by all sequences of DMs for the h-level hypergame is expressed as $(\cup_{w \in \Sigma^{h*}} O_{iw}^R)$. Because $O_{iw}^R \subseteq O_{ii}^R$ and Σ^{h*} contains all of the strings of DMs including ii, $(\cup_{w \in \Sigma^{h*}} O_{iw}^R) = O_{ii}^R$.

A DM's Set of Imagined Options

A given DM's option that is not in O_{ii}^R but is assumed to exist by either the DM itself or its opponents in the dispute is considered as an imagined course of action. As detailed by Aljefri et al. (2017a), this option is not real, and implementing it in real life is not logical. The set of these options is defined below for any order of DMs' perception.

Definition 6.1.1.2 (***DM i's* Set of Imagined Options**). *Choose $i \in N$. Let O_{ii}^I and O_{ij}^I be DM i's sets of courses of action that are imagined by DM i itself or by any $j \in N - \{i\}$ (Aljefri et al., 2017a). For w, a sequence of DMs, let O_{iw}^I symbolize the set of DM i's options that are imagined by the sequence of DMs w. Then, the collection of DM i's imagined options for the h-level hypergame is expressed as $(\cup_{w \in \Sigma^{h*}} O_{iw}^I)$.*

A DM's Set of Misunderstood Options

Definition 6.1.1.3 (***DM i's* Set of Misunderstood Options**). *Choose $i \in N$, and denote by $O_i^{\bar{R}}$ the set of DM i's courses of action that hold in reality, but are misunderstood by DM i. Also, let O_{ii}^M stand for the set of DM i's options that are misunderstood by* *itself.* O^M_{ii} *is formed by the function* $\Psi_i : O^{\bar{R}}_i \longrightarrow O^M_{ii}$*, such that for each course of action in* $O^{\bar{R}}_i$ *there is a misunderstood option in* O^M_{ii}*. Also, the options in* $O^{\bar{R}}_i$ *and* O^R_{ii} *that are misinterpreted by* w*, a sequence of DMs, is symbolized as* O^M_{iw}*. Then, the set of DM i's misunderstood options for the h-level hypergame is represented by* $(\cup_{w\in\Sigma^{h*}} O^M_{iw})$*.*

After defining the sets of *DM i's* correct options, imagined options, and misunderstood options, one can formalize *DM i's* universal set of options for the h-level hypergame as given below:

Definition 6.1.1.4 (*DM i's* Universal Set of Options in an h-level Hypergame). *For* $i \in N$*, let* $\ddot{O}^h_i$ *stand for the universal set of options of DM i for an h-level hypergame* H^h*. Then,* $\ddot{O}^h_i = (\cup_{w\in\Sigma^{h*}} O^R_{iw}) \cup O^{\bar{R}}_i \cup (\cup_{w\in\Sigma^{h*}} O^I_{iw}) \cup (\cup_{w\in\Sigma^{h*}} O^M_{iw})$

Remark 6.1.1.2. *Recall that* O_i *symbolizes the set of DM i's options for the graph model with complete information. Because of DM i's misperception,* O_i *is partitioned into two sets,* O^R_{ii} *and* $O^{\bar{R}}_i$*. Keep in mind that* O^R_{ii} *includes i's real courses of action that are known to itself, but* $O^{\bar{R}}_i$ *has i's real options that are unknown to it because of its misunderstanding, as explained in detail by Aljefri et al. (2017a). Similar to a first-level hypergame in graph form, the assumed option relationships are as follows:*

- $(\cup_{w\in\Sigma^{h*}} O^R_{iw}) = O^R_{ii}$.
- $(\cup_{w\in\Sigma^{h*}} O^R_{iw}) \cap (\cup_{w\in\Sigma^{h*}} O^I_{iw}) = (\cup_{w\in\Sigma^{h*}} O^R_{iw}) \cap (\cup_{w\in\Sigma^{h*}} O^M_{iw}) = (\cup_{w\in\Sigma^{h*}} O^I_{iw}) \cap (\cup_{w\in\Sigma^{h*}} O^M_{iw}) = \emptyset$.
- $O^{\bar{R}}_i \cap (\cup_{w\in\Sigma^{h*}} O^I_{iw}) \neq \emptyset$ *may or may not exist.*
- $(\cap_{w\in\Sigma^{h*}} O^I_{iw}) \neq \emptyset$ *may or may not hold.*
- $(\cap_{w\in\Sigma^{h*}} O^M_{iw}) \neq \emptyset$ *may or may not hold.*

In a similar fashion, the other DMs' universal set of options for the h-level hypergame can be constructed. The collection of all DMs' universal set of options defines the universal set of options for the overall h-level hypergame as expressed below.

$$\hat{O}^h = \cup_{i \in N} \ddot{O}_i^h \tag{6.2}$$

$\hat{O}^h$ is known to the analyst, the external expert. Based on a DM's perception, options in $\hat{O}^h$ may be completely known or partially known to it. The analyst will utilize $\hat{O}^h$ to generate states for the overall hypergame. Based on this set of states, the expert can construct each DM's subjective hypergame and account for any type of misperception a DM may encounter in a real-life situation.

6.1.2 Universal Set of States in an *n*-DM *h*-Level Hypergame

After defining $\hat{O}^h$, a method similar to the one formalized to define states for the standard GMCR in Section 2.1 is implemented to generate the states for the overall hypergame. For $N = \{1, 2, .., i, ..., n\}$, recall that $\hat{O}^h = \ddot{O}_1^h \cup \ddot{O}_2^h \cup ... \cup \ddot{O}_i^h \cup ... \cup \ddot{O}_n^h$. For every $i \in N$, $\ddot{O}_i^h = \{o_1^i, o_2^i, ..., o_{\bar{k}}^i\}$, where $\bar{k} = 1, 2, ..., m_i$ and m_i is the total number of options in $\ddot{O}_i^h$. A course of action can either be implemented or not by the DM who owns it. A strategy for a given player is determined when the player chooses which of its courses of action to select or not. *DM i's* strategy is represented by the mapping $g_i : \ddot{O}_i^h \longrightarrow \{0, 1\}$, such that for $\bar{k} = 1, 2, ..., m_i$,

$$g_i(o_{\bar{k}}^i) = \begin{cases} 1, & \text{if } DM\ i \text{ implements } o_{\bar{k}}^i \\ 0, & \text{otherwise} \end{cases}$$

States are generated after all of the players in the dispute have chosen a strategy. Thus, a state is defined by the mapping $f : \hat{O}^h \longrightarrow \{0, 1\}$, such that $\forall\ o_{\bar{k}}^i \in \hat{O}^h$,

$$f(o_{\bar{k}}^i) = \begin{cases} 1, & \text{if } DM\ i \text{ implements } o_{\bar{k}}^i,\ for\ i =\ 1,\ 2, ...,\ n \\ 0, & \text{otherwise} \end{cases}$$

Each state is represented by a column vector in which the number of entries is equal to the total number of options in $\hat{O}^h$. Each state is a vector in the fashion of $(f(o_1^1), f(o_2^1),$ $..., f(o_{m_1}^1), ...,\ f(o_1^n), f(o_2^n), ..., f(o_{m_n}^n))^T$ (Aljefri et al., 2017a). Let $\hat{S}^h$ be the universal set of the states for the overall h-level hypergame. If the total number of options in $\hat{O}^h$ is λ, the total number of states in $\hat{S}^h$ is equal to 2^λ. Note that for $s \in \hat{S}^h$, $s = ((g_1^s)^T, (g_2^s)^T, ..., (g_i^s)^T, ..., (g_n^s)^T)^T$, where g_i^s is *DM i's* strategy related to s. The analyst knows all the states in $\hat{S}^h$. The set of the states in each of the DMs' subjective hypergames is a subset of $\hat{S}^h$.

6.1.3 Modeling of a Given DM's Subjective Hypergame

As stated in Section 3.1.1, $H^h = \langle H_i^{h-1} : \forall i \in N,\ h = 1, 2, 3, ...\rangle$. Each DM's subjective hypergame is constructed in a hierarchical fashion to illustrate not only a DM's viewpoint of the situation under investigation but also how it views the opponents' perceptions of the dispute. Each DM's subjective hypergame is a system of subjective games, each of which is constructed by the sets of perceived DMs, states, state transitions, and preferences. Figure 6.2 shows the hierarchical structure of H_i^{h-1} from level zero to level $h-1$.

As can be seen at the top of Figure 6.2, H_i^0 includes only *DM i's* subjective game $G_i = \langle N_i, S_i, \{A_{ji} : j \in N_i\}, \{\succsim_{ji}: j \in N_i\}\rangle$ that represents his viewpoint of the conflict situation. In H_i^1, *DM i* is aware of the other players' subjective games. That is, the DM tries to perceive what the others' games look like before making a decision. Hence, H_i^1 includes G_i and G_{ji}, $\forall j \in N_i - \{i\}$. Note that in the hierarchical structure of H_i^{h-1} , G_i is always at the top of the branch and $G_{q...kji}$ is at the bottom of the branch. $G_{q...kji}$ stands for the way *DM i's* understands *DM j's* perception about how *DM k's* sees ... *DM q's* game. To simplify the notation, a string of DMs w can be used as a part of the subscript. w is defined as follows:

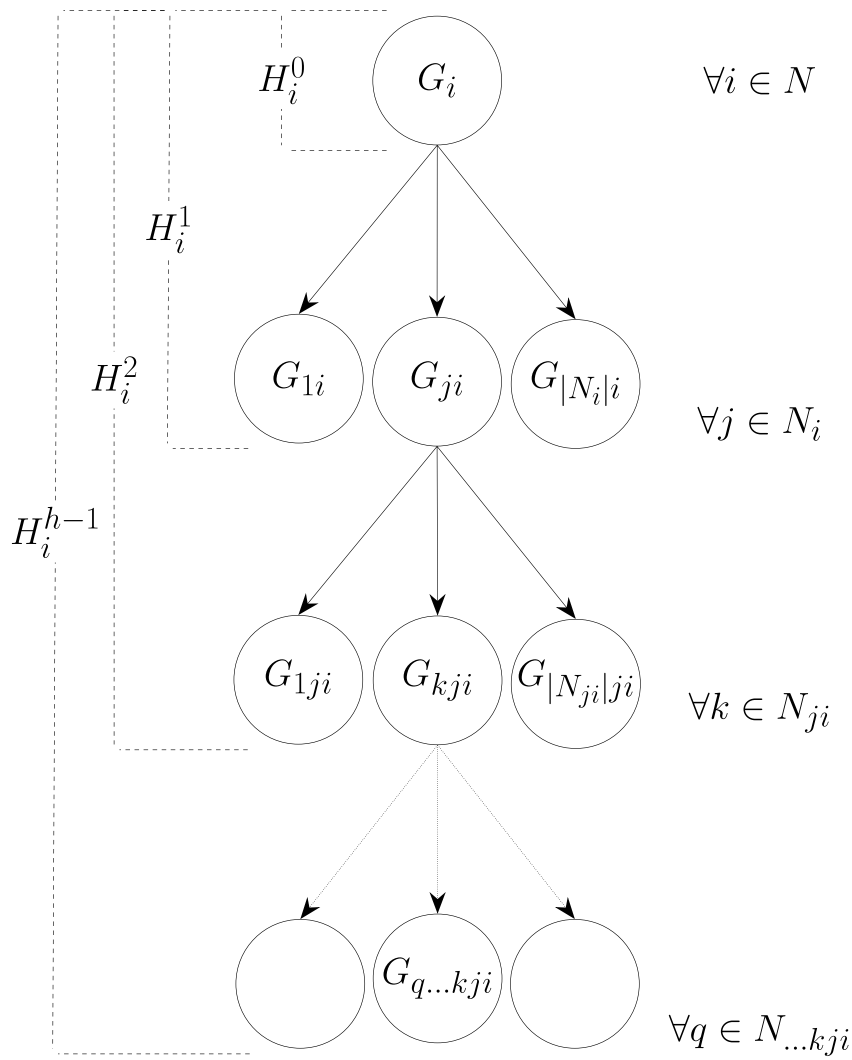


Figure 6.2: Hierarchical Strucuture of H_i^{h-1} in Graph Form

Definition 6.1.3.1 (**String of Decision Makers in a Hypergame**). *Recall that* $w = i_1i_2i_3...i_h$ *is an ordered string of decision makers in a hypergame, where* i_1, i_2, i_3,..., i_h $\in N$, *and* $i_j \in N_{i_{j+1}i_{j+2}...i_h}$, *where* $i_j \neq i_{j+1}$. *The length of* w *is equal to* $h \geqslant 1$.

Remark 6.1.3.1. *For example, if* $w = i$, *then the length of* w *is equal to 1 and* $i \in N$. *If* $w = kji$, *then the length of* $w = 3$, *and* $k \in N_{ji}$ *and* $j \in N_i$. *By using* w, *a string of DMs,* *a game at any level of perception can be constructed. For example, in G_{ji}, one can replace the subscript ji with the string w as G_w. If any DM k is perceived by w, then DM k's game as seen by w is denoted as G_{kw}, $\forall k \in N_w$. Keep in mind that G_w is always one level above G_{kw}.*

6.1.4 Mathematical Modeling of a DM's Subjective Game within H_i^{h-1}

In Section 6.1.3 the hierarchical design of H_i^{h-1} is provided as well as the concept of the DMs' ordered string. In this subsection, one can mathematically construct any of the subjective games in H_i^{h-1}. Similar to a graph model, each subjective game within H_i^{h-1} is constructed by the perceived sets of DMs, states, state transitions, and preferences. For any given string w, G_{kw} can be defined by 4-tuple as furnished below:

$$\langle N_{kw}, S_{kw}, \{A_{qkw} : q \in N_{kw}\}, \{\succsim_{qkw}: q \in N_{kw}\}\rangle \tag{6.3}$$

where, N_{kw} is the set of DMs as perceived by *DM k* and then by w, the string of DMs; $S_{kw} \subseteq \hat{S}^h$ is the set of states perceived by *DM k* as contemplated by w; and A_{qkw} and $\succsim_{qkw}$ are the state transitions and the preference relations, respectively, of *DM q* as seen by k and then by the string of DMs in w.

To formally define S_{kw}, the universal set of options for the h-level hypergame $\hat{O}^h$ must be partitioned based on the perception of k as seen by w into two categories: (1) the group of options recognized by *DM k* as assumed by the string of DMs in w and (2) the collection of options unknown to *DM k* as perceived by w.

Recall that O^R_{qkw}, O^I_{qkw}, and O^M_{qkw} denote the sets of *DM q's* options that are correctly perceived, imagined, and misunderstood, respectively, by *DM k* as contemplated by the string of DMs in w. Then, the group of options that are known to *DM k* as perceived by w can be expressed as $(\cup_{q\in N_{kw}} O^R_{qkw}) \cup (\cup_{q\in N_{qkw}} O^I_{qkw}) \cup (\cup_{q\in N_{kw}} O^M_{qkw})$. Also, the collection of options that are unknown to *DM k* as assumed by the string of DMs w can be formally defined as $[\hat{O}^h \backslash ((\cup_{q\in N_{kw}} O^R_{qkw}) \cup (\cup_{q\in N_{qkw}} O^I_{qkw}) \cup (\cup_{q\in N_{kw}} O^M_{qkw}))]$. With the groups of options mentioned above, the set of states S_{kw} in G_{kw} can be defined formally as described below:

Definition 6.1.4.1 (Set of States in G_{kw}). *Choose $k \in N_w$. Let $S_{kw} \subseteq \hat{S}^h$ stand for the set of states in G_{kw} as assumed by the string of DMs in w, where $\hat{S}^h$ is the universal set of states for the h-level hypergame. Then, a state $s \in S_{kw} \iff$ there is a mapping $f : \hat{O}^h \rightarrow \{0,1\}$ satisfying $f(o) = 0$,$\forall\ o \in [\hat{O}^h \setminus ((\cup_{q\in N_{kw}} O^R_{qkw}) \cup (\cup_{q\in N_{qkw}} O^I_{qkw}) \cup (\cup_{q\in N_{kw}} O^M_{qkw}))]$.* (Note that $f(\acute{o})$ = either 1 or 0, $\forall\ \acute{o} \in (\cup_{q\in N_{kw}} O^R_{qkw}) \cup (\cup_{q\in N_{qkw}} O^I_{qkw}) \cup (\cup_{q\in N_{kw}} O^M_{qkw})$).

6.1.5 Analysis of the *h*-Level Hypergame

Similar to a second-level hypergame in graph form (Aljefri et al., 2017b), the analysis of the *h*-level hypergame is performed in two phases. In the first phase, one analyzes each DM's subjective hypergame to predict the equilibria perceived by each DM. In the second phase, one combines all DMs' subjective equilibria to ascertain the overall equilibria for the *h*-level hypergame.

The Analysis of a DM's Subjective Hypergame

As discussed in Section 6.1.3, a DM's subjective hypergame is organized in a hierarchical order to depict a particular DM's different levels of perception. The hierarchical design of *DM i's* subjective hypergame, H_i^{h-1} is shown in Figure 6.2. The analysis of H_i^{h-1} starts by analyzing the games, $G_{q\ldots kji}, \forall q \in N_{\ldots kji}$, at the bottom of the branch of Figure 6.2 as simple games using standard GMCR solution concepts to identify the equilibria in $G_{q\ldots kji}$. This procedure is called Process 1. After that, one uses the equilibria in $G_{q\ldots kji}$ to identify the equilibria in the games which are one level above $G_{q\ldots kji}$. This method is called Process 2.

Note that Process 1 is performed once; whereas Process 2 is repeated as many times as required to reach *DM i's* subjective game G_i. Hence, Process 2 is applied $(h-1)$ times

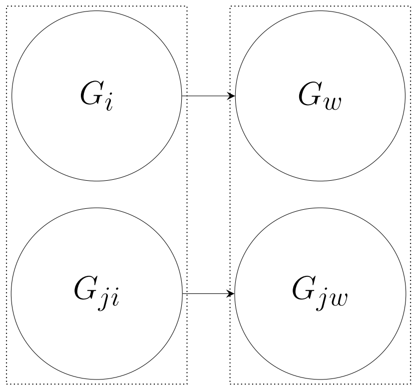


Figure 6.3: The Use of w Between G_i and G_{ji}

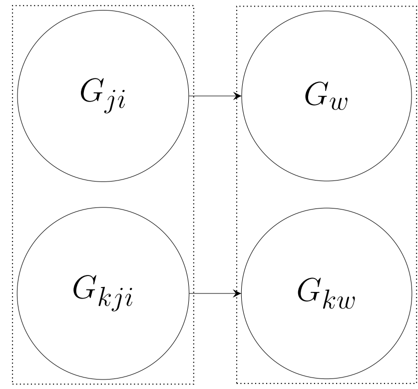


Figure 6.4: The Use of w Between G_{ji} and G_{kji}

within H_i^{h-1}. The stability analysis of either Process 1 or 2 achieved between G_i and G_{ji} is exactly identical to the one performed between G_{ji} and G_{kji}. The string of DMs w is used to illustrate the stability analyses of Process 1 and Process 2 that are performed among G_i and G_{ji} as G_w and G_{jw}, respectively, as illustrated in Figure 6.3. Also, it is used btween G_{ji} and G_{kji} as G_w and G_{kw}, respectively, as displayed in Figure 6.4. G_w is always one level above G_{jw}. Until otherwise specified, it is assumed that there is no level of DMs' perception beyond G_{jw}. In other words, there is no *DM k* perceived by *DM j* and then by the string of DMs in w.

Process 1:

In this process, $G_{jw}, \forall j \in N_w$ is analyzed as a simple game using the following solution concepts: Nash, SEQ, GMR, and SMR to identify the equilibria in G_{jw}. Since these solution concepts investigate each DM's possible moves and counter moves in a conflict situation according to specified rules, one must define first the concepts of uniliteral moves (UM) and unilateral improvement moves (UI) based on *DM j's* perception as contemplated by a string of DMs in w.

Recall that the set of DMs in G_{jw} is N_{jw}. Let $H \subseteq N_{jw}$ be any any group of players.

For $s_1 \in S_{jw}$, let $R_{qjw}(s_1)$ and $R^+_{qjw}(s_1)$ be the sets of *DM q's* UMs and UIs, respectively, from the initial state s_1 as seen by *DM j* and then contemplated by w, the string of DMs. Also, let $R_{Hjw}(s_1)$ denote the possible moves available from s_1 by any DMs in H via a legal sequence of moves. Within a legal sequence of moves, a DM is allowed to move more than once but not make two successive moves. For $s_2 \in R_{Hjw}(s_1)$, let $\Omega_{Hjw}(s_1,\ s_2)$ stand for the set of all last players in H in the legal sequences of moves from s_1 to s_2. Additionally, let $R^+_{Hjw}(s_1)$ stand for the possible UIs available from s_1 by any DMs in H via a legal sequence of UIs. For $s_2 \in R^+_{Hjw}(s_1)$, let $\Omega^+_{Hjw}(s_1,\ s_2)$ denote for the set of all last players in H in the legal sequences of UIs from s_1 to s_2. The sets $R_{Hjw}(s_1)$ and $R^+_{Hjw}(s_1)$ are defined as follows.

The list of UMs by $H \subseteq N_{jw}$ is defined as follows:

Definition 6.1.5.1 (Unilateral Moves by $H \subseteq N_{jw}$). *For $s_1 \in S_{jw}$, $R_{Hjw}(s_1)$ is inductively defined as shown below:*

- *If $q \in H$ and $s_2 \in R_{qjw}(s_1)$, then $s_2 \in R_{Hjw}(s_1)$ and $q \in \Omega_{Hjw}(s_1,\ s_2)$;*
- *If $s_2 \in R_{Hjw}(s_1)$, $q \in H$, and $s_3 \in R_{qjw}(s_2)$, then*
  1. *if $|\Omega_{Hjw}(s_1,\ s_2)| = 1$ and $q \notin \Omega_{Hjw}(s_1,\ s_2)$, then $s_3 \in R_{Hjw}(s_1)$ and $q \in \Omega_{Hjw}(s_1,\ s_3)$.*
  2. *if $|\Omega_{Hjw}(s_1,\ s_2)| > 1$, then $s_3 \in R_{Hjw}(s_1)$ and $q \in \Omega_{Hjw}(s_1,\ s_3)$.*

The induction ends when there exist no s_3 that can be included in $R_{Hjw}(s_1)$ and no change occurs from $|\ \Omega_{Hjw}(s_1,\ s_2)\ | = 1$ to $|\ \Omega_{Hjw}(s_1,\ s_2)\ | > 1$ for any $s_2 \in R_{Hjw}(s_1)$. All the states in $R_{Hjw}(s_1)$ are UMs achieved by any number of DMs in H.

Definition 6.1.5.2 (Unilateral Improvement List by $H \subseteq N_{jw}$). *For $s_1 \in S_{jw}$, $R^+_{Hjw}(s_1)$ is inductively constructed as follows:*

- *If $q \in H$ and $s_2 \in R^+_{qjw}(s_1)$, then $s_2 \in R^+_{Hjw}(s_1)$ and $q \in \Omega^+_{Hjw}(s_1,\ s_2)$;*

- *If* $s_2 \in R^{+}_{Hjw}(s_1)$, $q \in H$, *and* $s_3 \in R^{+}_{qjw}(s_2)$, *then*

  1. *if* $\mid \Omega^{+}_{Hjw}(s_1,\ s_2) \mid = 1$ *and* $q \notin \Omega^{+}_{Hjw}(s_1,\ s_2)$, *then* $s_3 \in R^{+}_{Hjw}(s_1)$ *and* $q \in \Omega^{+}_{Hjw}(s_1,\ s_3)$,
  2. *if* $\mid \Omega^{+}_{Hjw}(s_1,\ s_2) \mid > 1$, *then* $s_3 \in R^{+}_{Hjw}(s_1)$ *and* $q \in \Omega^{+}_{Hjw}(s_1,\ s_3)$.

  The induction finishes when no s_3 *can be included in* $R^{+}_{Hjw}(s_1)$ *and there are no changes from* $\mid \Omega^{+}_{Hjw}(s_1,\ s_2) \mid = 1$ *to* $\mid \Omega^{+}_{Hji}(s_1,\ s_2) \mid > 1$ *for any* $s_2 \in R^{+}_{Hjw}(s_1)$*. All the states in* $R^{+}_{Hjw}(s_1)$ *are UIs initiating from* s_1 *by any number of players in* H*.*

Definition 6.1.5.3 (**Nash Stability**). *A state* $s_1 \in S_{jw}$ *is Nash stable (Nash) for DM* $q \in N_{jw}$ *in* $G_{jw} \iff R^{+}_{qjw}(s_1) = \emptyset$*. The group of Nash stable states for DM* q *in* G_{jw} *is expressed as* $S_{jw}^{Nash_{qjw}}$*.*

Definition 6.1.5.4 (**SEQ Stability**). $s_1 \in S_{jw}$ *is sequentially stable (SEQ) for DM* $q \in N_{jw}$ *in* $G_{jw} \iff$ *for each* $s_2 \in R^{+}_{qjw}(s_1)$, $\exists\ s_3 \in R^{+}_{(N_{jw}-\{q\})jw}(s_2)$ *such that* $s_3 \precsim_{qjw} s_1$*. The collection of all SEQ stable states for DM* q *in* G_{jw} *is represented as* $S_{jw}^{SEQ_{qjw}}$*.*

Definition 6.1.5.5 (**GMR Stability**). $s_1 \in S_{jw}$ *is general metarational stable (GMR) for DM* $q \in N_{jw}$ *in* $G_{jw} \iff$ *for each* $s_2 \in R^{+}_{qjw}(s_1)$, $\exists\ s_3 \in R_{(N_{jw}-\{q\})jw}(s_2)$ *such that* $s_3 \precsim_{qjw} s_1$*. The group of GMR stable states for DM* q *in* G_{jw} *is denoted by* $S_{jw}^{GMR_{qjw}}$*.*

Definition 6.1.5.6 (**SMR Stability**). $s_1 \in S_{jw}$ *is SMR stable for DM* $q \in N_{jw}$ *in* $G_{jw} \iff$ *for each* $s_2 \in R^{+}_{qjw}(s_1)$, $\exists\ s_3 \in R_{(N_{jw}-\{q\})jw}(s_2)$ *such that* $s_3 \precsim_{qjw} s_1$, *and* $s_4 \precsim_{qjw} s_1$, $\forall\ s_4 \in R_{qjw}(s_3)$*. The group of all SMR stable scenarios for DM* q *in* G_{jw} *is symbolized by* $S_{jw}^{SMR_{qjw}}$*.*

The relations among the solution concepts mentioned above were studied within the frameworks of GMCR (zero-level hyprgame) (Fang et al., 1993), the first-level hypergame in graph form (Aljefri et al., 2017a), and the second-level hypergame in graph form (Aljefri et al., 2017b). In this current research, the same relations of the solution concepts are examined within the paradigm of the h-level hypergame in graph form. The study shows that the same links among the stability definitions found in GMCR, the first-level hypergame in graph form, and the second-level hypergame in graph form can be established for the h-level hypergame in graph form.

Theorem 6.1.5.1. *Select $q \in N_{jw}$ and $s_1 \in S_{jw}$ in G_{jw}. If $s_1 \in S_{jw}^{Nash_{qjw}}$ for DM q in G_{jw}, then $s_1 \in S_{jw}^{SMR_{qjw}}$ for DM q in G_{jw}; if $s_1 \in S_{jw}^{SMR_{qjw}}$ for DM q in G_{jw}, then $s_1 \in S_{jw}^{GMR_{qjw}}$ for DM q in G_{jw}.*

Proof. Based on Definition 6.1.5.3, s_1 is Nash stable for $DM\ q$ in G_{jw} if $R_{qjw}^{+}(s_1) = \emptyset$. This intuitively fulfills the conditions in Definition 6.1.5.6. Thus, $S_{jw}^{Nash_{qjw}} \subseteq S_{jw}^{SMR_{qjw}}$. Next, according to Definition 6.1.5.6, $s_1 \in S_{jw}^{SMR_{qjw}} \iff$ for each $s_2 \in R_{qjw}^{+}(s_1)\ \exists\ s_3 \in R_{(N_{jw}-\{q\})jw}(s_2)$ such that $s_3 \precsim_{qjw} s_1$, and $s_4 \precsim_{qjw} s_1\ \forall\ s_4 \in R_{qjw}(s_3)$. The constraint for each $s_2 \in R_{qjw}^{+}(s_1)\ \exists\ s_3 \in R_{(N_{jw}-\{q\})jw}(s_2)$ such that $s_3 \precsim_{qjw} s_1$ in Definition 6.1.5.5, suggests that $s_1 \in S_{jw}^{GMR_{qjw}}$. That is, $S_{jw}^{SMR_{qjw}} \subseteq S_{jw}^{GMR_{qjw}}$. Therefore, $S_{jw}^{SMR_{qjw}} \subseteq S_{jw}^{GMR_{qjw}}$. □

Theorem 6.1.5.2. *For $q \in N_{jw}$ and $s_1 \in S_{jw}$ in G_{jw}, if $s_1 \in S_{jw}^{Nash_{qjw}}$ is Nash stable for DM q in G_{jw}, it satisfies that $s_1 \in S_{jw}^{SEQ_{qjw}}$. Also, if $s_1 \in S_{jw}^{SEQ_{qjw}}$ for DM q in G_{jw}, it implies that $s_1 \in S_{jw}^{GMR_{qjw}}$ for DM q in G_{jw}.*

Proof. If $s_1 \in S_{jw}^{Nash_{qjw}}$, $R_{qjw}^{+}(s_1) = \emptyset$, then by Definition 6.1.5.4 $s_1 \in S_{jw}^{SEQ_{qjw}}$. Hence, $S_{jw}^{Nash_{qjw}} \subseteq S_{jw}^{SEQ_{qjw}}$. Also, as can be seen from Definition 6.1.5.4, $R_{(N_{jw}-\{q\})jw}^{+}(s_2) \subseteq R_{(N_{jw}-\{q\})jw}(s_2)$. That is, $S_{jw}^{SEQ_{qjw}} \subseteq S_{jw}^{GMR_{qjw}}$. □

Definition 6.1.5.7 (Nash Equilibria in G_{jw}). *A state $s_1 \in S_{jw}$ that is Nash stable for all the players in G_{jw} is Nash equilibrium in G_{jw}. The set of all Nash equilibrium states is denoted by E_{jw}^{Nash}.*

Remark 6.1.5.1. *Note that if a state $s_1 \in S_{jw}$ is SEQ, GMR, or SMR stable for all the players in G_{jw}, then it is SEQ, GMR, or SMR equilbrium, respectively, in G_{jw}. The groups* *of all SEQ, GMR, and SMR equilibrium states in G_{jw} are represented as E_{jw}^{SEQ}, E_{jw}^{GMR}, E_{jw}^{SMR}, respectively.*

Process 2:

The subjective eqilibria in G_{jw} are utilized to calculate the equilibria in G_w. Recall that G_w is always one level above G_{jw}. Hence, Process 2 is applied to ascertain the perceptual resolutions in G_w. Note that Process 2 is an exclusive aspect of the hypergame, which is sensitive to the level of DMs' misperception. Within Process 2, four steps are taken to ascertain the Nash equilibria in G_w:

1. Identify *DM j's* Nash strategies from E_{jw}^{Nash}, $\forall j \in N_w - \{\bar{w}\}$, where $\bar{w}$ is the first element of the string of DMs w.

2. Calculate the Cartesian product of the sets of DMs' Nash strategies for all DMs $j \in N_w - \{\bar{w}\}$.

3. In G_w, identify the group of states in S_w that are related to $\bar{w}'s$ opponents' Nash strategies obtained from Step 2.

4. Calculate the Nash stability of the states in S_w for $\bar{w}$ within G_w, that are related to $\bar{w}'s$ opponents' Nash strategies obtained from Step 2. If a state is found to be Nash stable for $\bar{w}$, it constitutes a Nash equilibrium in G_w

Remark 6.1.5.2. *If $w = i_1 i_2 i_3$ is an ordered string of DMs, then the first element in w is denoted by $\bar{w} = i_1$. Also, if $w = i_1$, then $\bar{w} = i_1$.*

Definition 6.1.5.8 (Nash Equilibria in G_w). *For $j \in N_w - \{\bar{w}\}$, recall that E_{jw}^{Nash} symbolizes the set of Nash equilibria in G_{jw}. Assume $E_{jw}^{Nash} = \{e_{1jw}^{Nash_{jw}}, e_{2jw}^{Nash_{jw}}, ..., e_{\varepsilon_{jw}}^{Nash_{jw}}\}$, where ε_{jw} is the number of elements in E_{jw}^{Nash}. Let $g_{jw}^{*Nash_{jw}} = \{g_{jw}^{e_1^{Nash_{jw}}}, g_{jw}^{e_2^{Nash_{jw}}}, ..., g_{jw}^{e_{\varepsilon_{jw}}^{Nash_{jw}}}\}$ stand for the set of Nash strategies of DM j in G_{jw}, where $g_{jw}^{e_{1jw}^{Nash_{jw}}}$ is the strategy of DM j as viewed by w that is attained from $e_{1jw}^{Nash_{jw}}$ in G_{jw}. Then, all possible combinations of DMs'*

Nash strategies except for $\bar{w}$, each of which is determined from the set of Nash equilbria in that particular DM's game, can be expressed as $(\prod_{j\in N_w-\{\bar{w}\}} g_{jw}^{*Nash_{jw}})$. *Now, for* $s_1 \in S_w$ *within* G_w, *let* $s_1 = ((g_w^{s_1})^T, (g_{N_w-\{\bar{w}\}}^{s_1})^T)^T$, *where* $g_w^{s_1}$ *is w's strategy related to state* s_1 *and* $g_{N_w-\{\bar{w}\}}^{s_1}$ *is* $\bar{w}'s$ *opponents' strategies related to* s_1. *Then,* s_1 *is a Nash equilibrium for w in* $G_w \Leftrightarrow s_1 \in S_w^{Nash_w}$ *and* $g_{N_w-\{\bar{w}\}}^{s_1} \in (\prod_{j\in N_w-\{\bar{w}\}} g_{jw}^{*Nash_{jw}})$. *The set of Nash equilibrium states in* G_w *is symbolized as* E_w^{Nash}.

Remark 6.1.5.3. *In a similar fashion one can calculate the sets of SEQ, GMCR, and SMR equilibria in* G_w, *denoted as* E_w^{SEQ}, E_w^{GMR}, *and* E_w^{SMR}, *respectively. Note that Process 2 is iterated as many times as required until reaching DM i's actual subjective game* G_i, $\forall i \in N$.

The Overall Hypergame Analysis

Similar to the work detailed by Aljefri et al. (2017a) and Aljefri et al. (2017b), an overall hypergame analysis for the h-level hypergame can be performed. The Nash equilibria, HE^{hNash}, for the h-level hypergame can be calculated by identifying *DM i's* Nash strategies from E_i^{Nash} within G_i, $\forall i \in N$. Then, by taking the Cartesian product of all DMs' Nash strategies, HE^{hNash} is determined.

Definition 6.1.5.9 (Hyper Nash Equilibrium for the h-level Hypergame). *For* $i \in N$, *note that* E_i^{Nash} *denotes the set of Nash equilibria in* G_i. $E_i^{Nash} = \{e_{1i}^{Nash_i}, e_{2i}^{Nash_i}, ..., e_{\varepsilon_i}^{Nash_i}\}$, *where* ε_i *is all Nash equilibria in* G_i. *Let* $g_i^{*Nash_i} = \{g_i^{e_{1i}^{Nash_i}}, g_i^{e_{2i}^{Nash_i}}, ..., g_i^{e_{\eta_{ii}}^{Nash_i}}\}$ *stand for the set of distinct Nash strategies of DM i in* G_i, *where* $g_i^{e_{1i}^{Nash_i}}$ *is the strategy of DM i collected from* $e_{1i}^{Nash_i}$ *in* G_i. *The group of hyper Nash equilibria for the h-level hypergame is calculated in the manner of:*

$$HE^{hNash} = \prod_{i\in N} g_i^{*Nash_i} \tag{6.4}$$

HE^{hSEQ}, HE^{hGMR}, and HE^{hSMR} are formalized in a similar way.

6.2 Classification of the h-Level Hypergame Equilibria

Identical to a first- and a second-level hypergame in graph form (Aljefri et al., 2017a,b), the classification of h-level hypergame equilibria is the final step in the investigation of a dispute with misperception. It is performed by an external analyst who is conscious of the differences in understanding among the players. The analyst classifies the h-level hypergame equilibria based on the following principles. First, the expert investigates if the h-level hypergame equilibrium state is viewed as a possible state in a DM's actual subjective game. If yes, then, the expert checks if the state is also an equilibrium state in a DM's actual subjective game. Also, the analyst examines the source of misperception, option misperceptions and/or preference misperception, that provoke the hypergame situation. These investigations allow the expert to obtain more insightful conclusions than those observed earlier in the published hypergame analysis (Bennett, 1977; Bennett and Dando, 1979; Bennett, 1980; Bennett et al., 1981; Takahashi et al., 1984; Wang et al., 1988, 1989).

To perform the classification of h-level hypergame equilibria, the set of states in a DM's actual subjective game must be classified as detailed in the work of Aljefri et al. (2017a) and Aljefri et al. (2017b). Because a DM is not aware of its misunderstanding, the set of states in a DM's actual subjective game is partitioned by the analyst. For $i \in N$, recall that $G_i = \langle N_i, S_i, \{A_{ji} : j \in N_i\}, \{\succsim_{ji} : j \in N_i\}\rangle$ represents *DM i's* actual subjective game and S_i is the set of states perceived by *DM i*. According to different option misperceptions, S_i is grouped into five disjoint classes: the collection of states that are (1) free from any misperception and known across all the players, S^R, (2) correctly perceived by *DM i* and maybe by some of its opponents but not by all, S_i^P, (3) imagined by *DM i* and perhaps by its opponents, S_i^I, (4) misunderstood by *DM i* and possibly by its opponents, S_i^M, and (5) imagined and misunderstood by *DM i* and probably by its opponents, $S_i^{I,M}$. For a detailed discussion of the five sets of states, the reader is referred to the work described by Aljefri et al. (2017a) and Aljefri et al. (2017b). In Table 6.1, the five groups of states are defined

Table 6.1: Partitioning of S_i in G_i within H^h

Group Type	Definition
S^R	A state $s \in S_i \subseteq \hat{S}^h$ is correctly considered by all players, i.e., $s \in S^R \subseteq S_i \iff$ there is $f : \hat{O}^h \to \{0,1\}$ satisfying $f(o) = 0,\ \forall\ o \in \hat{O}^h \setminus (\cap_{j\in N}\ O_{ij}^R)$.
S_i^P	A state $s \in S_i \subseteq \hat{S}^h$ is correctly recognized by *DM* i and perhaps by some of its competitors but not by all of them. That is, $s \in S_i^P \iff$ there is $f : \hat{O}^h \to \{0,1\}$ fulfilling $\exists\ o \in \cup_{j\in N}(O_{ji}^R \setminus O_j^R)$, $f(o) = 1$ and $f(o^{'}) = 0, \forall\ o^{'} \in \hat{O}^h \setminus (\cup_{j\in N} O_{ji}^R)$.
S_i^I	A state $s \in S_i$ is imagined by *DM* i in G_i, such that, $s \in S_i^I \iff$ there is $f : \hat{O}^h \to \{0,1\}$ satisfying $\exists\ o \in \cup_{j\in N} O_{ji}^I$, $f(o) = 1$ and $f(o^{'}) = 0, \forall\ o^{'} \in \hat{O}^h \setminus [(\cup_{j\in N} O_{ji}^I) \cup (\cup_{j\in N} O_{ji}^R)]$.
S_i^M	A state $s \in S_i \subseteq \hat{S}^h$ is misunderstood by *DM* i in G_i, that is $s \in S_i^M \iff$ there is $f : \hat{O}^h \to \{0,1\}$ satisfying $\exists\ o \in \cup_{j\in N} O_{ji}^M$, $f(o) = 1$ and $f(o^{'}) = 0, \forall\ o^{'} \in \hat{O}^h \setminus [(\cup_{j\in N} O_{ji}^M) \cup (\cup_{j\in N} O_{ji}^R)]$.
$S_i^{I,M}$	A state $s \in S_i \subseteq \hat{S}^h$ is referred to as an imagined and misunderstood state in G_i, such that, $s \in S_i^{I,M} \iff$ there is $f : \hat{O}^h \to \{0,1\}$ satisfying $\exists\ o \in \cup_{j\in N} O_{ji}^I$, $f(o) = 1$, $\exists\ o^{'} \in \cup_{j\in N} O_{ji}^M$, $f(o^{'}) = 1$, and $f(o^{''}) = 0, \forall\ o^{''} \in \hat{O}^h \setminus [(\cup_{j\in N} O_{ji}^I) \cup (\cup_{j\in N} O_{ji}^M) \cup (\cup_{j\in N} O_{ji}^R)]$.

within the framework of the h-level hypergame.

Identical to the first- and second-level hypergames in graph form, eight classes of the hyper Nash equilibria are put forward in this section within an h-level hypergame in graph form. For examples and a comprehensive discussion of the eight classes, see the research explained by Aljefri et al. (2017a) and Aljefri et al. (2017b). Table 6.2 shows the definitions of the eight classes of the hyper Nash equilibrium for the h-level hypergame as well as the types of misperception associated with each category.

As can be seen in Table 6.2, a hyper Nash equilibrium state that is known to all DMs as a correct scenario and predicted by them as a Nash equilibrium within their actual

Table 6.2: Classification of the h-level Hypergame Nash Equilbria

Group Name	Definition	Misperception	Known by	Equilibrium by
$SHNash^h$	A state $s \in HE^{hNash}$ is considered to be an $SHNash^h$ equilibrium iff $s \in S^R$ and $s \in \cap_{i \in N} E_i^{Nash}$.	No	All	All
$UHNash^h$	A state $s \in HE^{h\ Nash}$ is referred to as a $UHNash^h$ equilibrium iff $s \in S^R$ and $s \notin \cap_{i \in N} E_i^{Nash}$.	Preference misperception	All	Some
$STHNash^h$	A state $s \in HE^{h\ Nash}$ is an $STHNash^h$ equilibrium iff $\exists\ i \in N$, $s \in S_i^P \cap E_i^{Nash}$.	Preference misperception and unknown real options.	Some	All
$USTHNash^h$	An outcome $s \in HE^{h\ Nash}$ is categorized as a $USTHNash^h$ equilibrium iff $s \in S_i^P \setminus E_i^{Nash}$ for at least one $i \in N$.	Preference misperception and unknown real options.	Some	Some
$CHNash^h$	A state $s \in HE^{h\ Nash}$ is referred to as a $CHNash^h$ equilibrium $\Longleftrightarrow$ either $s \in \cap_{i \in N} S_i^I$, $s \in \cap_{i \in N} S_i^M$, or $s \in \cap_{i \in N} S_i^{I,M}$ and $s \in \cap_{i \in N} E_i^{Nash}$.	Preference misperception and option misperceptions	All	All
$UCHNash^h$	A state $s \in HE^{h\ Nash}$ is considered to be an $UCHNash^h$ equilibrium $\Longleftrightarrow$ either $s \in \cap_{i \in N} S_i^I$, $s \in \cap_{i \in N} S_i^M$, or $s \in \cap_{i \in N} S_i^{I,M}$ and $s \notin \cap_{i \in N} E_i^{Nash}$.	Preference misperception and option misperceptions	All	Some
$SCHNash^h$	A state $s \in HE^{h\ Nash}$ is an $SCHNash^h$ equilibrium iff there is a $DM\ i \in N$ such that $s \in (S_i^I \cup S_i^M \cup S_i^{I,M}) \setminus \cap_{j \in N}(S_j^I \cup S_j^M \cup S_j^{I,M})$.	Preference misperception and option misperceptions.	Some	N/A
$EHNash$	An outcome $s \in HE^{h\ Nash}$ is referred to as an $EHNash$ equilibrium iff $s \notin \cup_{i \in N} S_i$.	Each DM's options are known to itself but hidden to its opponents	None	N/A

subjective game, is referred to as a steady hyper Nash ($SHNash^h$) equilibrium for the h-level hypergame. This equilibrium is free from any misperception. Please notice that the superscript h in $SHNash^h$ indicates that it is an equilibrium for an h-level hypergame.

However, if at least one of the DMs misses predicting the hyper Nash equilibrium state as a Nash equilibrium in its subjective game, then it is called an unsteady hyper Nash ($UHNash^h$) equilibrium for the h-level hypergame. The source of misperception associated with this equilibrium state is preference misperception.

Next, a hyper Nash equilibrium state is referred to as a stealthy hyper Nash ($STHNash^h$) equilibrium for the h-level hypergame if (1) perceived by at least one DM as a correct scenario, (2) unknown to the other DMs, and (3) predicted as a Nash equilibrium in all of the DMs' subjective games for which the state is recognized. This category occurs when at least one of the DMs is unaware of some of its opponents' correctly perceived options. This equilibrium represents the intentional use of strategic surprise in a conflict situation. However, if at least one of the DMs who is aware of the state in its subjective game does not predict it as a Nash equilibrium, it is classified as an unsteady stealthy hyper Nash ($USTHNash^h$) equilibrium for the h-level hypergame.

Furthermore, if a hyper Nash equilibrium state is known to all DMs as a misperceived state and predicted as a Nash equilibrium by all of them within their subjective games, it is called a contingent hyper Nash ($CHNash^h$) equilibrium for the h-level hypergame. The sources of misperception, in this case, are preference misperception, and option misperceptions (imagined and misunderstood). Nevertheless, if the state is not a Nash equilibrium in at least one DM's subjective game, it is known as unsteady CHNash^h ($UCHNash^h$) for the h-level hypergame.

Moreover, a hyper Nash equilibrium state that is assumed by some DMs as a misperceived state and unknown to the other DMs is referred to as a self-CHNash^h ($SCHNash^h$) equilibrium for the h-level hypergame. Preference misperception and option misperceptions are associated with this category.

Finally, a hyper Nash equilibrium state that is unknown to all DMs in a hypergame is referred to as an emergent hyper Nash ($EHNash^h$) equilibrium for the h-level hypergame. This state occurs when each DM chooses to exercise courses of action in the dispute that are hidden to its opponents. That is, this state is a surprise to all DMs in the dispute.

In a similar fashion, the hyper SEQ, GMR, and SMR equilbria for the h-level hypergame can be classified.

6.3 Chapter Summary

The research described in this chapter outlines a novel method for modeling and analyzing a general hypergame of any level or number of DMs within the structure of a graph model. The procedures are created in such a way that a DM can hold misperceptions about itself and also its opponents. Moreover, it takes into account a DM's different levels of perception. It also contains generalized stability analysis methods to analyze any level of hypergame. Finally, it provides definitions to classify the overall hypergame equilibria into eight classes, each of which provides unique strategic insights about the source of misperceptions that provoke the hypergame situation.

Chapter 7

Strategic Analyses of the Hydropolitical Conflicts Surrounding the Grand Ethiopian Renaissance Dam

7.1 Introduction

Globally, there are a number of international rivers on which large water resources development facilities have been constructed in both upstream and downstream countries. The Colorado River, for example, is an international river shared by the United States and Mexico (MIT, 2014). This river has multiple storage facilities in both the upstream and downstream countries with an international agreement that coordinates their operation. What makes the Nile River situation unique is that, in the near future, two hydraulic dams, the Grand Ethiopian Renaissance Dam (GERD) and Aswan High Dam (AHD), each with a sufficient storage capacity to hold the annual flow of the Nile River, will be working without any international agreement to coordinate their operations (MIT, 2014). With a length of 6,800 km, the Nile River is one of the longest river systems on earth and is shared by 11 African countries: Burundi, Democratic Republic of Congo, Egypt, Eritrea, Ethiopia, Kenya, Rwanda, South Sudan, Sudan, Tanzania, and Uganda, as shown on the map in Figure 7.1. Disputes have arisen with the decline in water resources due to rapid population increases, development growth in Nile Basin countries, inequitable allotment of the Nile River water, and inequitable hydraulic development on the Nile River (Salman, 2013; Cascão and Alan, 2016).

The Nile Valley covers 3.18 million km^2 of Eastern Africa, which represents approximately 10.3% of the total area (Craig, 1991). As can be seen in Figure 7.1, the Nile River is fed by two main tributaries: the White Nile and the Blue Nile River. Lake Victoria, which is located in east central Africa on the frontiers of Uganda, Kenya, and Tanzania, is the primary water source of the White Nile. This lake is the second largest freshwater reservoir on earth. The Blue Nile River, on the other hand, is formed by Lake Tana in the Ethiopian highlands. The White and Blue Niles converge in Sudan to form the Nile River, which flows from south to north through Egypt and discharges into the Mediterranean Sea. Some of the water from the Nile River is stored in Egypt by AHD in the artificial Lake Nasser (Shahin, 1985). The White and Blue Niles, respectively, contribute 30% and 57% of the total water in the Nile River (Craig, 1991). The remaining 13% comes from a number of small rivers.

The most recent conflict regarding the Nile Basin erupted on April 11, 2011, when Ethiopia publicly announced the launch of its federal hydroelectric dam project, called the Grand Ethiopian Renaissance Dam (GERD). As will be mentioned later, Ethiopia's ambition to build a hydroelectric dam in the Ethiopian highlands within the Blue Nile River near the eastern Sudanese border, (see Figure 7.1), goes back to 1958. Ethiopia's unilateral decision to violate the 1929 and 1959 agreements and start constructing the dam on the Blue Nile River without prior notification to or approval from Egypt and Sudan has been the cause of a series of conflicts that began just before April 2011 (Blackmore

Figure 7.1: The Nile River Basin

and Whittington, 2008; Cascão, 2012; Salman, 2013; MIT, 2014; Abdelhady et al., 2015; Cascão and Alan, 2016).

Water disputes have been extensively studied during the last decades, and different methods have been utilized to model and analyze them (Madani, 2010). For example, a game theoretical approach, the Graph Model for Conflict Resolution (GMCR), has been utilized to study a generic version of the ongoing Jordan River dispute (Madani and Hipel, 2007) and the Nile River Basin conflict before the Egyptian revolution, which commenced on January 25, 2011 (Madani et al., 2011). Within this technique, complete information and common perception among the participating decision makers (DMs) are assumed. There is a stream of articles in the literature that examined the conflict over GERD. For instance, the potential scenarios of the hydroplitical game between Ethiopia, Sudan, and Egypt over GERD were explored by Sammaan (2014). Moreover, Cascão and Alan (2016) argued that the establishment of GERD will promote possible cooperation between the Eastern Nile countries in light of the geopolitical and economic changes.

The purpose of this research is to investigate in depth the disputes between the Eastern Nile countries – Egypt, Ethiopia, and Sudan – over GERD, in order to provide strategic insights and predict resolutions. The hypergame method in graph form, which models and analyzes real-world disputes under different levels of perception among the participating DMs (Aljefri et al., 2017a,b), will be used to study these conflicts. This technique is designed to be applied when there are discrepancies in DMs' perceptions of a dispute, perhaps because of the asymmetry of knowledge or a misunderstanding of the actual environment of the conflict among the participating DMs. In this case, GMCR standard solution concepts cannot be applied; hypergame stability analysis is introduced as a new theoretical procedure that extends GMCR's existing solution concepts to circumstances when DMs have a different interpretation of the real-life conflict. The overriding purpose of hypergame analysis in graph form is to foretell the possible equilibria of the dispute when DMs are not playing the same game.

The GERD dispute since the Egyptian revolution of January 2011 is analyzed at three points in time: the conflict just before April 11, 2011, which involves the use of strategic surprise by the Ethiopian government, the negotiation in early January 2014, and the negotiation in late August 2014, as shown in Figure 7.2.

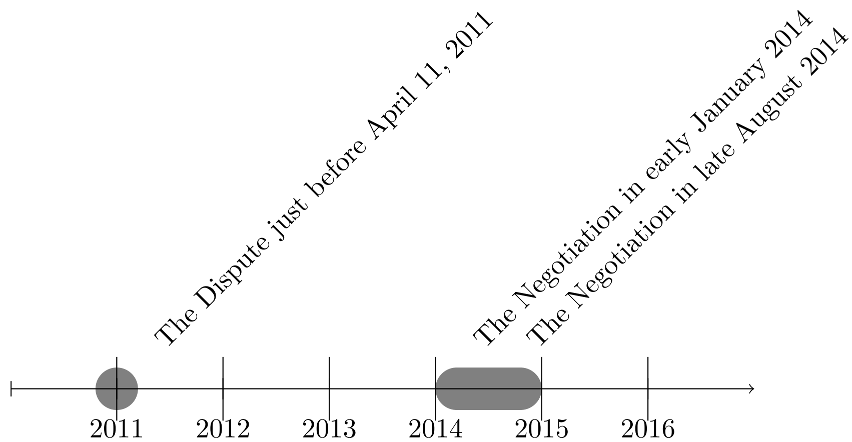


Figure 7.2: The Hydropolitical Conflict Timeline

The chapter is structured as follows. First, an overview of the Nile Basin treaties, related initiatives, and Eastern Nile countries' political and economic changes is provided. Next, the modeling and analysis of the dispute just before April 11, 2011, the negotiation in early January 2014, and the negotiation in late August 2014 are conducted. The conclusions and key insights are discussed at the end of the chapter.

7.2 Background

In this section the historical Nile Basin treaties are reviewed first. Next, a discussion about the Nile Basin Initiative is provided. Lastly, the geopolitical and economic changes in the Eastern Nile countries are highlighted to understand the cause of the conflicts.

7.2.1 Nile Basin Treaties

During the British colonial period, many agreements were made regarding the Nile River water allotment among the countries of the Nile Basin. These protocols were designed to protect Britain's interest in downstream states, ensuring that both Egypt and Sudan received a significant and sustainable flow of water from the Nile River, for agricultural and industrial production (Odidi, 1994; Swain, 1997; Degefu, 2003; Madani et al., 2011; Salman, 2013, 2016). However, these agreements resulted in inequitable rights regarding the use of the Nile River water by the countries in the region.

The 1902 Nile treaty between the United Kingdom (UK) (on behalf of Sudan) and Ethiopia aimed to establish a border between Ethiopia and Sudan. This agreement stipulated that Ethiopia could not implement any hydraulic project in the Blue Nile River, or Lake Tana, that would capture the natural flow of the Blue Nile River without first reaching an agreement with Britain. Based on Ethiopia's understanding of the agreement, this country could use the water in Lake Tana and the Blue Nile River as long as it did not stop the flow of water. Hence, Ethiopia did not interpret the UK's understanding of the agreement as being preventive to using the water in Lake Tana or the Blue Nile River. Therefore, Ethiopia claimed that its understanding of the agreement was valid, and continued to dispute the validity of the 1902 agreement (Odidi, 1994; Swain, 1997; Degefu, 2003; Madani et al., 2011; Salman, 2013, 2016).

After Egypt achieved its independence from UK in 1922, Britain (on behalf of Britain's colonies of Kenya, Uganda, Tanganyika, and Sudan) signed the Nile Water agreement with Egypt in 1929. None of the upstream countries except Ethiopia was independent at the time. This agreement granted Egypt an annual flow of 48 billion cubic meters (BCM) of the Nile River water, the right to develop any project on the Nile River without notifying upstream countries, and the right to stop any hydraulic project by upstream countries that would alter the flow of the Nile River. Moreover, due to Britain's interest in Sudan, the agreement granted Sudan an annual flow of 4 BCM of the Nile River water. The agreement thus left 32 BCM of Nile River water unallocated (Odidi, 1994; Swain, 1997; Degefu, 2003; Madani et al., 2011; Salman, 2013, 2016).

After Sudan gained its independence in 1956, it requested to renegotiate the 1929 agreement with Egypt to gain access to additional water that would satisfy Sudan's needs. Therefore, in 1959, Egypt and Sudan signed the Nile River water treaty for full utilization of the Nile River water. According to this agreement, the annual water allotments of Egypt and Sudan increased from 48 BCM to 55.5 BCM and from 4 BCM to 18.5 BCM, respectively. In addition, the agreement permitted Sudan to construct hydraulic projects on the Nile River that could regulate its flow. Egypt maintained all the rights that were given to it by the 1929 agreement. Upstream countries were prohibited from building any hydraulic infrastructure and from using the Nile River water (Odidi, 1994; Swain, 1997; Degefu, 2003; Madani et al., 2011; Salman, 2013, 2016).

The upstream countries did not accept either the 1929 or 1959 agreement, yet they were unwilling to actively oppose them due to their political instability and poor economic situations (Odidi, 1994; Swain, 1997; Degefu, 2003). Soon after the 1959 agreement had been signed, Ethiopia criticized the agreement, stressing its sovereignty over the water in Lake Tana and the Blue Nile River that flows in its territory (Odidi, 1994; Swain, 1997; Degefu, 2003; Madani et al., 2011; Salman, 2013, 2016). Therefore, Ethiopia, with the support of the United States Bureau of Reclamation (USBR), investigated the possible construction of hydropower dams on Ethiopia's Blue Nile River between 1958 and 1965 (USBR, 1964; Swain, 1997; Blackmore and Whittington, 2008; Cascão and Alan, 2016). The USBR decided to support Ethiopia after Egypt began building the AHD with huge support from the Soviet Union (Shupe et al., 1980; Wright et al., 1980). The studies had identified possible sites for constructing a hydropower dam and for implementing irrigation projects. However, between 1958 and 1999, Ethiopia was unable to acquire the necessary funds to implement the plans for these projects, due to its political instability, severe poverty, and harsh civil war (USBR, 1964; Swain, 1997; Blackmore and Whittington, 2008; Cascão and Alan, 2016; AfDB et al., 2016; Yihdego et al., 2016).

With the absence of colonial powers in Africa, it became evident that both the 1929 and 1959 agreements were unsustainable. Believing in the unfair agreements between upstream and downstream countries, the nations in the region began to establish a cooperative institution that promoted fair use of the Nile River water in 1992. Their efforts resulted in the formation of the Nile Basin Initiative (NBI) in 1999, which is discussed in the next subsection.

7.2.2 The Nile Basin Initiative

The NBI was launched in 1999 for the purpose of promoting sustainable development through cooperative and fair allotment of the Nile River water among countries in the region (Salman, 2013). This important initiative brought upstream and downstream countries together to investigate mutually beneficial projects in the Nile Basin. International organizations such as the World Bank and United Nations Development Program (UNDP) facilitated the establishment of the NBI. Indeed, the NBI was the first undertaking to garner strong international support. It aimed to identify possible regional investment opportunities in different sub-regions of the Basin that would provide mutual benefits for the countries therein. One of the first studies done by the NBI was conducted by the Joint Multipurpose Project (JMP) of the Eastern Nile countries in 2008. This study concluded that the Blue Nile River in the Ethiopian highlands provides a good investment opportunity for developing a large hydroelectric dam that has mutual benefits for Egypt, Sudan, and Ethiopia (Blackmore and Whittington, 2008). This proposed project was expected to reduce the amount of water loss, manage floods, and improve agricultural production in Egypt, Ethiopia, and Sudan (Brunnée and Toope, 2002; FAO, 2002; The World Bank, 2009).

Ethiopia, a country with huge ambitions to construct hydroelectric dams on the Blue Nile River, viewed the project proposed by the JMP of the Eastern Nile countries as the first real opportunity to construct such a dam on the Blue Nile River, with the benefit of jointly funding the project with the Eastern Nile countries, Egypt and Sudan, through the Nile Basin Trust Fund (NBTF) and with substantial aid from the international community (USBR, 1964; Blackmore and Whittington, 2008; Cascão and Alan, 2016). However, after the JMP report was released in 2008, Egypt disputed the validity of the study and rejected the proposals for building a dam on the Blue Nile River, because it believed that the dam would reduce the volume of water reaching Egypt (Blackmore and Whittington, 2008; Cascão, 2012; Salman, 2013; Cascão and Alan, 2016). Ethiopia and Sudan, on the other hand, praised the findings, viewing the project as an excellent opportunity for power trade, flood control, and irrigation projects that would benefit all the Eastern Nile countries. Therefore, from 2008 to 2009, Ethiopia and Sudan tried to convince Egypt to cooperate in the JMP of the NBI, but their efforts did not lead to any result. Until 2010, the NBI was not a legally binding agreement. Thus, the parties could walk away from the initiative without suffering any negative consequences.

To make the NBI a legally binding agreement for all Nile Basin countries, the parties involved worked from 1991 to 2010 to draft a Cooperative Framework Agreement (CFA). The objectives of the CFA are to give the right to each Nile country to use the Nile River water within its borders and to specify a number of factors that determine the equitable utilization of the Nile River water among the countries of the region. However, the situation of ratifying the CFA intensified when both Egypt and Sudan refused to sign the CFA of the NBI in 2010 due to Article 14b regarding water security (Dahan, 2009; Nile Basin Initiative, 2010). This article required all Nile Basin countries to have a fair use of the Nile River water. Egypt and Sudan wanted the CFA to maintain their historical rights, which had been granted to them by the 1929 and 1959 treaties. As a result, this window of opportunity for Ethiopia, Sudan, and Egypt to engage in any mutually beneficial and cooperative hydraulic projects was shut (Blackmore and Whittington, 2008; Cascão, 2012; Salman, 2013; Cascão and Alan, 2016).

At this point, Ethiopia realized that the development of a hydraulic project within the cooperative framework of the JMP through NBI would not be an option. Hence, Ethiopia returned to considering its national projects on its own and decided to construct a hydroelectric dam on the Blue Nile River, as identified by the USBR in 1964, but larger and with greater capacity (Blackmore and Whittington, 2008; Cascão, 2012; Salman, 2013; Cascão and Alan, 2016). In April 2011, Ethiopia publicly announced the launching of its federal hydroelectric dam project, GERD, on the Blue Nile River near the Sudanese eastern border (see Figure 7.1). The economic and political changes occurring in all the Eastern Nile nations allowed Ethiopia to commence the construction of this massive project, which is the first of its kind for Ethiopia. These changes are discussed in the next subsection.

7.2.3 Eastern Nile Countries: Political and Economic Changes

The geopolitical and economic changes in the Eastern Nile countries set the stage for building GERD. Egypt, for example, which had once been the most stable country in Africa economically and politically, suffered from dramatic political instability due to the Egyptian revolution, which began on January 25, 2011, and continued until the election of President Abdel Fattah el-Sisi on June 8, 2014, as outlined in Figure 7.3. As a result of the Egyptian revolution, Egypt's key decision makers were changed more than four times, with each having different views about the country's internal and international policies (Cascão and Alan, 2016).

Sudan also experienced significant political and economic transformations. The Comprehensive Peace Agreement (CPA), which was signed in 2005, granted South Sudan its independence from Sudan on July 9, 2011, with South Sudan receiving 48% of Sudan's total oil revenue (Oil and Energy Trends, 2011; Aljefri et al., 2014b). To compensate for this loss of oil revenue, Sudan worked to diversify its economy, focusing on investments in agriculture and irrigation projects within the Blue Nile River. Thus, Sudan was supportive of the construction of large hydroelectric dams on the Blue Nile River within the Ethiopian highlands as proposed by the JMP of the Eastern Nile River countries. Sudan expected that the proposed dams would grant it extra water that could be used for its ambitious

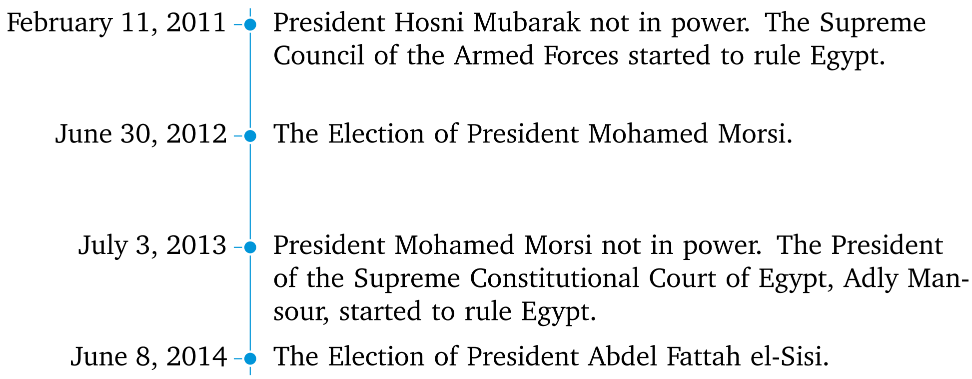


Figure 7.3: Egypt's Recent Political Changes

agriculture and irrigation projects (Cascão and Alan, 2016; Yihdego et al., 2016).

Unlike other countries in the region, Ethiopia has enhanced its political stability during the last two decades, improved its economy, attracted foreign investments, and conducted business trade with China. According to the World Bank, Ethiopia was ranked as the twelfth-fastest growing economy in the world in 2012 (The World Bank, 2013). As can be seen in Figure 7.4, Ethiopia's annual gross domestic product (GDP) growth increased from 6.1% in 2000 to 12.6% in 2010, slightly declining to 10.3% in 2014 (The World Bank, 2016). In comparison, the world average GDP growth dropped from 4.3% in 2000 to 4.1% in 2010, and further to 2.5% in 2014. These numbers clearly demonstrate that Ethiopia had one of the fastest-growing GDPs in the world during this period . On the other hand, the annual GDP growth in Egypt and Sudan was in line with the international trend (The World Bank, 2016). The sustained development of Ethiopia's agricultural and service sectors was the primary reason for its GDP growth (CIA, 2016). These factors allowed Ethiopia to commence construction of GERD in April 11, 2011 as a national project that was claimed not to have any real foreign investments (Cascão and Alan, 2016).

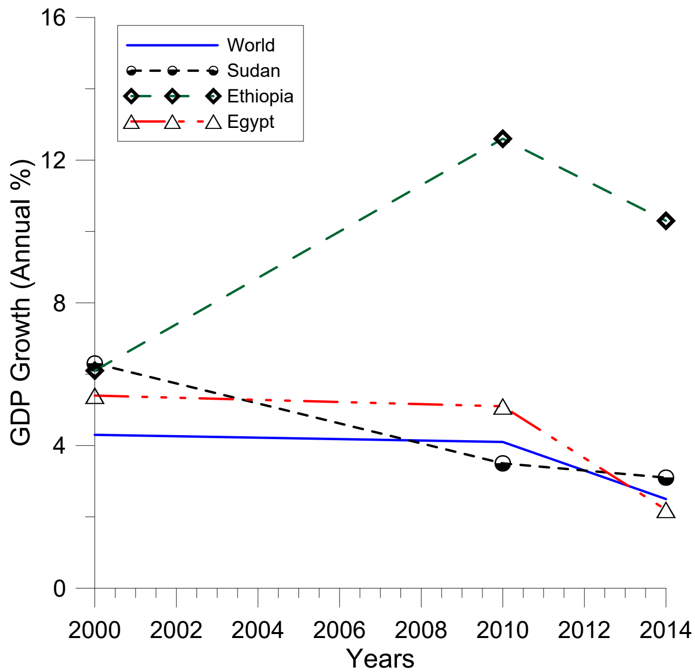


Figure 7.4: The Annual GDP Growth of the Eastern Nile Countries in Comparison With Global Overall Annual GDP Growth

7.3 The Conflict just before April 11, 2011

The Eastern Nile countries' conflicts over the construction of GERD took a critical turn when Ethiopia publicly announced on April 11, 2011 its decision to build GERD on the Blue Nile River without giving the downstream nations Egypt and Sudan, any prior notification, and without gaining their approval (Blackmore and Whittington, 2008; Cascão, 2012; Salman, 2013; Cascão and Alan, 2016). GERD includes a reservoir that is estimated to hold up to 70 BCM of water, and a power generation capacity of 6,000 megawatts. As such, it is the largest hydraulic dam in Africa in terms of power generation capacity. Ethiopia tendered the construction of the dam to an Italian company at a total cost of US $4.7 billion and the project is expected to be completed in 2017. As of 2016, 70% of the dam construction was completed (International Rivers, 2014; Abbas, 2016; Ministry of Water, Irrigation, and Electricity, 2016; Wheeler et al., 2016). The primary purpose of the dam is claimed to be hydroelectricity generation. Ethiopia secured the financing of the project locally by issuing diaspora bonds (Davison, 2011). International investors were not motivated to fund the project with the Ethiopian government due to Egypt's strong opposition to any projects on the Blue or White Nile.

To avoid any direct and severe confrontation with Egypt, Ethiopia released its decision to commence construction of GERD in the middle of the Egyptian revolution, which commenced on January 25, 2011 (Blackmore and Whittington, 2008; Cascão, 2012; Salman, 2013; Cascão and Alan, 2016). Although Ethiopia was adamant that it will implement the project, with or without cooperation from Egypt and Sudan, the speech delivered by the then Prime Minister of Ethiopia, Meles Zenawi, on April 11, 2011, emphasized that Egypt and Sudan would benefit from the dam; and, as a result, invited them to co-fund it.

Both Egypt and Sudan expressed their mistrust and rejection of GERD. Egypt received the news of Ethiopia's unilateral decision to construct GERD while in the midst of a critical political situation. As a result, significant courses of action such as political retaliation were not considered. Instead, Egypt emphasized its historical water rights that had been granted to it by the 1929 agreement and later by the 1959 accord (Blackmore and Whittington, 2008; Cascão, 2012; Salman, 2013; Cascão and Alan, 2016). Moreover, Egypt declared that GERD would reduce the volume of water flow from the Blue Nile River to the Nile River in Egypt, would reduce the hydroelectric capacity of AHD, and would turn some of Egypt's irrigated fields into desert. Hence, Egypt demanded that all research on GERD be provided so that the negative implications of GERD on Egypt could be accurately assessed. It is worth noting that the same concerns were raised by Egypt when, in 2008, the JMP of the Eastern Nile countries proposed constructing a dam in the Ethiopian highlands within the Blue Nile River (Ramadan et al., 2013; Arjoon et al., 2014; Whittington et al., 2014; Cascão and Alan, 2016).

Sudan also rejected Ethiopia's decision to start building GERD. The construction safety of GERD was of prominent concern to Sudan as any breaking, slipping, or collapsing of the dam would topple and destroy many Sudanese villages and cities, including the capital city of Khartoum (Arjoon et al., 2014; Whittington et al., 2014; Cascão and Alan, 2016). Despite its strong opposition in 2011, Sudan had supported the construction of the dam in the Ethiopian highlands within the Blue Nile River in 2008, when such a dam was proposed by JMP of the Eastern Nile countries. Sudan backed JMP's proposal due to its overwhelming desire to obtain additional water for its ambitious irrigation and agriculture projects that would enhance the state growth plan.

The Eastern Nile countries' dispute over GERD encountered a special type of misperception. In particular, Egypt and Sudan were unaware of Ethiopia's intention to commence GERD as announced on April 11, 2011 without any prior notification or approval, while Ethiopia, on the other hand, was aware of Egypt and Sudan's misperception (Blackmore and Whittington, 2008; Cascão, 2012; Salman, 2013; Cascão and Alan, 2016). Therefore, the structure of the second-level hypergame in graph form (SLHG) (Aljefri et al., 2017b) is used to model and analyze the conflict just before April 11, 2011. The modeling of the universal set of states for a second-level hypergame is first addressed.

7.3.1 Modeling the Universal Set of States for a Second-Level Hypergame

The DMs and their courses of action for the hydropolitical conflict just before April 11, 2011 are given in Table 7.1. Note that three DMs are participating in the dispute over GERD: Egypt (denoted by EGY), Sudan (denoted by SU), and Ethiopia (denoted by ETH). As can be seen, Egypt has two options: (1) maintain the status quo by adhering to the 1959 agreement or (2) agree to implement a cooperative hydraulic project within JMP of the Eastern Nile countries. Sudan, on the other hand, has the same two options as Egypt. Ethiopia, which is the only upstream country in this dispute, has three options: (1) obey the 1959 agreement, (2) implement a cooperative hydraulic project with the Eastern Nile countries within the framework of JMP, or (3) implement an independent national hydraulic project. In this conflict, Egypt and Sudan were unaware of Ethiopia's intention to construct the dam on the Blue Nile River, while Ethiopia was aware of this misperception on Egypt and Sudan's part. Hence, Ethiopia's option to act independently and start building a hydroelectric dam on the Blue Nile River was hidden from both Egypt and Sudan and will not be considered in their subjective games (Aljefri et al., 2017a,b).

Table 7.1: DMs and Options in the Hydropolitical Conflict just before April 11, 2011

DM	Options
Egypt (EGY)	1. Maintain the status quo of the 1959 treaty (Maintain)
	2. Cooperate with hydraulic development (Cooperate)
Sudan (SU)	3. Maintain the status quo of the 1959 treaty (Maintain)
	4. Cooperate with hydraulic development (Cooperate)
Ethiopia (ETH)	5. Obey the 1959 treaty (Obey)
	6. Cooperate with hydraulic development (Cooperate)
	7. Commence independently (Commence)

The options in Table 7.1 are used to mathematically define the universal set of states for a second-level hypergame, $\hat{S}^2$. Since a DM can decide to select an option or not, there are $2^7 = 128$ mathematically possible states for this dispute. Some of the states in $\hat{S}^2$ are infeasible and need to be eliminated. Because Egypt and Sudan cannot maintain the status quo of the 1959 agreement and implement a cooperative hydraulic project within the NBI framework, options 1 and 2 as well as options 3 and 4 are mutually exclusive (Fang et al., 1993; Kilgour and Hipel, 2005). The states in which Egypt and/or Sudan choose these options together are removed from the model. This constraint removes 56 states. Furthermore, Ethiopia cannot obey the 1959 agreement, implement a cooperative hydraulic project, and commence an independent national project together since they are mutually exclusive. Thus, this removes 18 states further. Moreover, the situation in which Ethiopia takes no action is highly unlikely to ever be taken, the states containing this combination of options are infeasible, thus removing 9 more states. Finally, the circumstance in which Egypt and Sudan cooperate and Ethiopia obeys the 1959 agreement is infeasible, which removes one additional state. Hence, for the dispute just before April 11, 2011, 26 states were found to be feasible as shown in Table 7.2.

Table 7.2: The Universal Set of States for a Second-Level Hypergame

DM	Option	States																									
EGY	1. Maintain	N	Y	N	N	Y	N	N	Y	N	Y	N	N	Y	N	N	Y	N	N	Y	N	N	Y	N	N	Y	N
	2. Cooperate	N	N	Y	N	N	Y	N	N	N	N	Y	N	N	Y	N	N	Y	N	N	Y	N	N	Y	N	N	Y
SU	3. Maintain	N	N	N	Y	Y	Y	N	N	N	N	N	Y	Y	Y	N	N	N	N	N	N	Y	Y	Y	N	N	N
	4. Cooperate	N	N	N	N	N	N	Y	Y	N	N	N	N	N	N	Y	Y	Y	N	N	N	N	N	N	Y	Y	Y
ETH	5. Obey	Y	Y	Y	Y	Y	Y	Y	Y	N	N	N	N	N	N	N	N	N	N	N	N	N	N	N	N	N	N
	6. Cooperate	N	N	N	N	N	N	N	N	Y	Y	Y	Y	Y	Y	Y	Y	Y	N	N	N	N	N	N	N	N	N
	7. Commence	N	N	N	N	N	N	N	N	N	N	N	N	N	N	N	N	N	Y	Y	Y	Y	Y	Y	Y	Y	Y
Label		1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26

Each option or course of action in Table 7.2 is marked with a number and can be either chosen (Y for yes) or not (N for no) by the DM who controls it. Each state in Table 7.2 accounts for a possible real-life scenario (Howard, 1971; Kilgour et al., 1987; Fang et al., 1993; Kilgour and Hipel, 2005). These states are then used to formulate states in each DM's subjective first-level hypergame (Aljefri et al., 2017a,b). State 13 is the status quo for the conflict, the state in which the conflict started just before April 11, 2011.

As mentioned earlier, SLHG is a structure consisting of subjective first-level hypergames, each of which represents not only a DM's viewpoint of the conflict situation but also its opinion on its opponents' subjective games (Aljefri et al., 2017a,b). Mathematically, the structure of a second-level hypergame, H^2, for the dispute just before April 11, 2011 is provided as follows:

$$H^2 = \{H^1_{EGY}, H^1_{SU}, H^1_{ETH}\} \tag{7.1}$$

where, H^1_{EGY}, H^1_{SU}, and H^1_{ETH} stand for Egypt's, Sudan's, and Ethiopia's subjective first-level hypergames, respectively. In the dispute between Egypt, Sudan, and Ethiopia, it has been noted that Egypt and Sudan share the same misperception about Ethiopia (i.e., unaware of Ethiopia's intention to commence building a dam on the Blue Nile without first reaching an agreement with Egypt and Sudan). Additionally, the investigation reveals that both Egypt and Sudan correctly capture each other's options and preferences in the dispute. These insightful results allow the authors to assume that Egypt's subjective first-level hypergame is identical to Sudan's subjective first-level hypergame (i.e., $H^1_{EGY} = H^1_{SU}$). Therefore, one can analyze H^1_{EGY} only and obtain both of Egypt and Sudan's stable strategies that are associated with the equilibrium states in H^1_{EGY}.

7.3.2 Stability Analysis and Equilibrium Results for Egypt's and Sudan's Subjective First-Level Hypergame

Egypt's subjective first-level hypergame H^1_{EGY} can be defined as follows.

$$H^1_{EGY} = \{G_{EGY\ EGY}, G_{SU\ EGY}, G_{ETH\ EGY}\} \tag{7.2}$$

where, $G_{EGY\ EGY}, G_{SU\ EGY}$, and $G_{ETH\ EGY}$ are Egypt's, Sudan's, and Ethiopia's subjective games, respectively, as seen by Egypt. Egypt assumes that its subjective game is the actual one for the dispute and all the engaging DMs see it in this manner. That is, $G_{EGY\ EGY} = G_{SU\ EGY} = G_{ETH\ EGY}$. Therefore, one needs to analyze $G_{EGY\ EGY}$ only.

The set of feasible states in $G_{EGY\ EGY}$ is $S_{EGY\ EGY} = \{1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17\}$. Note that some states in the universal set of states $\hat{S}^2$ are unknown to Egypt. The states in which Ethiopia decided to act independently and chose to start building a hydroelectric dam on the Blue Nile River were unknown to Egypt. Therefore, the set of hidden states in Egypt's subjective game is $S^H_{EGY\ EGY} = \{18, 19, 20, 21, 22, 23, 24, 25, 26\}$. Egypt will not consider these states in its subjective game (Aljefri et al., 2017a,b).

To conduct a stability analysis in $G_{EGY\ EGY}$, states are put in the order of preference for each DM as perceived by Egypt. DMs' ordinal preferences are given in Table 7.3 and explained as follows:

1. Egypt's first preference is to do nothing while Sudan either maintains the 1959 agreement or does nothing; and Ethiopia obeys the 1959 agreement. Egypt's second pref-

erence is to maintain the 1959 agreement to influence Ethiopia to follow the 1959 treaty. The least preferred scenarios for Egypt are when it cooperates under the influence of Sudan and Ethiopia (Blackmore and Whittington, 2008; Cascão, 2012; Salman, 2013; Cascão and Alan, 2016). As can be seen in Table 7.3, a line above or below a group of states means that they are equally preferred. For example, states 15 and 11 are equally preferred for Egypt.

2. Sudan's first preference is to engage in a cooperative water development project with Egypt and Ethiopia and maintain an excellent relationship with both Egypt and Ethiopia; second, Sudan prefers to boost its economy by establishing a cooperative project with Ethiopia, regardless of its relationship with Egypt; third, to maintain a strong relationship with Egypt and act according to Egypt's desire. The least preferred scenarios for Sudan are for it to cooperate and for both Egypt and Ethiopia to choose to maintain the 1959 treaty (Blackmore and Whittington, 2008; Cascão, 2012; Salman, 2013; Cascão and Alan, 2016).

3. Ethiopia's first preference is to cooperate with Egypt and Sudan with respect to constructing a hydraulic project on the Blue Nile River. Ethiopia's second preferences are that Egypt and Sudan do nothing while Ethiopia goes ahead with the project. The least preferred scenarios for Ethiopia are to obey the 1959 treaty. This would prevent Ethiopia from constructing any project on the Blue Nile River and from having fair use of the Blue Nile River water (Blackmore and Whittington, 2008; Cascão, 2012; Salman, 2013; Cascão and Alan, 2016).

After ordering the states based on each DM's preferences, one can analyze $G_{EGY\ EGY}$ by using the standard GMCR solution concepts (Nash, 1950, 1951; Howard, 1971; Fraser and Hipel, 1979, 1984; Fang et al., 1993; Aljefri et al., 2017a,b) to investigate DMs' possible moves and counter moves for the purpose of identifying the subjective equilibria in $G_{EGY\ EGY}$. The decision support system GMCR II (Fang et al., 2003a,b) is used to perform the analysis and predict the equilibrium states for the dispute.

Table 7.3: Ranking of States for the DMs in the Conflict just before April 11, 2011 as Seen by Egypt

DM	States																
Egypt	17	15	11	14	16	9	10	12	13	5	2	4	8	6	3	7	1
Sudan	11	17	15	1	16	5	13	9	2	10	7	8	12	6	3	4	14
Ethiopia	1	14	12	9	7	5	2	13	10	8	6	3	14	11	16	15	17
	Most preferred												Least preferred				

DM's individual stability results and the overall equilibria in $G_{EGY\ EGY}$ are furnished in Table 7.4. Since states 2 and 5 are stable for all DMs under Nash, SEQ, GMR, and SMR solution concepts, they are Nash, SEQ, GMR, and SMR equilibria for $G_{EGY\ EGY}$. States 2 and 5 are strong equilibria in $G_{EGY\ EGY}$ because they are resolutions within all the solution concepts. States 9 and 17 are also strong resolutions because they constitute states that are equilibria under SEQ and GMR. Moreover, states 6, 8, 10, and 13 are weak equilibrium states for the dispute because they are resolutions under GMR and SMR, in which a DM may have sanctions that are detrimental to itself.

Table 7.4: Stability Analysis and Equilibrium Results in $G_{EGY\ EGY}$

States		1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17
EGY	Nash	NO	YES	NO	NO	YES	NO	NO	YES	NO	NO	YES	NO	NO	YES	NO	NO	YES
	SEQ	NO	YES	NO	NO	YES	NO	NO	YES	YES	YES	YES	YES	YES	YES	YES	NO	YES
	GMR	NO	YES	NO	YES	YES	YES	NO	YES	YES	YES	YES	YES	YES	YES	YES	YES	YES
	SMR	NO	YES	NO	YES	YES	YES	NO	YES	YES	YES	YES	YES	YES	YES	YES	YES	YES
SU	Nash	YES	YES	YES	NO	YES	YES	NO	NO	NO	NO	YES	NO	NO	NO	YES	YES	NO
	SEQ	YES	YES	YES	NO	YES	YES	YES	NO	YES	YES	YES	NO	YES	NO	YES	YES	YES
	GMR	YES	YES	YES	NO	YES	YES	YES	YES	YES	YES	YES	NO	YES	NO	YES	YES	YES
	SMR	YES	YES	YES	NO	YES	YES	YES	YES	NO	NO	YES	NO	YES	NO	YES	YES	YES
ETH	Nash	YES	YES	YES	YES	YES	YES	YES	YES	NO	NO	NO	NO	NO	NO	NO	NO	YES
	SEQ	YES	YES	YES	YES	YES	YES	YES	YES	YES	NO	NO	YES	NO	NO	NO	NO	YES
	GMR	YES	YES	YES	YES	YES	YES	YES	YES	YES	YES	NO	YES	YES	NO	NO	NO	YES
	SMR	YES	YES	YES	YES	YES	YES	YES	YES	YES	YES	NO	YES	YES	NO	NO	NO	YES
Equilibrium	Nash	/	E	/	/	E	/	/	/	/	/	/	/	/	/	/	/	/
	SEQ	/	E	/	/	E	/	/	/	E	/	/	/	/	/	/	/	E
	GMR	/	E	/	/	E	E	/	E	E	E	/	/	E	/	/	/	E
	SMR	/	E	/	/	E	E	/	E	/	/	/	/	E	/	/	/	E

Having identified the equilibrium states, one needs to determine Egypt and Sudan's strategies that are related to the equilibrium states in $G_{EGY\ EGY}$. Normally, one obtains each DM's strategies from the equilibrium states in its subjective game. In this case, Egypt and Sudan's strategies are obtained from $G_{EGY\ EGY}$ because $H^1_{EGY} = H^1_{SU}$.

Egypt's strategy related to states 2, 5, 8, 10, and 13 is $g^2_{EGY} = g^5_{EGY} = g^8_{EGY} = g^{10}_{EGY} = g^{13}_{EGY} = (YN)^T$, its strategy related to states 6 and 17 is $g^6_{EGY} = g^{17}_{EGY} = (NY)^T$, and its strategy related to state 9 is $g^9_{EGY} = (NN)^T$. Hence, one can determine Egypt's set of Nash strategies g^{*Nash}_{EGY} as follows:

- $g^{*Nash}_{EGY} = \{g^2_{EGY}, g^5_{EGY}\} = \{(YN)^T\}$.

Egypt's sets of SEQ, GMR, and SMR strategies, $g^{*SEQ}_{EGY}, g^{*GMR}_{EGY}$, and g^{*SMR}_{EGY}, respectively, can be obtained analogously as follows:

- $g^{*SEQ}_{EGY} = \{g^2_{EGY}, g^5_{EGY}, g^9_{EGY}, g^{17}_{EGY}\} = \{(YN)^T, (NN)^T, (NY)^T\}$,
- $g^{*GMR}_{EGY} = \{g^2_{EGY}, g^5_{EGY}, g^6_{EGY}, g^8_{EGY}, g^9_{EGY}, g^{10}_{EGY}, g^{13}_{EGY}, g^{17}_{EGY}\} = \{(YN)^T, (NY)^T, (NN)^T\}$, and
- $g^{*SMR}_{EGY} = \{g^2_{EGY}, g^5_{EGY}, g^6_{EGY}, g^8_{EGY}, g^{13}_{EGY}, g^{17}_{EGY}\} = \{(YN)^T, (NY)^T\}$.

Sudan's strategy related to states 2, 9, and 10 is $g^2_{SU} = g^9_{SU} = g^{10}_{SU} = (NN)^T$, its strategy connected to states 5, 6, and 13 is $g^5_{SU} = g^6_{SU} = g^{13}_{SU} = (YN)^T$, and its strategy related to states 8 and 17 is $g^8_{SU} = g^{17}_{SU} = (NY)^T$. Sudan's sets of Nash, SEQ, GMR, and SMR strategies, $g^{*Nash}_{SU}, g^{*SEQ}_{SU}, g^{*GMR}_{SU}$, and g^{*SMR}_{SU}, respectively, can be obtained as follows:

- $g^{*Nash}_{SU} = \{g^2_{SU}, g^5_{SU}\} = \{(NN)^T, (YN)^T\}$.
- $g^{*SEQ}_{SU} = \{g^2_{SU}, g^5_{SU}, g^9_{SU}, g^{17}_{SU}\} = \{(NN)^T, (YN)^T, (NY)^T\}$,
- $g^{*GMR}_{SU} = \{g^2_{SU}, g^5_{SU}, g^6_{SU}, g^8_{SU}, g^9_{SU}, g^{10}_{SU}, g^{13}_{SU}, g^{17}_{SU}\} = \{(NN)^T, (YN)^T, (NY)^T\}$, and
- $g^{*SMR}_{SU} = \{g^2_{SU}, g^5_{SU}, g^6_{SU}, g^8_{SU}, g^{13}_{SU}, g^{17}_{SU}\} = \{(NN)^T, (YN)^T, (NY)^T\}$.

7.3.3 Stability Analysis and Equilibrium Results for Ethiopia's First-Level Hypergame

Ethiopia's subjective first-level hypergame H^1_{ETH} is defined as follows.

$$H^1_{ETH} = \{G_{EGY\ ETH}, G_{SU\ ETH}, G_{ETH\ ETH}\} \tag{7.3}$$

where, $G_{EGY\ ETH}$, $G_{SU\ ETH}$, and $G_{ETH\ ETH}$ are Egypt's, Sudan's, and Ethiopia's subjective games, respectively, as seen by Ethiopia.

Ethiopia correctly perceived the conflict situation and was aware not only of its own subjective game but also those of Egypt and Sudan. Ethiopia knows that Egypt and Sudan play the same game and have the same misperception about Ethiopia. Accordingly, in this dispute, Ethiopia utilized this extra insight to its benefit. Also, Ethiopia knows that $G_{EGY\ ETH} = G_{SU\ ETH} = G_{EGY\ EGY}$.

Analysis of H^1_{ETH} starts by first analyzing $G_{EGY\ ETH}$ by using a range of GMCR solution concepts of human behavior under conflict. That is, the set of equilibrium states in $G_{EGY\ ETH}$ is calculated and DMs' strategies that are associated with the equilbrium states are determined. Second, in $G_{ETH\ ETH}$, one identifies the states associated with Egypt's and Sudan's strategies arising from the equilibrium states in $G_{EGY\ ETH}$. If a state is stable for Ethiopia according to the particular solution concept in $G_{ETH\ ETH}$, it constitutes an equilibrium in $G_{ETH\ ETH}$. These equilibrium states also comprise resolutions for H^1_{ETH} (Aljefri et al., 2017a,b). Note that the equilibrium states in $G_{EGY\ EGY}$ and Egypt's and Sudan's strategies related to the equilibrium states in $G_{EGY\ EGY}$ are the same for $G_{EGY\ ETH}$ and $G_{SU\ ETH}$. Hence, one only needs to model and analyze Ethiopia's subjective game $G_{ETH\ ETH}$.

The modeling of $G_{ETH\ ETH}$ starts by identifying the set of feasible states as perceived by Ethiopia in its subjective game, denoted by $S_{ETH\ ETH}$. Because Ethiopia correctly captured the conflict situation, it perceived all the states in $\hat{S}^2$. Hence, $S_{ETH\ ETH} =$

$\hat{S}^2 = \{1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26\}$. However, based on Ethiopia's perception, $S_{ETH\ ETH}$ is partitioned into two disjoint sets: the group of states that are correctly perceived by Ethiopia as well as Egypt and Sudan (Aljefri et al., 2017a,b), denoted as $S^R = \{1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17\}$; and the collection of states that are correctly perceived by only Ethiopia and hidden to Egypt and Sudan, expressed as $S^P_{ETH\ ETH} = \{18, 19, 20, 21, 22, 23, 24, 25, 26\}$ (Aljefri et al., 2017a,b).

Egypt's and Sudan's preferences as perceived by Ethiopia are identical to their preference in $G_{EGY\ EGY}$. Ethiopia's preferences as perceived by itself are explained as follows (Blackmore and Whittington, 2008; Cascão, 2012; Salman, 2013; Cascão and Alan, 2016). Ethiopia's first preferences are to implement an independent water development project on the Blue Nile River. Of course, Ethiopia wishes to do so as Egypt and Sudan decide to take no action. However, Ethiopia is adamant about building a dam on the Blue Nile River and, as a result, would also prefer to pursue its water development even if Egypt and Sudan maintain the 1959 agreement. The second preferences for Ethiopia are to cooperate with Egypt and Sudan regarding building a mutually beneficial water development project on the Blue Nile River, whereas the least favored situations for Ethiopia are to obey the 1959 agreement. Maintaining the status quo means that Ethiopia cannot build a dam on the Blue Nile River and continues to have an unfair share of the Blue Nile River water.

Table 7.5 shows the ranking of states from most to least preferred for Egypt, Sudan, and Ethiopia as perceived by Ethiopia in $G_{ETH\ ETH}$. Because Ethiopia correctly understands the conflict situation and was also aware of Egypt's and Sudan's misperception, Egypt's and Sudan's preference relationships in Table 7.5 are identical to their preference relationships as presented in Table 7.3.

To identify the equilibria in $G_{ETH\ ETH}$, the group of states that are related to Egypt's and Sudan's winning strategies obtained from the equilibrium states in Table 7.4 needs to be checked for stability. Note that all the states in $S_{ETH\ ETH}$ are related to Egypt's and Sudan's winning strategies. Therefore, Ethiopia's individual stability analysis needs to be carried out over all the states in $S_{ETH\ ETH}$. The results of Ethiopia's individual stability

Table 7.5: Ranking of States for the DMs in the Conflict just before April 11, 2011 as Seen by Ethiopia

DM	States
Egypt	17 15 11 14 16 9 10 12 13 5 2 4 8 6 3 7 1
Sudan	11 17 15 1 16 5 13 9 2 10 7 8 12 6 3 4 14
Ethiopia	18 22 19 21 23 25 20 24 26 17 15 11 14 16 9 10 12 13 5 2 4 6 3 7 1 8
	Most preferred … Least preferred

analysis and equilibrium results in $G_{ETH\ ETH}$ are presented in Table 7.6.

Table 7.6: Ethiopia's Individual Stability Analysis and Equilibrium Results in $G_{ETH\ ETH}$

States		1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26
EGY	g^{*Nash}	YES	NO	NO	NO	YES	NO	NO	YES	NO	YES	NO	NO	YES	NO	NO	YES	NO	NO	YES	NO	NO	YES	NO	NO	YES	NO
	g^{*SEQ}	YES	YES	YES	YES	YES	YES	YES	YES	YES	YES	YES	YES	YES	YES	YES	YES	YES	YES	YES	YES	YES	YES	YES	YES	YES	YES
	g^{*GMR}	YES	YES	YES	YES	YES	YES	YES	YES	YES	YES	YES	YES	YES	YES	YES	YES	YES	YES	YES	YES	YES	YES	YES	YES	YES	YES
	g^{*SMR}	YES	YES	YES	NO	YES	YES	NO	YES	NO	YES	YES	NO	YES	YES	YES	YES	YES	NO	YES	YES	NO	YES	YES	YES	YES	YES
SU	g^{*Nash}	YES	YES	YES	YES	YES	YES	NO	NO	YES	YES	YES	YES	YES	YES	NO	NO	NO	YES	YES	YES	YES	YES	YES	NO	NO	NO
	g^{*SEQ}	YES	YES	YES	YES	YES	YES	YES	YES	YES	YES	YES	YES	YES	YES	YES	YES	YES	YES	YES	YES	YES	YES	YES	YES	YES	YES
	g^{*GMR}	YES	YES	YES	YES	YES	YES	YES	YES	YES	YES	YES	YES	YES	YES	YES	YES	YES	YES	YES	YES	YES	YES	YES	YES	YES	YES
	g^{*SMR}	YES	YES	YES	YES	YES	YES	YES	YES	YES	YES	YES	YES	YES	YES	YES	YES	YES	YES	YES	YES	YES	YES	YES	YES	YES	YES
ETH	Nash	NO	NO	NO	NO	NO	NO	NO	NO	NO	NO	NO	NO	NO	NO	NO	NO	NO	YES	YES	YES	YES	YES	YES	YES	YES	YES
	SEQ	NO	NO	NO	NO	NO	NO	NO	NO	NO	NO	NO	NO	NO	NO	NO	NO	NO	YES	YES	YES	YES	YES	YES	YES	YES	YES
	GMR	NO	NO	NO	NO	NO	NO	NO	NO	NO	NO	NO	NO	NO	NO	NO	NO	NO	YES	YES	YES	YES	YES	YES	YES	YES	YES
	SMR	NO	NO	NO	NO	NO	NO	NO	NO	NO	NO	NO	NO	NO	NO	NO	NO	NO	YES	YES	YES	YES	YES	YES	YES	YES	YES
Equilibrium	Nash	/	/	/	/	/	/	/	/	/	/	/	/	/	/	/	/	/	/	E	/	/	E	/	/	/	/
	SEQ	/	/	/	/	/	/	/	/	/	/	/	/	/	/	/	/	/	E	E	E	E	E	E	E	E	E
	GMR	/	/	/	/	/	/	/	/	/	/	/	/	/	/	/	/	/	E	E	E	E	E	E	E	E	E
	SMR	/	/	/	/	/	/	/	/	/	/	/	/	/	/	/	/	/	/	E	E	/	E	E	E	E	E

As can be seen in Table 7.6, states 18, 19, 20, 21, 22, 23, 24, 25, and 26 are Nash stable for Ethiopia as no unilateral improvements (UIs) are available for Ethiopia, beginning from these states and moving to any other more preferred states. These states are also stable under SEQ, GMR, and SMR by definition (Nash, 1950, 1951; Howard, 1971; Fraser and Hipel, 1979, 1984; Fang et al., 1993; Aljefri et al., 2017a,b). The other states are unstable for Ethiopia because there is at least a UI from them from which Egypt and Sudan have no deterrent sanctioning moves. For example, Ethiopia can move from state 12 to a more preferred state 21. Egypt and Sudan are not aware of state 21; as a result, they have no credible deterrent. Therefore, Ethiopia will take advantage of Egypt and Sudan's misperception and move to state 21.

As mentioned earlier, if a state that is individually stable for Ethiopia in $G_{ETH\ ETH}$ under a particular solution concept and Egypt's and Sudan's strategies related to that state are found to be stable under the same stability definition in $G_{EGY\ ETH}$, then the state is considered as an equilibrium in $G_{ETH\ ETH}$ within that specific solution concept.

For example, states 19 and 22 are individually stable for Ethiopia in $G_{ETH\ ETH}$ under Nash, SEQ, GMR, and SMR solution concepts. Also, Egypt's and Sudan's strategies related to states 19 and 22 are found to be stable in $G_{EGY\ ETH}$ under all the solution concepts. Thus, states 19 and 22 are Nash, SEQ, GMR, and SMR equilibria in $G_{ETH\ ETH}$. Furthermore, by investigating the data in Table 7.6 one can see that states 18 and 21 are SEQ and GMR equilibria in $G_{ETH\ ETH}$. Additionally, states 20, 23, 24, 25, and 26 are SEQ, GMR, and SMR equilibria in $G_{ETH\ ETH}$. Keep in mind that $G_{EGY\ ETH} = G_{EGY\ EGY}$.

Having identified the equilibria in $G_{ETH\ ETH}$, one needs to determine Ethiopia's strategies that are associated with these equilibrium states. Ethiopia's strategy related to states 18, 19, 20, 21, 22, 23, 24, 25, and 26 is $g^{18}_{ETH} = g^{19}_{ETH} = g^{20}_{ETH} = g^{21}_{ETH} = g^{22}_{ETH} = g^{23}_{ETH} = g^{24}_{ETH} = g^{25}_{ETH} = g^{26}_{ETH} = (NNY)^T$. Ethiopia's sets of Nash, SEQ, GMR, and SMR strategies are defined as $g^{*Nash}_{ETH} = g^{*SEQ}_{ETH} = g^{*GMR}_{ETH} = g^{*SMR}_{ETH} = \{g^{18}_{ETH}, g^{19}_{ETH}, g^{20}_{ETH}, g^{21}_{ETH}, g^{22}_{ETH}, g^{23}_{ETH}, g^{24}_{ETH}, g^{25}_{ETH}, g^{26}_{ETH}\} = \{(NNY)^T\}$. In the next section, the stability analysis and equilibrium results for the second-level hypergame are put forward.

7.3.4 Stability Analysis and Equilibrium Results for the Second-Level Hypergame just before April 11, 2011

The overall equilibria for a second-level hypergame can be determined by taking the Cartesian product of Egypt's and Sudan's strategies that are related to the equilibrium states in $G_{EGY\ EGY}$ within H^1_{EGY} with Ethiopia's strategies that are associated with the equilibrium states in $G_{ETH\ ETH}$ within H^1_{ETH}. The results are furnished in Table 7.7. As can be seen, states 18, 19, 20, 21, 22, 23, 24, 25, and 26 are found to be possible equilibrium states for the second-level hypergame. States 19 and 22 are Nash, SEQ, GMR, and SMR equilibria for the second-level hypergame because Egypt's, Sudan's, and Ethiopia's strategies linked with states 19 and 22 are stable under the same solution concepts. These two states are the strongest resolutions to the dispute because they are resolutions under all four of the solution concepts. Further, states 20, 23, 25, and 26 are equilibria under SEQ, GMR, and SMR solution concepts for the conflict. Finally, states 18, 21, and 24 are found to be SEQ and GMR equilibria for the dispute because DMs' strategies related to these states are stable under SEQ and GMR solution concepts.

These equilibrium states are classified as steady stealthy hypergame equilibria for a second-level hypergame because they (1) are only recognized by Ethiopia, (2) constitute resolutions in $G_{ETH\ ETH}$, and (3) are unknown states to both Egypt and Sudan (Aljefri et al., 2017a,b). A steady stealthy hyper equilibrium state for a second-level hypergame demonstrates the planned use of a strategic surprise by at least one DM in a conflict situation (Aljefri et al., 2017a,b).

Historically, state 22, the steady stealthy hyper Nash equilibrium state, comprised the equilibrium of the conflict. State 22 is the situation in which both Egypt and Sudan decide not to cooperate with Ethiopia regarding building a dam on the Blue Nile River and to maintain their historical right, as granted to them by the 1959 treaty. It also represents the circumstance in which Ethiopia violated the agreement and surprisingly announced its decision to build a hydraulic dam within the Blue Nile River in the Ethiopian heights, without any prior notification or approval from Egypt.

The evolution of the conflict just before April 11, 2011 is outlined in Table 7.8. As can be seen, the actual historical evolution of the dispute began by moving from state 13, the status quo of the dispute, on the left to the final resolution, state 22, on the right. Recall that in 2010 both Egypt and Sudan refused to sign the CFA and emphasized their historical water shares as provided under the 1959 agreement. At the same time, Ethiopia lost hope of developing a hydraulic project within a cooperative framework with Egypt and Sudan. Due to Egypt and Sudan's misperception, state 13 is predicted by them as a possible final resolution of the dispute on April 11, 2011 as can be seen in their games $G_{EGY\ EGY}$ and $G_{SU\ SU}$, respectively, that state 13 is a GMR and SMR equilibrium. Egypt and Sudan underestimated Ethiopia's capability to individually build a dam on the Blue Nile River.

Table 7.7: Equilibrium Results for the Second-Level Hypergame

	Winning Strategy									
EGY	Stability	YN	YN	YN	NN	NN	NN	NY	NY	NY
	Nash	YES	YES	YES	NO	NO	NO	NO	NO	NO
	SEQ	YES	YES	YES	YES	YES	YES	YES	YES	YES
	GMR	YES	YES	YES	YES	YES	YES	YES	YES	YES
	SMR	YES	YES	YES	NO	NO	NO	YES	YES	YES
	Winning Strategy									
SU	Stability	NN	YN	NY	NN	YN	NY	NN	YN	NY
	Nash	YES	YES	NO	YES	YES	NO	YES	YES	NO
	SEQ	YES	YES	YES	YES	YES	YES	YES	YES	YES
	GMR	YES	YES	YES	YES	YES	YES	YES	YES	YES
	SMR	YES	YES	YES	YES	YES	YES	YES	YES	YES
	Winning Strategy									
ETH	Stability	NNY	NNY	NNY	NNY	NNY	NNY	NNY	NNY	NNY
	Nash	YES	YES	YES	YES	YES	YES	YES	YES	YES
	SEQ	YES	YES	YES	YES	YES	YES	YES	YES	YES
	GMR	YES	YES	YES	YES	YES	YES	YES	YES	YES
	SMR	YES	YES	YES	YES	YES	YES	YES	YES	YES
States		19	22	25	18	21	24	20	23	26
Second-Level Hypergame Equilibrium	Nash	HE	HE	/	/	/	/	/	/	/
	SEQ	HE	HE	HE	HE	HE	HE	HE	HE	HE
	GMR	HE	HE	HE	HE	HE	HE	HE	HE	HE
	SMR	HE	HE	HE	/	/	/	HE	HE	HE
Classification of the Second-Level Hypergame Equilibria	Nash	STHNash	STHNash	/	/	/	/	/	/	/
	SEQ	STHSEQ	STHSEQ	STHSEQ	STHSEQ	STHSEQ	STHSEQ	STHSEQ	STHSEQ	STHSEQ
	GMR	STHGMR	STHGMR	STHGMR	STHGMR	STHGMR	STHGMR	STHGMR	STHGMR	STHGMR
	SMR	STHSMR	STHSMR	STHSMR	/	/	/	STHSMR	STHSMR	STHSMR

As a result, both Egypt and Sudan were faced with a strategic surprise when Ethiopia announced its decision, on April 11, 2011, to construct a massive hydroelectric dam on the Blue Nile River as a national project. Ethiopia was aware of Egypt and Sudan's political instability and announced its decision at a very critical time for both countries. While Egypt received the news of GERD in the midst of the Egyptian revolution, Sudan became aware of Ethiopia's decision when South Sudan was about to receive its independence from Sudan.

After Egypt and Sudan became aware of their misperception, the intensity of the conflict

Table 7.8: Evolution of the Conflict just before April 11, 2011

DM	Option	Status Quo	Equilibrium State
EGY	1. Maintain	Y	Y
	2. Cooperate	N	N
SU	3. Maintain	Y	Y
	4. Cooperate	N	N
ETH	5. Obey	N	N
	6. Cooperate	Y →	N
	7. Commence	N →	Y
Label		13	22

between the Eastern Nile countries declined. In May 2011, Egypt, Ethiopia, and Sudan agreed to establish an international panel of experts (IPoE) for the purpose of assessing the engineering and construction plans for the dam. The board consisted of 10 experts: two specialists from each country and four international experts. The IPoE was given one year to conduct its study and was required to submit its report to the three countries by May 2013. The possible confrontation between the Eastern Nile countries over the release of the IPoE's report is addressed in the investigation of the negotiation between the Eastern Nile countries during the third tripartite meeting of the ministers of water resources that took place from January 4 to 5, 2014, which will be furnished in the next section (IPoE, 2013; Cascão and Alan, 2016; Tawfik, 2016; Wheeler et al., 2016).

7.4 The Conflict just before January 4, 2014

On May 28, 2013, Ethiopia diverted the natural flow of the Blue Nile River in order to start building the GERD structure. Egypt expressed its disapproval of Ethiopia's actions and asked the country to halt construction until the IPoE's report had been released (Cascão and Alan, 2016; Tawfik, 2016; Wheeler et al., 2016). A few days later, on May 31, 2013, the

IPoE published its report, which recommended that Ethiopia conduct in-depth studies on the impacts of the GERD project. It also suggested that Ethiopia modify the structural measures of the dam to ensure that its foundation would be stable and safe. Further information about the IPoE's recommendations can be found in IPoE (2013). Egypt and Sudan reacted differently to the release of the IPoE's report.

Sudan, a country that would benefit significantly from the dam, publicly announced its approval of GERD. Sudan supported the construction of GERD for economic but not for political reasons. Therefore, it clearly stated that it would act as a mediator between Egypt and Ethiopia to try to bridge the gap between them (Sudan Tribune, 2013, 2014; Cascão and Alan, 2016; Tawfik, 2016; Wheeler et al., 2016).

Egypt, which from June 24, 2012 to July 2013 was under the leadership of the former president Mohammed Morsi, disputed the validity of the IPoE's report and stressed the water security granted to it by the 1959 agreement (IPoE, 2013; Cascão and Alan, 2016; Tawfik, 2016; Wheeler et al., 2016). The meeting led by the former president Mohammed Morsi in June 2013 recommended deterring Ethiopia from constructing the dam by threatening to use military power there (Ahramonline, 2013). However, Ethiopia stressed its good relationships with its neighboring countries and clearly stated that it would not go to war with Egypt over GERD (IPoE, 2013; Cascão and Alan, 2016; Tawfik, 2016; Wheeler et al., 2016).

From November 2013 to January 2014, Egypt, Ethiopia, and Sudan held three tripartite ministerial meetings in the Sudanese capital city of Khartoum. The purpose of the meetings was to negotiate how to implement the IPoE's recommendations. Egypt proposed forming an international expert committee to conduct the studies suggested by the IPoE (Cascão and Alan, 2016; Tawfik, 2016; Wheeler et al., 2016). It also suggested halting the construction of GERD until the investigations had been completed. Ethiopia, on the other hand, rejected Egypt's request, stating that the IPoE recommended that Ethiopia have the authority to conduct the studies without suspending the construction of GERD. As a result of the strong disagreement between Egypt and Sudan, the negotiation process between the Eastern Nile countries stopped after the third meeting from January 4 to 5, 2014, yet the construction of GERD continued (Cascão and Alan, 2016; Tawfik, 2016; Wheeler et al., 2016).

Before the January 2014 negotiation, DMs are completely aware of each other's options and preferences. Therefore, the structure of a zero-level hypergame in graph form, H^0 (Aljefri et al., 2017a), which models and analyzes real-life disputes under the assumption of complete information, is utilized to model and analyze the hydropolitical conflict.

7.4.1 Decision Makers, Options, and States for the Conflict just before January 4, 2014

The DMs and courses of actions for the hydropolitical dispute just before January 4, 2014 are given in Table 7.9. As can be seen, Egypt has three options: (1) accept the IPoE's recommendations, (2) request Ethiopia to modify GERD based on Egypt's recommendations, or (3) require Ethiopia to amend GERD based on Egypt's reduced terms. Sudan, which decided in this dispute to act as a third party, has one single course of action: to act or not. Ethiopia, which is the only upstream country in this dispute, has three options: (1) accept modification of GERD based on the IPoE's recommendations, (2) accept modification of GERD based on Egypt's conditions, or (3) accept modification of GERD based on Egypt's reduced terms. The descriptions of these courses of actions are shown in Table 7.9.

Each option in Table 7.9 can be either selected (Y for Yes) or not selected (N for No) by the DM who possesses it. Therefore, the total number of mathematically possible states for this dispute is $2^7 = 128$ states. Some of these states are infeasible and need to be removed from the model. Egypt's options are mutually exclusive since it cannot choose more than one of its three options at a time. This removes 64 states. Similarly, Ethiopia can only modify GERD based on one recommendation. Hence, the situations in which Ethiopia accepts modification of GERD based on more than one recommendation are infeasible. This removes 32 states. Hence, for this dispute, 32 states are found to be feasible as shown

Table 7.9: DMs, Options, and Descriptions for the Conflict just before January 4, 2014

DM	Option	Choice	Description
Egypt	1. Accept the IPoE's recommendations	Y	Allows Ethiopia to proceed with the construction of the GERD based on the IPoE's recommendations.
		N	Disputes the validity of the IPoE's report.
	2. Request Ethiopia to modify the GERD based on Egypt's recommendations	Y	Demands that Ethiopia halts the construction of GERD and requests an international committee to conducts the studies recommended by the IPoE.
		N	The option is not taken.
	3. Require Ethiopia to amend the GERD based on Egypt's reduced terms	Y	Permits Ethiopia to continue building the GERD while the international committee conducts the studies.
		N	The option is not taken.
Sudan	4. Act	Y	Acts as a third party to mediate between Egypt and Ethiopia for reconciliation.
		N	Does not act.
Ethiopia	5. Accept modifications to the GERD based on the IPoE's recommendations	Y	Proceeds with building the GERD and modifies the project based on the IPoE's requirements.
		N	Continues building the GERD based on Ethiopia's original plans.
	6. Accept modifications to the GERD based on Egypt's conditions	Y	Stops building the GERD and allows an international committee to conduct the IPoE's recommendations.
		N	Continues building the GERD based on Ethiopia's original plans.
	7. Accept changing the GERD based on Egypt reduced terms	Y	Continues building the GERD and allows an international committee to conduct the IPoE's recommendations.
		N	Continues building the GERD based on Ethiopia's original plans.

in Table 7.10.

7.4.2 Stability Analysis and Equilibrium Results for the Dispute just before January 4, 2014

To conduct a stability analysis for H^0, states are put in order of preference for each DM. The ranking of states from most to least preferred for Egypt, Sudan, and Ethiopia is given in Table 7.11. Note that a line above or below a group of states means that they are equally preferred. Based on the preference statements below, states are ranked with respect to each

Table 7.10: Set of Feasible States for the Conflict just before January 4, 2014

DM	Option	States																															
EGY	1. IPoE's Terms	N	Y	N	N	N	Y	N	N	N	Y	N	N	N	Y	N	N	N	Y	N	N	N	Y	N	N	N	Y	N	N	N	Y	N	N
	2. EGY's Terms	N	N	Y	N	N	N	Y	N	N	N	Y	N	N	N	Y	N	N	N	Y	N	N	N	Y	N	N	N	Y	N	N	N	Y	N
	3. EGY's Reduced Terms	N	N	N	Y	N	N	N	Y	N	N	N	Y	N	N	N	Y	N	N	N	Y	N	N	N	Y	N	N	N	Y	N	N	N	Y
SU	4. Act	N	N	N	N	Y	Y	Y	Y	N	N	N	N	Y	Y	Y	Y	N	N	N	N	Y	Y	Y	Y	N	N	N	N	Y	Y	Y	Y
ETH	5. Accept IPoE Terms	N	N	N	N	N	N	N	N	Y	Y	Y	Y	Y	Y	Y	Y	N	N	N	N	N	N	N	N	N	N	N	N	N	N	N	N
	6. Accept EGY Terms	N	N	N	N	N	N	N	N	N	N	N	N	N	N	N	N	Y	Y	Y	Y	Y	Y	Y	Y	N	N	N	N	N	N	N	N
	7. Accept EGY Reduced Terms	N	N	N	N	N	N	N	N	N	N	N	N	N	N	N	N	N	N	N	N	N	N	N	N	Y	Y	Y	Y	Y	Y	Y	Y
		1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27	28	29	30	31	32

DM in this dispute.

1. Egypt first preference is to request Ethiopia to stop construction of GERD and to modify it based on Egypt's requirements. Next, Egypt prefers that Ethiopia modify GERD based on its reduced terms. These terms would allow Ethiopia to continue building GERD until the international committee had completed its studies. After that, Egypt prefers that Ethiopia modify GERD based on the IPoE's recommendations. The least preferred scenarios for Egypt are when it does nothing and Ethiopia continues building GERD based on Ethiopia's original plans.

2. Sudan prefers, first, to act as a third party to facilitate a deal between Egypt and Sudan. Sudan's second preferences are not to interfere as long as Egypt and Ethiopia continue to negotiate a resolution to GERD. The least preferred scenarios for Sudan are when it decides not to act and both Egypt and Ethiopia halt the negotiation process.

3. Ethiopia's first preference is to modify the construction of GERD based on the IPoE's recommendations; its second, to continue with construction of GERD based on its original plans; and its third, to modify GERD founded on Egypt's reduced terms. The least preferred scenarios for Ethiopia are when it modifies GERD based on Egypt's

original terms. That means, Ethiopia halts construction of the dam and allows an international committee to conduct the studies recommended by the IPoE's report.

Table 7.11: Ranking of States for the DMs in the Conflict just before January 4, 2014

DM	States																															
Egypt	23	19	31	27	24	20	22	18	21	17	32	28	30	26	29	25	15	11	16	12	14	10	13	9	3	7	8	4	6	2	5	1
Sudan	14	15	16	22	23	24	30	31	32	10	11	12	18	19	20	26	27	28	6	7	8	13	21	29	2	3	4	9	17	25	5	1
Ethiopia	14	10	13	9	16	12	15	11	1	5	2	6	8	4	7	3	32	28	31	27	30	26	29	25	23	19	24	20	22	18	21	17
	Most preferred																										Least preferred					

After ordering the states based on each DM's preferences, one can analyze H^0 using the standard GMCR solution concept. GMCR II (Fang et al., 2003a,b) is used to perform the analysis and predict the equilibria for the dispute. The results are shown in Table 7.12. As can be seen, state 15 comprises the strong equilibrium for the conflict because it is a resolution under Nash, SEQ, GMR, and SMR. States 9, 10, 11, 12, 13, 14, and 16 are weak equilibria for the dispute because they comprise resolutions under GMR and SMR.

Historically, state 15 comprised the equilibrium of the conflict. State 15 is the situation in which Egypt request Ethiopia to stop GERD and form an international committee of experts to conduct in-depth studies about the GERD construction safety. It also represent the situation in which Ethiopia rejects Egypt's demand and continues building GERD taking into account the original IPoE's recommendations. Hence, the negotiation process between Egypt, Sudan, and Ethiopia stops as a result of their failure to achieve an agreement, and Ethiopia continues building GERD based on the IPoE's recommendations. The negotiation process between Egypt, Ethiopia, and Sudan stopped from January 2014 to August 2014 (IPoE, 2013; Cascão and Alan, 2016; Tawfik, 2016; Wheeler et al., 2016).

Table 7.12: Stability Analysis and Equilibrium Results for the Negotiation in January 2014

States		9	10	11	12	13	14	15	16
EGY	Nash	NO	NO	YES	NO	NO	NO	YES	NO
	SEQ	NO	NO	YES	NO	NO	NO	YES	NO
	GMR	YES	YES	YES	YES	YES	YES	YES	YES
	SMR	YES	YES	YES	YES	YES	YES	YES	YES
SU	Nash	NO	NO	NO	NO	YES	YES	YES	YES
	SEQ	NO	NO	NO	NO	YES	YES	YES	YES
	GMR	YES	YES	YES	YES	YES	YES	YES	YES
	SMR	YES	YES	YES	YES	YES	YES	YES	YES
ETH	Nash	YES	YES	YES	YES	YES	YES	YES	YES
	SEQ	YES	YES	YES	YES	YES	YES	YES	YES
	GMR	YES	YES	YES	YES	YES	YES	YES	YES
	SMR	YES	YES	YES	YES	YES	YES	YES	YES
Equilibrium	Nash	\	\	\	\	\	\	E	\
	SEQ	\	\	\	\	\	\	E	\
	GMR	E	E	E	E	E	E	E	E
	SMR	E	E	E	E	E	E	E	E

Table 7.13: Evolution of the Conflict just before January 4, 2014

DM	Option	Status Quo	Transitional State I	Transitional State II	Equilibrium
EGY	1. IPoE's Terms	N	N	N	N
	2. EGY's Terms	N →	Y	Y	Y
	3. EGY's Reduced Terms	N	N	N	N
SU	4. Act	N	N →	Y	Y
ETH	5. Accept IPoE's Terms	N	N	N →	Y
	6. Accept EGY's Terms	N	N	N	N
	7. Accept EGY's Reduced Terms	N	N	N	N
Label		1	3	7	15

The evolution of the conflict in early January 2014 is outlined in Table 7.13. As can be seen, the dispute started by moving from state 1, the status quo of the dispute, on the left via a transitional state, state 3, to another transitional state, state 7, to the final resolution, state 15, on the right. Egypt disputed the validity of the IPoE's report and requested Ethiopia to halt construction of GERD, and also form an international committee of experts to conduct in-depth analysis on the dam. To bridge the gap between Ethiopia and Egypt, Sudan acted as a third party to try to find a solution to the problem around the negotiation table. However, after three rounds of fruitless negotiations, Egypt and Ethiopia failed to reach an agreement. As a result, the negotiation process stopped and Ethiopia continued to build GERD based on the IPoE's recommendations. In the next section, one can see how the negotiation process continued after President Abdel Fattah el-Sisi was elected on June 8, 2014 and began handling the case of GERD (IPoE, 2013; Cascão and Alan, 2016; Tawfik, 2016; Wheeler et al., 2016).

7.5 The Conflict just before August 25, 2014

The negotiations between Egypt, Ethiopia, and Sudan resumed after the election of Egyptian President Abdel Fattah el-Sisi on June 8, 2014. In a meeting held in Khartoum on August 25, 2014, the Eastern Nile countries agreed to form an international committee of experts to conduct the studies recommended by the IPoE (IPoE, 2013; Cascão and Alan, 2016; Tawfik, 2016; Wheeler et al., 2016). Furthermore, the three nations nominated experts from their own countries to supervise the international committee's work. In this agreement, Egypt decided to drop its request to stop construction of GERD until the studies had been concluded, and Ethiopia accepted the formation of an international committee to conduct these investigations. This agreement facilitated the signing of a Declaration of Principles (DoP) (Ahramonline, 2015) among the Eastern Nile countries in March 2015. This disclosure provides some general guidelines on how to operate GERD after its construction is completed in 2017. The modeling and analysis of the conflict just before August 25, 2014 is given in the next section.

7.5.1 Modeling and Analyzing the Conflict just before August 25, 2014

The dispute between the Eastern Nile countries in August 2014 is a continuation of their negotiation that occurred in January 2014 (Matbouli et al., 2013). For two conflicts to be connected, the equilibrium state in the first round must be the status quo for the new round. State 15, the equilibrium state for the dispute that took place in January 4, 2014, was the status quo for the conflict just before August 25, 2014. Therefore, the parameters of the conflict that remained the same are the DMs and their options as shown in Table 7.9. However, DMs' preferences over the states, in Table 7.11, are changed because the DMs change their objectives. Their new preference statements are explained below:

- In this dispute, Egypt shows some willingness to cooperate with Ethiopia by dropping its request to stop construction of GERD. Hence, Egypt's first preference is to request Ethiopia to modify GERD based on Egypt's reduced terms; its second, for Ethiopia to modify the project based on its original terms. The least preferred scenarios for Egypt are when it does nothing or requests Ethiopia to modify GERD based on the IPoE's terms, and Ethiopia decides to continue building GERD (IPoE, 2013; Cascão and Alan, 2016; Tawfik, 2016; Wheeler et al., 2016).
- Ethiopia has also shown some eagerness to cooperate with Egypt in this dispute. In particular, it has displayed some willingness to accept Egypt's reduced terms. Therefore, Ethiopia's first preference is to accept modification of the construction of GERD based on Egypt's reduced terms; and second, to modify the building of GERD based on the IPoE's original report. The least preferred scenarios for Ethiopia are when it decides to modify GERD based on Egypt's original conditions (IPoE, 2013; Cascão and Alan, 2016; Tawfik, 2016; Wheeler et al., 2016).
- Sudan continued to act as a third party without any change in its preferences.

Based on the above preference statements, states are ranked from most to least preferred with respect to each DM as shown in Table 7.14. A range of solution concepts are used to investigate the dispute and predict the possible compromise resolutions for the conflict. For this analysis, a decision support system, GMCR II, was used to perform the calculations. The results are depicted in Table 7.15. As can be seen, state 32 comprises the strong equilibrium for the conflict because it is a resolution under Nash, SEQ, GMR, and SMR. States 25, 26, 27, 28, 29, 30, and 31 comprise weak equilibria for the dispute because they are resolutions under GMR and SMR.

Table 7.14: Ranking of States for the DMs in the Conflict just before August 25, 2014

DM	States																															
Egypt	32	28	30	26	29	25	23	19	31	27	24	20	22	18	21	17	15	11	16	12	14	10	13	9	3	7	8	4	6	2	5	1
Sudan	14	15	16	22	23	24	30	31	32	10	11	12	18	19	20	26	27	28	6	7	8	13	21	29	2	3	4	9	17	25	5	1
Ethiopia	32	28	31	27	30	26	29	25	14	10	13	9	16	12	15	11	1	5	2	6	8	4	7	3	23	19	24	20	22	18	21	17
	Most preferred																										Least preferred					

Historically, state 32 comprised the equilibrium of the conflict. State 32 is the situation in which both Egypt and Ethiopia agree to cooperate, with Egypt requesting Ethiopia to modify GERD based on Egypt's reduced terms to which Ethiopia agrees.

The evolution of the conflict that occurred in August 2014 is outlined in Table 7.16. As can be seen, the dispute began by moving from state 15, the status quo of the dispute and the equilibrium state for the conflict in early January 2014, on the left, via a transitional state, state 31, to the final resolution, state 32, on the right. The conflict evolved after both Egypt and Ethiopia showed some willingness to cooperate and solve the conflict. Egypt reduced its terms by allowing Ethiopia to continue building GERD simultaneously with the international committee of experts conducting their in-depth studies. Ethiopia, on the other hand, agreed to form an international committee of experts to conduct the studies and continued building GERD. As a result of this understanding between Egypt and Ethiopia, the Eastern Nile countries signed the DoP in March 2015.

Table 7.15: Stability Analysis and Equilibrium Results for the Negotiation on August 25, 2014

States		25	26	27	28	29	30	31	32
EGY	Nash	NO	NO	NO	YES	NO	NO	NO	YES
	SEQ	NO	NO	NO	YES	NO	NO	NO	YES
	GMR	YES	YES	YES	YES	YES	YES	YES	YES
	SMR	YES	YES	YES	YES	YES	YES	YES	YES
SU	Nash	NO	NO	NO	NO	YES	YES	YES	YES
	SEQ	NO	NO	YES	NO	YES	YES	YES	YES
	GMR	YES	YES	YES	YES	YES	YES	YES	YES
	SMR	YES	YES	YES	YES	YES	YES	YES	YES
ETH	Nash	YES	YES	YES	YES	YES	YES	YES	YES
	SEQ	YES	YES	YES	YES	YES	YES	YES	YES
	GMR	YES	YES	YES	YES	YES	YES	YES	YES
	SMR	YES	YES	YES	YES	YES	YES	YES	YES
Equilibrium	Nash	\	\	\	\	\	\	\	E
	SEQ	\	\	\	\	\	\	\	E
	GMR	E	E	E	E	E	E	E	E
	SMR	E	E	E	E	E	E	E	E

Table 7.16: Evolution of the Conflict just before August 25, 2014

DM	Option	Status Quo	Transitional State I	Equilibrium
EGY	1. IPoE's Terms	N	N	N
	2. EGY's Terms	Y	Y →	N
	3. EGY's Reduced Terms	N	N →	Y
SU	4. Act	Y	Y	Y
ETH	5. Accept IPoE's Terms	Y →	N	N
	6. Accept EGY's Terms	N	N	N
	7. Accept EGY's Reduced Terms	N →	Y	Y
Label		15	31	32

7.6 Chapter Summary

The overall evolution of the Eastern Nile countries' disputes over GERD is depicted in Table 7.17. As can be seen, the conflict over the Nile River water intensified when Ethiopia publicly announced, on April 11, 2011, the beginning of construction on GERD on the Blue Nile River without giving Egypt and Sudan prior notification. Egypt and Sudan were not aware of Ethiopia's intention to build GERD independently; as a result, they encountered a strategic surprise in the dispute. However, after both Egypt and Sudan became aware of their misperception, they expressed their rejection of Ethiopia's decision and requested Ethiopia to respect their respective historic water rights that had been granted to them by the 1959 agreement. Egypt and Sudan took no aggressive deterrent actions to halt Ethiopia from continuing construction of GERD. Instead, they agreed with Ethiopia to form the IPoE for the purpose of studying the adverse impacts of GERD on Egypt and Sudan. The three countries agreed to give the IPoE one year to conduct its analysis and also permitted Ethiopia to continue building GERD. However, after the release of the IPoE report in May 2013, Egypt disputed the validity of the report and requested Ethiopia to stop construction of GERD. Furthermore, Egypt asked for an international committee of experts to conduct an in-depth analysis regarding the negative impacts of the dam. Because Egypt and Ethiopia could not reach an agreement, the negotiations between them stopped, but the construction of GERD continued as shown in the second column in Table 7.17. The situation improved during the negotiation in August 25, 2014 when Egypt and Ethiopia agreed to form an international committee of experts to conduct some studies on GERD without stopping the construction of the dam. This scenario is depicted in the far right column of Table 7.17. This agreement facilitated the signing of the DoP in March 2015.

The analysis of the hydropolitical conflict between the Eastern Nile countries over GERD provided the following insights. Firstly, river agreements that allocate unfair allotment among riparian states may create conflict (Tir and Stinnett, 2011). Recall that, in 2010, Egypt and Sudan refused to sign the CAF due to the possible implications of this agreement on the volume of water each country would receive from the Nile River. Secondly, powerful states, militarily, economically, and politically, may influence the negotiations process in their own interest (Priscoli and Wolf, 2009). Since 1959, Egypt has

Table 7.17: Overall Evolution of the Eastern Nile Countries' Dispute from April 11, 2011 to August 25, 2014

DM	Option	Just Before April 11, 2011	Just Before January 4, 2014	Just Before August 25, 2014
EGY	1. Maintain the 1959 Treaty	Y →	N	N
	2. Cooperate	N	N	N
	3. IPoE's Terms	N	N	N
	4. EGY's Terms	N →	Y	N
	5. EGY's Reduced Terms	N	N →	Y
SU	6. Maintain the 1959 Treaty	Y →	N	N
	7. Cooperate	N	N	N
	8. Act	N →	Y	Y
ETH	9. Obey the 1959 Treaty	N	N	N
	10. Cooperate	N	N	N
	11. Commence Independent	Y	Y	Y
	12. Accept the IPoE's Terms	N →	Y →	N
	13. Accept EGY's Terms	N	N	N
	14. Accept EGY's Reduced Terms	N	N →	Y

controlled all the negotiations regarding the use of the Nile River water to its favor. It has also prevented any upstream countries from conducting any water resources development on the Nile River. Thirdly, geopolitical and economic changes in countries may be the reason for a new era of collaboration. As explained earlier, GERD was a cause of political and economic change in the Eastern Nile countries. These changes meant the GERD project became a reality, and cooperation between Egypt, Ethiopia, and Sudan was the only way to move forward. Because of the tumultuous geopolitical changes in Egypt, the government was not ready to capably address the conflict over GERD. If Egypt wanted to prevent GERD from becoming a reality, it should have stopped the progress of the GERD project at its earlier stages. Ethiopia, on the other hand, utilized Egypt's political instability and made significant progress on the construction of GERD. Hence, it became impossible for Egypt to prevent Ethiopia from removing GERD after Ethiopia had already completed more than 60% of the construction as of 2016. The fourth lesson that can be obtained from the case presented in this paper is the important role of the utilization of strategic surprise by a DM to achieve better results. The 2011 dispute was modeled as a second-level hypergame because Egypt and Sudan did not anticipate that Ethiopia would start building GERD without prior notification and Ethiopia was aware of Egypt and Sudan's misperception in this respect. The historical equilibrium state for the 2011 dispute, state 22, was predicted under the definition of the stealthy hyper Nash equilibrium state for a second-level hypergame (Aljefri et al., 2017b). This definition demonstrates the intended use of strategic surprise by Ethiopia to achieve results in the conflict. This equilibrium is considered to constitute unstable equilibrium because, as one saw in the analysis of the 2011 conflict, Egypt and Sudan challenged the resolution after they became aware of it. Hence, the conflict between the Eastern Nile countries continued until March 2015.

Chapter 8

Contributions and Future Opportunities

8.1 Summary of Contributions

A new and encompassing approach is developed for systematically incorporating hypergames within the framework of the graph model for any finite number of DMs and any level of perception. The methodology allows for a given DM to have misunderstandings not only about one or more of its opponents, but also about itself. Within this flexible approach, each DM perceives the set of DMs, states, state transitions, and preferences in a subjective way that reflects the DM's viewpoint of the situation under investigation. This technique provides definitions to categorize the overall hypergame equilibria into eight classes, each of which provides a unique strategic insight about the sources of misperceptions that provoke the hypergame situation. The key contributions to incorporate misperception into GMCR are explained below.

1. A first-level hypergame for the case of two- and n-DM disputes is developed in Chapter 3 to handle conflict situations having misperceptions among the participating

DMs. Specifically,

- The concept of a universal set of options for a first-level hypergame is introduced to capture all possible (correct or incorrect) options within first-level hypergame situations.
- The idea of a universal set of options for a first-level hypergame is extended to define the universal set of states for a first-level hypergame. These states cover all possible scenarios for a conflict situation, both real and factitious.
- The mathematical modeling of each DM's subjective game as well as the overall first-level hypergame in graph form are developed.
- Formal definitions are developed to classify the universal set of states for a first-level hypergame based on a DM's perception into two groups: (1) recognizable states, and (2) hidden states. Hidden states are those states that are not considered by a DM in its subjective game. Recognizable states, on the other hand, are those states that are considered by a DM in its subjective game. Furthermore, definitions are developed to partition a DM's recognizable set of states into five groups based on the type of option perception. These five classes are used in the analysis of the overall first-level hypergame.
- The four basic stability definitions for the standard graph model – Nash, SEQ, GMR, and SMR – are generalized to calculate the stability of states in each DM's subjective game within the first-level hypergame.
- An overall first-level hypergame stability analysis procedure in graph form is developed to calculate the first-level hypergame equilibria.
- Definitions are designed and implemented to categorize the first-level hypergame equilibria into eight classes to provide unique strategic insights about the sources of the misperceptions that cause the hypergame situation.

2. In Chapter 4, two real-life case studies are investigated within the architecture of a

first-level hypergame in graph form. In particular,

- The 2011 conflict between North and South Sudan over South Sudanese oil exports is modeled and analyzed as a first-level hypergame in graph form. Because of the realistic design of the new first-level hypergame in graph form, the author obtained valuable strategic insights about why the dispute evolved into another round after reaching the equilibrium state.
- The 1956 Suez Canal nationalization dispute between Egypt and Britain/the US partnership is modeled and analyzed as a first-level hypergame. The author contributed new strategic insights beyond those found earlier in the first published work by Shupe et al. (1980) which were also reported in the book of Fraser and Hipel (1984).

3. A second-level hypergame for the case of n-DM disputes is developed in Chapter 5 to handle conflict situations having misperceptions among the participating DMs and at least one DM possessing knowledge of the other DMs' misperceptions. More specifically,

   - The concepts of universal sets of options and states for a first-level hypergame are extended to define the universal sets of options and states for a second-level hypergame.
   - The mathematical modeling of a second-level hypergame in graph form is developed.
   - Definitions to partition the universal set of states for a second-level hypergame into two sets based on a DM's perception are developed.
   - Stability analysis procedures are developed to analyze each DM's subjective first-level hypergame within the second-level hypergame in graph form.
   - Stability analysis procedures are developed to analyze the overall second-level hypergame.

- The classification of a first-level hypergame equilibria is extended to categorize the second-level hypergame equilibria into eight classes.

4. In Chapter 6, first-level hypergame in graph form is extended to accommodate any level of DMs' perception in real-life conflicts. In particular,

   - The concepts of the universal sets of options and states for a first-level hypergame are extended to any h level of DMs' perception.
   - The structure of a DM's subjective hypergame is developed in a hierarchical fashion.
   - The mathematical modeling of each DM's subjective game as well as the overall hypergame are developed in graph form.
   - A stability analysis method is developed to analyze each DM's subjective hypergame.
   - An overall stability analysis procedure is introduce to analyze the overall hypergame.
   - The overall hypergame equilibria are classified into eight groups as done for the first-level hypergame.

5. In Chapter 7, an application to a real-world conflict is investigated within the structure of a hypergame in graph form. In particular, a second-level hypergame stability analysis is carried out on the 2011 conflict among Egypt, Ethiopia, and Sudan over the unexpected construction of GERD by the Ethiopian government. The author employs the second-level hypergame stability definitions for an n-DM graph model. This analysis predicts a strong equilibrium, which is a stealthy hyper Nash equilibrium for the dispute. It demonstrates the significant utilization of strategic surprise by the Ethiopian government to achieve a firm outcome in the dispute. Because Egypt and Sudan underestimated Ethiopia's capability to independently build a dam on the

Blue Nile River, they faced strategic surprise when Ethiopia announced its decision, on April 11, 2011, to construct a massive hydroelectric dam on the Blue Nile River as a national project. Also, because Ethiopia was aware of Egypt and Sudan's misperception as well as their political instability, it was successful in launching the GERD project without any harsh response from Egypt and Sudan. The categorization of the overall second-level hypergame equilibria assists the author in obtaining valuable strategic insights about this dispute.

8.2 Future Work

The hypergame method in graph form constitutes a comprehensive approach for modeling and analyzing misperceptions within the structure of GMCR. In fact, a hypergame expressed in graph form, introduced in this dissertation, is a fresh concept and may therefore be combined with recent expansions within the paradigm of GMCR. A number of ideas for future research are listed below:

- *Learning within the Hypergame Approach in Graph Form*: The eight classes of the overall hypergame equilibria could be initially studied to identify learning situations that will increase a DM's ability to correct its perception, and thereby take appropriate actions in the process of making an informed decision.

- *Matrix Representation of the Hypergame Analysis in Graph Form*: Xu et al. (2009) developed a matrix representation of GMCR's stability definitions for utilization in computer coding. A hypergame could be formulated using matrix methods for possible software development.

- *Robustness of the Hypergame Equilibria:* Matbouli et al. (2015) developed a concept that measures the robustness of equilibria in the standard GMCR. This concept can be appropriately revised for employment in hypergame theory in graph form to investigate if the overall hypergame equilibria are final or temporary.

- *Inverse Hypergame Analysis in Graph Form*: Kinsara et al. (2015a) developed an inverse GMCR technique to identify DMs' preferences at a given desired outcome. This technique can be used as a negotiation support tool. Currently, the inverse GMCR approach assumes common perception among the engaging DMs. Since many real-life situations possess misperceptions among the participating DMs, it is useful to combine the hypergame methodology in graph form, developed herein, with the inverse GMCR approach.

References

Abbas, A. (2016). Ethiopia Close to Finishing 70% of Grand Ethiopian Renaissance Dam. *Awrambatimes*. `http://www.awrambatimes.com/?p=14880`. Accessed August 11, 2016.

Abdelhady, D., Aggestam, K., Andersson, D.-E., Beckman, O., Berndtsson, R., Palmgren, K. B., Madani, K., Ozkirimli, U., Persson, K. M., and Pilesjö, P. (2015). The Nile and the Grand Ethiopian Renaissance Dam: Is there a Meeting Point between Nationalism and Hydrosolidarity? *Journal of Contemporary Water Research and Education*, 155(1):73–82.

AfDB, OECD, and UNDP (2016). *African Economic Outlook 2016: Sustainable Cities and Structural Transformation*. OECD Publishing, Paris.

Ahramonline (2013). President Morsi Calls for Egyptian 'Unity' in Face of Threats to Nile Water. *Ahramonline*. `http://tinyurl.com/jkpr68z`. Accessed August 11, 2016.

Ahramonline (2015). Full Text of 'Declaration of Principles' Signed by Egypt, Sudan and Ethiopia. *Ahramonline*. `http://tinyurl.com/zrf9mp7`. Accessed August 11, 2016.

Aljefri, Y. M., Abul Bashar, M., Fang, L., and Hipel, K. W. (2017a). First-Level Hypergame for Investigating Misperception in Conflicts. `http://dx.doi.org/10.1109/TSMC.2017.2690619`. *IEEE Transactions on Systems, Man and Cybernetics: Systems*, 18 pages.

Aljefri, Y. M., Abul Bashar, M., Hipel, K. W., and Fang, L. (2015). Generating Hypergame States within the Paradigm of the Graph Model for Conflict Resolution. *Presented at*

GDN (Group Decision and Negotiation) 2015, Warsaw, Poland, June 22-26, 2015. Refereed extended abstract published in the Proceedings of the 15th International Conference on Group Decision and Negotiation, pages 249–252.

Aljefri, Y. M., Fang, L., and Hipel, K. W. (2014a). Misperception of Preferences in the Graph Model for Conflict Resolution. *Presented at GDN (Group Decision and Negotiation) 2014, Toulouse, France, June 10-13, 2014*, pages 200–207.

Aljefri, Y. M., Fang, L., and Hipel, K. W. (2014b). Modeling Misperception of Options and Preferences in the Graph Model for Conflict Resolution. In *Proceedings of the 2014 IEEE International Conference on Systems, Man, and Cybernetics*, pages 1592–1597, San Diego, California, October 5 to 8.

Aljefri, Y. M., Hipel, K. W., and Fang, L. (2013). Oil Export Pipeline Conflict between North and South Sudan. *Presented at GDN (Group Decision and Negotiation), Stockholm, Sweden, June 17-20, 2013. Extended abstract published in the Proceedings of the International Conference on Group Decision and Negotiation (GDN) 2013*, pages 31–34.

Aljefri, Y. M., Hipel, K. W., and Fang, L. (2017b). Second-level Hypergame within the Graph Model Framework. *Department of Systems Design Engineering, University of Waterloo, Waterloo, ON, Canada.*

Aljefri, Y. M., Hipel, K. W., Fang, L., and Abul Bashar, M. (2016a). Misperception in Nationalization of the Suez Canal. In *Proceedings of the 2016 IEEE International Conference on Systems, Man, and Cybernetics*, pages 355–360, Budapest, Hungary, October 9 to 12.

Aljefri, Y. M., Hipel, K. W., Fang, L., and Abul Bashar, M. (2016b). Second-level Hypergame with Two Decision Makers in Graph Form. *Presented at GDN (Group Decision and Negotiation) 2016, Western Washington University, June 20-24, 2016. Refereed extended abstract published in the Proceedings of the 16th International Conference on Group Decision and Negotiation.*

Aljefri, Y. M., Hipel, K. W., Fang, L., and Madani, K. (2016c). The Conflict Between the Upstream and Downstream Countries Over the Construction of the Grand Ethiopian

Renaissance Dam. *Presented at The 7th International Conference on Water Resources and Environment Research (ICWRER2016) 2016, Kyoto TERRSA, Kyoto, Japan, June 5-9,* pages s05-04-1 to s05-04-2.

Arjoon, D., Mohamed, Y., Goor, Q., and Tilmant, A. (2014). Hydro-Economic Risk Assessment in the Eastern Nile River Basin. *Water Resources and Economics*, 8:16–31.

Aumann, R. J. and Maschler, M. (1995). *Repeated Games with Incomplete Information.* MIT Press, Cambridge, MA.

Ben-Zvi, A. (1995). Perception, Misperception and Surprise in the Yom Kippur War: A Look at the New Evidence. *Journal of Conflict Studies*, 15(2):5–29.

Bennett, P. G. (1977). Toward a Theory of Hypergames. *Omega*, 5(6):749–751.

Bennett, P. G. (1980). Hypergames: Developing a Model of Conflict. *Futures*, 12(6):489–507.

Bennett, P. G. and Dando, M. (1979). Complex Strategic Analysis: A Hypergame Study of the Fall of France. *Journal of the Operational Research Society*, 30(1):23–32.

Bennett, P. G., Huxham, C., and Dando, M. (1981). Shipping in Crisis: A Trial Run for 'Live' Application of the Hypergame Approach. *Omega*, 9(6):579–594.

Betts, R. K. (2000). Is Strategy an Illusion? *International Security*, 25(2):5–50.

Blackmore, D. and Whittington, D. (2008). Opportunities for Cooperative Water Resources Development on the Eastern Nile: Risks and Rewards. *Report to the Eastern Nile Council of Ministers, Nile Basin Initiative, Entebbe, Uganda*.

Bluth, C. (2004). The British Road to War: Blair, Bush and the Decision to Invade Iraq. *International Affairs (Royal Institute of International Affairs)*, 80(5):871–892.

Brams, S. J. (1977). Deception in 2×2 Games. *Conflict Management and Peace Science*, 2(2):171–203.

Brosché, J. (2008). CPA–New Sudan, Old Sudan or Two Sudans? A Review of the Implementation of the Comprehensive Peace Agreement. *Dahre, U. J. Ed., Post-Conflict Peace-Building in the Horn of Africa*, pages 231–252, Department of Social Anthropology, Political Science, Lund University, Sweden.

Brunnée, J. and Toope, S. J. (2002). The Changing Nile Basin Regime: Does Law Matter? *Harvard International Law Journal*, 43:105–110.

Cascão, A. E. (2012). Nile Water Governance. *The Nile River Basin: Water, Agriculture, Governance and Livelihoods, Routledge: UK*, pages 229–252.

Cascão, A. E. and Alan, N. (2016). GERD: New Norms of Cooperation in the Nile Basin? *Water International*, 41(4):550–573.

CIA (2016). The World Fact Book (Ethiopia). *Central Intelligence Agency (CIA).*

Craig, G. M., editor (1991). *The Agriculture of the Sudan*. Oxford University Press, Oxford, UK.

Dahan, M. E. (2009). Egypt Says Historic Nile River Rights Not Negotiable, *Reuters*. `http://www.reuters.com/article/us-egypt-nile-framework-idUSTRE56Q3LZ20090727`. Accessed August 11, 2016.

Davison, W. (2011). Ethiopia Compels Private Banks to Buy Bonds to Fund Development. *Bloomberg*. `http://tinyurl.com/zwye93u`. Accessed August 11, 2016.

Degefu, G. T. (2003). *The Nile: Historical, Legal and Developmental Perspectives*. Trafford Publishing, Victoria, Canada.

Fang, L., Hipel, K. W., and Kilgour, D. M. (1993). *Interactive Decision Making: The Graph Model for Conflict Resolution*. Wiley, New York.

Fang, L., Hipel, K. W., Kilgour, D. M., and Peng, X. (2003a). A Decision Support System for Interactive Decision Making-Part I: Model Formulation. *IEEE Transactions on Systems, Man, and Cybernetics, Part C: Applications and Reviews*, 33(1):42–44.

Fang, L., Hipel, K. W., Kilgour, D. M., and Peng, X. (2003b). A Decision Support System for Interactive Decision Making-Part II: Analysis and Output Interpretation. *IEEE Transactions on Systems, Man, and Cybernetics, Part C: Applications and Reviews*, 33(1):56–66.

FAO (2002). The Nile Basin Initiative Act, *Food and Agriculture Organization of the United Nations*. Accessed August 11, 2016.

Fraser, N. M. and Hipel, K. W. (1979). Solving Complex Conflicts. *IEEE Transactions on Systems, Man and Cybernetics*, 9(12):805–816.

Fraser, N. M. and Hipel, K. W. (1984). *Conflict Analysis: Models and Resolutions*. North-Holland, New York.

Fraser, N. M., Hipel, K. W., and Monte, J. R. D. (1983). Approaches to Conflict Modeling: A Study of a Possible USA-USSR Nuclear Confrontation. *Journal of Policy Modeling*, 5(3):397–417.

Gharesifard, B. and Cortes, J. (2011). Learning of Equilibria and Misperceptions in Hypergames with Perfect Observations. In *Proceedings of the 2011 American Control Conference*, pages 4045–4050, San Francisco, California, June 29 to July 1.

Gharesifard, B. and Cortes, J. (2012). Evolution of Players' Misperceptions in Hypergames Under Perfect Observations. *IEEE Transactions on Automatic Control*, 57(7):1627–1640.

Gharesifard, B. and Cortes, J. (2014). Stealthy Deception in Hypergames Under Informational Asymmetry. *IEEE Transactions on Systems, Man, and Cybernetics*, 44(6):785–795.

Harsanyi, J. C. (1967). Games with Incomplete Information Played by "Bayesian" Players, Part I. The Basic Model. *Management Science*, 14(3):159–182.

Harsanyi, J. C. (1968a). Games with Incomplete Information Played by "Bayesian" Players, Part II. Bayesian Equilibrium Points. *Management Science*, 14(5):320–334.

Harsanyi, J. C. (1968b). Games with Incomplete Information Played by 'Bayesian' Players, Part III. The Basic Probability Distribution of the Game. *Management Science*, 14(7):486–502.

Hipel, K. W., editor (2009a). *Conflict Resolution, Encyclopedia of Life Support Systems*, volume 1. Eolss Publishers, Oxford, UK.

Hipel, K. W., editor (2009b). *Conflict Resolution, Encyclopedia of Life Support Systems*, volume 2. Eolss Publishers, Oxford, UK.

Hipel, K. W., Wang, M., and Fraser, N. M. (1988). Hypergame Analysis of the Falkland/Malvinas Conflict. *International Studies Quarterly*, 32(3):335–358.

Howard, N. (1971). *Paradoxes of Rationality: Theory of Metagames and Political Behavior*. MIT Press, Cambridge, MA.

Inohara, T., Takahashi, S., and Nakano, B. (1997). Integration of Games and Hypergames Generated from a Class of Games. *The Journal of the Operational Research Society*, 48(4):423–432.

International Rivers (2014). The Grand Ethiopian Renaissance Dam Fact Sheet, *International Rivers*. `http://tinyurl.com/hc7szpx`. Accessed August 11, 2016.

IPoE (2013). International Panel of Experts (IPoE) on Grand Ethiopian Renaissance Dam Project (GERDP) Final Report. `http://tinyurl.com/jfly28p`. Accessed August 11, 2016.

Jervis, R. (1968). Hypotheses on Misperception. *World Politics*, 20(3):454–479.

Jervis, R. (1976). *Perception and Misperception in International Politics*. Princeton University Press, Princeton, New Jersey.

Kaufmann, C. (2004). Threat Inflation and the Failure of the Marketplace of Ideas: The Selling of the Iraq War. *International Security*, 29(1):5–48.

Kilgour, D. M. and Eden, C., editors (2010). *Handbook of Group Decision and Negotiation.* Springer, Dordrecht, New York.

Kilgour, D. M. and Hipel, K. W. (2005). The Graph Model for Conflict Resolution: Past, Present, and Future. *Group Decision and Negotiation*, 14(6):441–460.

Kilgour, D. M., Hipel, K. W., and Fang, L. (1987). The Graph Model for Conflicts. *Automatica*, 23(1):41–55.

Kinsara, R. A., Kilgour, D. M., and Hipel, K. W. (2015a). Inverse Approach to the Graph Model for Conflict Resolution. *IEEE Transactions on Systems, Man, and Cybernetics: Systems*, 45(5):734–742.

Kinsara, R. A., Petersons, O., Hipel, K. W., and Kilgour, D. M. (2015b). Advanced Decision Support for the Graph Model for Conflict Resolution. *Journal of Decision Systems*, 24(2):117–145.

Kuang, H., Bashar, M. A., Hipel, K. W., and Kilgour, D. M. (2015). Grey-based Preference in a Graph Model for Conflict Resolution with Multiple Decision Makers. *IEEE Transactions on Systems, Man, and Cybernetics: Systems*, 45(9):1254–1267.

Leary, M. R. (1995). *Self-Presentation: Impression Management and Interpersonal Behavior.* Brown & Benchmark Publishers.

Levy, J. S. (1983). Misperception and the Causes of War: Theoretical Linkages and Analytical Problems. *World Politics*, 36(1):76–99.

Luce, R. D. and Raiffa, H. (1957). *Games and Decisions: Introduction and Critical Surveys.* Wiley, New York.

Madani, K. (2010). Game Theory and Water Resources. *Journal of Hydrology*, 381(3):225–238.

Madani, K. and Hipel, K. W. (2007). Strategic Insights Into the Jordan River Conflict. In *World Environmental and Water Resources Congress 2007: Restoring Our Natural Habitat*, pages 1–10.

Madani, K., Rheinheimer, D., Elimam, L., and Connell-Buck, C. (2011). A Game Theory Approach to Understanding the Nile River Basin Conflict. *A Water Resource" Festschrift in Honor of Professor Lars Bengtsson, Division of Water Resources Engineering*, pages 97–114.

Matbouli, Y. T., Hipel, K. W., and Kilgour, D. M. (2013). Characterization of a Conflict. In *2013 IEEE International Conference on Systems, Man, and Cybernetics (SMC)*, pages 2050–2054. IEEE.

Matbouli, Y. T., Kilgour, D. M., and Hipel, K. W. (2015). Robustness of Equilibria in the Graph Model for Conflict Resolution. *Journal of Systems Science and Systems Engineering*, 24(4):450–465.

Ministry of Water, Irrigation, and Electricity (2016). Grand Ethiopian Renaissance Dam Project Overview, *Ethiopia's Ministry of Water, Irrigation, and Electricity*, August 11, 2016.

MIT (2014). The Grand Ethiopian Renaissance Dam: An Opportunity for Collaboration and Shared Benefits of the Eastern Nile Basin. *An Amicus brief to the Riparian Nations of Ethiopia, Sudan and Egypt from the International, Non-partisan Eastern Nile Working Group Convened at the Massachusetts Institute of Technology on 13–14 November 2014 by the MIT Abdul Latif Jameel World Water and Food Security Lab. Cambridge, MA: MIT and Abdul Lateef Jameel World Water and Food Security Lab.*

Nash, J. F. (1950). Equilibrium Points in n-Person Games. *Proceedings of the National Academy of Sciences of the United States of America*, 36:48–49.

Nash, J. F. (1951). Noncooperative Games. *Annals of Mathematics*, 2:286–295.

Nile Basin Initiative (2010). Agreement on the Nile River Basin Cooperative Framework. *Nile Basin Initiative.*

Obeidi, A., Hipel, K. W., and Kilgour, D. M. (2005). Perception and Emotion in the Graph Model for Conflict Resolution. In *Proceedings of the 2005 IEEE International*

Conference on Systems, Man, and Cybernetics, pages 1126–1131, Waikoloa, HI, October 10 to 12.

Obeidi, A., Kilgour, D. M., and Hipel, K. W. (2009). Perceptual Stability Analysis of a Graph Model System. *IEEE Transactions on Systems, Man and Cybernetics, Part A: Systems and Humans*, 39(5):993–1006.

Odidi, O. C. (1994). History of the Nile and Lake Victoria Basins Through Treaties. *Cambridge University Press* , pages 321–350.

Oil and Energy Trends (2011). Focus: South Sudan Seeks New Role as Oil Exporter. *Oil and Energy Trends*, 36(6):3–6.

Okada, N., Hipel, K. W., and Oka, Y. (1985). Hypergame Analysis of the Lake Biwa Conflict. *Water Resources Research*, 21(7):917–926.

Passer, M., Smith, R., Atkinson, M., Mitchell, J., and Muir, D. (2011). *Psychology: Frontiers and Applications, 4^{th} Canadian edition*. McGraw Hill Ryerson, Whitby, Ontario.

Pollack, K. M. (2004). Spies, Lies, and Weapons: What Went Wrong. *The Atlantic Monthly*, 293(1):78–92.

Priscoli, J. D. and Wolf, A. T. (2009). *Managing and Transforming Water Conflicts (International Hydrology Series)*. Cambridge University Press.

Ramadan, S., Negm, A., Smanny, M., and Helmy, A. (2013). Environmental Impacts of Great Ethiopian Renaissance Dam on the Egyptian Water Resources Management and Security. In *The 23rd International Conference On: Environmental Protection is a Must*, pages 11–13.

Rêgo, L. C. and dos Santos, A. M. (2015). Probabilistic Preferences in the Graph Model for Conflict Resolution. *IEEE Transactions on Systems, Man, and Cybernetics: Systems*, 45(4):595–608.

Rosenhead, J., editor (1989). *Rational Analysis for a Problematic World: Structuring Methods for Complexity, Uncertainty and Conflict*. Wiley, Chichester, UK.

Rosenhead, J. and Mingers, J., editors (2001). *Rational Analysis for a Problematic World Revisited*. Wiley, Chichester, UK.

Rummel, R. J. (1975). *Understanding Conflict and War: The Dynamic Psychological Field*. Wiley, New York.

Salman, S. M. (2013). The Nile Basin Cooperative Framework Agreement: A Peacefully Unfolding African Spring? *Water International*, 38(1):17–29.

Salman, S. M. A. (2016). The Grand Ethiopian Renaissance Dam: The Road to the Declaration of Principles and the Khartoum Document. *Water International*, 41(4):512–527.

Sammaan, M. (2014). *The Win-Win-win Scenario in the Blue Nile's Hydropolitical Game: Application on the Grand Ethiopian Renaissance Dam*. Forschungsberichte aus dem Institut für Sozialwissenschaften (ISW): Institut für Sozialwissenschaften. ISW.

Sasaki, Y., Hämäläinen, R. P., and Saarinen, E. (2015). Modeling Systems of Holding Back as Hypergames and their Connections with Systems Intelligence. *Systems Research and Behavioral Science*, 32(6):593–602.

Sasaki, Y. and Kijima, K. (2008). Preservation of Misperceptions - Stability Analysis of Hypergames. In *Proceedings of the* 52^{nd} *Annual Conference of the International Society for the Systems Sciences*, volume 2, pages 615–629, Madison, Wisconsin, July 13-18.

Sasaki, Y., Kobayashi, N., and Kijima, K. (2007). Mixed Extension of Hypergames and its Applications to Inspection Games. *Proceedings of the* 51^{st} *Annual Meeting of the International Society for the Systems Sciences, Tokyo, Japan, 2007*, pages 1035–1043.

Shahin, M. M. (1985). *Hydrology of the Nile Basin*, volume 21. Elsevier, Amsterdam, The Netherlands.

Shupe, M. C., Wright, W. M., Hipel, K. W., and Fraser, N. M. (1980). Nationalization of the Suez Canal: A Hypergame Analysis. *The Journal of Conflict Resolution*, 24(3):477–493.

Song, Y., Li, W., and Zeng, X. (2009). λ-Equilibrium of Hypergame with Fuzzy Strategy and Preference Perceptions. *Proceedings of the 2009 IEEE International Conference on Intelligent Computing and Intelligent Systems, Shanghai, China, November 20 to 22, 2009, Volume 2*, pages 641–645.

Stein, A. A. (1982). When Misperception Matters. *World Politics*, 34(4):505–526.

Stoessinger, J. G. (2008). *Why Nations Go to War.* Thomson/Wadsworth, Belmont, California.

Sudan Tribune (2011a). S. Sudan Warns Oil Companies Over Illegal Deals With Sudan.

Sudan Tribune (2011b). South Sudan Threatens to Suspend Oil Production if North Imposes Charges Unilaterally.

Sudan Tribune (2013). Sudan Reiterates Support of Ethiopian Dam Plans. *Sudan Tribune*, August 11, 2016. `http://tinyurl.com/o2t3xnf`.

Sudan Tribune (2014). Egypt Wants Sudan to Mediate in Nile Water Tripartite Meeting. *Sudan Tribune*. `http://www.sudantribune.com/spip.php?article52155`. Accessed August 11, 2016.

Swain, A. (1997). Ethiopia, the Sudan, and Egypt: The Nile River Dispute. *The Journal of Modern African Studies*, 35(04):675–694.

Takahashi, M. A., Fraser, N. M., and Hipel, K. W. (1984). A Procedure for Analyzing Hypergames. *European Journal of Operational Research*, 18(1):111–122.

Tawfik, R. (2016). The Grand Ethiopian Renaissance Dam: A Benefit-Sharing Project in the Eastern Nile? *Water International*, 41(4):574–592.

The World Bank (2009). The Nile Basin Initiative (NBI): Building a Cooperative Future, *The World Bank*, August 11, 2016.

The World Bank (2013). Ethiopia Economic Update: Laying the Foundation for Achieving Middle Income Status, *The World Bank*, August 11, 2016.

The World Bank (2016). The Little Data Book, *The World Bank*, August 11, 2016.

Tir, J. and Stinnett, D. M. (2011). The Institutional Design of Riparian Treaties The Role of River Issues. *Journal of Conflict Resolution*, 55(4):606–631.

Trivers, R. (2000). The Elements of a Scientific Theory of Self-Deception. *Annals of the New York Academy of Sciences*, 907(1):114–131.

Tuchman, B. W. (1985). *The March of Folly: From Troy to Vietnam*. Random House, New York.

USBR (1964). *United States Bureau of Reclamation, Land and Water Resources of the Blue Nile Basin, Ethiopia*. Washington, DC: United States Department of Interior; US Government Printing Office.

Von Neumann, J. and Morgenstern, O. (1944). *Theory of Games and Economic Behavior, 1st edition*. Princeton University Press, Princeton, NJ.

Walker, S. B., Hipel, K. W., and Xu, H. (2013). A Matrix Representation of Attitudes in Conflicts. *IEEE Transactions on Systems, Man, and Cybernetics: Systems*, 43(6):1328–1342.

Wang, M., Hipel, K. W., and Fraser, N. M. (1988). Modeling Misperceptions in Games. *Behavioral Science*, 33(3):207–223.

Wang, M., Hipel, K. W., and Fraser, N. M. (1989). Solution Concepts in Hypergames. *Applied Mathematics and Computation*, 34(3):147–171.

Wheeler, K. G., Basheer, M., Mekonnen, Z. T., Eltoum, S. O., Mersha, A., Abdo, G. M., Zagona, E. A., Hall, J. W., and Dadson, S. J. (2016). Cooperative Filling Approaches for the Grand Ethiopian Renaissance Dam. *Water International*, 41(4):1–24.

Whittington, D., Waterbury, J., and Jeuland, M. (2014). The Grand Renaissance Dam and Prospects for Cooperation on the Eastern Nile. *Water Policy*, 16(4):595–608.

Wohlstetter, R. (1962). *Pearl Harbor: Warning and Decision*. Stanford University Press, Stanford, California.

Wright, W. M., Shupe, M. C., Fraser, N. M., and Hipel, K. W. (1980). A Conflict Analysis of the Suez Canal Invasion of 1956. *Conflict Management and Peace Science*, 5(1):27–40.

Xu, H., Hipel, K. W., and Kilgour, D. M. (2009). Matrix Representation of Solution Concepts in Multiple-Decision-Maker Graph Models. *IEEE Transactions on Systems, Man, and Cybernetics-Part A: Systems and Humans*, 39(1):96–108.

Xu, H., Hipel, K. W., Kilgour, D. M., and Fang, L. (2017). *Conflict Resolution Using the Graph Model: Strategic Interactions in Competition and Cooperation*. Book Manuscript, Department of Systems Design Engineering, University of Waterloo, Waterloo, ON, Canada, 2017.

Yihdego, Z., Rieu-Clarke, A., and Cascão, A. E. (2016). How has the Grand Ethiopian Renaissance Dam Changed the Legal, Political, Economic and Scientific Dynamics in the Nile Basin? *Water International*, 41(4):503–511.