# The Economics of Waste Clean-Up from Resource Extraction Projects: Environmental Bonds versus Strict Liability

by

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## Declaration

This thesis consists of material all of which I authored or co-authored: see Statement of Contributions included in the thesis. This is a true copy of the thesis, including any required final revisions, as accepted by my examiners.

I understand that my thesis may be made electronically available to the public.

## Statement of Contribution

Chapters 1 and 2 are co-authored with my supervisor, Professor Margaret Insley. I contributed to all stages of research in both of these chapters including the development of the research question, formulation of the economic model, implementation of the numerical solution, and interpretation of the results. Chapter 3 is written solely by myself.

**Abstract**

This thesis contains three essays spanning the fields of environmental economics and investment in a non-renewable resource under uncertainty. All essays relate to the analysis of the clean-up of hazardous waste resulting from natural resource extraction.

The first essay addresses the problem of inadequate hazardous waste clean-up by resource extraction firms. It compares the impacts of an environmental bond and a strict liability rule on a firm's ongoing waste abatement and eventual site clean-up decisions. The firm's problem is modeled as a stochastic optimal control problem that results in a system of Hamilton Jacobi Bellman equations. The model is applied to a typical copper mine in Canada. The resource price is modeled as a stochastic differential equation, which is calibrated to copper futures prices using a Kalman filtering approach. A numerical solution is implemented to determine the optimal abatement and extraction rates as well as the critical levels of copper prices that would motivate a firm to clean up the accumulated waste under each policy. We have found that the effect of an environmental bond relative to the strict liability rule depends on certain key characteristics of the bond - in particular whether the bond pays interest and whether the firm borrows at a premium above the risk-free rate to fund the bond. If the firm can borrow at the risk-free rate, and if the government pays the risk-free interest rate on the bond, the value of the mine prior to construction, optimal abatement rates, and optimal operating decisions are the same under the bonding policy and strict liability rule. In contrast, if no interest is paid on the bond, the value of the project is reduced compared to the strict liability rule and the firm undertakes a larger amount of waste abatement under the bond. Because the mine is less profitable, it is less likely that the firm will invest in this mine. In the more realistic case that the firm borrows to fund the bond at a premium over the risk-free rate, the value of mine is reduced further and waste abatement levels are increased. The prospect of investment in the mine is even less likely compared to the previous case.

The model developed in the first essay allows that the firm temporarily mothballs the project, but eventually clean-up must occur at the end of the project life. However, the possibility of firm bankruptcy was not explicitly included in that model, and thus mothballing is the only option available to the firm to delay waste clean-up. The second

essay contributes to our previous study by considering another important option available to the firm, i.e., the possibility of declaring bankruptcy. A firm's decision to declare bankruptcy is specified as a Poisson process that treats bankruptcy as an exogenous, risky event governed by a hazard rate. The hazard rate at a project level depends on waste stock and output prices, while at the company wide level depends on the commodity prices only. For both default scenarios, the paper demonstrates that the firm operating under a bonding policy, that covers the full cost of waste clean-up, is less able to avoid its liability costs, particularly if the bond is financed from retained earnings. If the firm borrows to finance the bond, it is possible that the firm avoids clean-up costs by defaulting on the loan following a bankruptcy. In contrast to the results of the first essay, if the firm finances the bond out of its retained earnings, and if the government pays the risk-free rate of interest on the bond, the bond and the strict liability rule do not give the same outcome when bankruptcy is possible. Such a bond encourages a higher abatement rate and makes site clean-up more likely compared to the strict liability rule. Firms operating under the liability rule have stronger incentives to delay their clean-up costs by sitting idle and they may eventually go bankrupt at the mothballed stage. Therefore, the possibility of bankruptcy makes the firm worse off under the bonding policy, while benefits the firm under the strict liability rule.

Modelling uncertain commodity prices is a key component of the analysis of optimal firm behavior in hazardous waste clean-up. The third essay investigates the dynamics of copper prices by comparing and contrasting three different stochastic models, which are a one-factor mean-reverting model, a two-factor model, and a one-factor long-term model. These models are calibrated to copper futures prices using a Kalman filtering approach. The first model assumes spot prices are mean reverting in drift. The second model defines two correlated stochastic factors that are spot prices and convenience yield. The third model transforms the two-factor price model into a single factor model. We have found that the first model fails to describe the term structure of copper futures prices with long maturities. In contrast, the two-factor and the long-term models are shown to provide a reasonable fit of the term structure of copper futures prices and can be applied to long-term investment projects. The results highlight the importance of stochastic convenience yield in copper price formation.

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To the most beautiful persons in my life

my parents, Zahra and Ghasem

my husband, Soroosh

and

my son, Shervin

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# Chapter 1

# Optimal Timing of Hazardous Waste Clean-Up under an Environmental Bond and a Strict Liability Rule

## 1.1 Introduction

Hazardous waste production is a significant consequence of large natural resource projects such as mines. Such waste is often disposed of into local ecosystems and can impose high risks on society during mining operations and after a mine is abandoned. Without appropriate regulations, profit maximizing firms are likely to generate more waste than is desirable and are unlikely to undertake adequate waste clean-up. This problem is commonly dealt with through the imposition of a strict liability rule,[1] whereby the agent is held legally responsible for waste clean-up or restoration upon project termination. An obligation for restoration under the strict liability rule increases the cost of mine abandonment, which may cause some firms to choose to remain inactive as a way to escape restoration

[1]Strict liability refers to the imposition of liability on a firm regardless of whether the firm has adhered to accepted standards of care. This may be contrasted with the negligence standard, where a firm is only liable if it has acted been negligently.

costs,[2] even when there is no hope for reactivation (Muehlenbachs, 2015). Restoration requirements also increase the risk of default on environmental obligations due to insolvency or bankruptcy. Surveys reveal that large numbers of mining operations in the US and Canada have been abandoned due to bankruptcy resulting in significant environmental damages and clean-up costs. In the case of firm bankruptcy, the environmental liability may fall to government with restoration costs funded out of general tax revenue, leading to a dead-weight loss (Campbell and Bond, 1997). For various reasons, the clean-up cost to government may be higher than for the firm, including the need to hire outside contractors (Ferreira et al., 2004). For governments, the potential for highly negative media coverage and public outrage is another undesirable consequence of firms shirking their clean-up obligations.

In practice, environmental bonds, as a complement to the strict liability rule, have been widely used to address these issues by attempting to ensure adequate funds are available for end-of-activity restoration.[3] Under an environmental bond, a firm estimates and reports its expected future clean-up costs based on current knowledge and deposits a bond of an equivalent amount. The amount deposited for the bond may be updated over time as the firm's expected clean-up costs are revised. The government releases the funds upon successful closure and restoration; otherwise it retains them. Environmental bonds are intended to simulate all future adverse effects, consider them in present terms, and internalize the associated clean-up costs (Perrings, 1989).[4] However, without a specific template for cost estimations and also the absence of a third-party verification, firms may underestimate their clean-up costs. If the bond amount is inadequate and if a firm walks away from its obligation, clean-up costs will be transferred to the government. In 2009, over 10,000 mines operating under an environmental bonding regulation in Canada were

[2] Waste clean-up costs are mine specific, can range from millions to billions of dollars for a single mine (Boyd, 2002, Grant et al., 2009), and depend on the extent of activity, the expected difficulty of restoration, etc (Grant et al., 2009).

[3] See World Bank (2009) for a survey of bonding practices in extractive industries worldwide. This document is available at `http://siteresources.worldbank.org/EXTOGMC/Resources/COCPObrochureFINAL.pdf?resourceurlname=COCPObrochureFINAL.pdf`.

[4] Peck and Sinding (2009) note that environmental bonds can be deposited through a variety of mechanisms such as cash deposited in a trust fund, letters of credit, and pledge of assets. The current practice of such different mechanisms are surveyed by Miller (2005), and the incentives for environmental protection by US hazardous waste managers under each mechanism are compared in Zhou (2014).

classified as abandoned without being cleaned up and with insufficient funds for restoration (Grant et al., 2009). For instance, the Faro Mine in the Yukon Territory set aside $93.8 million for restoration resulting in a $356 million government's liability, and the Giant Mine in the Northwest Territories deposited only $400,000 environmental bonds and transferred $399 million uncompensated clean-up costs to society (Grant et al., 2009). An adequate level of environmental bond increases the likelihood that a firm will meet its obligation to clean up a contaminated site, rather than shirking their clean-up obligations through project mothballing or declaring bankruptcy. This fact is confirmed by an empirical study for the US oil and gas producers (Boomhower, 2014).

Given the empirical importance of clean-up costs, it is surprising that the literature has devoted little attention to a deep analysis of their likely impacts on mining firms' operating decisions. Some studies assume zero costs for site restoration at the termination date (Almansour and Insley, 2016, Brennan and Schwartz, 1985), while abandonment is completely overlooked in some other research (Dixit, 1989, 1992, Mason, 2001, Slade, 2001). More recently, one study has examined optimal extraction of a non-renewable resource with the resource price modeled as regime switching stochastic process and assuming a positive restoration cost, and has found that abandonment timing depends on the level of reserves and the profitability of the project – which is affected by the price process (Insley, 2017). As noted, an empirical analysis has shown that the main motivation behind temporary closures in the Canadian oil and gas industry is to avoid high costs of environmental liabilities, and not to keep the option to reactivate alive (Muehlenbachs, 2015). A recent study explores optimal regulation in a deterministic model of a firm that extracts resources and generates waste (Lappi, 2018). The socially optimal policies are found to include a pollution tax, shut-down date, and firm deposited bond. In this setting the timing of when the bond is paid is irrelevant.

This study contributes to the current literature by introducing a dynamic mechanism for an environmental bond into a model of optimal decision making by a firm whose activities generate hazardous waste. The goal of the bond is to fully collateralize the government from the possibility that a firm may be unable or unwilling to clean up its waste. The bond is a further regulatory requirement for the firm, in addition to strict liability for clean-up. The main objective of this investigation is to compare the impacts of an environmental bond (plus liability for clean-up) versus the strict liability rule on its own, on the firm's optimal timing of the clean-up of hazardous waste generated by its operations.[5] In the paper we refer to the bond plus liability as the 'bonding policy' or just 'the bond', to contrast with the strict liability rule on its own.

We abstract from any environmental damages caused by waste creation during the production process, in order to focus on the clean-up of the stock of waste. The payment required to the bond in each time period is determined so that clean-up costs would be fully covered should the mine be closed immediately. It is further assumed that the bond is in the form of a cash deposit, which is a common form of environmental bond in practice. We consider the impact of different features of the bond, such as whether interest is paid,the inclusion of extra (third-party) costs that the government would incur if it undertakes the clean-up, and the risk premium the firm may pay to finance the bond.

We develop a stochastic optimal control model of a firm's decisions regarding the construction, operation, and abandonment of a mining project in an environment of uncertain commodity prices. The price of the mine's output is modeled as an Ito process. The mine owner chooses the optimal timing to build, operate, mothball, and eventually abandon the project. During operations, the mine produces waste that accumulates and by legal requirement must be cleaned up when the firm ceases operations permanently (abandonment) or by a the end of the project life, whichever comes first.[6] The firm can undertake abatement during the project to reduce the waste flow. The firm chooses the amount of ore produced and the level of waste abatement to maximize the value of the mining operation. The optimal control model results in a system of Hamilton Jacobi Bellman equations, solved using a numerical approach. The results allow us to contrast the firm's optimal decisions under an environmental bond compared to a strict liability rule.

Note that in this paper we do not model the risk of bankruptcy. Rather, we assume that

[5]In this study, we are not concerned about the clean-up of environmental accidents associated with a waste disposal facility, such as accidental release of chemicals. However, environmental bonds and liability rules have been widely used to control environmental risks. Torsello and Vercelli (1998) provides a critical assessment of these policies for risk control, and Poulin and Jacques (2007), Gerard and Wilson (2009), Smith (2012), and Davis (2015) highlight their practical challenges for different case studies relevant to environmental risks.

[6]The project has a fixed end date, $T$, and by government regulation the waste stock must be cleaned up at this time.

the firm operates as a going concern and does not consider the possibility of bankruptcy in its optimal choices regarding production, waste abatement, and timing of operations. We believe this is a reasonable assumption for many firms due to the high costs of declaring bankruptcy, including loss of goodwill and reputation. The government nevertheless requires an environmental bond for all mining operations in order to be fully collateralized against any possible losses.[7]

To preview our results, we find that the effect of the bonding policy relative to the strict liability rule on its own depends on certain key characteristics of the bond - in particular whether the bond pays interest and whether the firm borrows at a premium above the risk free rate to fund the bond. Note that current practice regarding payment of interest on environmental bonds varies across jurisdictions, with some but not all, paying some amount of interest. Also, it would normally be the case that any firm borrowing to finance bond payments would need to pay a risk premium above the risk-free rate.[8] If the firm does not have to pay a risk premium (i.e., it can borrow at the risk-free rate) and if the government pays the risk-free interest rate on the bond, then the value of the mine prior to construction is the same under the bonding policy and strict liability rule. The optimal abatement rates are also the same under the two policies, as the bond imposes no extra costs on the firm. In contrast, if no interest is paid on the bond, the value of the project is reduced compared to the strict liability policy and the firm undertakes a larger amount of waste abatement under the bond. This implies a smaller stock of waste at the termination of the project and lower final clean-up costs. Because the mine is less profitable, it is less likely that the firm will invest in this mine. In the more realistic case that the firm borrows to fund the bond at a premium over the risk-free rate, the value of mine is reduced further and waste abatement levels are increased. The prospect of investment in the mine is even less likely compared to the previous case.

A bond that covers the full cost of immediate clean-up of mine waste ensures that the government will never have to bear the cost of mine waste clean-up. However certain characteristics of the bond noted above (i.e., risk premium and payment of interest by

[7] In Chapter 2, we address different approaches to modelling bankruptcy risk and examine the impact on firm behaviour.

[8] Or if the bond is funded from retained earnings or by issuing shares, the cost of capital would also exceed the risk-free rate.

the government) make it more costly than a strict liability rule for the firm. Whether the added costs imposed on the firm are worth it from society's point of view depends on the size of the extra costs incurred when governments are forced to take on this environmental liability.

The next section explores the existing literature about restoration, environmental bonds, and resource-valuation models in the context of optimal decisions under uncertainty. Section 1.3 develops the theoretical model. The dynamic programming solution of the model and optimal strategies for extraction and abatement are in Section 1.4. The case of borrowing to finance the bond is explained in Section 1.5. Section 1.6 presents a numerical solution approach. An application of the model to the copper industry is discussed in Section 1.7. An analysis of results is provided in Section 1.8. The last section summarizes results and conclusions.

## 1.2 Literature review

### 1.2.1 Restoration and environmental bonds

Policy instruments which put a price on emissions, such as a Pigouvian tax or pollution permits, are well suited to deal with damages from flow or stock externalities (Baumol and Oates, 1988, Farzin, 1996, among others). With an emissions tax set appropriately to reflect the marginal environmental damages from one more unit of extraction, firms will choose an optimal rate of pollution abatement that equates marginal social benefits of reducing pollution with its marginal costs. In practice, the main application of these policy instruments has been to control for externalities associated with air and water pollution, for which the quality can be maintained only through abatement because the clean-up of the stock of the pollutant is either technologically infeasible or prohibitively expensive.

Some types of environmental damage can be controlled both through abatement of emissions and through clean-up and restoration of environmental quality some time after the pollution initially created. Abatement and restoration may be substitutes for each other, whereby more abatement today implies the need for less restoration in future. Mine waste is a prime example. In such cases it is inefficient to focus solely on pollution abatement. Keohane et al. (2007) have shown that when site restoration is feasible, it is not optimal to depend only on abatement to improve the quality of the environment, as at some low levels of environmental quality or high social damage costs, abatement may become more expensive than restoration. Keohane et al. (2007) analyze the optimal trade-off between abatement and restoration in a model in which the quality of the environment fluctuates randomly, and there are economies of scale in restoration in the sense that restoration entails significant fixed costs that do not increase as the environmental quality decreases. Under these circumstances it is optimal to clean up the stock of damage only when the quality falls to a sufficiently low level that abatement becomes costlier than restoration. In their model, the policy maker sets a tax rate equal to the marginal value of abatement to ensure that firms choose the optimal amount of abatement given the possibility of periodic restoration of environmental quality. Site restoration is assumed to be the responsibility of the government which is supported by funds raised by the tax. Keohane et al. (2007) show that the funds raised will generally be close to the required clean-up cost, depending on the realization of the path of uncertain environmental quality.

While an appropriate time-variable tax rate can, in theory, deal with flow and stock externalities, in practice it may be difficult to implement, given the political realities in many jurisdictions where taxes are strongly resisted. Command and control regulations specifying allowed pollution levels may be more politically acceptable. However, neither a pollution tax nor limitations on polluting emissions can by themselves deal with the problem of ensuring firms meet their clean-up obligations in a timely manner. As noted in the introduction, firms delaying or failing to undertake restoration is a significant problem in many jurisdictions. Liability rules and environmental bonding requirements focus specifically on this problem.

Perrings (1989) first developed the mechanism of bonds as a means to encourage firms to invest in research to examine the potential adverse effects of their activities. Under an environmental bond each firm is required to conduct periodic investigation about potential damage of its current activity, and to *deposit* a bond equivalent to its own best estimations of clean-up costs for the "worst case" environmental outcome. This "worst case" outcome is called the "focus loss"[9] of the activity, given current knowledge about the future. At each point in time, if the firm can prove that the restoration costs are lower than estimated, the policy maker refunds a portion of the firm's deposit. Therefore, firms have the private incentives to increase their research or investigations of all potential consequences of their current activities until the costs of research[10] equal the resulting benefits of reduction in the value of bonds. At the end of the activity, the policy maker completely *refunds* the firm's deposit if the firm cleans up all damages. This bonding mechanism guarantees the availability of funds for future restoration should a firm default on its environmental obligation. Moreover, Shogren et al. (1993) highlighted the fact that with this bonding system firms become aware of potential environmental costs of their current actions, and take required measures to minimize their compliance costs.

The bond mechanism developed by Perrings (1989) mimics the idea of a deposit-refund system,[11] which dates back to 1971 when Solow (1971) developed the idea of the "materials use fee". According to Solow (1971),[12] this fee is equivalent to *"the social cost to the environment if the material were eventually returned to the environment in the most harmful way possible. The fees would be refunded to anyone who could certify that he had disposed of the material"*. Cornwell and Costanza (1994) explained one simple application of a refundable deposit – the fee on glass bottles. This fee is intended to encourage the most socially desirable method of waste disposal – recycling as opposed to littering. In their study, environmental bonding is identified as a variation of the deposit-refund system. Unlike the former, environmental impacts of each disposal method are known in a deposit-refund system, and also the fee is often set lower than the cost of choosing the worst method of disposal but high enough to encourage the best method, i.e., returning the bottle for recycling. In contrast, with the bonding system all known and predicted unknown future impacts of the activity determine the exact value of the bonds.

An environmental bond, aimed at raising funds for future restoration projects, is a

[9] Focus loss does not represent the worst case scenario one can "imagine", but it is the least surprising severity of damage that may occur (Costanza and Perrings, 1990, Perrings, 1989).

[10] Research costs include expenditures on investigations of a mine site so that firms are better able to predict damages.

[11] The theory of deposit-refund systems are comprehensively surveyed by Bohm (1981).

[12] p. 502.

complement to the strict liability rule. Kaplow and Shavell (1996) argued that the strict liability rule may not be a good alternative to induce firms with limited liability to pay up-front for their potential clean-up costs due to liquidity constraints and litigation difficulties. Limited liability firms are not required to pay for any damage beyond their asset value, which has two implications. The first implication is that social welfare is compromised as a result of damage residuals potentially being extended to society. The second implication suggests that limited liability firms may simply ignore any damage costs above their asset value, which dilutes damage prevention incentives.[13] Moreover, strict liability requires costly and lengthy processes to sue a responsible party through litigation. In this context, several authors highlighted some advantages of environmental bonds over the strict liability rule (Cornwell and Costanza, 1994, Costanza and Perrings, 1990, Gerard, 2000, Shogren et al., 1993). One important advantage is that bonds shift the "burden of proof" from governments and society to firms, reducing the need for litigation. Instead of taking the firm to court to prove the damage caused by the firm's activities, now the firm is legally bound to reveal the true costs of its activity and deposits the equivalent amount of money with the government. Another advantage is that environmental bonds mitigate the impacts of liquidity constraints and ensure that funds are available if the constraint binds.

Bonds have disadvantages, as well, which are thoroughly discussed by Shogren et al. (1993). The most important disadvantage is moral hazard which exists for both firms and government. In the private sector, since the value of bonds relies on firms' self-reporting of expected future damage costs, firms with incentive to reduce their compliance costs may not truthfully report their environmental costs. To deal with this issue, estimations can be audited and verified by a third party such as an environmental authority to ensure that firms comply with estimations standards and truthfully report the costs of their activity. In the government sector, there is also a financial incentive to claim that firms have shirked their environmental obligations, thus justifying seizing the bond. However, false bond reports have reputation costs for firms, just as false claims have repercussions for

[13] Shavell (2005) shows that such impacts can be dealt with through a minimum asset requirement combined with liability insurance. The minimum asset reduces individuals' incentives to engage too often in activities with potential harmful impacts, and if they have engaged, the liability insurance improves damage prevention incentives. However, Poulin and Jacques (2007) argue that if a firm has not enough assets, no insurance company will participate, restricting the firm's engagement in such activities.

governments. Reputation costs mitigate the incentive for any false claims in both sectors because cheating by a firm increases the cost of bonds to start future activities, if the government realizes that the firm has not been truthful. Cheating by governments discourages innovative activities, and to offset this effect, non-credible governments can offer subsidies to investors to encourage investments.

### 1.2.2 Investment in non-renewable resources

Studies of environmental bonds are largely limited to conceptualizing and describing the theory of environmental bonds and their mechanisms. There is little analysis of the effect of this policy on firm investment decisions and its optimal choices of site clean-up, especially in a dynamic setting. Igarashi et al. (2010) studied the effect of environmental bonds on firms' exploration and extraction decisions in the oil and gas sector, through a theoretical model. They demonstrated that internalizing restoration costs only slows the resource extraction, without affecting the level of exploration and thus increases the in-ground reserve-to-well ratio. White et al. (2012) showed that insolvency leads to a deadweight loss because the restoration costs must be raised from general tax revenue, and thus with environmental bonds, the government liability will be reduced by the payout of the bonds. Lappi (2018) examines optimal policies for a resource extraction operation that causes pollution during production and also requires remediation of the pollution stock when production terminates. He sets up a theoretical model of the production and remediation stages of operations and characterizes the socially optimal extraction and rehabilitation rates. The optimal regulation is described in the form of a pollution tax, an optimal shut down date, and a deposit from the producer to cover remediation costs.

These studies have analyzed firms' extraction and restoration behavior in the absence of output price uncertainty, and thus the timing of initial investment and final restoration are deterministic. Price volatility affects the optimal timing of project stages, including the temporary mothballed stage and the restoration phase, and will thus affect waste accumulation and clean-up. In addition, in these studies, there is no need for the government to require the remediation funds be deposited prior to production being undertaken. In contrast to these studies, our focus is on examining the impact of different liability and bonding policies on the firm over the life cycle of an operation, from construction to abandonment, if the government is to be fully protected from the risk that the firm does not meet its clean-up obligations. Our analysis also incorporates uncertain commodity prices.

The existing literature on real options deals with price uncertainty and its implications for optimal investment decisions in non-renewable resources. An early study by Brennan and Schwartz (1985) used a real options approach to analyze a firm's optimal policies for managing natural resource projects in an uncertain environment. Their study allows for three discrete choices after the original investment decisions have been made, i.e., to activate, mothball, or shut down a project, and examines the impact of the sunk costs of mothballing and reopening on the optimal timing of operation and the value of investment. Slade (2001) looked at the managerial flexibility this choice set provides for firms, and found that this flexibility increases the value of project. Flexibility in this case means that the option to become inactive should a current operation incur loss, and the option to reopen a mine when operation becomes profitable. Therefore, idle firms are willing to tolerate some losses with the expectation that commodity prices will rise in the future and make the operation profitable again. However, according to Dixit (1992), when the incurred loss in the phase of inactivity exceeds the option value of reactivation, permanent closure becomes optimal. Dixit (1989, 1992) demonstrated that once an activity is mothballed, a firm could remain in the state of hysteresis, reflecting the fact that the critical prices for reopening the activity tend to be higher than the prices that triggered inactivation in the first place. The source of hysteresis is the existence of switching costs that motivate firms to delay such irreversible costs. This phenomenon also exists when firms decide to close an already opened activity in the sense that closure occurs at lower prices than the original ones that caused the decision maker to open the mine. Mason (2001) extended Brennan and Schwartz (1985) by considering that the resource stock is exhaustible, and observed hysteresis even with low sunk costs.

The research to date has tended to evaluate the value of natural resource investment under irreversible costs of project suspension and reactivation, and firms' optimal response to clean-up costs associated with permanent closure has been largely overlooked. Brennan and Schwartz (1985) and Almansour and Insley (2016) assumed that abandonment entails no costs, and Dixit (1989), Mason (2001) and Slade (2001) did not include the option of permanent closure in their choice set. However, shutting down a mine requires costly investments to perform restoration and remediation of all disturbed areas. More recently, Insley (2017) examined optimal extraction of a non-renewable resource with the resource price modeled as regime switching stochastic process and assuming irreversible restoration costs for abandonment. She found that the critical prices that trigger abandonment depend on the stock of reserves and the profitability of the project: for low reserves and low operational profits, firms abandon the project before the lease is expired, while when the level of reserve is high firms keep the reopening option alive as there is an opportunity benefit to waiting. At the end of the lease, it is assumed that all firms were required to abandon the mine and remediate the entire site regardless of the level of remaining reserves. Muehlenbachs (2015) has studied the effect of abandonment costs for the Canadian oil and gas sector, and observes that such costs may motivate firms to strategically exercise the option to suspend operation even if the future reopening option has zero value. This phenomenon increases the likelihood that firms will walk away from their environmental obligations due to bankruptcy or insolvency. Therefore, becoming mothballed as a way to escape the environmental obligations is unambiguously welfare reducing and extends the costs of damage clean-up to society.

This study contributes to the current literature by analyzing the impacts of restoration costs on a resource extraction firm's decisions. We develop a simple mine valuation model that accounts for environmental quality, in terms of the stock of waste, as an additional state variable and is capable of analyzing the influence of an environmental bonding mechanism on abatement decisions. Clearly, abatement reduces the stock of waste and thus the final environmental liability costs. The model is also used to examine the strict liability rule where no bond is required but the firm must undertake site restoration at the end of the project. To the best of our knowledge, this is the first attempt to understand the impacts of environmental bonds, as a complement to the strict liability rule, on a firm's optimal investment strategies and its project value under price uncertainty, in a dynamic setting.

## 1.3 Model formulation

### 1.3.1 Description of the decision problem

Consider a risk-neutral firm which extracts a non-renewable resource and thereby generates hazardous waste disposed of into a landfill. A government regulator requires the waste be cleaned up when the operation is terminated. This study assumes that two policies can be implemented: 1. the strict liability rule, and 2. an environmental bond combined with liability for clean-up. We refer to the latter as the bonding policy. For simplicity, we have assumed that there is no risk of accidental release of pollution from the landfill. Therefore, the only environmental obligation is the clean-up of the landfill.

As noted in the introduction, the objective of the environmental bond is to fully collateralize the government for the clean-up cost. The bond addresses any inefficiencies that arise when the government is left to clean up a mine site, including any dead weight loss if the cost is funded by general tax revenues, as well as the extra costs involved because the government has less expertise and experience than the mining firm. In this paper, we assume that there are no damages from the flow of waste production or the build up of the waste stock prior to a fixed date, denoted by $T$. By government regulation, waste clean-up must happen before or on this date. This assumption is made in order to focus on the bond, but could be relaxed through the addition of a damage function which depends on the stock of waste.

Before being allowed to develop the mine, the firm enters into an environmental contract with the government which specifies the firm's clean-up obligations. Once the firm enters into the environmental contract, it can decide the optimal timing of its initial investment to develop the project, which entails significant capital costs. After the project is launched, the firm manages the level of reserve and the stock of waste by choosing the optimal rates of extraction and abatement, respectively. In addition, the firm maximizes its project value by determining the optimal timing of production, mothballing, reopening the operation, and abandoning the facility and site restoration.

The firm's optimal decisions depend on four state variables: the price of the commodity, $P(t)$, the stock of the resources, $S(t)$, the amount of waste in the land fill, $W(t)$, and the stage of operation, $\delta_i,\ i = 1, 2, 3, 4$. Stage 1 ($i = 1$) is pre-construction, Stage 2 ($i = 2$) is active extraction, Stage 3 ($i = 3$) is mothball or temporary shut down, and Stage 4 ($i = 4$) is abandonment and landfill restoration. The firm has three control variables: the rate of resource extraction, $q$, the rate of waste abatement, $a$, and the decision to move to a new stage of operation, $\delta$. We note that $\delta$ serves as both a state variable and a control variable in the model. Cash flows depend on the current $\delta$ at a particular time $t$, but as described later, the firm makes choices at discrete times as to whether to move to a different stage.

The commodity price, $P(t)$, is assumed to be described by a simple one-factor Ito process, which is mean-reverting in the drift term. As is discussed in Section 1.7.1, this model has been used by other researchers to describe commodity prices (Schwartz, 1997).

$$
\begin{aligned}
dP(t) &= \kappa(\hat{\mu} - \ln P)P\ dt + \sigma P dz;\ \ P(0) = p_0\ given \\
P\ &\in [p_{\min}, p_{\max}]
\end{aligned}
\tag{1.1}
$$

where $\kappa, \hat{\mu}, \sigma$ are parameters reflecting the speed of mean-reversion, the long run mean of $\ln(P)$, and volatility, respectively. $t$ denotes time where $t \in [0, T]$, and $dz$ is the increment of a Wiener process. The estimation of the parameters is described in Section 1.7.1. Parameters are estimated for the risk-neutral world, so that the term $\kappa(\hat{\mu} - \ln P)P$ represents a risk-adjusted drift rate.

The level of resource stock, $S(t)$, falls over time at the extraction rate $q$. The dynamic path of resource stock is given as

$$dS(t) = -qdt; \quad S(0) = s_0\ given. \tag{1.2}$$

The waste stock, $W(t)$, as a by-product of the operation, is assumed to be disposed of into a landfill with a known, maximum capacity denoted by $\bar{w}$. By assumption, $\bar{w}$ is specified by regulation and is optimal from society's point of view. During the operation phase, each unit of resource extracted adds to the stock of waste at the constant rate $\phi$, and abatement at the rate $a$ reduces the waste flow. This dynamic continues until the capacity of landfill is exhausted. Therefore, the rate of change in the volume of waste or in the stock of landfill is given by

$$dW(t) = (\phi q - a)dt; \; W(0) = w_0 \tag{1.3}$$

in which $\phi q \lesseqgtr a$, and $w_0$ represents the initial level of waste that is required to be cleaned up at the end of operations, where $0 \leq w_0 \leq \bar{w}$.

For intuition, we define the environmental quality in terms of the stock of waste so that waste accumulation deteriorates the environmental quality. Therefore, $\phi q$ can be thought of as the flow rate of the environmental deterioration, assuming zero natural decay for the waste. The abatement effort is any action, such as recycling the waste, that occurs during the operation phase. Consistent with the model of Keohane et al. (2007), the abatement rate could be higher than the environmental deterioration rate (i.e., $\phi q < a$). It follows that waste abatement could affect the previously generated waste and contributes to a positive rate of change in the environmental quality. Abatement is restricted by the installed capital and cannot exceed its maximum value, $\bar{a}$, at each point of time. By assumption, this upper bound does not change over time.

We now specify admissible sets for $\delta$, $q$, and $a$. Let $Z_\delta$ denote the admissible set for $\delta$ where

$$Z_\delta = \{\delta_1, \delta_2, \delta_3, \delta_4\}. \tag{1.4}$$

We define an admissible set for the extraction rate $q$, which depends on both the resource stock and stage of operation. Denote this admissible set as $Z_q(S, \delta)$, which is given as follows

$$\begin{aligned} & q \in Z_q(S, \delta) \\ & Z_q = [0, \bar{q}], \quad \text{if} \;\; S > 0, \; \delta = \delta_2. \\ & Z_q = 0, \quad \text{if} \;\; S = 0, \; \delta = \delta_2. \\ & Z_q = 0, \quad \text{if} \;\; \delta = \delta_i, \; i = 1, 3, 4, \; \forall S. \end{aligned} \tag{1.5}$$

By assumption, the extraction rate cannot exceed its maximum rate $\bar{q}$. This upper bound is known as the capacity constraint and is assumed to remain constant during the operation.

Similarly, we define an admissible set for $a$, denoted $Z_a(w,q,\delta)$, as follows

$$\begin{aligned} a &\in Z_a(w,q,\delta) \\ Z_a &= [0,\bar{a}], \quad \text{if} \quad W < \bar{w},\ \delta = \delta_2 \\ Z_a &= [\phi q,\bar{a}], \quad \text{if} \quad W = \bar{w},\ \delta = \delta_2 \\ Z_a &= 0, \quad \text{if} \quad \delta = \delta_i,\ i = 1,3,4,\ \forall W. \end{aligned} \tag{1.6}$$

It is assumed that $\bar{a} > \phi\bar{q}$, implying that the firm can abate at a rate that exceeds the waste level generated when extraction is at the maximum $\bar{q}$. Note that Equations (1.2)–(1.6) imply that

$$\begin{aligned} 0 &\le W \le \bar{w} \\ 0 &\le S \le s_0. \end{aligned} \tag{1.7}$$

The characteristics of extraction costs are given in Assumption 1.

**Assumption 1** *The extraction cost function $C^q(q)$ is linear in the extraction rate so that $C^q(0) = 0$, $C'^q(\cdot) \ge 0$, and $C''^q(\cdot) = 0$.*

Assumption 2 gives the cost of abatement as a convex function, implying that removing each additional unit of pollution with abatement is increasingly difficult and more costly to the firm.

**Assumption 2** *The abatement cost function, $C^a(a)$, is assumed to be twice differentiable with $C^a(\cdot) \ge 0$, $C^a(0) = 0$, $C'^a(\cdot) \ge 0$, $C''^a(\cdot) \ge 0$, and $C'''^a(\cdot) = 0$.*

Restoration improves the quality of the environment by affecting the stock of waste, rather than the flow. To ease the analysis, it is assumed that periodic restoration is not possible, and thus abatement is the only way to maintain the quality of the environment during the active life of the project.

### An environmental bond

To model the mechanism of an environmental bond, we assume that the firm must deposit an amount with the government prior to project commencement sufficient to cover clean-up costs of waste generated during construction. The value of the environmental bond has to be adjusted periodically during the life of the project based on the firm's estimated restoration costs. Therefore, at the end of each period, the firm submits a revised cost estimate and the government adjusts the amount of deposited bonds according to these estimates. The value of the environmental bond in any period must completely cover the closure costs if the firm were to abandon the mine at the end of the current period. We assume that the appropriate level of restoration and the associated cost are correctly determined and thus the bond level is adequate.

One important policy consideration is that the firm is required to estimate the closure costs based on the fact that a third party will do the restoration should the firm default. It has been found in practice that it is more costly for a third party to clean up environmental damages than for the firm itself by 15% to 30% (Ferreira et al., 2004). This additional amount internalizes third-party costs such as mobilization costs (Peck and Sinding, 2009, White et al., 2012). Therefore, requiring restoration cost estimates to be made on the basis of expenses to a third party ensures sufficient funds for the required clean-up should the firm walk away from its obligations (Grant et al., 2009, Otto, 2010). This study assumes a convex cost function for clean-up given by Assumption 3. As the stock of waste increases, it becomes increasingly more difficult to return the land to its pristine state. Therefore, additional waste requires additional costs for removing a greater volume of waste and, depending on the degree of toxicity, requires greater safety precautions for workers during restoration. Moreover, the cost of stabilizing the waste to prevent geographical expansion can increase with waste volume (Phillips and Zeckhauser, 1998). As a result, more waste requires more clean-up effort which becomes more costly at the margin.

**Assumption 3**

- *We define the firm's clean-up costs by $C^f(W)$ and the third party's clean-up costs by $C^{tp}(W)$, so that $C^{tp}(\cdot) = \nu C^f(\cdot)$ where $\nu > 1$ is a constant.*

- *The firm's cost of cleaning up the accumulated waste and improving the quality from the state $W$ to zero waste is given by $C^f(W)$ with $C'^f(\cdot) \geq 0$, $C''^f(\cdot) \geq 0$, and $C'''^f(\cdot) = 0$.*

- *It is assumed that $C^f(W)$ is truthfully estimated and reported by the firm.*

Let $B(t)$ denote the total value of the bond at each point of time. This value varies according to rate of change in the firm's restoration costs adjusted by potential expenses to the third party, $\frac{dC^{tp}(W)}{dt}$. In fact, the variation of the environmental bonds over each period (i.e., the annual cost of bonds to the firm) denoted by $\frac{dB(t)}{dt}$, is equal to the rate of change in the restoration cost, and can be written as

$$\begin{aligned} \frac{dB}{dt} &= \frac{dC^{tp}(W)}{dt} \\ &= \frac{dC^{tp}}{dW}\frac{dW}{dt} \\ &= \theta(W)(\phi q - a) \end{aligned} \tag{1.8}$$

where $\frac{dC^{tp}}{dW} \equiv \theta(W)$, and $\frac{dW}{dt} = \phi q - a$ is given by Equation (1.3). $\theta(W)$ is defined as the marginal restoration cost or the marginal rate of fine that the government collects on the waste flow over a given time interval. Therefore, the firm's rate of payment on bonds to the government at each time (i.e., $\frac{dB}{dt}$) is given by $\theta(W)(\phi q - a)$, which could be positive, negative, or zero depending on $\phi q \gtreqless a$. If the abatement rate is such that the extraction activities add to the stock of waste ($\phi q > a$), the firm will have to update the deposited bond accordingly, representing an increase in bond value. In contrast, if abatement dominates the deterioration rate ($\phi q < a$) and reduces the waste accumulation, the bond value will decline, indicating that the firm has been reimbursed an amount equal to the reduction in clean-up costs. If abatement fully offsets the current deterioration ($\phi q = a$), the net change in the stock of waste and thus the compliance cost with the bonding regulation are zero. Therefore, there is a trade-off between the decision to abate today and to post bonds for clean-up at the terminal time. Note that $\theta(W)$ increases linearly in $W$, and is determined based on the company's estimate of the change in restoration costs to a third party as $W$ changes.

Let $B_0 = C^{tp}(W(0))$ cover the potential clean-up cost of the initial waste. $B_0$ has to be deposited with the government before the operation starts. According to Peck and Sinding (2009), this mechanism provides adequate assurance for the existence of funds for future clean-up because it "raises money according to the initial footprint and [is] linked to marginal increases or decreases in mine footprint over its life". Since the estimated restoration costs are higher than the costs to the firm by an amount $\nu$ (see Assumption 3), the difference will be returned to the firm at project termination. We refer to this saving as restoration benefit defined by Assumption 4.[14]

**Assumption 4** *Under bonding requirements, the firm's benefit (saving) from restoration at the terminal point, $T$, is $(\nu - 1)C^f(W)$,*[15] *which is the difference between the firm's estimated restoration costs to a third party and its actual costs of restoration, and $\nu > 1$.*

We allow for the possibility that interest is paid on the bond at the risk-free rate, $r$.[16] While the project is operating, the annual compliance cost with the environmental bond has three components: 1) the cost of abatement effort, 2) the expected bond payment, and 3) any interest paid on the bond. Therefore, the annual compliance cost is defined by

$$\Omega = C^a(a) + \mathbf{1}_{b=true}\theta(W)(\phi q - a) - \mathbf{1}_{b=true} rB \tag{1.9}$$

where $\mathbf{1}$ is the indicator function and $b = true$ under the environmental bonding policy and is false otherwise.

### The strict liability rule

Under the strict liability rule, the regulator requires the firm to clean-up the stock of waste once the project terminates, and does not require *ex ante* payments for associated

[14] Note that in practice, a firm may go bankrupt, and would thereby forfeit the bond refund. This case is not considered in this paper.

[15] According to Assumption 3, the benefit of restoration is $[C^{tp}(W) - C^f(W)] = [\nu C^f(W) - C^f(W)] = (\nu - 1)C^f(W)$ where $\nu > 1$.

[16] Note that in Section (1.5) we also consider the case where the firm borrows at rate $\rho > r$ to finance the bond. For clarity in describing the model, we ignore this possibility at the moment.

costs. Moreover, termination entails sunk costs to the firm. Therefore, we can adjust Assumption 4 as follows

**Assumption 5** *Under liability requirements, the firm's restoration cost at the terminal point, $T$, is $C^f(W)$.*

While the project is operating, the annual compliance cost with the strict liability rule is associated with abatement efforts. In Equation (1.9) the last two terms on the right hand side disappears as $b =$ false.

#### Instantaneous cash flow

The firm's objective is to choose controls to maximize the discounted sum of risk neutral expected cash flows. Cash flows at any time $t$ will depend on the firm's stage of operations, $\delta$, rate of abatement, $a$, and extraction, $q$. Instantaneous cash flows are given as follows

$$\pi(t) = P(t)q - C^q(q) - \left[C^a(a) + \mathbf{1}_{b=true}\theta(W)(\phi q - a) - \mathbf{1}_{b=true} rB\right] - C_i^m, \quad \text{if} \quad \delta = \delta_i, \quad i = 1, 2, 3 \tag{1.10}$$

$$\pi(t) = 0, \quad \text{if} \quad \delta = \delta_4$$

in which the term in square brackets is the compliance cost, $\Omega$, as previously given by Equation (1.9). $C_i^m$ refers to fixed costs under both the bond and strict liability policies in stage $i$. Because the tax treatment of bonds varies across jurisdictions, we have chosen to ignore taxes in our model specification.[17]

### 1.3.2 Defining state and control variables, and the value function

The resource price, $P(t)$, resource stock $S(t)$, waste stock, $W(t)$, and stage of operation, $\delta(t)$, all represent state variables in the decision problem. The value of the firm's operations is a function of these state variables and time, $t$, denoted as $V(P, S, W, \delta, t)$.

[17] Tax issues include whether the money paid into the bond is deductible for income taxes, whether interest paid to finance the bond is deductible, and whether the bond refund is taxable. See World Bank, 2009.

It is assumed that at specific fixed times, the firm makes a decision about whether to move to another stage of operation. These fixed decision times are given as follows

$$\mathcal{T}_d \equiv \{t_0 = 0 < t_1 < ... < t_m <, ..., t_M = T - 1\} \tag{1.11}$$

where we assume that the optimal decision to move to another stage of operation occurs instantaneously at $t \in \mathcal{T}_d$. Note that at the end of the project life, $T$, the firm's only option is to terminate the operations. Therefore, time $T$ is excluded from the firm's optimal decisions dates in the above set. Choices regarding optimal rates of abatement, $a$, and extraction, $q$, are made in continuous time at time intervals given as follows

$$\mathcal{T}_c \equiv \{(t_0, t_1), ..., (t_{m-1}, t_m), ..., (t_{M-1}, t_M)\}. \tag{1.12}$$

Since we search for the closed loop control, we assume the controls are in feedback form, i.e., functions of the state variables. Control variables can be specified as: $q(P, S, W, \delta, t)$, $a(P, S, W, \delta, t);\ t \in \mathcal{T}_c$, and $\delta^+(P, S, W, \delta, t);\ t \in \mathcal{T}_d$. Admissible sets for $q$, $a$ and $\delta$ are given as $Z_q$, $Z_a$ and $Z_\delta$, specified in Equations (1.5) and (1.6), and (1.4). We specify a control set which contains the controls for all $t_0 \leq t \leq t_M$ as follows

$$K = \{(\delta^+)_{t\in\mathcal{T}_d}\ ;\ (q, a)_{t\in\mathcal{T}_c}\}. \tag{1.13}$$

Regardless of the controls chosen, the value function can be written as the risk neutral expected discounted value of the integral of cash flows, given the state variables, with the expectation taken over the controls

$$\begin{aligned} V(p, s, w, \bar{\delta}, t) \ &= \\ &\mathbb{E}_K \left[ \int_{t'=t}^{t'=T} e^{-r(t'-t)}\ \pi(P(t'), S(t'), W(t'), \delta)\ dt' \ + e^{-r(T-t)} V(P(T), S(T), W(T), \delta(T), T) \right. \\ &\qquad \left. \Big| P(t) = p, S(t) = s, W(t) = w, \delta(t) = \bar{\delta} \right] \end{aligned} \tag{1.14}$$

where $(p, s, w, \bar{\delta})$ denote realizations of the random and path dependent variables $(P, S, W, \delta)$.

$r$ is the risk free interest rate, and $\mathbb{E}[\cdot]$ is the expectation operator. The value in the final time period, $T$, is assumed to be the expected net benefits from closing and restoring the mine. This is described as a boundary condition in Appendix A.1.

## 1.4 Dynamic Programming Solution

Equation (1.14) is solved backwards in time using dynamic programming. For a particular $t_m \in \mathcal{T}_d$, we define $t_m^-$ and $t_m^+$ to represent the moments just before and after $t_m$. Specifically $t_m^- = t_m - \epsilon$ and $t_m^+ = t_m + \epsilon$, $\epsilon \to 0^+$. As a visual aid, the times around $t_m$ and $t_{m+1}$ are depicted below, going forward in time

$$t_m^- \;\rightarrow\; t_m^+ \;\rightarrow\; t_{m+1}^- \;\rightarrow\; t_{m+1}^+ \;. \tag{1.15}$$

At $t_m$ we determine the optimal control $\delta^+$, while in the interval $(t_m^+, t_{m+1}^-)$. We solve for the optimal controls $q$ and $a$ in continuous time.

### 1.4.1 Determining optimal rates of abatement, $a$, and extraction, $q$, from $t_{m+1}^- \rightarrow t_m^+$

We define $\mathcal{L}V$ as the differential operator as follows

$$\mathcal{L}V = \frac{1}{2}\sigma p^2 \, \frac{\partial^2 V}{\partial p^2} + \kappa(\hat{\mu} - \ln p)p \, \frac{\partial V}{\partial p} + rV. \tag{1.16}$$

Using a standard contingent claims approach (Dixit and Pindyck, 1994), we can derive a system of partial differential equations that describe the value of the resource, $V$, in the interval $(t_m^+, t_{m+1}^-)$ for all operating stages except for abandonment.

$$\frac{\partial V}{\partial t} + \mathcal{L}V + \max_{q,a} \, \Big\{ - q \, \frac{\partial V}{\partial s} + (\phi q - a) \, \frac{\partial V}{\partial w} + \pi(t) \Big\} = 0, \quad \text{for } \delta = \delta_i, \quad i = 1, 2, 3 \tag{1.17}$$

where we maximize with respect to the control variables $a$ and $q$, and $\pi(t)$ refers to net cash flows as defined in Equation (1.10).

Once the project is in Stage 4, the project value goes to zero.

$$V(p, s, w, \delta = \delta_4, t) = 0. \tag{1.18}$$

### 1.4.2 Determining optimal operating stage, $\delta$ at $t_m$

For $t_m \in \mathcal{T}_d$, the firm checks to determine whether it is optimal to switch to a different operating stage. The firm will choose the operating stage which yields the highest value net of any costs of switching. Let $C(\delta^-, \delta')$ denote the cost of switching from stage $\delta^-$ to $\delta'$. Recall that $t = t^-$ represents the moment before $t_m$ and $t = t^+$ denote the instant after $t_m$. Solving going backward in time, and noting the optimal stage is denoted as $\delta^+$, the value at $t_m^-$ is given by

$$\begin{aligned} V(p, s, w, \delta^-, t_m^-) &= V(p, s, \delta^+, t_m^+) - C(\delta^-, \delta^+) \\ \delta^+ &= \arg\max_{\delta'} [V(p, s, w, \delta', t_m^+) - C(\delta^-, \delta')]. \end{aligned} \tag{1.19}$$

Switching costs differ under the bond and strict liability policies for project commencement as well as for mine abandonment. Opening the mine under the bond requires the investment cost and initial bond payment, whereas the latter is absent under the liability rule. Denoting the investment cost with $I$, the cost of opening the mine is given as

$$C(\delta_1, \delta_2) = I + \mathbf{1}_{b=true} B(w_0). \tag{1.20}$$

The cost to switch to Stage 4 (abandonment) from either Stage 2 (operating) or Stage 3 (mothballed) is given by

$$C(\delta_i, \delta_4) = -[\mathbf{1}_{b=true} C^{tp}(w) - C^f(w)] \qquad i = 2, 3. \tag{1.21}$$

Under strict liability this is just the firm's own clean-up cost $C(\delta_i, \delta_4) = C^f(w) > 0,\ i =$

$2, 3$. Under the bonding policy, the firm will receive a refund of the bond equal to $C^{tp}(w)$ which exceeds the expenditures required for the firm to implement the clean-up, $C^f(w)$. Hence under the bond policy $C(\delta_i, \delta_4)$ will be a negative cost, i.e., it is a restoration benefit to the firm.

### 1.4.3 Optimal extraction and abatement policies

The decision problem specified in Equations (1.17)–(1.19) has no closed form solutions and is solved using a numerical approach, which is discussed in the next section. In this section, we examine the first order conditions for extraction and abatement which hold during in Stage 2, $\delta = \delta_2$, when the firm is actively producing the ore. These first order conditions reveal the nature of the optimal extraction and abatement rates, denoted $a^*$ and $q^*$, and in particular whether the solutions are bang-bang.

#### An environmental bond

The optimal extraction rate, $q^*$, and the optimal abatement rate, $a^*$, under bonding requirements are obtained by maximizing Equation (1.17) with respect to the terms that contain $q$ and $a$. The optimal extraction rate for a firm that actively extracts under a bonding policy satisfies

$$P - C'^q - \frac{\partial V}{\partial s} + \phi\Big[\frac{\partial V}{\partial w} - \mathbf{1}_{b=true}\ \theta(w)\Big] \begin{cases} \geq 0 & \Rightarrow q^* = \bar{q} \\ < 0 & \Rightarrow q^* = 0. \end{cases} \tag{1.22}$$

The first three terms in Equation (1.22) are the marginal revenue from extraction, marginal cost of extraction, and marginal value of the reserve to the firm. We have called the term in square brackets the firm's *marginal cost of environmental deterioration* which has two components: 1) the marginal value of the waste stock to the firm, $\frac{\partial V}{\partial w}$, and 2) the marginal restoration cost, $\mathbf{1}_{b=true}\ \theta(w)$. The total marginal cost of extracting a reserve is captured in the terms to the right of $P$.

**Remark:** Since both the profit function and the resource stock are linear in extraction rate, the optimal extraction rate, $q^*$, is either zero or at capacity, hence this is a bang-bang solution.

It follows that given an optimal abatement rate, the firm extracts at capacity as long as the marginal effect is positive. For zero marginal effect, the firm remains indifferent between extracting at capacity or not extracting at all, and thus it is reasonable to extract at capacity. Therefore, the firm extracts at capacity as long as the marginal revenue of extraction is not lower than its total marginal costs.

The optimal abatement under a bonding policy is given by

$$-C'^{a}(a^*) = \frac{\partial V}{\partial w} - \mathbf{1}_{b=true}\,\theta(w) \Rightarrow \begin{cases} 0 \leq a^* \leq \bar{a} & \text{if} \quad w < \bar{w} \\ \phi\bar{q} \leq a^* \leq \bar{a} & \text{if} \quad w = \bar{w}. \end{cases} \tag{1.23}$$

Along the optimal abatement path, the marginal cost of environmental degradation, $\frac{\partial V}{\partial w} - \theta(w)$, is equal to the marginal abatement cost. If abatement is costlier than environmental degradation at the margin, the polluter reduces its abatement effort and posts environmental bonds instead, until it remains indifferent between abating and polluting. In contrast, if the costs of environmental degradation are larger than the abatement costs at the margin, the optimal strategy is to increase abatement until the equality in Equation (1.23) holds. Thus according to the optimal policy rule, pollution should be abated up to the point that the marginal costs of abatement equal the potential marginal costs of the environmental degradation. Once the landfill capacity is reached, the lowest optimal abatement rate equals the environmental deterioration rate. This condition ensures that the landfill does not receive waste beyond its capacity.

A comparison between optimal criteria in Equation (1.22) and the optimal extraction policy with no environmental interaction in Brennan and Schwartz (1985) and the subsequent studies reveals that in our study the firm has to take into account costs of waste accumulation including the cost of clean-up (i.e., $\phi\Big[\frac{\partial V}{\partial w} - \mathbf{1}_{b=true}\,\theta(w)\Big]$) when choosing the optimal extraction rate. These terms were zero in such previous studies. Therefore, the firm may require a relatively higher price to start its operation, because accounting for costs of waste accumulation in the profit function increases the operational costs and reduces the private net benefit of extraction.

### The strict liability rule

The optimal rules for extraction and abatement are also given in Equations (1.22) and (1.23) but the indicator function will be zero. The firm's *marginal cost of environmental deterioration* under the liability rule is simply the marginal value of the waste stock. Similar to the bond, the firm operating under the liability rule extracts at either zero or at capacity, because both the profit function and the resource stock are linear in extraction rate. A comparison between optimal criteria for extraction and abatement under the bond and the liability rule reveals that the payment for the marginal restoration cost, $\theta(w)$, does not appear under the latter.

## 1.5 The case of borrowing

As noted, the value of the mine is specified in the Q-measure, which means that risk due to uncertain commodity prices is taken into account via the risk-adjusted parameters in the commodity price model. The solvent firm is assumed to operate as a going concern, and hence does not consider the possibility that it might not meet its clean-up obligation due to bankruptcy. Using standard contingent claims arguments, it is therefore appropriate to use the risk-free discount rate in our valuation model and this fully accounts for the opportunity cost of the bond to the firm.

However, in reality, the model as outlined in Section 1.3 for a solvent firm is unlikely to adequately reflect the true cost of the bond to the firm. As is discussed by White et al. (2012) and White (2015), a firm will typically be subject to additional costs, such as a bond service charge (largely a risk premium) assessed by a surety company. The risk premium would reflect the market's assessment that a firm might not meet its clean-up obligations, even though the firm fully intends to do so. To take account of this extra cost of the bond, we consider a case in which the solvent firm is assumed to borrow to finance the bond and must pay a premium over the risk-free rate, which accounts for the market's perception of risk and other bond service charges.

The model can easily be adjusted for the case where the firm borrows at rate $\rho > r$ to finance the bond. Assume that the government does not pay interest on the bond. Hence, Equation (1.9) which specifies annual compliance costs becomes

$$\Omega = C^a(a) - \mathbf{1}_{b=true}\rho B \tag{1.9b}$$

in which $\rho B$ denotes the interest payments the firm makes on the loan at each period prior to abandonment.

At the time of project commencement, the firm borrows $B(w_0)$ which is deposited into the bond. Since there will be no net cash outflow associated with the bond, the cost to move from Stage 1 to Stage 2 is just the construction cost. Equation (1.20) is adjusted to become

$$C(\delta_1, \delta_2) = I. \tag{1.20b}$$

If the firm chooses to close the project (i.e. go to Stage 4), the firm receives a refund of the bond from the government which is used to pay off the loan. Hence the net cash flow at closure reflects the clean-up cost. Equation (1.21) becomes:

$$C(\delta_i, \delta_4) = -C^f(w) \qquad i = 2, 3. \tag{1.21b}$$

## 1.6 Numerical solution approach

Equations (1.17)–(1.19) represent a stochastic optimal control problem which must be solved using numerical methods. The computational domain of Equation (1.17) is $(p, s, w, \bar{\delta}, t) \in \Gamma$ where $\Gamma \equiv [p_{min}, p_{max}] \times [0, s_0] \times [0, \bar{w}] \times Z_\delta \times [0, T]$. More details are given in Appendix A.1 where boundary conditions are specified for the PDEs. $\mathcal{L}V$ in Equation (1.17) can be discretized using a standard finite difference approach. The other terms in the equation are discretized using a semi-Lagrangian scheme as described in Chen and Forsyth (2007) and will not be described further here.

Recall that the optimal control for $q$ which we denote by $q^*$ is bang-bang so that $q^* \in \{0, \bar{q}\}$. To determine the optimal control we search over the set $(q, a) \in \{0, \bar{q}\} \times Z_a$. We discretize the controls $a \in Z_a$ and determine the optimal control by exhaustive search at each point in the state space $(p, s, w, t)$.

| | | | |
|---|---|---|---|
| $\kappa$ | 0.0264 (0.001) | Root Mean Square Error | 0.07 |
| $\mu$ | 2.7051 (0.079) | Mean Absolute Error | 0.05 |
| $\eta$ | 2.7845 (0.026) | Log-likelihood function | 9652 |
| $\sigma^2$ | 0.0458 (0.002) | Number of observation | 937 |

Table 1.1: *Estimation results for the one-factor copper price model using Kalman Filter. RMSE, MAE, $\mu$, and $\eta$ are in terms of US \$/lb. Standard errors are in parenthesis. Weekly futures data from Aug 1st, 1997 to Jul 13th, 2015.*

## 1.7 An application to the copper industry

To illustrate the impact of an environmental bond versus the strict liability rule on optimal firm decisions, this study considers the case of investment decisions for a copper mine. A numerical example is developed based on available data from an open-pit copper mine in British Columbia, supplemented by researcher assumptions when data is lacking. The parameters of the stochastic model assumed for copper prices are estimated using copper futures contracts. We will use these estimated parameter values to solve the mine valuation problem.

### 1.7.1 Estimating the parameters of the price process

The parameters of Equation (1.1) are estimated in the risk-neutral world. We define the parameter $\hat{\mu} = \mu - \eta$ so that the market price of risk, $\eta$, is deducted from $\mu$ which is the long-run mean of $ln(P)$ before adjusting for the price risk. The market price of risk reflects additional returns that the market demands over the risk-free interest rate per each unit of price volatility, $\sigma$. Note that in the stochastic price process $\kappa > 0$, $\mu > 0$, $\sigma > 0$, and $\eta > 0$. These parameters are estimated using data for copper futures prices, reflecting current market expectations. Estimation results are provided in Table (1.1).

To obtain estimates, we have used a Discrete Kalman Filtering approach and a Maximum Likelihood Function.[18] This study uses weekly data for copper futures contracts traded on the London Metal Exchange (LME).[19] The estimation is done for six futures

[18]These methods are explained in Schwartz (1997).

[19]Data for this study were collected from Datastream.

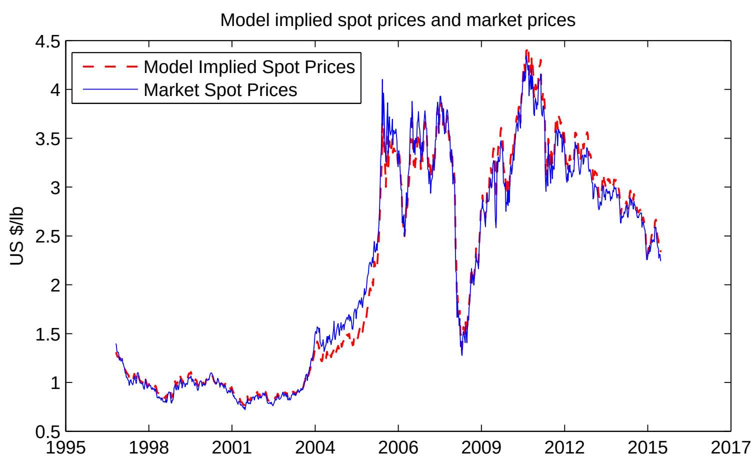


Figure 1.1: *Model implied copper spot prices and market copper prices. Weekly data from Aug 1st, 1997 to Jul 13th, 2015. Nominal prices are deflated by the US Consumer Price Index, base year=2007.*

contracts dated from August 1997 to July 2015, with 1, 6, 11, 16, 21, and 24 months to maturity.[20] To find real copper prices, futures prices are deflated by the US Consumer Price Index. Due to the lack of data on copper spot prices, futures contracts closest to maturity proxy the market spot prices (Schwartz, 1997). The Root Mean Square Error (RMSE) and Mean Absolute Error (MAE) of the estimates of log futures prices are 7 cents per pound and 5 cents per pound, respectively. Moreover, the standard errors of all parameter estimates are small. All estimates are significant. These findings suggest that the one-factor model provides a good tracking of the copper market prices as shown in Figure (1.1).

### 1.7.2 Project specification

The numerical example is based on data from Copper Mountain which is an open-pit mine located in south-western British Columbia which had an expected mine life of 15 years when it was first proposed. In 2007, the Copper Mountain project proceeded to a feasibility study

[20]Long maturity contracts are of most interest as the goal of this study is to value a long-term investment project.

to construct an open-pit mine at the estimated cost of US $380 million. An additional US $5 million for the feasibility study, environmental testing and geological consulting increased the construction cost to US $385 million. This mine had a production target of 78.2 million pounds of copper per year, starting from June 2011, with an estimated average production cost of US $1.35 per pound of copper. The fixed cost of sustaining capital are estimated to be US $1.66 million per year. The mine's average strip ratio (i.e., waste/ore) is 1.5 pounds of waste per each pound of ore extracted.

Additional assumptions required for the numerical example are described below. By assumption, the maximum amount of waste that is allowed to be generated during the life of project is 2200 million pounds. The parameter of the clean-up cost function is calibrated based on the data provided by the Financial Assurance Guideline for determining the closure cost of a landfill provided by the government of Ontario (2011).[21] It is further assumed that the maximum feasible rate of abatement can be twice as high as the environmental deterioration rate, i.e., $\bar{a} = 2\phi\bar{q}$.[22] This assumption allows for the possibility that the abatement rate may exceed the deterioration rate.

Launching the project with liability requirements entails fixed costs of US $385 million, whereas the bonding policy imposes an additional cost on the firm that is the initial amount of the bond adjusted by the third-party expenses. The third-party cost that reflects administrative costs, mobilization costs, etc is assumed to be 30% of the firm's restoration cost. Either mothballing the mine or resuming operations after mothballing are assumed to entail an up-front cost of $5 million. It is further assumed that remaining in the mothballed stage costs $1 million per year for environmental monitoring and maintenance. Note that in Equation (1.10) in the production phase, $C_2^m$ equals the fixed costs of sustaining capital, while at the mothballed stage $C_3^m$ is the summation of costs for sustaining capital, $C_3^{m_1}$, as well as for environmental monitoring and maintenance, $C_3^{m_2}$. Table (1.2) summarizes the parameter values used for the numerical example. Recall that taxes are not included

[21] In this guideline, the estimated closure cost of a landfill with $60,000$ tonnes capacity is around US $3 million. After transforming tonnes to pounds, we have calculated the total closure cost of a landfill with 2200 capacity equivalent to US $49.895 million, i.e., $C^f(\bar{w}) = 49.895$. Then, $\beta = 49.895/\bar{w}^2 \simeq 10^{-5}$. This estimate is intended to provide a rough order of magnitude for clean-up costs. This guideline is available at https://www.ontario.ca/document/f-15-financial-assurance-guideline-0.

[22] This study sets the abatement ceiling high enough so that the likelihood it binds is small, because abating at high rates is prohibitively expensive.

| | | | |
|---|---|---|---|
| Life of project | | $T = 15$ | years |
| Risk-free rate* | | $r = 0.02$ | per year |
| Bond service charges* | | $\rho = 0.05$ | per year |
| Initial reserve | | $s_0 = 1173$ | million lb |
| Strip ratio (waste:ore) | | $\phi = 1.5 : 1$ | |
| Production capacity | | $\bar{q} = 78.2$ | million lb/year |
| Abatement ceiling* | | $\bar{a} = 2\phi\bar{q}$ | million lb/year |
| Landfill capacity* | | $\bar{w} = 2200$ | million lb |
| Extraction cost parameter | $C^q(q) = \gamma q$ | $\gamma = 1.35$ | \$/lb |
| Abatement cost parameter* | $C^a(a) = \alpha a^2$ | $\alpha = 10^{-3}$ | |
| Firm's clean-up cost parameter** | $C^f(w) = \beta w^2$ | $\beta = 10^{-5}$ | |
| 3rd party cost adjustment factor*** | | $\nu = 1.30$ | |
| Project stages | | $\delta_1,\ \delta_2,\ \delta_3,\ \delta_4$ | |
| Fixed decision time* | | $\tau_d$ | every year |
| Construction cost | $I$ | \$385 | million |
| Cost to mothball and reactivate* | $C(\delta_2, \delta_3), C(\delta_3, \delta_2)$ | \$5 | million |
| Fixed costs of sustaining capital | $C_2^m,\ C_3^{m_1}$ | \$1.66 | million/year |
| Fixed monitoring costs while mothballed | $C_3^{m_2}$ | \$1 | million/year |

Table 1.2: *Parameter values and functional forms for the prototype open-pit copper mine. All dollar values are based on 2007 US dollars. *Assumed by the authors. **$\beta$ is calibrated based on landfill closure costs provided by the Government of Ontario 2011. ***From Ferreira et al. (2004).* Other values are from 2007 feasibility study conducted by the Copper Mountain Mining Corporation.

in the analysis.

## 1.8 Results analysis

This section compares the impacts of the environmental bond and the strict liability rule on the firm's optimal investment decisions as indicated by critical prices. In addition, we compare the project value and optimal abatement decisions under each policy. Note that the quantitative results at each stage of the project are dependent on the current values of the state variables – the resource stock, resource price, level of the waste stock, and time. To depict the results graphically, we must choose representative values for the state variables. However, the numerical solution is available over the full ranges of the state variables.

The results for the bonding policy depend on the particular characteristics of the bond. In this paper, we focus on whether government pays interest on the bond and whether the firm pays a risk premium on loans to finance the bond. Regarding the interest paid by the government, we contrast cases when the bond pays either zero interest or the risk-free interest rate. In the analysis, we examine the case where the bond must cover the full costs of clean-up to the government, which are assumed to be 30% higher than if the firm did the clean-up itself (i.e., $\nu = 1.3$ in Assumption 3). We do not show the results for cases where the bond is set at the firm's own estimated clean-up costs (i.e., $\nu = 1$) as these results are quantitatively close to $\nu = 1.3$, but the government is not fully collateralized for the cost of clean-up, which is the objective of the bond.

This paper focuses on the impact of these policies on the firm's optimal behaviour. In Appendix A.2, we briefly discuss whether the bonding polices would give a first best outcome.

### 1.8.1 Valuation results

We begin by showing how the value of the mining project varies with the price of copper, the stock of waste, and the level of copper reserves. This is depicted in Figure (1.2) for the case of strict liability. Diagrams for the bond paying the risk-free interest are similar, and are not shown. The value of the investment project is depicted prior to construction at the initial time, $t = 0$. The left-hand panel of Figure (1.2) shows the value of the project across different starting prices and different levels of reserve prior to initial investment, when the starting level of waste is at 500 million pounds.[23] We observe, as expected, that there is an increasing trend in the value of the project with respect to prices and reserve levels.

The right-hand panel in Figure (1.2) represents the value of the project across different resource stock levels and different levels for the starting value of waste as a result of

[23] This initial level of waste is chosen for the purpose of illustration only. Changing the initial level of waste changes the project value but the intuition remains the same.

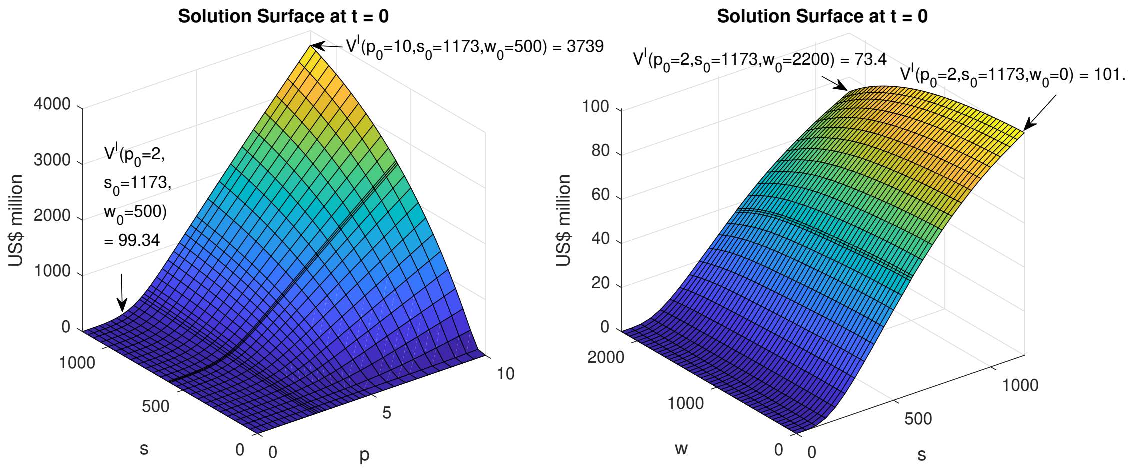


Figure 1.2: *Project value prior to construction under the strict liability rule. In the left-hand panel, the level of waste is fixed at* $w_0 = 500$ *million pounds, and in the right-hand panel, the price is fixed at \$2/pounds.* $s_0$ : *million pounds,* $p$ : *US\$/pound,* $w$ : *million pounds.*

construction, when the price of copper is \$2/pound. At a given level of initial reserve, generating a larger amount of waste during the construction phase reduces the project value by increasing the firm's cost of complying with the strict liability rule during the extraction phase and at project termination. A larger initial waste implies that the landfill capacity will bind faster during the operation, and thus once the construction is completed, the operating firm will have to exercise more abatement to maintain space in the landfill. In addition, the firm's liability costs at project termination date rise as more waste builds up. For the prototype project with $s_0 = 1173$ million pounds, the project value prior to construction ranges from \$73.4 million to \$101.1 million depending on the severity of damage during the initial construction.

Figure (1.3) compares the project value prior to construction and during the extraction phase under the strict liability rule and the bonding policy. This figure is shown for one realization of the copper price and full reserves at the initial time $t = 0$. Three cases are analyzed for the bond:

- **Case I:** The firm receives the risk-free rate of interest on the bond ($r = 2\%$).

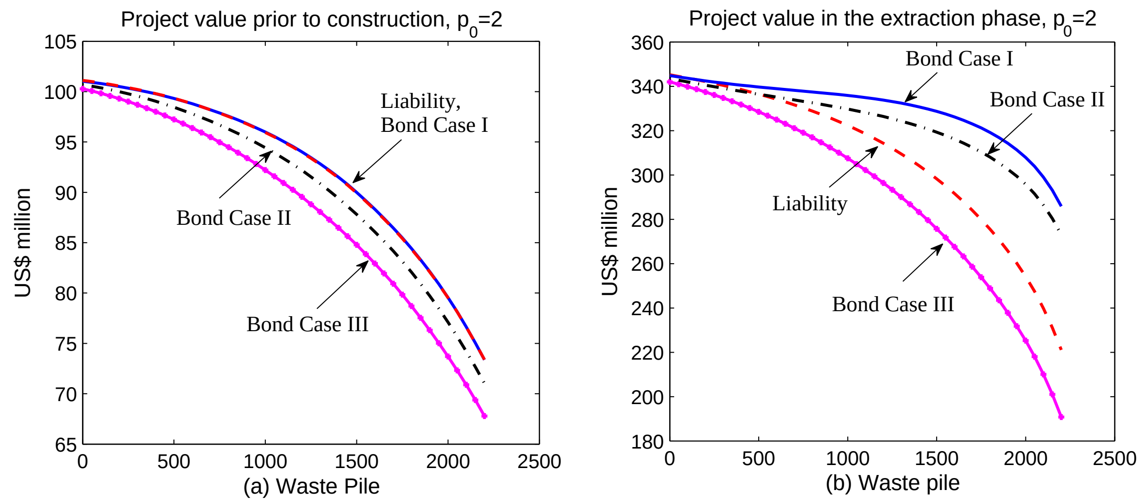


Figure 1.3: *A comparison of project value (a) prior to construction and (b) during the active extraction under the strict liability rule and the three bond cases, for* $p_0$ *=\$2/pound and* $s_0 = 1173$ *million pounds.*

- **Case II :** The government pays no interest on the bond.
- **Case III:** The firm borrows at a premium over the risk-free rate ($\rho = 5\%$) and the government pays no interest on the bond.

Panel (a) of Figure (1.3) shows that the value of the project under the bond for Case I (interest paid at the risk-free rate) is identical to the strict liability rule. This follows because the interest paid on the bond is the same as the discount rate, with the implicit assumption that the firm can borrow or lend at the risk-free rate. As long as the firm receives the risk-free rate on the bond, it will be indifferent between paying clean-up costs via the bond as waste accumulates, or delaying payment of clean-up costs to the end of the project. If no interest is paid on the bond (Case II ), the bond is more burdensome to the firm, reducing the value of project compared to Case I. Under the realistic assumption in Case III that the firm can only borrow at rate $\rho$, which is higher than the risk-free rate, the project value is reduced even further.

Panel (b) of Figure (1.3) compares the values of project in the production phase (Stage 2) when operations have commenced. To reach Stage 2, the firm must pay an initial amount into the bond, which reflects the waste stock generated to initiate operations. For a firm in Stage 2, this initial payment is a sunk cost, and thus does not influence the project value in the extraction phase. It follows that the value of the project under the Case I bond will be higher than for the strict liability rule, where all clean-up costs will be paid at project termination. Hence, we see in panel (b) that the curve reflecting the bond in Case I lies above the liability curve. If the firm is paid no interest on the bond as in Case II , the value of the project is reduced compared to the Case I, but still higher than the liability. Under Case III, when the firm must borrow at a risk premium, the value of the project is reduced even further and falls below that of the strict liability rule.

### 1.8.2 Optimal abatement rates

The left-hand panel of Figure (1.4) compares the optimal abatement rate in the production phase (Stage 2) versus the waste stock at time zero, for the strict liability rule and the three different bond cases. Note that the optimal abatement rates are all the same when the landfill is at capacity, because at this point the only way that the firm can continue production is to abate all of the waste as it is created.

For intuition about abatement rates, it is helpful to consider the optimal abatement condition expressed in Equation (1.23), whereby the marginal cost of increasing abatement by one unit on the left hand side is set equal to the marginal cost of environmental deterioration on the right hand side. The marginal cost of environmental deterioration to the firm differs across the different cases. For the strict liability rule, the marginal cost of environmental deterioration consists solely of $\frac{\partial V}{\partial w}$, which reflects the cost of using up capacity in the landfill as well as adding to future clean-up costs. For the bond in Case I, the marginal cost of environmental deterioration includes $\frac{\partial V}{\partial w}$ as well as the marginal cost of posting the bond, $\theta(w)$. Since costs of clean-up are paid immediately by posting the bond, $\frac{\partial V}{\partial w}$ in Case I reflects only the cost of using up landfill capacity net of the marginal restoration benefit and any interest paid on the bond. As long as the firm receives the risk-free interest rate on the bond, it will be indifferent between posting the bond when the waste is created or paying for clean-up at project termination. Hence, the abatement rates for Case I and the liability rule are identical.

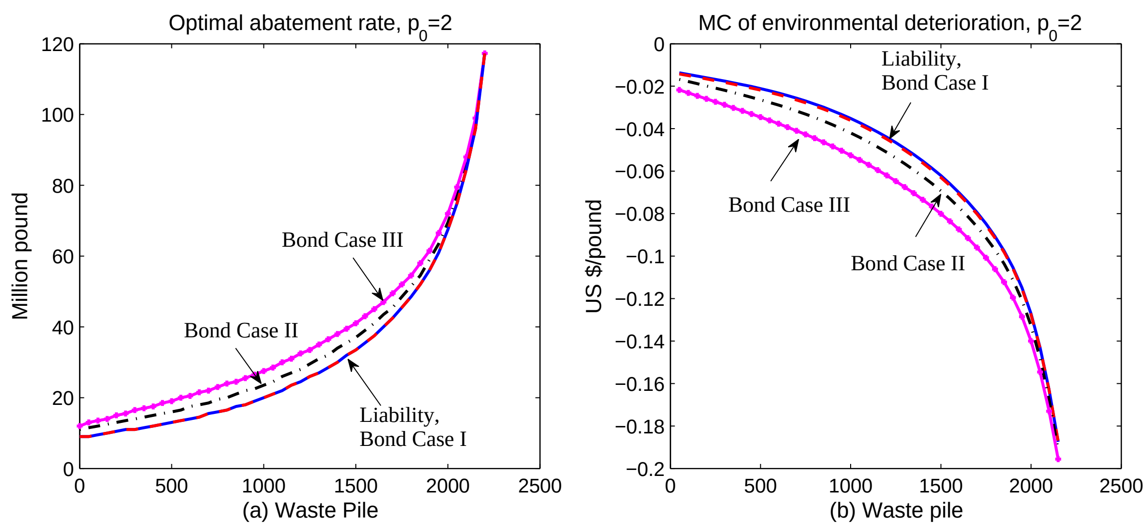


Figure 1.4: *A comparison of (a) optimal abatement incentives and (b) marginal environmental deterioration costs, at each level of initial waste under the strict liability rule versus three bond cases, at* $p_0$ *=\$2/pound and* $s_0 = 1173$ *million pounds.*

If there is no interest paid on the bond (Case II ), $\frac{\partial V}{\partial w}$ becomes more negative relative to Case I, because the foregone interest represents an additional cost of paying clean-up costs up-front. Hence the marginal cost of environmental deterioration is higher in Case II than Case I. This motivates the firm to abate at a higher rate than under strict liability (or the bond in Case I) over all waste levels, as can be seen in panel (a) of Figure (1.4). The marginal cost of environmental deterioration for the firm in Case III is even higher than in Case II. The requirement to borrow to finance the bond at a risky interest rate, combined with the fact that no interest is paid by government on the bond, means that increasing the waste stock becomes even more costly for the firm. $\frac{\partial V}{\partial w}$ is more negative as a result, motivating additional abatement compared to the strict liability rule or the other bond cases.

The right-hand panel of Figure (1.4) shows the marginal environmental deterioration cost of the strict liability rule and the three bond cases. A more negative marginal environmental deterioration cost at a given level of waste implies a higher optimal abatement rate. Therefore, this diagram is the mirror image of the left-hand panel of Figure (1.4).

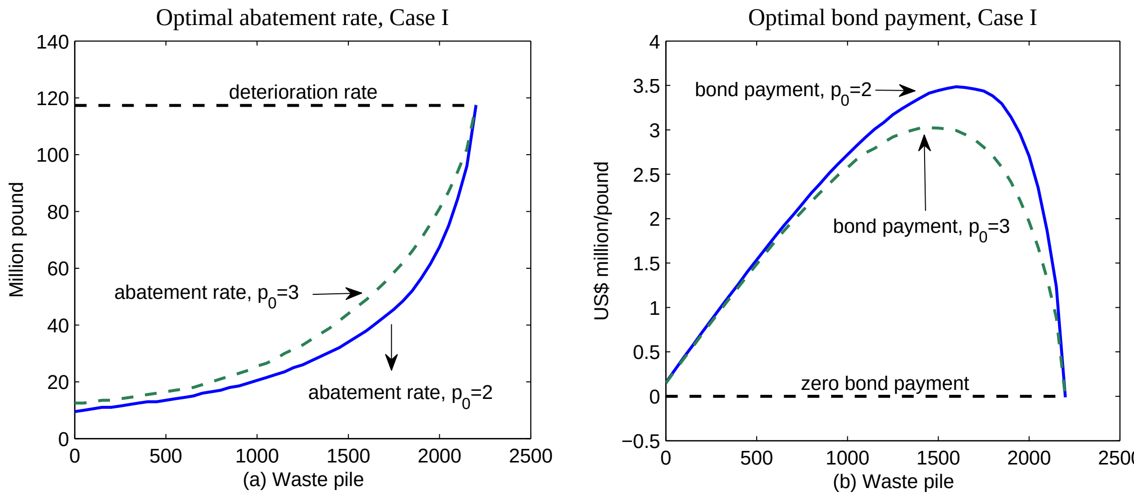


Figure 1.5: *Optimal rate of abatement and optimal expected bond payment across waste accumulation at the operating stage under the bond in Case I, for two price levels and* $s_0 = 1173$ *million pounds, at time zero.*

Another interesting result that is the trade-off between abatement and the bond payment in Stage 2, as shown in Figure (1.5) for the Case I bond, panel (a) in the figure shows the optimal abatement rate versus the waste stock for two different copper prices. Panel (b) shows the corresponding payments into the bond. At low levels of waste, the optimal abatement rate increases as more waste accumulates but is not high enough to create a significant change in the stock of waste. Consequently, the firm's payment to the bond increases with waste accumulation. At higher levels of waste, when the landfill is reaching its capacity, the firm's optimal abatement effort progressively increases. As a result, waste accumulates at a slower rate and thus the payments to the bond gradually diminish. Once the landfill capacity is reached, the only way to continue operations is to abate at least at the deterioration rate. If abatement fully offsets the deterioration rate so that the level of waste does not change, the expected bond payment is zero. This trade-off does not exist under the liability rule, because restoration costs are not required until the project terminates. At the higher price, the project is more profitable which motivates the firm to maintain more capacity in landfill by abating at a higher rate compared to the lower copper price. Note that at a zero level of initial waste, the firm exercises a positive abatement rate because the extraction activity is creating a flow of waste. We have not shown Cases II and III, as the intuition is the same. However, the expected bond payments for Case II and Case III are lower than Case I, due to higher abatement.

### 1.8.3 Optimal choice of project stages

We examine the lowest copper prices at which it is optimal to switch from one stage to another, which we refer to as critical prices. Critical prices are optimally determined based on Equation (1.19) and change with the level of reserve, size of waste stock, and time. For example, for $s_0 = 1173$ and $w_0 = 500$ million pounds, Figure (1.6) illustrates the value of the project in Case I prior to the initial investment (Stage 1) and once the operation has started (Stage 2) net of the up-front costs of construction and initial bond payment. These values are plotted across copper prices up to US \$10 per pound. It is optimal to start extraction activities once the value in Stage 2 less switching costs exceeds the value in Stage 1. Therefore, it is not optimal to incur the construction cost until copper prices increase to US \$2.6 per pound. Before this threshold, the net present value prior to incurring switching costs is positive and higher than the net present value of the operating option. Thus, there is an opportunity benefit to waiting for a higher price before beginning operations.

The first column of Table (1.3) shows the critical prices to move from one stage to another stage at time zero when switching entails up-front costs, given $s_0 = 1173$ and $w_0 = 500$ million pounds, for the bond in Case I. If per pound copper prices become as low as US \$1.39, the optimal strategy is to mothball current activity and to remain idle until the prices increase to US \$1.51. This is the lowest price that encourages reactivation. The inactive firm can also choose to terminate the project and carry out the restoration work. Critical prices that trigger termination from the mothballed stage tend to be as low as US \$0.71 per pound.

The critical prices in Table (1.3) shown for $t = 0$ imply that the firm would not directly abandon the project from the operating stage at the early life of the project. This can be seen by noting that the critical prices to mothball from Stage 2 are above the prices for abandonment from Stage 2. This means that the firm will always go through the mothballed stage first before abandoning. However these critical prices reflect optimal

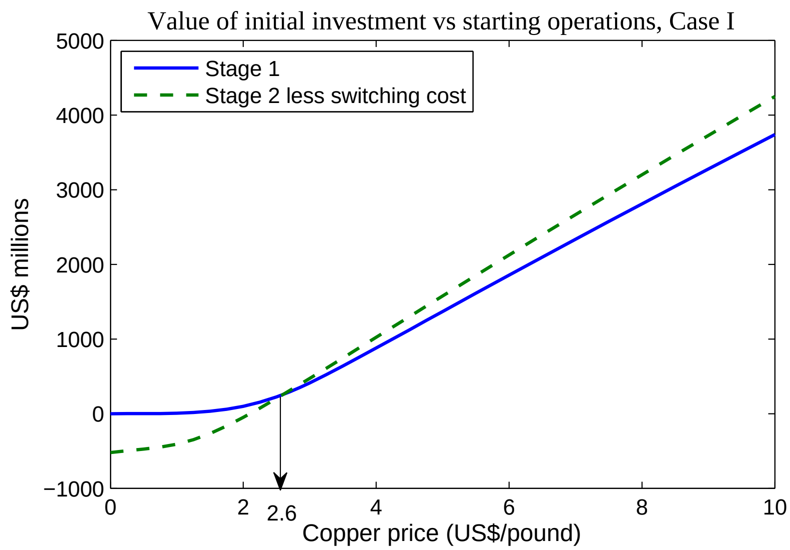


Figure 1.6: *Value of initial investment and begining production (Stage 1 to 2) under the bond in Case I (identical to the strict liability rule), given* $s_0 = 1173$ *and* $w_0 = 500$ *million pounds.*

decisions for an operating project at time zero. We would expect that if the time left in the life of the project is small and mothballing costs are non-negligible, it may be optimal to abandon directly from the operating stage. In this numerical example, we find that when there are two years left in the life of the project, it is optimal to abandon without first mothballing over all levels of waste.[24] Appendix A.3 discusses critical prices for abandoning and mothballing the extraction activities for different decision dates prior to $T$ and different levels of waste accumulation, under the bond in Case I.

Critical prices are sensitive to the level of reserve, which has previously been described by Insley (2017). The second column of Table (1.3) shows that critical prices, under the bond that pays interest, are higher at all stages if half of the reserve is used up, assuming $w_0 = 500$ million pounds. The initial investment occurs at higher prices for lower initial reserves due to the sizable fixed construction costs. After the project is launched, as the reserve depletes and becomes more scarce, its shadow value increases (i.e., a larger $\partial V/\partial s$ in Equation (1.17)). Thus, the firm needs to obtain higher prices to reopen or mothball

[24]Recall that the firm that is actively extracting has an obligation to terminate the project at $T = 15$ directly from the extraction phase regardless of prices.

| Transition from: | $s_0 = 1173$ | $s_0 = 587$ |
|---|---|---|
| Stages 1 to 2: Begin production | 2.60 | 3.00 |
| Stages 2 to 3: Mothball | 1.39 | 1.48 |
| Stages 3 to 2: Reactivate | 1.51 | 1.63 |
| Stages 2 to 4: Abandon | 1.09 | 1.17 |
| Stage 3 to 4: Abandon | 0.71 | 0.77 |

Table 1.3: *Critical prices (US$/lb) at time zero under the bond in Case I (identical to the strict liability rule), for* $s_0 = 1173$ *and* $s_0 = 587$ *million lb, given* $w_0 = 500$ *million lb.*

the activity. Finally, the abandonment of the mine when half of the reserve is depleted will happen at a higher price, which indicates that the mine with the lower reserve is more likely to be abandoned.

What is interesting in the current study is how the critical prices vary in response to changes in the size of the waste stock under each policy, at a given level of reserve. We examine the extent to which each policy affects critical prices, in particular, to launch the project and to abandon the mine. The decision to move from Stage 1 to Stage 2 depends on the benefits of delaying the up-front investment costs versus the costs of delay in gaining the value of the project commencement. In Figure (1.7), we observe that critical prices to commence operations are identical for the strict liability policy and the Case I bond. Referring to Equation (1.19), recall that the cost of moving to Stage 2 is higher under the bond policy than the strict liability rule by the amount that must be paid into the bond. As noted earlier, the value of the project in Stage 2 is higher under the bond than under the strict liability rule. These two differences offset each other and result in the same critical prices for the Case I bond and the strict liability rule to move from Stage 1 to Stage 2.

The bond without interest income in Case II raises the critical prices to start extraction activities compared to the strict liability rule and thus fewer projects will be undertaken. Clearly, critical prices to begin the project are even higher for Case III, when the firm borrows at a risk premium. These results are also shown in Figure (1.7). Note that more waste accumulation raises the critical prices to begin operations due to higher costs of complying with both policies, making project commencement less likely under both policies.

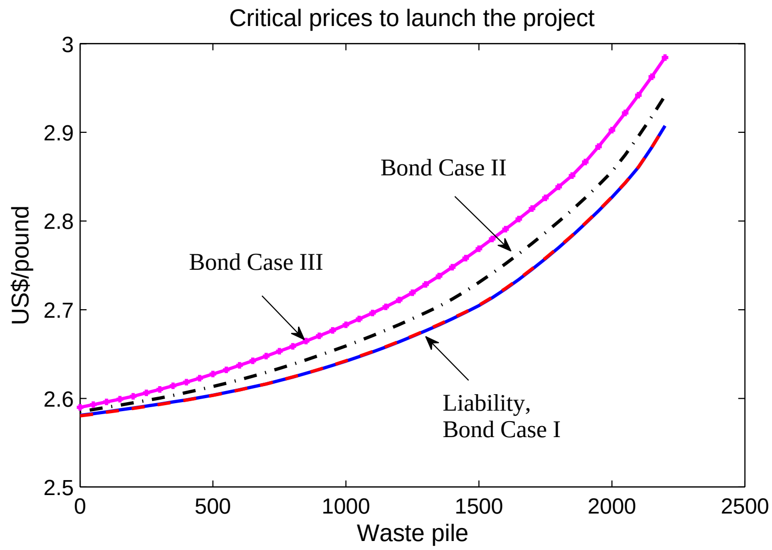


Figure 1.7: *Critical prices to launch the project versus the waste stock under the strict liability rule and the three bond cases, at time zero, given* $s_0 = 1173$ *million pounds.*

Decisions to abandon the mine are compared in Figure (1.8) and some interesting results emerge. First, both the bond and liability policies lead to the same optimal abandoning decisions from the mothballed stage if the bond pays interest (Case I). This follows from similar logic as described above for moving from Stage 1 to Stage 2.[25] As already noted, the Case II and Case III bonds are more burdensome to the firm, yielding lower project values. Hence we observe it is optimal to abandon the project at higher prices than for the Case I bond or strict liability rule.

Under the Case I bond and strict liability rule, there is a decreasing pattern for critical abandoning prices, which implies that the firm's motivation to abandon the site becomes weaker as more waste builds up. Under the strict liability rule, the sizable restoration cost at large quantities of waste provides incentives for the firm to delay paying for such costs

[25]The relevant equations are $V(p, s_0, w_0, \delta_3) = V(p, s_0, w_0, \delta_4) + [\mathbf{1}_{b=true}C^{tp}(w_0) - C^f(w_0)]$ with $V(p, s_0, w_0, \delta_4) = 0$. With simple algebra we can derive $V^b(p^b, s_0, w_0, \delta_3) - V^l(p^l, s_0, w_0, \delta_3) = C^{tp}(w_0)$ in which $p^b$ and $p^l$ denote critical prices under the bond and the liability, respectively. If the gap between the two values at a given level of reserve and waste are equal to $C^{tp}(\cdot)$, the same critical prices satisfy $V^b(p^b, s_0, w_0, \delta_3) - V^l(p^l, s_0, w_0, \delta_3) = C^{tp}(w_0)$, and thus $p^b = p^l$. The Case II bond yields a relatively lower value in Stage 3 compared to Case I, and thus a higher $p^b$ eliminates the gap, resulting in $p^b > p^l$.

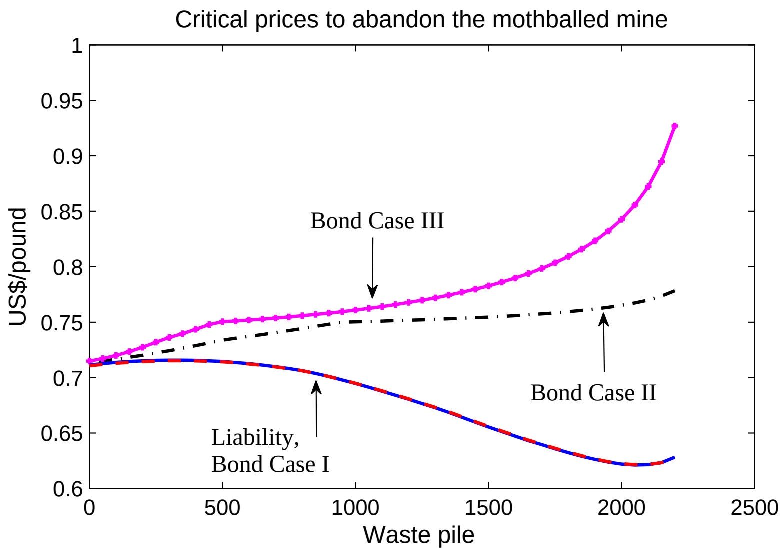


Figure 1.8: *Critical prices to abandon the mothballed project versus the waste stock under the strict liability rule and the three bond cases, at time zero, given* $s_0 = 1173$ *million pounds.*

by remaining at the mothballed stage. In contrast, with bonding requirements in Case I, the higher interest income on the deposited money at larger waste stock motivates the firm to sit idle longer. However, if the bond does not generate interest income (Case II), the longer the firm sits idle the higher the opportunity cost of the bond. Consequently, increasing critical prices are observed in Figure (1.8) for the Case II bond, indicating that as waste accumulates and the amount of bond grows, the firm's motivation to abandon the mine becomes stronger. For the Case III bond, as the waste stock grows, the firm will have to repay a large loan at termination. Because the cost of the loan exceeds the risk-free rate, the larger is the waste stock, the more costly it is to the firm to delay clean-up. This results in an increasing trend in critical prices with a higher waste stock, and critical prices are higher than in the other cases, implying mine abandonment is more likely.

Critical prices to mothball and reactivate operations are also higher for the mine with greater waste as shown in Figure (1.9) under both policies and for all bond cases. More waste accumulation results in a lower project profitability due to a higher cost of compliance during operations with both policies. Therefore, the project with larger quantity of waste is more likely to be mothballed following a decrease in copper price. Similarly, the idle firm

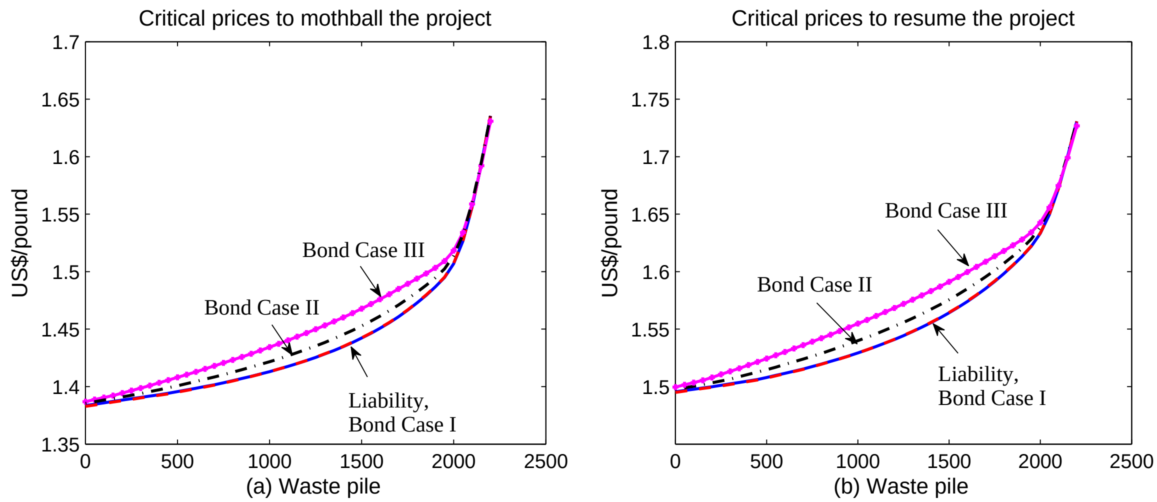


Figure 1.9: *Critical mothballing and resuming prices versus the waste stock under the strict liability rule and the three bond cases, at time zero, given* $s_0 = 1173$ *million pounds.*

facing more waste is less likely to reopen its mine as the anticipated profits are smaller.

## 1.9 Conclusions

This paper is motivated by the observation that many resource extraction projects leave behind a toxic legacy and taxpayers are left to fund the clean-up. Firms may walk away from their clean-up obligations or may simply let projects sit idle, even when there is no intent to restart operations. If designed appropriately, an environmental bond is one mechanism to ensure that adequate funds are set aside by private firms to undertake site clean-up. In practice, it has been observed that funds set aside in environmental bonds are often less than needed to cover actual clean-up costs.

This study formulates a stochastic optimal control problem to examine the incentives for waste creation and clean-up with and without an environmental bond designed to fully cover estimated clean-up costs. The firm is obliged to clean up any waste left at the termination of a project (strict liability), but under the bonding policy, the firm must deposit funds up-front equal to estimated clean-up costs of a third party. These funds are reimbursed to the firm as waste reduction or abatement occurs. The optimal control model is analyzed for a representative copper mine in Canada. We compare the strict liability rule (liability for clean-up) versus the bonding policy, which also includes liability for final site clean-up.

The objective of the bond is to fully collateralize the government from any liability for clean-up costs. We do not model the risk of bankruptcy. Rather it is assumed the firm chooses its optimal production and abatement, assuming it will continue to be a going concern in the future. Our objective is to examine the impact of the bonding policy on a firm which will remain solvent. The impact of possible bankruptcy on a firm's decisions is examined in Chapter 2.

Under both the strict liability and environmental bonding policies, there is no requirement for waste clean-up until the termination of the project. However, two factors, other than the bond, may give an incentive for waste abatement during the life of the project. First, there is an upper limit on the permitted size of the waste stock and when that limit is reached, firms must abate their waste in order to maintain production. Second, abatement costs and eventual clean-up costs are convex with respect to waste. Depending on the specifics of the cost functions, firms may find it beneficial to do some waste abatement during project operations rather than leave it all to the end. The environmental bond, depending on its characteristics, provides a third incentive to abate waste during the life of the project and also provides a greater incentive to abandon the project early (before $T$) which triggers final restoration of the accumulated waste.

The bond requires the firm to pay clean-up costs as the waste is generated, rather than delay until project termination. Provided the government pays the risk-free rate on the bond deposit, and if it is assumed that the firm can borrow or lend at the risk-free rate, this early payment for clean-up is not detrimental to the firm. However, markets would normally demand a risk premium from firms to finance bond payments which increases the cost of the bond to the firm. Further, if the government does not pay interest on the bond deposit, this imposes an additional cost on the firm. In our numerical example, our main findings are as follows for the Case III bond in which the firm must pay a risk premium and receives no interest income on the bond.

- The bond has a significant effect on the operations of a prototype copper mine. The bonding requirements in Case III reduces the project value and thus increases the threshold price needed for the project to go ahead, making the project commencement less likely.

- Since refunding the bond following a restoration yields a cash payment to the firm equal to the third-party costs, the firm is more likely to abandon the mine and undertake the required clean-up. In the absence of a bond (strict liability rule) the firm is more likely to leave the mine inactive, rather than abandoning and cleaning up the mine.

- The bond also causes the firm to abate more during the life of the mine, and the final accumulated wasted is reduced.

The bonding policy we analyze is demanding of the firm in that the full cost of clean-up must be deposited with the government and this cost must be updated over time as the waste stock changes. This policy avoids any risk to the government of being left to clean up mine waste. However the policy is costly to firms, and will reduce the number of projects that are developed. Whether the additional costs imposed on firms are worthwhile depends on the extent of avoided costs from firms not fulfilling clean-up obligations. However, given the number of orphan waste sites in North American and elsewhere, a more stringent bonding policy seems long overdue.

# Chapter 2

# Bankruptcy Risk and Optimal Hazardous Waste Clean-Up Decisions under an Environmental Bond and a Strict Liability Rule

## 2.1 Introduction

Inadequate hazardous waste clean-up by mining firms has been one of the most pressing environmental issues in industrialized countries such as the US and Canada. A common approach to dealing with such issues is based on a strict liability rule that assigns a liability for clean-up to responsible firms. However, evidence reveals that many resource extraction projects undertake inadequate clean-up mainly due to the large costs that this policy imposes on firms at project termination[1]. They may go bankrupt due to sizable clean-up costs or may simply let projects mothball or sit idle as a way to escape such costs. Figure 2.1 shows a significant increase in the number of mothballed oil and gas wells in

[1]Clean-up costs are mine-specific, range from millions to billions of dollars for a single mine (Boyd, 2002, Lemphers et al., 2010, Parente et al., 2006), and depend on several factors such as the extent of activity and the probable difficulty of reclamation (Gerard, 2000).

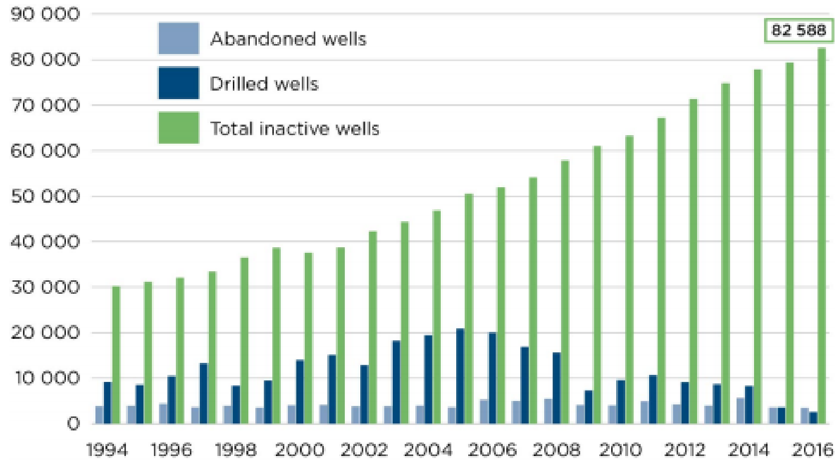


Figure 2.1: *Alberta's inactive wells from 1994 t0 2016. Source: Alberta Energy Regulator*

Alberta over the last 20 years.[2] Once a project is mothballed due to low prices and large levels of waste it may remain idle for an extended period of time, as discussed in Chapter 1. This phenomenon is observed by a recent study of Alberta's oil and gas industry. Between 1993 and 2007, the average and maximum inactivity durations for Alberta's oil producers have been around 8 years and 73 years, respectively (Muehlenbachs, 2015). Remaining at the mothballed stage to avoid clean-up costs can increase the risk of bankruptcy, especially if commodity prices are low (Dachis et al., 2017, Kahn et al., 2001, Muehlenbachs, 2017).

These observations affirm that a strict liability rule does not guarantee that firms clean up their sites. However, combining a strict liability rule with an appropriate bonding requirement helps ensure clean-up, even when bankruptcy is possible. An environmental bond specifies a payment by a firm to the government before a project commences and/or during its operation. The firm is obligated to periodically report its amount of waste creation along with the required clean-up costs and deposits bonds of an equivalent value (Perrings, 1989). This deposit will be refunded should the firm be solvent at project termination date and do the required clean-up. Otherwise, the entire bond will be used by government for clean-up.

[2]This figure is obtained from a report by the Alberta Energy Regulator (AER).

The impacts of an environmental bond on firms' clean-up decisions have been analyzed by several studies, as reviewed in Chapter 1. Our study in Chapter 1 models the dynamic effects of an environmental bond plus strict liability for clean-up (the bonding policy) versus the strict liability rule alone and demonstrates that the optimal site clean-up decisions by a solvent firm depend on the characteristics of the bond. If it is assumed that the firm can borrow and lend at the risk-free rate and if the government pays interest at the risk-free rate on the deposited money, the solvent firm's optimal clean-up strategies under the bonding policy and the strict liability rule are equivalent.[3] As more waste accumulates, the critical price that triggers abandonment and site clean-up falls under both policies, meaning that site clean-up becomes less likely and the firm is more likely to sit idle in a mothballed state.

Chapter 1 also highlights that, in practice, a firm would not be able to finance the bond at the risk-free rate, but would have to pay a risk premium. Further, government may pay no interest on the bond (or at less than the risk-free rate). In these cases, the bonding policy makes the firm worse off than under the strict liability rule alone and optimal firm strategy is affected. Notably, under these more costly bonds, critical prices that trigger abandonment and site clean-up are increased, implying that site clean-up may happen sooner than under the strict liability rule. Further, the probability of site clean-up increases as the waste stock builds up, in contrast to what happens under the strict liability rule.

The model developed in Chapter 1 allows that the firm temporarily mothball the project but eventually clean-up must occur at the end of the project life. However, the possibility of firm bankruptcy was not explicitly included in that model, and thus mothballing is the only option available to the firm to delay waste clean-up. Chapter 2 contributes to our previous study by considering another important option available to the firm, i.e., the possibility of declaring bankruptcy. Bankruptcy is a real possibility faced by all firms, and firms involved in commodity industries are particularly susceptible. A firm's ability to avoid paying for waste clean-up following bankruptcy will depend on current environmental

[3]As discussed in Chapter 1, this result still holds if the clean-up costs are increased by mobilization costs to a third-party, who will perform site restoration following a default. The reason is that the solvent firm receives interest on the higher amount of the deposited bond over the project life and eventually receives back the third-party's additional costs following a successful restoration.

policies. Under a strict liability rule, a bankrupt firm might entirely avoid waste clean-up costs, transferring that responsibility to the government. A bond is intended to prevent this possibility. Provided that the bond covers the full cost of clean-up, the government will not find itself liable for any portion of the clean-up costs. The firm has to pay the bond amount as the waste is created, and hence is less able to avoid these costs, particularly if the firm finances the bond out of retained earnings. If the firm borrows to finance the bond, it is possible that the firm will avoid clean-up costs by defaulting on the loan in the event of a bankruptcy.

In Chapter 1 it was assumed that the firm acts as a going concern, ignoring the possibility of bankruptcy. It was found that if the government pays the risk-free rate of interest on the bond, and if the firm finances the bond out of its retained earnings, the bonding policy and strict liability rule give the same outcome. This will no longer be the case once bankruptcy risk is taken into account. Instead, we would expect the firm to behave differently under the strict liability rule and bonding policy, particularly if the firm finances the bond out of retained earnings. The differences in a firm's optimal decisions between the liability rule and bonding policy under the risk of bankruptcy will be studied in this chapter. To contrast with the results of Chapter 1, we focus on the case in which the government pays the risk-free rate on the bond and the firm finances the bond from retained earnings.

The problem of how to model the potential for bankruptcy associated with environmental clean-up has been discussed in previous studies (Dionne and Spaeter, 2003, Larson, 1996, Merolla, 1998, Schmitt and Spaeter, 2005, Shavell, 1984, 2005, Van't Veld and Shogren, 2012, among others). These studies concern bankruptcy associated with environmental accidents and analyze private incentives relative to the social optimum for environmental accident prevention and for clean-up after an accident occurs. Although we are not modeling environmental accidents, familiarity with the relevant literature is useful to understanding the main determinants of related bankruptcies. According to these studies, a firm is bankrupt or "judgment proof" if its value is not sufficient to completely cover its liability costs. Relevant liability costs include those related to random environmental damages conditional on an accident occurring. Most studies assume that a firm is solvent if its asset value covers its liability costs (e.g. Merolla, 1998, Shavell, 1984, 2005, Van't Veld and

Shogren, 2012); Others use static models whereby operating cash flow is used as a proxy for a firm's value (e.g. Dionne and Spaeter, 2003, Larson, 1996, Schmitt and Spaeter, 2005).

Although a firm's asset value and cash flows are attractive modeling frameworks in principle, for several reasons, these may not be good indicators of the potential for bankruptcy. If a firm expects potentially large damage costs, imposing liability may give the firm an incentive to hold only a few assets to minimize losses in the event of bankruptcy (Larson, 1996). In addition, a firm might choose to lease most of its assets, leaving it little ability to pay for its future environmental obligations. This problem is addressed in a study through modeling a firm's asset value as depending on the ownership structure of its working capital (Van't Veld and Shogren, 2012). Cash flow is not considered a good indicator of bankruptcy, in particular in dynamic models, because a significant fraction of the eventual stock of damage of mining activities often occurs in the construction phase, prior to the commencement of cash flow (Peck and Sinding, 2009). Linking a firm's probability of bankruptcy only to its cash flow implies that the firm is always bankrupt with any size of damage generated during construction.

Similar to studies related to environmental risk, the literature on the valuation of debt with default risk assumes that the basic underlying state variable in a model of bankruptcy is a firm's value (Black and Cox, 1976, Longstaff and Schwartz, 1995, Merton, 1974, among others). According to these studies, bankruptcy occurs when a firm's value becomes lower than its financial obligations. The more recent literature on risky-debt valuation has proposed a framework in which the basic underlying factor associated with bankruptcy is a firm's stock price, which is continuously observable in a market.[4] Evidence suggests that "in most cases, default is better characterized as involving a gradual erosion of the stock price prior to the event" (Ayache et al., 2003). Therefore, the issuing firm's stock prices gradually decline prior to a bankruptcy, followed by a significant decrease upon bankruptcy. In this setting, the risk of default is exogenous, the result of a jump loss event that occurs with a probability over the next short interval (Madan and Unal, 2000). This default probability is specified as a decreasing function of stock prices (Ayache et al., 2003).

[4]See the literature on the valuing of convertible bonds such as Ayache et al. (2003) and Kovalov and Linetsky (2008).

The fortunes of mining firms and other firms involved in producing and selling basic commodities as well as their stock prices are closely tied to commodity prices. Numerous studies have examined the relationship between commodity prices and the stock prices of firms in different natural resource industries. For example, the findings of a study of 50 North American firms operating in the gold mining industry between 1990 and 1994 indicate that a 1% increase in gold price would increase these mines' stock prices by 3% to 10% (Tufano, 1998). Another study reports correlated movement in the same direction for Canadian oil stock prices and the price of crude oil, based on monthly data that covers the period of 1983 to 1999 (Sadorsky, 2001). This correlation supports the argument that low commodity prices are an important factor that contribute to bankruptcy of firms in commodity related industries.

In addition to commodity prices, another important determinant of bankruptcy at a project level is a firm's liability costs. For a small firm, the build-up of the waste stock combined with low commodity prices might cause bankruptcy. However, the probability that site clean-up costs will bankrupt larger firms with multiple projects is small. In addition, generating more waste increases site clean-up costs, which might motivate a firm to declare bankruptcy strategically to avoid its liabilities (White et al., 2012).

The objective of this chapter is to further our understanding of the firm's optimal ongoing abatement and eventual clean-up decisions under both the bond and the liability policies, when there is a risk for bankruptcy. This study explicitly models a firm's decision to declare bankruptcy specified as a Poisson process that treats bankruptcy as an exogenous, risky event governed by a hazard rate[5]. The determinants of the bankruptcy depend on our assumptions about the size of the firm relative to the resource project. We first examine a case in which the resource project is assumed to be owned by a large firm that also owns multiple other projects. In this case, the firm might go bankrupt even if the value of the individual project is positive. We assume that there is an exogenous probability of bankruptcy that depends solely on the price of the commodity. We also examine a second scenario in which the probability of bankruptcy depends on the stock of

[5]A Poisson process has been widely used by authors to model risks in different contexts. For example, this process is used by Insley and Lei (2007) to specify the risk of fire in an optimal tree harvesting model, by Nkuiya (2015) to capture jumps in climate change damage, and by Ayache et al. (2003) to model a default risk for valuing risky convertible debt.

waste, which determines the firm's clean-up liability, as well as on commodity prices. In this case, the project is a large enough component of the firm's portfolio of projects that a large waste stock for this one project, combined with low commodity prices, could trigger in bankruptcy. Under this scenario, the firm can influence the probability of bankruptcy through its production and abatement decisions.

We analyze abatement and waste clean-up decisions for a prototype copper mine based on data from the Copper Mountain mine in B.C. We implement a numerical solution similar to the approach used in Chapter 1. Our findings reveal that the bond and liability policies have the same impacts on the solvent firm's optimal decisions. However, when bankruptcy is allowed, these two policies are not equal. Bankruptcy results in a lower project value under the bond than the liability and thus creates a gap between the abatement rates and critical prices obtained under the two policies. The firm exercises a lower abatement rate under the liability rule than the bonding policy. In addition, the bonding policy requires higher critical prices for project commencement, and project termination is more likely under the bond as more waste accumulates.

This paper is organized as follows: the next section presents the theoretical model. Section 2.2 explains the theoretical model. The dynamic programming solution of the model is in Section 2.3 and Section 2.4 presents a numerical solution approach. Section 2.5 shows an application of the model to the copper industry. An analysis of results is provided in Section 2.6. The last section concludes.

## 2.2 Model formulation

This section models the optimal decision of a resource extraction firm under an environmental bonding policy and the strict liability rule, when there is risk of bankruptcy. The problem formulation is developed based on that described in Chapter 1 and incorporates the risk of bankruptcy into the firm's optimal operating decisions. For the convenience of reader and completeness of the paper, we briefly overview the model in this chapter, while formulating the risk of bankruptcy and explaining our contributions.

Consider a risk-neutral firm whose extraction from a non-renewable resource generates hazardous waste disposed of into a landfill. The landfill is required to be cleaned up when the project terminates or at the end of the project life. To ensure site clean-up or restoration, a government regulator can implement one of two policies: 1. the strict liability rule, and 2. an environmental bond combined with liability for clean-up. For simplicity, we have assumed that there is no risk of accidental release of pollution from the landfill. Therefore, the only environmental obligation is the clean-up of the landfill.

The firm's optimal operating decisions depend on four state variables: the commodity price, $P(t)$, the stock of the resource, $S(t)$, the amount of waste in the landfill, $W(t)$, and the stage of operation, $\delta_i,\ i = 1, 2, 3, 4$. We define the four stages as follows

- Stage 1 ($i = 1$) is pre-construction
- Stage 2 ($i = 2$) is active extraction
- Stage 3 ($i = 3$) is mothball or temporary shut down
- Stage 4 ($i = 4$) is abandonment.

The firm has three control variables: the resource extraction rate, $q$, the waste abatement rate, $a$, and the decision to move to a new stage of operation, $\delta$.

We have assumed the paths governing the commodity prices, the level of reserve, and the stock of waste are respectively given by

$$dP(t) = \kappa(\hat{\mu} - \ln P)P\ dt + \sigma P dz;\ P(0) = p_0\ given \tag{2.1}$$

$$dS(t) = -qdt; \quad S(0) = s_0\ given \tag{2.2}$$

$$dW(t) = (\phi q - a)dt;\ W(0) = w_0. \tag{2.3}$$

Equation (2.1) represents the risk-adjusted one-factor price model in which $\kappa, \hat{\mu}, \sigma$ are parameters reflecting the speed of mean reversion, the long run mean of $\ln(P)$, and volatility, respectively. $t$ denotes time where $t \in [0, T]$, and $dz$ is the increment of a Wiener process. In Equations (2.2) and (2.3), $q$ is the extraction rate, $a$ represents the abatement rate, and $\phi$ denotes the waste flow rate or landfill deterioration rate. We have assumed that $w_0$ represents the initial level of waste generated during the construction phase. Recall from Chapter 1 that $q \in [0, \bar{q}]$ and $a \in [0, \bar{a}]$ in which the upper bounds on extraction and abatement represent the maximum feasible rates based on the best available technology (Roan and Martin, 1996). Consistent with Keohane et al. (2007), $\bar{a} > \phi\bar{q}$, implying that the firm can abate at a rate that exceeds the waste level generated when extraction is at the maximum rate, $\bar{q}$. In addition, we have $0 \leq W \leq \bar{w}$ and $0 \leq S \leq s_0$ where $\bar{w}$ is landfill capacity and $s_0$ denotes the initial level of reserve.

Assumptions 6 and 7 give the characteristics of extraction cost and abatement cost functions, respectively.

**Assumption 6** *The extraction cost function $C^q(q)$ is linear in the extraction rate so that $C^q(0) = 0$, $C'^q(\cdot) \geq 0$, and $C''^q(\cdot) = 0$.*

**Assumption 7** *The abatement cost function, $C^a(a)$, is assumed to be twice differentiable with $C^a(\cdot) \geq 0$, $C^a(0) = 0$, $C'^a(\cdot) \geq 0$, $C''^a(\cdot) \geq 0$, and $C'''^a(\cdot) = 0$.*

### 2.2.1 Bankruptcy as a Poisson process

Restoration improves the quality of the environment by affecting the stock of waste, rather than the flow. It is assumed that restoration occurs only at the termination date, and thus abatement is the only way to maintain the quality of the environment during the project life. However, the firm may go bankrupt before the restoration phase starts. We specify the risk of bankruptcy, captured by $d\varphi$, as a Poisson default process. Thus

$$d\varphi = \begin{cases} 1 & \text{with probability} \quad \lambda(\cdot)dt \\ 0 & \text{with probability} \quad 1 - \lambda(\cdot)dt \end{cases} \tag{2.4}$$

where $\lambda(\cdot)$ represents the hazard rate over the infinitesimal interval $dt$, and $\lambda(\cdot)$ always takes a positive number between zero and infinity. Clearly, low commodity prices play a key role in putting a firm in the state of bankruptcy. Other drivers of a bankruptcy and thus the specification of the hazard function depend on our assumption about the firm's size. To develop the functional form of $\lambda$, we have considered two scenarios

- **Scenario I:** The mine owner is a large multinational company and would not be bankrupted by this one project. The risk of bankruptcy depends only on commodity prices. Therefore, the company-wide hazard function is specified as

$$\lambda(P) = \frac{k_0}{P} \tag{2.5}$$

in which $k_0$ is a positive constant.

- **Scenario II:** The mine owner is a single small firm, so that the probability of bankruptcy depends on commodity prices and waste accumulation in this particular project. Therefore, the project-level hazard function can be defined as

$$\lambda(P, W) = \frac{k_1 + k_2 W}{P} \tag{2.6}$$

in which $k_1$ and $k_2$ are positive constants. Adding $k_1$ rules out the possibility that the firm is always solvent with no waste creation.

**Assumption 8** *To make the outcomes of these two scenarios quantitatively comparable, we have assumed that $k_0 = k_1$. Therefore, the firm in Scenario II compared to Scenario I has a similar hazard rate at zero waste and experiences a relatively higher hazard rate as waste accumulates.*

The hazard function in both scenarios is defined for all $P > 0$ and $0 \leq W \leq \bar{w}$, where $\bar{w}$ can be significantly large. Equation (2.5) implies that regardless of the project liability costs, the firm is always bankrupt at very low prices (if $P \to 0$), and is never bankrupt at significantly high prices (if $P \to \infty$).

According to Equation (2.6), the probability of bankruptcy decreases as commodity prices rise and increases with waste accumulation. The firm's decision to declare bankruptcy in Equation (2.6) depends on the commodity prices and waste accumulation. The former, which tends to be volatile, influences the firm's financial situation in each period, whereas the latter affects its environmental liability costs. We begin the intuition by analyzing the extreme cases. The firm is assumed to be solvent at all times when prices are extremely high, regardless of the level of waste. This assumption implies that as $P \to \infty$, $\lambda(P, W)dt \to 0 \;\; \forall W$. In addition, it is assumed that extremely low prices lead to a bankruptcy at a given, positive level of $W$. Mathematically, $\forall 0 < W \leq \bar{w}$, if $P \to \epsilon$, where $\epsilon$ is a small positive infinitesimal quantity, $\lambda(P, W) \to \infty$ and thus $\lambda(P, W)dt \to 1$.[6] Any other combination of $P$ and $W$ results in a different likelihood for bankruptcy between zero and 1. The lower the prices are and the larger the accumulated waste and thus the associated clean-up costs, the higher the probability of bankruptcy will be.

We continue the descriptions of the model for the strict liability rule and an environmental bond.

### 2.2.2 Modeling an environmental bond and the strict liability rule

To model the mechanism of an environmental bond, we assume that the firm must deposit an amount with the government sufficient to cover clean-up costs of waste deposited into the landfill. We assume that the appropriate level of restoration and the associated cost are correctly determined and thus the bond level is appropriate. To mitigate the impacts of bankruptcy, the firm is required to estimate the closure costs based on the fact that a third party will do the restoration should the firm default (Grant et al., 2009, Otto, 2010). It has been found in practice that it is more costly for a third party to clean up environmental damages than for the firm itself by 15% to 30% (Ferreira et al., 2004) due to mobilization costs (Peck and Sinding, 2009, White et al., 2012). Assumption 9 defines the clean-up cost function.

**Assumption 9** *The firm's clean-up cost function at each level of waste is given by $C^f(W)$ with $C'^f(\cdot) \geq 0$, $C''^f(\cdot) \geq 0$, and $C'''^f(\cdot) = 0$. The third party's clean-up cost function is given by $C^{tp}(W) = \nu C^f(W)$ where $\nu > 1$ is a constant.*

Let $B(t)$ denote the total value of the bond at each point of time. Prior to project commencement, if the firm does not go bankrupt during the construction, it deposits the

[6] These results hold even if we assume that $\bar{w}$ is an extremely large number or infinity, $\bar{w} \to \infty$.

initial bond $B(0) = C^{tp}(W(0))$ with the government to cover the potential clean-up cost of the initial waste. During the extraction phase, the firm updates the value of the bond according to rate of change in its restoration costs adjusted by potential expenses to the third party, $\frac{dC^{tp}(W)}{dt}$. Thus, the firm's expected bond payment at each period, $\frac{dB(t)}{dt}$, is

$$\begin{aligned}\frac{dB}{dt} = \frac{dC^{tp}(W)}{dt} &= \frac{dC^{tp}}{dW}\frac{dW}{dt} \\ &= \theta(W)(\phi q - a)\end{aligned} \tag{2.7}$$

where $\frac{dC^{tp}}{dW} \equiv \theta(W)$, and $\frac{dW}{dt} = \phi q - a$ is given by Equation (2.3). The firm's expected bond payment (i.e., the flow of revenue to the government) at each time can be positive, negative, or zero depending on $\phi q \gtreqless a$. Note that $\theta(W)$ increases linearly in $W$.

Project termination can happen as an optimal decision of the firm prior to time $T$. If the project is still in operation at time $T$, it must be terminated according to government order. Once the project terminates, refunding the deposited bond following a successful restoration yields a restoration benefit to the firm. If the firm goes bankrupt, no clean up is undertaken and the entire fund will be forfeited. Under the strict liability rule, the regulator requires the firm to clean up the stock of waste once the project terminates (either through the firm's choice or at time $T$), and does not require *ex ante* payments for associated costs. Moreover, project termination entails a significant clean-up cost to the firm should the firm be solvent. Assume that $\mathbf{1}$ is the indicator function and $b = true$ under the environmental bonding policy and is false otherwise. Assumption (10) defines the firm's restoration benefit under the bond and the restoration cost under the liability, at project termination given a solvent firm.

**Assumption 10** *The firm's benefit/cost from restoration at the project termination date under each policy is* $\left[\mathbf{1}_{b=true}C^{tp}(W) - C^{f}(W)\right]$.[7]

In the operating stage, the annual compliance cost with the environmental bond has two components: 1) the cost of abatement effort, and 2) the expected bond payment.

[7]According to Assumption 9, the benefit of restoration under the bond is: $C^{tp}(W) - C^{f}(W) = \nu C^{f}(W) - C^{f}(W) = (\nu - 1)C^{f}(W)$ where $\nu > 1$.

However, the latter is zero under the liability rule. Therefore, the annual compliance cost under each policy is defined by

$$\Omega = C^a(a) + \mathbf{1}_{b=true}\big(\theta(W)(\phi q - a) - rB\big) \tag{2.8}$$

implying that the interest income earned under the bond, $rB$, reduces the annual compliance costs.

### 2.2.3 Instantaneous cash flow

The firm's objective is to choose controls to maximize the discounted sum of risk neutral expected stream of future cash flows. Cash flows at any time $t$ will depend on the firm's stage of operations, $\delta$, and rates of abatement, $a$, and extraction, $q$. Instantaneous cash flows are given as follows

$$\pi(t) = P(t)q - C^q(q) - \big[C^a(a) + \mathbf{1}_{b=true}\big(\theta(W)(\phi q - a) - rB\big)\big] - C_i^m, \quad \text{if} \quad \delta = \delta_i, \quad i = 1, 2, 3 \tag{2.9}$$

$$\pi(t) = 0, \quad \text{if} \quad \delta = \delta_4$$

in which the term in square brackets is the compliance cost, $\Omega$, as previously given by Equation (2.8). $C_i^m$ refers to fixed costs under both the bond and strict liability policies.

### 2.2.4 Defining the value function

The value of the firm's operations, denoted by $V(P, S, W, \delta, t)$, is a function of the state variables and time, $t$. It is assumed that at specific fixed times, the firm makes a decision about whether to move to another stage of operation. These discrete decision times are given as follows

$$\mathcal{T}_d \equiv \{t_0 = 0 < t_1 < ... < t_m <, ..., t_M = T - \Delta t\} \tag{2.10}$$

where we assume that the decision to move to another stage of operation occurs instantaneously at $t \in \mathcal{T}_d$. Excluding time $T$ from the decision dates implies that bankruptcy cannot happen in the final instant. If bankruptcy has not occurred during the project life and the firm reaches time $T$, it has to clean-up the site.

Choices regarding optimal rates of abatement, $a$, and extraction, $q$, are made in continuous time at time intervals given as follows

$$\mathcal{T}_c \equiv \{(t_0, t_1), ..., (t_{m-1}, t_m), ..., (t_{M-1}, t_M)\}. \tag{2.11}$$

Since we search for the closed loop control, we assume the controls are in feedback form, i.e., functions of the state variables. Control variables can be specified as: $q(P, S, W, \delta, t)$, $a(P, S, W, \delta, t);\ t \in \mathcal{T}_c$, and $\delta^+(P, S, W, \delta, t);\ t \in \mathcal{T}_d$. We specify a control set which contains the controls for all $t_0 \leq t \leq t_M$ as follows

$$K = \{(\delta^+)_{t \in \mathcal{T}_d}\ ;\ (q, a)_{t \in \mathcal{T}_c}\}. \tag{2.12}$$

Regardless of the controls chosen, the value function can be written as the risk neutral expected discounted value of the integral of cash flows, given the state variables, with the expectation taken over the controls:

$$\begin{aligned} V(p, s, w, \bar{\delta}, t) \ &= \\ \mathbb{E}_K &\left[ \int_{t'=t}^{t'=T} e^{-r(t'-t)}\ \pi(P(t'), S(t'), W(t'), \delta)\ dt' \ + e^{-r(T-t)} V(P(T), S(T), W(T), \delta(T), T) \right. \\ &\qquad \left. \Big| P(t) = p, S(t) = s, W(t) = w, \delta(t) = \bar{\delta} \right] \end{aligned} \tag{2.13}$$

where $(p, s, w, \bar{\delta})$ denote realizations of the random and path dependent variables $(P, S, W, \delta)$. $r$ is the risk free interest rate, and $\mathbb{E}[\cdot]$ is the expectation operator. The value in the final time period, $T$, is assumed to be the net benefits from restoring and closing the mine. This is described as a boundary condition in Appendix B.1.

## 2.3 Dynamic Programming Solution

Equation (2.13) is solved backwards in time using dynamic programming. For a particular $t_m \in \mathcal{T}_d$, we define $t_m^-$ and $t_m^+$ to represent the moments just before and after $t_m$. At $t_m$, we determine the discrete optimal control $\delta^+$, while in the interval $(t_m^+, t_{m+1}^-)$. We solve for the optimal controls $q$ and $a$ in continuous time.

### 2.3.1 Determining optimal rates of abatement, $a$, and extraction, $q$, from $t_{m+1}^- \to t_m^+$

Using a standard contingent claims approach (Dixit and Pindyck, 1994), we can derive a system of partial differential equations that describe the value of the resource, $V$, in the interval $(t_m^+, t_{m+1}^-)$ for all operating states except for abandonment.

$$\begin{aligned}&\frac{\partial V}{\partial t} + \frac{1}{2}\sigma^2 p^2 \,\frac{\partial^2 V}{\partial p^2} + \kappa(\hat{\mu} - \ln p)p \,\frac{\partial V}{\partial p} \\ &+ \max_{q,a} \,\Big\{ - q \,\frac{\partial V}{\partial s} + (\phi q - a) \,\frac{\partial V}{\partial w} + \pi(t) \Big\} \\ &- \lambda(\cdot)\big(V_{\text{bankrupt}} - V\big) + rV = 0, \quad \text{for } \delta = \delta_i, \quad i = 1,2,3\end{aligned} \tag{2.14}$$

where we maximize with respect to the control variables $a$ and $q$. The hazard function, $\lambda(\cdot)$, is given by Equations (2.5) and (2.6). We set the value of the project after bankruptcy to zero – i.e., $V_{\text{bankrupt}} = 0$. Therefore, $V$ represents the project value prior to bankruptcy. Let $\mathcal{L}V$ be the differential operator as follows

$$\mathcal{L}V = \frac{1}{2}\sigma^2 p^2 \,\frac{\partial^2 V}{\partial p^2} + \kappa(\hat{\mu} - \ln p)p \,\frac{\partial V}{\partial p} + \big(r + \lambda(\cdot)\big)V. \tag{2.15}$$

Substituting $\mathcal{L}V$ in Equation (2.14) gives

$$\frac{\partial V}{\partial t} + \mathcal{L}V + \max_{q,a} \,\Big\{ - q \,\frac{\partial V}{\partial s} + (\phi q - a) \,\frac{\partial V}{\partial w} + \pi(t) \Big\} = 0, \quad \text{for } \delta = \delta_i, \quad i = 1,2,3 \tag{2.16}$$

Once the project is in stage 4, the project value goes to zero.

$$V(p, s, w, \delta = \delta_4, t) = 0. \tag{2.17}$$

Note that we analyze the firm's investment decisions under the $\mathbb{Q}$ measure which allows us to use a risk-free interest rate as well as risk-adjusted parameters for the commodity price and for the probability of bankruptcy. The risk due to price volatility can be hedged by deducting the market price of price risk from the drift rate (see Section (2.5)). This market price reflects an additional return over the risk-free interest rate that the firm demands per unit of price volatility.

As is discussed in Insley and Lei (2007) and in Ayache et al. (2003), there are two approaches to hedging a jump risk due to bankruptcy (or other causes). One is to assume the risk is fully diversifiable in a portfolio of assets. In this case the asset would generate no extra return for an investor due to bankruptcy risk and it can be assumed that the real world probability of bankruptcy is equal to the risk-neutral probability. The second approach is to assume that the risk of bankruptcy can be hedged by trading another asset which faces the same risk. In this case, the market price of a jump-related risk (i.e., bankruptcy risk in our study) will be used in the valuation model instead of the actual probability (i.e., the historical probability of bankruptcy in our study). This implies that, in our study, $\lambda(\cdot)$ in Equation (2.14) should be replaced by the market price of bankruptcy risk reflecting an additional return over the risk-free interest rate that the firm requires to obtain per each unit of potential loss in the project value due to bankruptcy.

The market price of bankruptcy risk reflects the market's perception of the bankruptcy risk and could be higher from the historical bankruptcy risk. It has been observed that the corporate bond yields exceed the risk-free rate by an amount greater than what is justified by historical default rates (Amato and Remolona, 2003).

This study assumes that the risk from price volatility can be hedged. In Chapter 1, the risk-adjusted parameters of the commodity price including the market price for the price risk are estimated using futures prices. In Section (2.5), we have used the same estimated values for the price parameters in this study. However, estimating the market price of bankruptcy risk is beyond the scope of this paper and instead we examine the sensitivity of our results to the parameters of the hazard function in Appendix B.2.

### 2.3.2 Determining optimal operating stage, $\delta$ at $t_m$

For $t_m \in \mathcal{T}_d$, the firm checks to determine whether it is optimal to switch to a different operating stage. The firm will choose the operating stage which yields the highest value net of any costs of switching. Let $C(\delta^-, \delta')$ denote the cost of switching from stage $\delta^-$ to $\delta'$. Recall that $t = t^-$ represents the moment before $t_m$ and $t = t^+$ denote the instant after $t_m$. Solving going backward in time, and noting the optimal stage is denoted as $\delta^+$, the value at $t_m^-$ is given by

$$\begin{aligned} V(p, s, \delta^-, t_m^-) &= V(p, s, \delta^+, t_m^+) - C(\delta^-, \delta^+) \\ \delta^+ &= \arg\max_{\delta'} \, [V(p, s, \delta', t_m^+) - C(\delta^-, \delta')]. \end{aligned} \tag{2.18}$$

Switching costs are the same under the bond or strict liability policies except for project commencement as well as when the mine is abandoned. Opening the mine under the bond requires investment cost and initial bond payment, whereas the latter is absent under the liability rule. In addition, recalling from Assumption 10, the abandonment cost with bonding requirements, $C(\delta_i, \delta_4), \ i = 2, 3$, simply equals the negative of reimbursement after clean-up has been completed. However, Assumption 10 indicates that under the strict liability rule, $C(\delta_i, \delta_4), \ i = 2, 3$, equals the firm's expected clean-up costs. Note that no waste is created in Stage 1.

## 2.4 Numerical solution approach

Equations (2.16) and (2.18) represent a stochastic optimal control problem which must be solved using numerical methods. The computational domain of Equation (2.16) is $(p, s, w, \bar{\delta}, t) \in \Gamma$ where $\Gamma \equiv [p_{min}, p_{max}] \times [0, s_0] \times [0, \bar{w}] \times \{\delta_1, \delta_2, \delta_3, \delta_4\} \times [0, T]$. More details are given in Appendix B.1 where boundary conditions are specified for the PDEs. $\mathcal{L}V$ in Equation (2.16) can be discretized using a standard finite difference approach. The

| | | | |
|---|---|---|---|
| $\kappa$ | 0.0264 (0.001) | Root Mean Square Error | 0.07 |
| $\mu$ | 2.7051 (0.079) | Mean Absolute Error | 0.05 |
| $\eta$ | 2.7845 (0.026) | Log-likelihood function | 9652 |
| $\sigma^2$ | 0.0458 (0.002) | Number of observation | 937 |

Table 2.1: *Estimation results for the one-factor copper price model using Kalman Filter. RMSE, MAE, $\mu$, and $\eta$ are in terms of US \$/lb. Standard errors are in parenthesis. Weekly futures data from Aug 1st, 1997 to Jul 13th, 2015.*

other terms in the equation are discretized using a semi-Lagrangian scheme as described in Chen and Forsyth (2007) and will not be described further here. Recall that the optimal control for $q$ which we denote by $q^*$ is bang - bang so that $q^* \in \{0, \bar{q}\}$. To jointly determine the optimal controls, $(q^*, a^*)$, we discretize the control $a \in [0, \bar{a}]$ and determine the optimal controls by exhaustive search at each point in the state space $(p, s, w, t)$.

## 2.5 An application to the copper industry

To compare the impacts of bankruptcy on the firm's optimal decisions under each policy with the results of Chapter 1 obtained for a solvent firm, this study considers the case of investment decisions for a copper mine similar to Chapter 1. The parameters of the stochastic model assumed for copper prices are already estimated in Chapter 1 using weekly data for copper futures contracts that are traded in London Metal Exchange Market from August 1997 to July 2015. These estimations are summarized in Table (2.1) and are obtained by adopting a Discrete Kalman Filtering approach and a Maximum Likelihood Function as explained in Schwartz (1997). Note that we define the parameter $\hat{\mu} = \mu - \eta$ so that the market price of price risk, $\eta$, is deducted from $\mu$ which is the long-run mean of $\ln(P)$ before adjusting for the price risk. The market price of price risk, $\eta$, reflects additional returns that the firm demands over the risk-free interest rate per each unity of price volatility. A numerical example is developed based on available data from an open-pit copper mine in British Columbia, supplemented by our assumptions when data is lacking, as given by Table (2.2).

| | | | |
|---|---|---|---|
| Life of project | | $T = 15$ | years |
| Risk-free rate | | $r = 0.02$ | per year |
| Initial reserve | | $s_0 = 1173$ | million lb |
| Strip ratio (waste:ore) | | $\phi = 1.5 : 1$ | |
| Production capacity | | $\bar{q} = 78.2$ | million lb/year |
| Abatement ceiling | | $\bar{a} = 2\phi\bar{q}$ | million lb/year |
| Landfill capacity | | $\bar{w} = 2200$ | million lb |
| Extraction cost | $C^q(q) = \gamma q$ | $\gamma = 1.35$ | \$/lb |
| Abatement cost | $C^a(a) = \alpha a^2$ | $\alpha = 10^{-3}$ | |
| Firm's clean-up cost | $C^f(w) = \beta w^2$ | $\beta = 10^{-5}$ | |
| Adjustment factor | | $\nu = 1.30$ | |
| Hazard function (Scenario I) | $\lambda(p, w) = \frac{k_0}{p}$ | $k_0 = 10^{-1}$ | |
| Hazard function (Scenario II) | $\lambda(p, w) = \frac{k_1 + k_2 w}{p}$ | $k_1 = 10^{-1}$ | |
| | | $k_2 = 1.5 \times 10^{-4}$ | |
| Project stages | | $\delta_1, \delta_2, \delta_3, \delta_4$ | |
| Fixed decision time | | $\tau_d$ | every year |
| Construction cost | $I$ | \$385 | million |
| Cost to mothball and reactivate | $C(\delta_2, \delta_3), C(\delta_3, \delta_2)$ | \$5 | million |
| Fixed costs of sustaining capital | $C_2^m, C_3^{m_1}$ | \$1.66 | million/year |
| Fixed monitoring costs while mothballed | $C_3^{m_2}$ | \$1 | million/year |

Table 2.2: *Parameter values and functional forms for the prototype open-pit copper mine. All dollar values are based on 2007 US dollars.*

## 2.6 Results analysis

This section compares the impacts of the environmental bond and the strict liability rule on the firm's optimal investment decisions indicated by critical prices, under each scenario. In addition, we compare the project value and optimal abatement decisions, under each policy and each scenario, at the initial time. Results are presented for Scenario I (bankruptcy risk depends only on commodity price) and Scenario II (bankruptcy risk depends on commodity prices and the level of waste), as well as for no bankruptcy risk. Recall that in the case examined, it is assumed the firm receives the risk-free rate from the government on the bond and also that the firm finances the bond from its retained earnings. This was referred to as Case I bond in Chapter 1. All figures in this section are shown for time zero and reserves at maximum level.

### 2.6.1 Valuation results

Figure (2.2) illustrates the value of the investment project prior to construction across different levels of starting prices and initial waste, when the reserve is fixed at its initial level. The top panels are for the strict liability rule and the bottom panels represent the bonding policy. Scenarios I and II are shown in the left-hand panels and the right-hand panels, respectively. Whether for the bonding policy or the strict liability rule, an increase in the initial waste generated through construction reduces the value of the project. As noted in Chapter 1, higher initial waste reduces the remaining capacity of the landfill. In addition, the waste will have to be cleaned up upon project termination. With no bankruptcy risk, this expected clean-up cost is the same under the either policy. Including the risk of bankruptcy reduces the project value compared to the no bankruptcy case over all values of the waste pile. Such lower values are to be expected since the exogenous bankruptcy risk may cause early termination of the project under the bond, and may increase the during of inactivity under the liability. This behaviour is discussed in Section (2.6.3).

Figure (2.3) compares the value of project with no bankruptcy versus Scenarios I and II for a particular price level. In this figure, we observe that the two policies no longer give the same result. Project values under both Scenarios I and II under the bond are less than for the liability policy. This result follows because under the bond the firm must pay the clean-up costs up-front. If bankruptcy occurs, the firm will not receive a refund on the bond. In contrast, under the strict liability rule, bankruptcy would allow the firm to avoid paying the clean-up costs. Under the possibility of bankruptcy, the bond is much more costly to the firm. This figure also highlights the difference in the firm value under the two scenarios. When the risk of bankruptcy depends on the waste stock under Scenario II, we observe lower values compared to Scenario I. We also observe, not surprisingly, that the value of the project decreases much more markedly as the waste stock builds up in Scenario II.

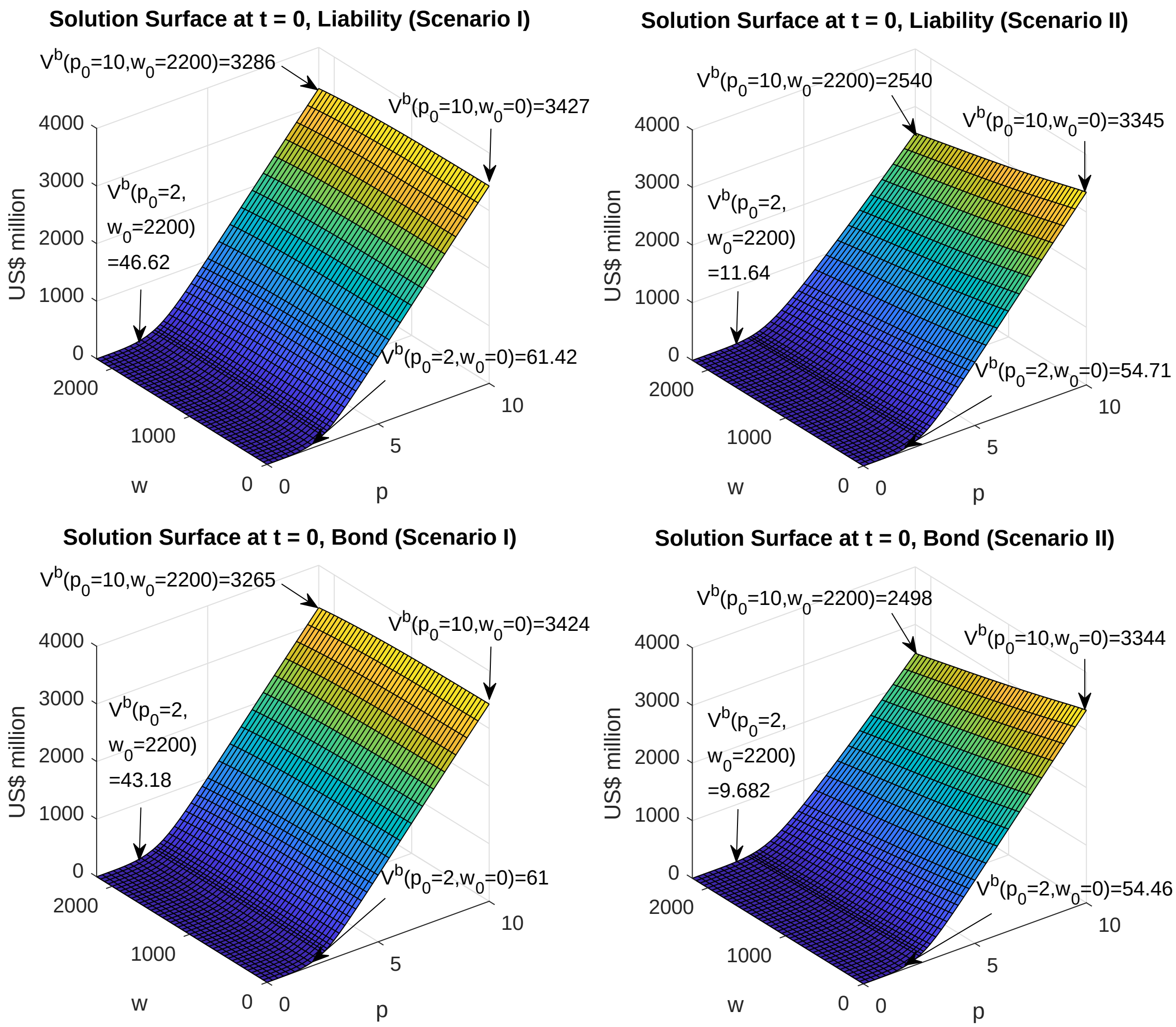


Figure 2.2: *Project value prior to construction, given* $s_0 = 1173$ *million pounds, for all price levels and waste piles, under the liability rule (the top panels) and the bonding policy (the bottom panels), for Scenario I (the left-hand panels) and Scenario II (the right-hand panels). w: million pounds and p: US\$/pound.*

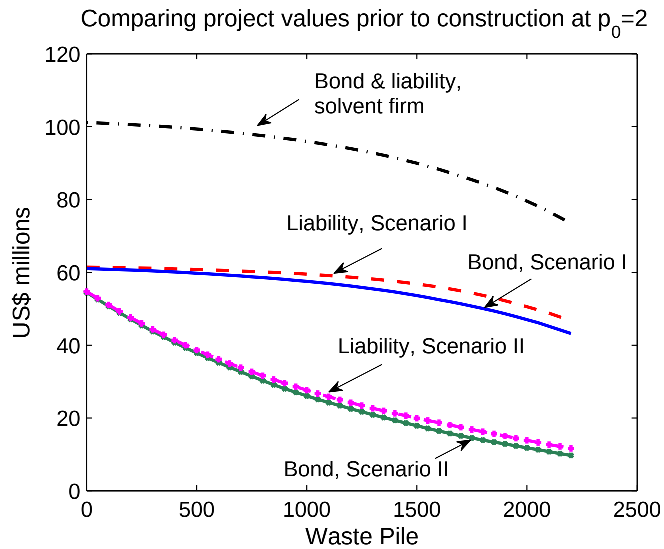


Figure 2.3: *Project values prior to construction across waste pile (million pounds), under Scenarios I and II versus no bankruptcy case, at* $p_0$ *=US$2/pound and* $s_0 = 1173$ *million pounds.*

### 2.6.2 Optimal abatement decisions

Figure (2.4) shows optimal abatement rates for Scenario I for the bonding and liability policies compared to the no bankruptcy case from Chapter 1. Recall that for the solvent firm, the bond and liability policies give identical results. The figure is plotted for $p =\$2$ and full reserves at time zero. We observe that optimal abatement rates increase with the waste stock for all three cases. As the level of waste gets closer to its maximum level, the firm must abate at a higher rate in order to maintain production capacity. The firm's abatement rates converge as the landfill capacity constraint binds. However, there is an obvious gap between such rates with the solvent firm having the highest abatement rates, followed by the bonding policy and then the strict liability rule under Scenario I. Recalling from Chapter 1, the marginal rule for abatement under each policy is given by

$$- C'^{a}(a^*) = \frac{\partial V}{\partial w} - \mathbf{1}_{b=true}\, \theta(w) \Rightarrow \begin{cases} 0 \leq a^* \leq \bar{a} & \text{if} \quad w < \bar{w} \\ \phi \bar{q} \leq a^* \leq \bar{a} & \text{if} \quad w = \bar{w}. \end{cases} \tag{2.19}$$

Intuitively, unlike the bond, the marginal rule for abatement under the strict liability

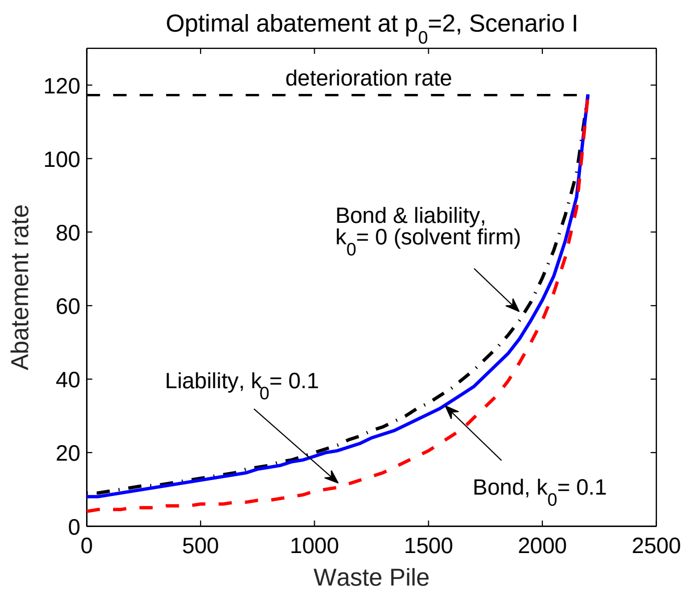


Figure 2.4: *Optimal abatement rates in Scenario I under the bonding and liability policies, at time zero,* $p_0 = \$2$*, and* $s_0 = 1173$ *million pounds.*

rule does not include the full restoration costs of waste generated by extraction as future clean-up costs may be avoided through bankruptcy. Consequently, the firm's marginal cost of environmental deterioration, $\frac{\partial V}{\partial w} - \mathbf{1}_{b=true}\, \theta(w)$, is higher under the bond than the liability, leading to higher abatement efforts with bonding requirements. This result is in contrast with equal abatement rates that we have observed in Chapter 1 for the solvent firm.

We have discussed in Chapter 1 that $\frac{dV}{dw}$ for a solvent firm operating under the liability rule reflects the cost of using up capacity in the landfill as well as adding to future clean-up costs. Under the bond, because the clean-up cost is paid immediately, this term for the solvent firm reflects the cost of using up capacity net of the marginal clean-up benefit and any interest paid. It follows that the solvent firm exercises the same abatement under both policies. When bankruptcy is possible, $\frac{dV}{dw}$ reflects the cost of using up capacity in the landfill as well as the impacts of bankruptcy on the eventual clean-up cost/benefit under each policy. The value of having spare capacity in the landfill is reduced as the firm might go bankrupt and be unable to use this capacity. It follows that abatement is less valuable

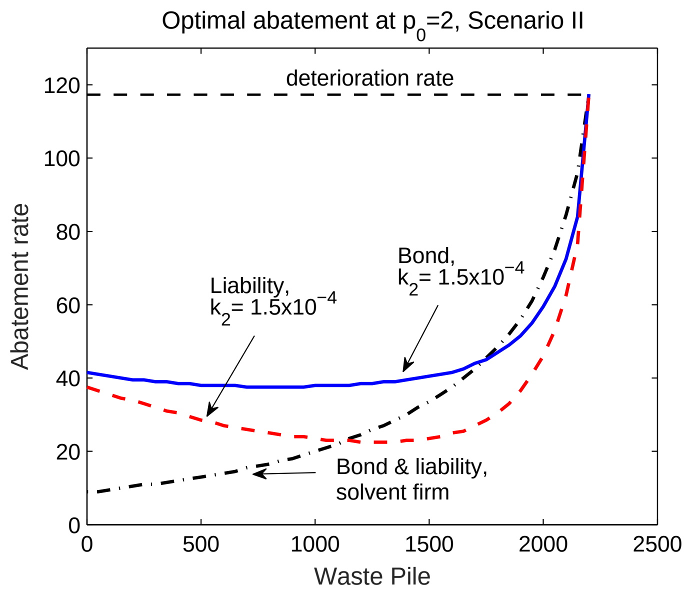


Figure 2.5: *Optimal abatement rates in Scenario II under the bonding and liability policies, at time zero,* $p_0 = \$2$*, and* $s_0 = 1173$ *million pounds.*

to the firm, and optimal abatement rates are reduced compared to a solvent firm. The lowest abatement rates are for the strict liability rule, because under bankruptcy the firm would avoid its eventual clean-up costs altogether.

Another way to interpret the optimal abatement decisions when bankruptcy is possible is to observe that the hazard rate, $\lambda(\cdot)$, increases the effective discount rate in Equation (2.15). This implies that the firm cares less about future benefits and costs. Under both policies, the firm generates more waste today as it puts less weight on the future impact of the loss in landfill capacity. For the liability case, it also puts less weight on the future clean-up costs triggered by project termination.

Optimal abatement under Scenario II for the bond and liability policies compared to the solvent firm are shown in Figure (2.5). The most obvious difference of Scenario II with Scenario I and with the solvent firm is that the optimal abatement levels first decline and then increase with the waste stock. In addition, at low waste levels, the optimal abatement rates are initially higher than for the solvent firm, but then drop below the solvent firm at higher waste levels below full capacity of $\bar{w}$.

For intuition we can identify two effects that account for the shape of the optimal abatement curve, depending on which effect dominates. First, the optimal abatement rate will tend to rise with the waste stock in order to maintain capacity in the landfill for future production. This effect explains why optimal abatement rises with the waste stock for the solvent firm, as well as for the Scenario I cases. In Scenario II, there is a second effect to consider in that the firm is able to reduce the probability of bankruptcy through abatement. In this case, the impact on optimal abatement choices depends on the level of waste stock. At low levels of waste, the value of the project is relatively high (under both the bond and liability cases) so it is beneficial for the firm to reduce bankruptcy risk. However, this benefit declines with waste accumulation as the effective discount rate increases with $\lambda(\cdot)$, meaning the firm is less concerned with value from future production. At low levels of waste, this second effect dominates. Thus, we see optimal abatement rates for the bond and liability cases decline as $w$ increases, but are higher than for the solvent firm. However at some point, the first effect starts to dominate and optimal abatement increases with $w$ and falls below the optimal rates for the solvent firm. Finally at the maximum $\bar{w}$, if the firm chooses to produce copper, the abatement level must be equal to the deterioration rate. As a result, the abatement choices of the three cases converge at $w = \bar{w}$. Similar to Scenario I, the optimal abatement rate for the liability case is always lower than for the bond since bankruptcy allows the firm to escape clean-up costs under the strict liability rule.

Note that a higher $\lambda(\cdot)$ due to a higher $k_0$ in Equation (2.5) and a higher $k_2$ in Equation (2.6) further reduces the optimal abatement rates, under both policies. These results are shown in Appendix B.2 using a sensitivity analysis on these parameters.

The decreasing pattern of optimal abatement under the bond in Scenario II depends on the parameters of the clean-up cost function. As the restoration becomes costlier at all levels of waste – i.e., a higher $\beta$ in Table 2.2, bankruptcy hurts the firm more than the base-case parameters. Consequently, the firm abates progressively as waste builds up to post a lower bond. Creating a smaller quantity of waste reduces the risk of bankruptcy through $\lambda(p, w)$ so that with a higher probability the firm will receive the clean-up benefit. Therefore, for a sizable restoration cost, the optimal abatement path has an increasing pattern at all levels of waste in Scenario II. This case is shown in Appendix B.3 using a

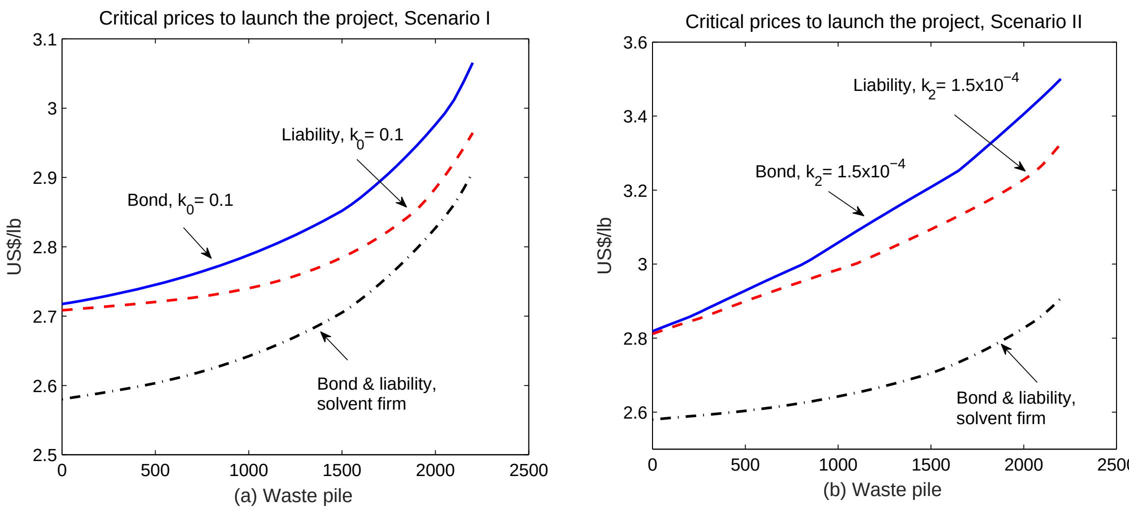


Figure 2.6: *Critical prices to launch the project across the waste pile (million pounds) for Scenario I (the left-hand panel) and for Scenario II (the right-hand panel), under the bonding and liability policies, given* $s_0 = 1173$ *million pounds.*

sensitivity analysis on $\beta$.

At higher prices, with both scenarios, the risk of bankruptcy declines and the project becomes less risky. In addition, higher prices increase the project profitability. Consequently, exercising a higher abatement at all levels of waste extends the life of landfill and the project does not terminate too early due to the exhaustion of landfill capacity (see Appendix B.4).

### 2.6.3 Optimal choices of project stages

This section examines the copper prices at which it is optimal to switch from one stage to another, which we refer to as critical prices. Critical prices vary in response to the level of remaining reserve at a given level of waste. As discussed by Insley (2017) and shown in Chapter 1, critical mothballing and reopening prices increase as the reserve depletes and becomes more scarce. In addition, with lower reserve, the project commencement and project termination occur at higher prices.

We are interested in the impacts of bankruptcy on the firm's operating decisions in response to waste accumulation, under each policy, as demonstrated by how critical prices change with the stock of waste under the risk of bankruptcy. According to Figure (2.6), the possibility of bankruptcy creates a gap between the critical starting prices under the two policies. The bond requires an upfront payment as well as subsequent payments during operations, whereas the clean-up costs under the liability might be avoided through bankruptcy. It follows that the increased compliance cost and thus the reduced profitability of the project under the bond compared to the liability rule increase critical prices to begin the project under the former. The gap becomes more significant for a higher stock of waste. The firm's lower project value in Scenario II leads to higher prices to commence the project compared to Scenario I. Critical prices to launch the project in both scenarios are higher under bankruptcy risk than for the solvent firm case.

Figure (2.7) shows the impact of each policy on critical prices for mothballing and resuming the project for Scenario I (the left-hand panels) and for Scenario II (the right-hand panels). If the landfill capacity is reached, the firm's optimal mothballing and reopening decisions are identical under the both policies. At other levels of waste, critical prices for mothballing and resuming the project under the bond are higher than the liability. Such higher prices imply that the project under the bond will more likely be mothballed, and once mothballed, the idle firm will less likely reopen its mine. The reason for this behaviour is lower anticipated profits due to higher annual compliance costs under the bond. Critical prices to mothball and resume the project in both scenarios are lower under bankruptcy risk than for the solvent firm case due to the increased effective discount rate in Equation (2.15) by $\lambda(\cdot)$ compared to the solvent case.

Figure (2.8) shows that critical prices to abandon the mothballed mine across the waste pile have an increasing trend under the bond and a decreasing trend under the liability. This follows because project termination and site clean-up under the bond yield restoration benefits that increase with waste, motivating the firm to carry out restoration projects rather than sitting idle in the mothballed stage. In contrast, under the liability, the last stage of operation entails restoration costs to the firm that rise with waste accumulation, motivating the firm to remain idle as a way to escape paying for such costs. Interestingly, with liability requirements, no critical prices are found for abandoning the mothballed

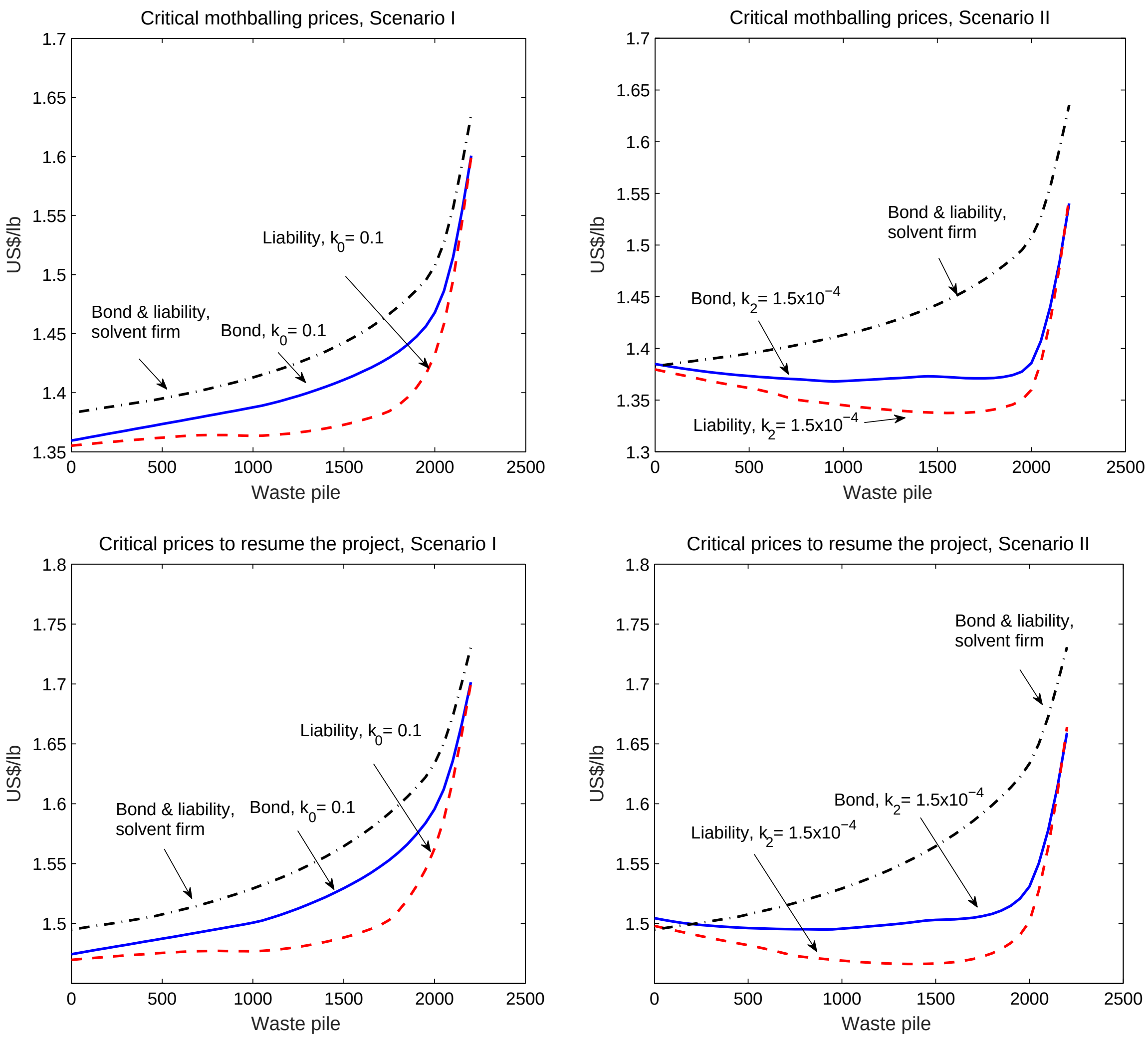


Figure 2.7: *Critical prices to mothball and resume the project across the waste pile (million pounds) for Scenario I (the left-hand panels) and for Scenario II (the right-hand panels), under the bonding and liability policies, given* $s_0 = 1173$ *million pounds.*

project for waste accumulation beyond 950 and 700 million pounds in Scenarios I and II, respectively. Beyond such waste thresholds, the idle firm facing low prices either goes bankrupt or remains inactive for an extended periods of time.[8]

[8]We have observed that these thresholds increase as time passes so that after 14 years such waste levels

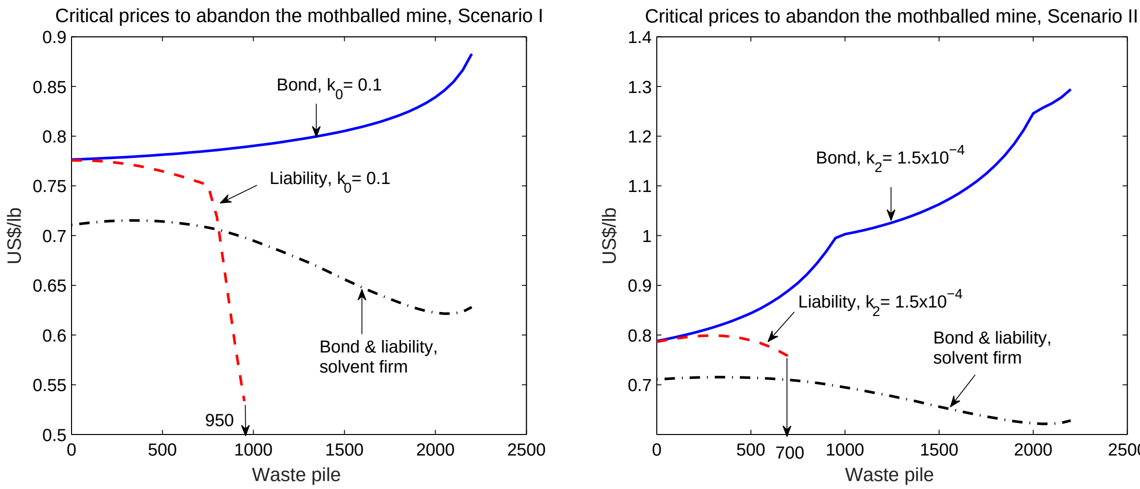


Figure 2.8: *Critical abandoning prices from the mothballed stage across the waste pile (million pounds) for Scenario I (the left-hand panels) and for Scenario II (the right-hand panels), under the bonding and liability policies, given* $s_0 = 1173$ *million pounds.*

Under the bonding policy, higher critical abandonment prices in both scenarios compared to the solvent firm case imply that the project will terminate sooner when bankruptcy is possible. As noted, the possibility of bankruptcy reduces the project value, making the project termination more probable. Under the strict liability rule, the critical prices in Scenario I falls below the solvency scenario at large levels of waste, implying a longer duration for inactivity under bankruptcy. For this reason, we have observed that in Scenario I, the critical prices under the strict liability rule at some levels of waste are lower than the solvent firm case. In Scenario II, we have observed no critical prices at large levels of waste.

increase to 1450 and 1000 million pounds in Scenarios I and II, respectively. If the firm has not declared bankruptcy at an instant prior to $T$, it will have to clean up the site at time $T = 15$, regardless of waste and price levels.

## 2.7 Conclusions

This study demonstrates the inclusion of bankruptcy risk in optimal waste clean-up decisions under an environmental bond and the strict liability rule. We have explicitly modeled a firm's decision to declare bankruptcy as a Poisson jump process, and analyzed its impacts on a firm's optimal operating decisions under each policy. We have examined the case when the firm can finance the bond from its retained earnings and the government pays the risk-free rate on the bond. The firm's problem is specified as a Hamilton Jacobi Bellman equation, which is solved numerically using a finite difference approach. This model is applied to a typical copper mine in Canada. The firm's clean-up decisions are based on critical levels of copper prices.

Our findings reveal that the bond and liability policies have the same impacts on the solvent firm's optimal decisions. However, when bankruptcy is allowed, these two policies are not equal anymore. Bankruptcy increases the probability of early project termination under both policies, in particular under the bond, and increases the duration of inactivity under the liability, which results in a lower project value prior to construction compared to a solvent firm case. In addition, when bankruptcy is allowed, the project value is higher under the strict liability rule than the bonding policy, because the firm operating under the liability rule may avoid paying for site clean-up costs through bankruptcy. For the same reason, the firm exercises a lower abatement rate under the liability rule than the bonding policy. We have also observed that the bonding policy requires higher critical prices for project commencement, and project termination is more likely under the bond as more waste accumulates.

We discuss that because the probability of bankruptcy is under the Q-measure, the risk of bankruptcy reflects the extra return demanded by the market to undertake a project with bankruptcy risk. In other words, the impact of the risk of bankruptcy is fully accounted for in the model, eliminating the need to include a risk premium for borrowing. If the firm can borrow the full clean-up cost and if the government pays the risk-free interest on the bond, the project value is similar to the liability rule, as the firm may default on the borrowed funds. Therefore, when bankruptcy is allowed, borrowing is relatively more beneficial to the firm than the bond financed from retained earnings, as bankruptcy extends the liability costs to financial institutions that have lent the funds to the firm. However, both of the bond cases fully collateralize the government.

Overall, the bond is very effective at protecting government from the liability of site clean-up. If this is the goal of the government, bonds should be used. However, bonds financed from retained earnings are costly for the firm, and will result in fewer investment in mining projects, in particular when bankruptcy is possible. This issue needs to be recognized. The private sector might solve the bonding problem for the firm by lending to cover the cost of bonds.

# Chapter 3

# Estimating the Stochastic Models of Copper Prices

## 3.1 Introduction

Copper is one of the main non-ferrous metals that is traded in major markets in the world (Lasheras et al., 2015). Specifically, the London Metal Exchange (LME) handles more than 90% of the world's copper trades and thus is the major futures exchange for copper at the global level (Li and Li, 2015). The prices discovered on the LME trading platforms have been used by investors to manage their exposure to risks, making the LME copper futures price the global reference price.[1] Copper prices play a key role in the global economy and are very sensitive to the level of economic growth.[2] This commodity has been widely used in electrical products, medical equipment, the construction industry, and many other industries. In addition, the economy and the national income of some countries - such as Chile, the world's top producer of copper, and Zambia - depend greatly on their copper industries (Buncic and Moretto, 2015, Lasheras et al., 2015). Not surprisingly,

[1] See https://www.lme.com/.

[2] The link between copper prices and economic growth has been covered in Media, highlighting that copper price depreciation is a sign for slowdown in the global economy. See https://www.marketwatch.com/story/copper-prices-slump-to-2009-levels-sparking-growth-concerns-2015-01-13.

factors affecting the level of demand by copper-dependent industries and supply by main copper producers influence the price of this commodity. As a result, fluctuations in copper prices reflect movements in global economic activity (Labys et al., 1998). This fact makes forecasting copper prices an interesting research topic.

Forecasting and modeling stochastic copper prices are important for valuing projects and assets that are contingent on the price of copper. For instance, investment decisions by copper mining firms, whose objective is to maximize their future cash flows and to maintain their projects value, depend significantly on the price of this commodity. An appropriate price model helps mine managers to make decisions and determine optimal investment strategies prior to the construction phase and during operations. The literature on modeling commodity prices uses futures contracts as an indication of the expected change in spot prices over time. A futures contract is an agreement to buy or sell a commodity at a certain date in future for a price determined today. *The storage theory* explains the relationship between futures prices and spot prices (Brennan and Schwartz, 1985, Kaldor, 1939, Working, 1949). According to this theory, agents who agreed to sell a commodity at a future date may buy and carry it until the delivery date. This process incurs storage costs as well as the opportunity cost of investing cash to purchase the commodity, but yields a benefit from being able to trade it until maturity, particularly when there is an unexpected increase in demand. This benefit is called *convenience yield* which refers to the benefit associated with stockpiling a commodity as opposed to holding futures contracts.

This theory implies an inverse relationship between the convenience yield and inventory level. When the level of inventory is scarce and demand is high, spot prices rise above futures prices, motivating firms to sell the commodity. In contrast, abundant inventory and low demand cause spot prices fall below futures prices, motivating firms to incur the costs of holding an inventory. As a result, two factors explain the difference between spot prices and futures prices: 1) the net convenience yield, which is the convenience yield net of the storage cost, and 2) the forgone interest income of buying and holding a commodity. This difference is known as *the basis*. If the former dominates the latter, the spot prices exceed the futures prices, highlighting the importance of the convenience yield to explain the basis. Otherwise, the forgone interest is a dominant factor to explain such a difference.

Since copper is an important production input for many industries and is an storable commodity with low storage costs,[3] the net convenience yield may play an important role in explaining the dynamics of copper prices. Therefore, we are interested to know the impact of net convenience yield as an stochastic factor that would affect copper price formation. This study investigates the ability of three price models to explain the stochastic behavior of copper prices. The first model is the one-factor mean-reverting model in which the only stochastic factor is associated with spot prices. In addition to this stochastic factor, the second model introduces a mean-reverting stochastic process for the convenience yield and is known as the two-factor model. The last model, is called the one-factor long-term model in which the convenience yield is constant but can explain the main characteristics of the two-factor model in terms of explaining copper futures prices. The first two models are developed by Schwartz (1997) and the third model is developed by Schwartz (1998) based on the mathematical transformation of the two-factor model. This latter model is much simpler in its application to asset valuation models.[4]

We have used the data of copper futures contracts traded in the London Metal Exchange and calibrated the parameters of the models using a Kalman filtering approach and a Maximum Likelihood function. Since we are interested in futures contracts with long maturities, we have used two sub-samples covering different periods and consisting of shorter-term futures contracts with a maximum maturity of two years and longer-term futures contracts with five years as the longest maturity. Results suggest that the estimated spot prices by the one-factor mean-reverting model cannot track the spot copper prices if the longer-term data set is used, and cannot explain the term structure of market futures prices for both sub-samples. In contrast, the other two models are able to mimic the dynamics of copper spot prices, implying the importance of convenience yield for copper price formation. We have also observed that the two-factor model is able to explain the term structure of market futures prices in both sub-samples. However, the term structure of future prices implied by one-factor long-term model can approximate that of the two-factor model after 5 years for shorter-term data and after 15 years for longer-term data. We have argued that before these thresholds, the difference between the results of these

[3] See Fama and French (1987).

[4] Chen (2010) has adopted these models to estimate the dynamics of lumber prices. Other mean-reverting price models are estimated by Almansour and Insley (2016) for the oil and gas industry.

two models is not very significant. Therefore, the one-factor long-term model would be an appropriate alternative to the two-factor model as it involves only one stochastic factor and thus is simpler to apply in valuation models.

This paper is organized as follows: The next section analyzes the LME copper futures data. Section (3.3) explains the price valuation models. Sections (3.4) and (3.5) provide the Kalman filter procedure and the state-space representation of each model, respectively. Model calibration results are discussed in Section (3.6). The ability of these models to explain the term structure of copper futures prices is compared in Section (3.7). The last section concludes.

## 3.2 LME copper spot and futures prices

This study uses a dataset that contains the price of copper futures contracts traded on the London Metal Exchange.[5] The LME futures contracts are designed with a "prompt-date structure" whereby such contracts can be traded daily up to three months, weekly from three months up to six months, and monthly afterward up to 123 months.[6]. Such flexibility in maturity dates enables investors to accurately hedge their risks on a daily basis, distinguishing the LME from other futures exchanges.

The data used in this study consist of weekly observations of copper futures prices. As stated in Schwartz, due to the lack of data on copper spot prices, futures contracts closest to maturity can be used to approximate the spot prices. Since we are interested in estimating the stochastic process for real copper prices, futures prices are deflated by the US consumer price index. Figure (3.1) plots weekly LME real futures prices per pound of copper with one month or less to maturity, from August 1997 to July 2015. As shown in Table (3.1), spot prices have been highly volatile, between \$0.72 and \$4.34, and the standard deviation indicates a considerable level of volatility. The Kurtosis of the distribution is less than 3 implying that the distribution of data has a lower and flatter peak compared to a normal

[5]The data were collected from Datastream.

[6]See `https://www.lme.com/Trading/Physical-market-services/Prompt-date-structure` Note that the delivery dates are daily for the first three months, every Wednesday for the next three months, and the third Wednesday of the month for time to maturity beyond 6 months.

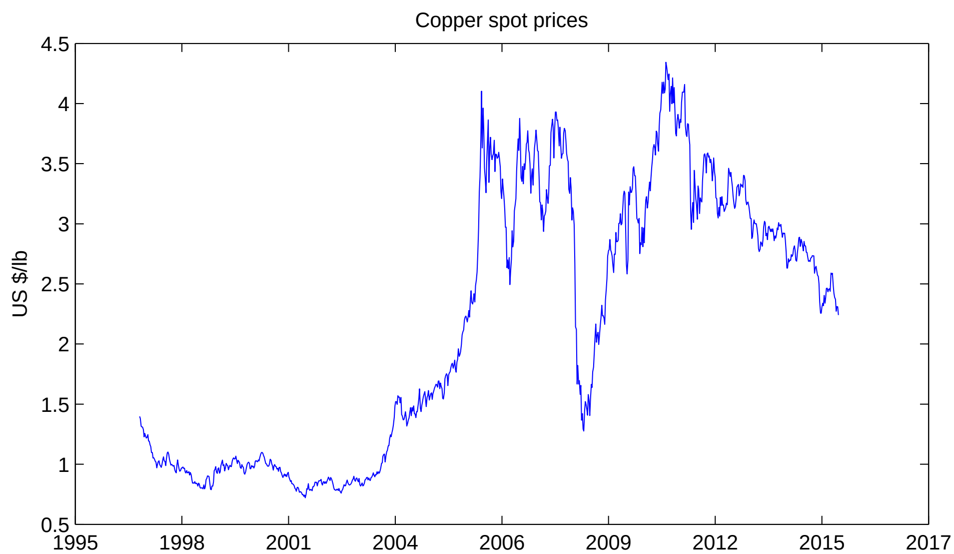


Figure 3.1: *Weekly LME copper futures prices with less than one month to maturity, from Aug 1st, 1997 to Jul 10th, 2015. Nominal prices are deflated by the Consumer Price Index, base year=2007.*

| Min | Max | Mean | S.D. | Skewness | Kurtosis | Num. of obs. |
|---|---|---|---|---|---|---|
| 0.72 | 4.34 | 2.15 | 1.09 | 0.15 | 1.49 | 937 |

Table 3.1: *Summary statistics for weekly copper spot prices (US $/lb) from Aug 1st, 1997 to Jul 10th, 2015.*

distribution. In addition, the near zero skewness indicates that the distribution of copper prices is approximately symmetric.

Among the data, we have chosen two sub-samples, each with 6 futures contracts that vary in terms of time to maturity across the sub-samples. The first sub-sample covers the period from 8/1/1997 to 7/10/2015 with complete data on contracts with 1 to 24 months to maturity. The second sub-sample includes data from 10/4/2002 to 7/10/2015, with 62 months as the maximum maturity. The selected futures for the first sub-sample have 1, 6, 11, 16, 21, and 24 months to maturity, and for the second sub-sample have 1, 24, 30, 40, 50, and 60 months to maturity.[7] The number of observations for each futures contracts in the first and second sub-samples are 937 and 667, respectively.

To understand whether convenience yield is an important factor in modeling copper prices, we analyze the theory of storage using our data. Let $P(t)$ be the spot price at time $t$, and $F(t,T)$ be the futures price at $t$ for delivery of copper at $T$. This theory defines the return from buying the commodity at $t$ and selling it for delivery at $T$ as follows[8]

$$F(t,T) - P(t) = r(t,T)P(t) - \delta(t,T) \tag{3.1}$$

in which $r(t,T)P(t)$ represents the forgone interest and $\delta(t,T)$ is the convenience yield from an additional unit of storage net of the marginal storage cost. As noted, the spread between futures and spot prices is called the basis in the literature.

According to Equation (3.1), the negative values for the basis indicate that the futures prices are lower than the spot prices at time $t$. This situation is referred to as *backwardation* which occurs when futures prices decrease with time to maturity so that the spot prices are higher than futures prices (Routledge et al., 2000). It is argued that the underlying cause of backwardation is a supply shortage that results in an increase in net convenience yield. This interpretation is consistent with the theory of storage whereby convenience yield has an inverse relationship with the level of inventory (Working, 1949). Such an inverse correlation suggests that a constant convenience yield, in general, does not hold in reality (Gibson and Schwartz, 1990). The market can also be in *contango* if the basis has positive values, implying that the commodity supply is plentiful. In this case, futures prices increase with time to delivery and remain higher than spot prices when the convenience yield is lower than the forgone interest.

Figure (3.2) shows the basis versus time to maturity for the two sub-samples of this study. Both contango and backwardation behaviour can be seen for the copper commodity. In both panels, the spread of the basis increases with time to maturity because the futures

[7] Note that our data set contains complete data on contracts, with 100 months as the longest maturity from 10/17/2008 to 7/10/2015. However, due to this short time interval and thus small sample size, we do not analyze those contracts. Other contracts with longer maturities than 100 months are not frequently traded and thus are not amenable to analysis.

[8] This formula is from Fama and French (1987).

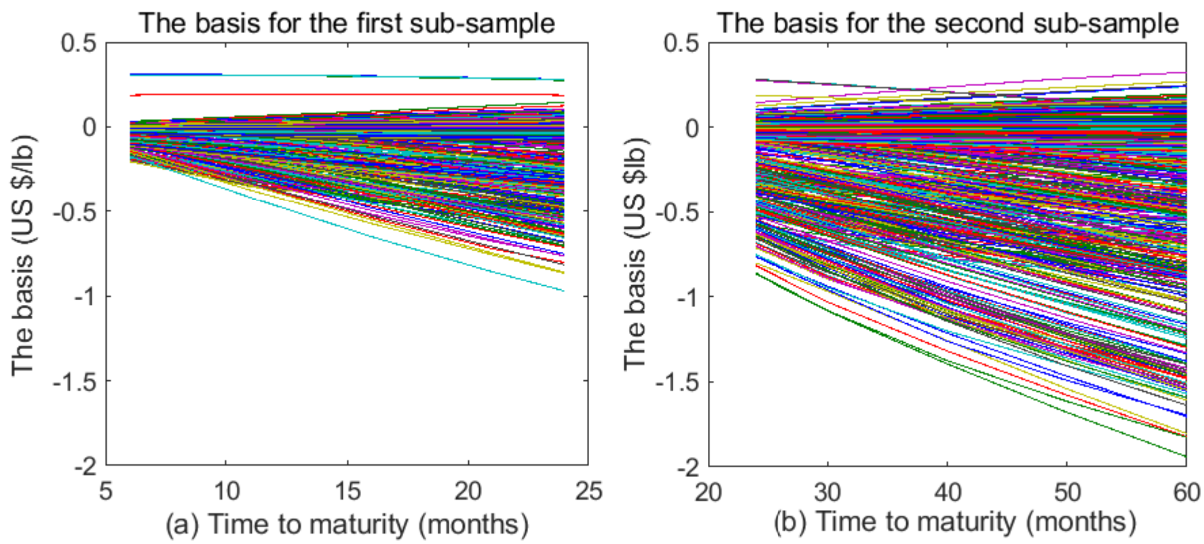


Figure 3.2: *The spread of the basis across time to maturity for (a) the first sub-sample and (b) the second sub-sample. Weekly data from Aug 1997 to Jul 2015 for panel (a) and from Nov 2002 to Jul 2015 for panel (b). Nominal prices are deflated by the Consumer Price Index, base year=2007.*

prices with shorter maturities are closer to the spot prices. A comparison between the two panels shows that this spread is relatively wider in panel (b) because of longer maturities of the futures contracts in the second sub-sample. This observation confirms the role of stochastic convenience yield in explaining futures prices, in particular, with longer maturities.

Tables (3.2) and (3.3) show detailed summary statistics of the basis for the first and second sub-sample, respectively. In these tables, $FT$ denotes the futures contract with $T$ months to maturity and $F1$ proxies the spot prices. The more negative values for the mean at longer maturity dates confirm the importance of convenience yield for the associated futures contracts, particularly for the second sub-sample. Furthermore, the standard deviation of the basis increases with maturity, which implies the wider spread of the basis at longer maturities, as noted before. Both of these statistics are stronger for the second sub-sample due to longer-term futures. We have also observed that around 47% of the data in the first sub-sample has a negative basis, whereas this number increases to 74%

| Item | Min | Max | Mean | S.D. | $> 0$ (%) | $= 0$ (%) | $< 0$ (%) | Maturity |
|---|---|---|---|---|---|---|---|---|
| $F6 - F1$ | -0.204 | 0.311 | -0.014 | 0.052 | 60.19 | 0.32 | 39.49 | 6 |
| $F11 - F1$ | -0.418 | 0.307 | -0.035 | 0.098 | 57.21 | 0 | 42.79 | 11 |
| $F16 - F1$ | -0.650 | 0.299 | -0.058 | 0.141 | 52.08 | 0.43 | 47.49 | 16 |
| $F21 - F1$ | -0.857 | 0.289 | -0.082 | 0.181 | 47.28 | 0.11 | 52.62 | 21 |
| $F24 - F1$ | -0.970 | 0.281 | -0.096 | 0.204 | 45.04 | 0 | 54.96 | 24 |
| **Entire data** | -0.970 | 0.311 | -0.057 | 0.135 | **52.36** | 0.172 | **47.47** | |

Table 3.2: *Summary statistics of the basis (i.e., $FT - F1$ where months to maturity is $T$) for the first sub-sample from Aug 1997 to Jul 2015. F1 proxies the spot prices. Sample size: 4,685*

| Item | Min | Max | Mean | S.D. | $> 0$ (%) | $= 0$ (%) | $< 0$ (%) | Maturity |
|---|---|---|---|---|---|---|---|---|
| $F24 - F1$ | -0.970 | 0.281 | -0.149 | 0.219 | 30.13 | 0 | 69.87 | 24 |
| $F30 - F1$ | -1.196 | 0.256 | -0.189 | 0.266 | 27.14 | 0 | 72.86 | 30 |
| $F40 - F1$ | -1.545 | 0.237 | -0.251 | 0.339 | 25.34 | 0 | 74.66 | 40 |
| $F50 - F1$ | -1.847 | 0.287 | -0.307 | 0.405 | 24.59 | 0 | 75.41 | 50 |
| $F60 - F1$ | -1.94 | 0.322 | -0.359 | 0.466 | 23.69 | 0 | 76.31 | 60 |
| **Entire data** | -1.94 | 0.322 | -0.251 | 0.339 | **26.18** | 0 | **73.82** | |

Table 3.3: *Summary statistics of the basis (i.e., $FT - F1$ where months to maturity is $T$) for the second sub-sample from Nov 2002 to Jul 2015. F1 proxies the spot prices. Sample size: 3,335*

in the second sub-sample. It follows that the market is in backwardation the majority of time in the second sub-sample and half of the time in the first sub-sample. As a conclusion, net convenience yield and its variations are important factors in modeling copper prices in both sub-samples.

## 3.3 Valuation models

In this section we present the specification of three models that we will use to explain the dynamics of copper spot and futures prices. The price models shown in this study are developed by Schwartz (1997) and Schwartz (1998).

### 3.3.1 One-factor mean-reverting model

The one-factor mean-reverting model developed by Schwartz (1997) assumes that the natural logarithm of the commodity price reverts to its long-run mean. Denoting spot prices by $P$, this process can be written as

$$dP = \kappa(\mu - \ln P)Pdt + \sigma P dz \tag{3.2}$$

in which $\kappa > 0$ is the speed of mean reversion, $\mu > 0$ denotes the expected long-run mean, and $\sigma > 0$ represents the constant price volatility. Let $S = lnP$ be the natural logarithm of spot prices. Applying Ito's lemma on $S$ reduces this model to the Ornstein-Uhlenbeck stochastic process given by

$$dS = \kappa(\mu - \frac{\sigma^2}{2\kappa} - S)dt + \sigma dz \tag{3.3}$$

The risk-adjusted version of this equation is obtained by deducting the market price of risk, $\lambda$, from the mean of log price, $\mu - \frac{\sigma^2}{2\kappa}$. Thus

$$dS = \kappa(\hat{\mu} - S)dt + \sigma d\hat{z} \tag{3.4}$$

where $\hat{\mu} = \mu - \frac{\sigma^2}{2\kappa} - \lambda$ is the risk-adjusted drift rate, and $d\hat{z}$ is the increment of a Wiener process under the equivalent risk-neutral measure.

The value of a futures contract with maturity $T$ is the expected value of the spot price in the risk-neutral world. Using the properties of the log-normal distribution, the value of a futures contract is given by

$$F(P, T) = E[P(T)] = e^{(E_0[S(T)] + \frac{1}{2} Var_0[S(T)])}. \tag{3.5}$$

Mastro (2013)[9] shows that the mean and the variance of $S(t)$ can be obtained by solving

[9] Ch.8.

Equation (3.4) using the variation of parameters method as follows

$$\begin{aligned} g(S(t),T) =& S(t)e^{\kappa t} = S(t) + \int_{t'=0}^{t'=t} e^{\kappa t'} \kappa \ \hat{\mu} \ dt' + \int_{t'=0}^{t'=t} e^{\kappa t'} \sigma \ d\hat{z}(t') \\ & S(t) = S(t)e^{-\kappa t} + \hat{\mu}(1 - e^{-\kappa t}) + \int_{t'=0}^{t'=t} e^{\kappa(t'-t)} \sigma \ d\hat{z}(t') \end{aligned} \tag{3.6}$$

where the first two terms are the expected value. Thus

$$E[S(t)] = S(t)e^{-\kappa t} + \hat{\mu}(1 - e^{-\kappa t}) \tag{3.7}$$

The variance can be found by

$$\begin{aligned} Var[S(t)] = E\big[S(t) - E[S(t)]\big]^2 &= E[\int_{t'=0}^{t'=t} e^{2\kappa(t'-t)} \sigma^2 \ d\hat{z}(t')] \\ &= \sigma^2 e^{-2\kappa t} E[\int_{t'=0}^{t'=t} e^{2\kappa(t')} d\hat{z}(t')] \\ &= \frac{\sigma^2}{2\kappa}(1 - e^{-2\kappa t}). \end{aligned} \tag{3.8}$$

Substituting the mean and variance from Equations (3.7) and (3.8) in Equation (3.5) in log form gives

$$\ln F(P,T) = e^{-\kappa T} \ln P + \hat{\mu}(1 - e^{-\kappa T}) + \frac{\sigma^2}{4\kappa}(1 - e^{-2\kappa T}) \tag{3.9}$$

This last equation shows the linear relationship between the log futures prices and the log spot prices and will be used to estimate the price parameters.

### 3.3.2 Two-factor model

The two-factor model presented in this section is developed by Schwartz (1997).[10] In this model, there are two stochastic state variables - i.e., the spot prices and the net

[10] Schwartz (1997)'s study is based on the model developed earlier by (Gibson and Schwartz, 1990).

instantaneous convenience yield[11] denoted by $\delta$ - which are explained by a joint stochastic process as follows

$$\begin{aligned} dP &= (\mu_{TF} - \delta)Pdt + \sigma_p P dz_p \\ d\delta &= \kappa_{TF}(\alpha - \delta)dt + \sigma_\delta dz_\delta \\ dz_p dz_\delta &= \rho dt \end{aligned} \tag{3.10}$$

in which $\mu_{TF}$ and $\alpha$ are the expected long-run mean of the two state variables, $\kappa_{TF}$ denotes the speed of mean reversion, and $\sigma_p$ and $\sigma_\delta$ represent the constant volatilities of the spot prices and the net convinced yield, respectively. In addition, $\rho$ captures the correlation between the increments of the Wiener process of the two paths. The subscript $TF$ refers to the two-factor model. Note that if we define $\delta(P) = \kappa \ln P$, the two-factor model reduces to the one-factor mean-reverting model.

The risk-adjusted version of spot price and convenience yield processes are given by

$$\begin{aligned} dP &= (r - \delta)Pdt + \sigma_p P d\hat{z}_p \\ d\delta &= [\kappa_{TF}(\alpha - \delta) - \lambda_{TF}]dt + \sigma_\delta d\hat{z}_\delta \\ d\hat{z}_p d\hat{z}_\delta &= \rho dt \end{aligned} \tag{3.11}$$

in which $r - \delta$ is the risk-adjusted drift rate of spot prices, $r$ is the risk-free interest rate, and $\lambda_{TF}$ denotes the constant market price of risk associated with the stochastic convenience yield. Note that the risk-adjusted price process is obtained through the no-arbitrage argument. Such a process for the net convenience yield is obtained by deducting the market price of $\delta$ risk from the drift.

The log form of spot prices, $S = \ln P$, after applying Ito's lemma is

$$dS = (\mu_{TF} - \frac{1}{2}\sigma_p^2 - \delta)dt + \sigma_p dz_p \tag{3.12}$$

As shown by Gibson and Schwartz (1990), futures prices, $F(P, \delta, T)$, should satisfy the

[11] The net convenience yield of holding an additional unit of inventory net of the storage cost of that additional unit.

partial differential equation (PDE) given by

$$\frac{1}{2}\sigma_p P^2 \frac{\partial^2 F}{\partial P^2} + (r-\delta)P \frac{\partial F}{\partial P} + \frac{1}{2}\sigma_\delta^2 \frac{\partial^2 F}{\partial \delta^2} + (\kappa_{TF}(\mu_{TF}-\delta)-\lambda_{TF}) \frac{\partial F}{\partial \delta} + \rho\sigma_p\sigma_\delta P \frac{\partial^2 F}{\partial P \partial \delta} - \frac{\partial F}{\partial t} = 0 \tag{3.13}$$

with a boundary condition of $F(P, \delta, 0) = P$, implying that the futures contract with the maturity date, $T$, approaching to zero proxies the spot prices.

According to Jamshidian and Fein (1990), the solution to this PDE in the log form is as follows

$$ln\ F(P, \delta, T) = ln\ P - \delta\ \frac{1 - e^{-\kappa_{TF} T}}{\kappa_{TF}} + B(T) \tag{3.14}$$

in which

$$\begin{aligned} B(T) = &\Big(r - \hat{\alpha} + \frac{1}{2}\frac{\sigma_p^2}{\kappa_{TF}^2} - \frac{\sigma_p\sigma_\delta\rho}{\kappa_{TF}}\Big)\ T + \frac{1}{4}\sigma_\delta^2\ \frac{1 - e^{-2\kappa_{TF} T}}{\kappa_{TF}^3} \\ &+ \Big(\hat{\alpha}\kappa_{TF} + \sigma_p\sigma_\delta\rho - \frac{\sigma_\delta^2}{\kappa_{TF}}\Big)\frac{1 - e^{-\kappa_{TF} T}}{\kappa_{TF}^2} \\ \hat{\alpha} = &\ \alpha - \frac{\lambda_{TF}}{\kappa_{TF}} \end{aligned} \tag{3.15}$$

Equation (3.14) is the linear relationship between the log futures prices and log spot prices which will be used to calibrate the model parameters.

### 3.3.3 One-factor long-term model

As we discussed earlier, the two-factor model introduces a stochastic process for the net convenience yield and thus more accurately explains spot and futures prices, compared to the one-factor mean-reverting model. However, applying the two-factor model to valuation problems adds one more dimension to the resulting partial differential equation (PDE), which adds complexity to the numerical solution.[12] To deal with this problem, Schwartz (1998) developed a simpler model than the two-factor model but with almost similar characteristics for pricing the term structure of futures prices, in particular, with

[12] Note that the numerical solution of problems with two stochastic factors is common in the literature, but a parsimonious one-factor model is preferred for simplicity, if it can adequately capture price uncertainty.

long maturities. This simpler model is known as one-factor long-term model and its results match as closely as possible to those of the two-factor model. The objectives of this model is to accurately estimate both the long-term futures prices and the volatility of futures prices.

The one-factor long-term model uses one single state variable called the shadow spot price and a constant convenience yield. Given the parameters of the two-factor model, the shadow spot price, $Z$, as a function of spot prices and net convenience yield, can be defined by[13]

$$Z(P, \delta) = Pe^{(\frac{c-\delta}{\kappa_{TF}} - \frac{\sigma_\delta^2}{4\kappa_{TF}^3})} \tag{3.16}$$

in which $c$ denotes the constant net convenience yield and is expressed as

$$c = \mu_{TF} - \frac{\lambda_{TF}}{\kappa_{TF}} - \frac{\sigma_\delta^2}{2\kappa_{TF}} + \frac{\rho\sigma_p\sigma_\delta}{\kappa_{TF}}. \tag{3.17}$$

The shadow spot price rises with spot prices and decreases with the net convenience yield. Schwartz (1998) has shown that the futures prices of this long-term model, denoted by $F(Z, T)$, converge to the two-factor model futures prices, $F(P, \delta, T)$, at a sufficiently long maturity date $T$ (i.e., when $T \to \infty$)[14] and is given by

$$ln\ F(Z, T) = ln\ Z + (r - c)T \tag{3.18}$$

in which the convenience yield is constant and is defined by Equation (3.17). Note that the stochastic process for $Z$ is

$$\frac{dZ}{Z} = (r - c)dt + \sigma_z(t)d\hat{z} \tag{3.19}$$

where $\sigma_z(t)$ represents the volatility of $Z$ which is a function of time. This volatility is

[13]This expression is taken from Chen (2010) and is slightly different from the corresponding equation in Schwartz (1998).

[14]Schwartz (1998) using the oil data obtained the close convergence of the futures prices of the two models after 3 years.

given by

$$\sigma_z^2(t) = \sigma_p^2 + \sigma_\delta^2 \frac{(1 - e^{-\kappa_{TF}t})^2}{\kappa_{TF}^2} - 2\rho\sigma_p\sigma_\delta \frac{(1 - e^{-\kappa_{TF}t})}{\kappa_{TF}}. \tag{3.20}$$

We will use Equations (3.16) and (3.18) to investigate the performance of this model for copper prices.

## 3.4 The Kalman filter estimation method

This section explains the Kalman filter estimation procedure and the maximum likelihood function, based on Harvey (1990)[15] and Hamilton (1994)[16].

### 3.4.1 The state-space form

To calibrate the models' parameters using the Kalman filter, the first step is to represent the model in the state-space form. The state-space representation of a multivariate time series model consists of two equations: 1) the measurement equation and 2) the transition equation. Assume that $y_t$ is an $N \times 1$ vector of observable variables and $x_t$ is an $m \times 1$ vector of unobservable state variables. The latter is known as the *state vector*.

The measurement equation relates $y_t$ to the state vector as follows

$$y_t = d_t' + Z_t' x_t + \epsilon_t, \qquad t = 1, ..., NT \tag{3.21}$$

in which $d_t'$ is an $N \times 1$ vector, $Z_t'$ is an $N \times m$ matrix, and $\epsilon_t$ is an $N \times 1$ vector of serially uncorrelated error terms so that

$$E(\epsilon_t) = 0 \quad and \quad Var(\epsilon_t) = H_t. \tag{3.22}$$

As noted, the elements of the state vector, $x_t$, are not observable. The transition

[15] Ch.3.
[16] Ch.13.

equation assumes that $x_t$ follows a first-order Markov process as follows

$$x_{t+1} = c_t + T_t x_t + \nu_{t+1}, \qquad t = 1, ..., T \tag{3.23}$$

where $c_t$ is an $m \times 1$ vector, $T_t$ is an $m \times m$ matrix, and $\nu_t$ is an $m \times 1$ vector of serially uncorrelated error terms so that

$$E(\nu_t) = 0 \quad \textit{and} \quad Var(\nu_t) = Q_t. \tag{3.24}$$

By assumption, the error terms of the measurement and transition equations are orthogonal in all periods. In addition, the system matrices - i.e., $d_t, Z_t, H_t, c_t, T_t$, and $Q_t$ - are assumed to be non-stochastic. Note that the system matrices change with time but are assumed to be predetermined at each period.

### 3.4.2 The Kalman filter procedure

The Kalman filter can be applied to the state-space model, specified by Equations (3.21) and (3.23), for optimally estimating the state vector at time $t$ given available information at that time, through a recursive procedure. The recursion starts at the initial time with a forecast of the state vector when there is no observation of $y$. Therefore, the forecast of the state vector at the current time given the past (unavailable) information, $\hat{x}_{1|0}$, is the unconditional mean of $x_1$.

$$\hat{x}_{1|0} = E(x_1) \tag{3.25}$$

The Mean Squared Error (MSE) of this forecast (i.e., the covariance matrix of the estimation error) is an $m \times m$ matrix given by

$$P_{1|0} = E\big[(x_1 - E(x_1))(x_1 - E(x_1))'\big] \tag{3.26}$$

$\hat{x}_{1|0}$ and $P_{1|0}$ are the forecasted mean and the covariance matrix of the initial state vector. These values can be computed using the transition and measurement equations. Hamilton (1994) has found a unique solution for each of these values that holds only if the eigenvalues of the matrix $Z$ in Equation (3.23) at the initial time are less than unity - i.e., inside the unit circle. These solutions are

$$\begin{aligned} &E(x_1) = 0 \\ &\text{vec}(P_{1|0}) = \left[I_{r^2} - (Z_t \otimes Z_t)\right]^{-1}. \text{ vec}(Q_t) \end{aligned} \tag{3.27}$$

in which the second expression expresses the elements of the $m \times m$ matrix of $P_{1|0}$ as a column vector. Note that if the eigenvalues of $Z$ are not inside the unit circle, the values of $\hat{x}_{1|0}$ and $P_{1|0}$ can be determined based on the researcher's best guess. The assigned values to the matrix of $P_{1|0}$ reflect the confidence in the guess so that larger values of the diagonal element of this matrix imply greater uncertainty about the true value of $x_t$.

The initial values will be used by the Kalman filter for the next period forecast of the state vector and the associated MSE. More generally, the objective for each step $t$ is to calculate the values of $\hat{x}_{t+1|t}$ and $P_{t+1|t}$ given $\hat{x}_{t|t-1}$ and $P_{t|t-1}$, respectively. The latter terms are now conditional on the observed data of $y$ up to time $t-1$. As it is shown by Hamilton (1994), the Kalman filter at time $t = 1, ..., T$ uses the information of time $t-1$ to produce a forecast for the state vector at time $t+1$ via the state equation given by

$$\hat{x}_{t+1|t} = T_t \hat{x}_{t|t-1} + K_t \eta_t \tag{3.28}$$

with

$$K_t = T_t P_{t|t-1} Z_t F_t^{-1} \tag{3.29}$$

$$F_t = Z_t' P_{t|t-1} Z_t + H_t \tag{3.30}$$

$$\eta_t = y_t - d_t' - Z_t' \hat{x}_{t|t-1} \tag{3.31}$$

in which $K_t$ is called the *gain matrix* and $\eta_t$ is the vector of prediction errors with an MSE equals to $F_t$.

The MSE associated with the forecast of the state equation is

$$P_{t+1|t} = T_t P_{t|t-1} T_t' - K_t Z_t' P_{t|t-1} T' + Q_t \tag{3.32}$$

in which $K_t$ is defined by Equation (3.29). In sum, the Kalman filter iterates on Equations (3.28) and (3.32) for all time steps until all the information set over $t = 1, ...T$ - i.e., $\{y_1, ..., y_t, ..., y_T\}$ - has been used in the iteration process. The output will be a smoothed estimate of the state vector.

### 3.4.3 Maximum likelihood function

The ultimate objective of using the Kalman filter in this study is to estimate the unknown parameters in the system matrices of $d_t, Z_t, H_t, c_t, T_t$, and $Q_t$. Such parameter estimations can be obtained by maximizing the sample log likelihood function. By using the prediction error decomposition, the log likelihood function is[17]

$$\ln\, L = -\frac{NT}{2} \ln\, 2\pi - \frac{1}{2}\sum_{t=1}^{T} \ln\, |F_t| - \frac{1}{2}\sum_{t=1}^{T} \eta_t' F_t^{-1} \eta_t \tag{3.33}$$

in which $F_t$ and $\eta_t$ are defined by Equations (3.30) and (3.31), respectively. The Kalman filter uses this function to estimate the unknowns through numerical optimization methods. The next section provides the state-space representation of the price valuation models of this study.

## 3.5 Valuation models: the state-space representation

### 3.5.1 One-factor mean-reverting model

The state vector of this model contains the elements of the spot prices, as they are not observed at all time steps. From Equation (3.9) and setting $m = 1$, the terms of the

[17] The origin of this equation is

$$L = \prod_{t=1}^{T} f(y_t|Y_{t-1})$$

in which $f(\cdot)$ denotes the distribution of $y_t$ conditional on the set of information up to $t-1$ given by $Y_{t-1} = \{y_1, ..., y_{t-2}, y_{t-1}\}$.

measurement equation at each time step $t$ can be expressed as

$$\begin{aligned}
d'_t &= \Big[\hat{\mu}(1-e^{-\kappa T_i}) + \frac{\sigma^2}{4\kappa}(1-e^{-2\kappa T_i})\Big]_{N\times 1} && i=1,...,N \\
Z'_t &= \Big[e^{-\kappa T_i}\Big]_{N\times 1} && i=1,...,N \\
y_t &= \Big[\ln F(T_i)\Big]_{N\times 1} && i=1,...,N \\
x_t &= S(t) \\
H_t &= \sigma^2_{\epsilon_i}\ I_N && i=1,...,N
\end{aligned} \tag{3.34}$$

in which $I_N$ is the identity matrix with the dimension of $N \times N$.

From Equation (3.4), the terms of the transition equation at each $t$ can be written as

$$\begin{aligned}
c_t &= \kappa(\mu - \frac{\sigma^2}{2\kappa})\Delta t \\
T_t &= 1 - \kappa\Delta t \\
Q_t &= \sigma^2\Delta t.
\end{aligned} \tag{3.35}$$

### 3.5.2 Two-factor model

In this model, in addition to the spot prices, the net convenience yield can not be observed in the market. Therefore, these two variables form the state vector. From Equation (3.14), the measurement equation at each time step $t$ can be written as

$$\begin{aligned}
d'_t &= \Big[B(T_i)\Big]_{N\times 1} && i=1,...,N \\
Z'_t &= \Big[1, \frac{1-e^{-\kappa_{TF} T_i}}{\kappa_{TF}}\Big]_{N\times 2} && i=1,...,N \\
y_t &= \Big[\ln F(T_i)\Big]_{N\times 1} && i=1,...,N \\
x_t &= \Big[S(t), \delta(t)\Big]'_{2\times 1} \\
H_t &= \sigma^2_{\epsilon_i}\ I_N && i=1,...,N.
\end{aligned} \tag{3.36}$$

From Equation (3.12), the terms of the transition equation at each $t$ can be written as

$$\begin{aligned}
c_t &= \left[(\mu_{TF} - \frac{\sigma_p^2}{2})\Delta t, \kappa_{TF}\alpha\Delta t\right]'_{2\times1} \\
T_t &= \begin{bmatrix} 1 & -\Delta t \\ 0 & 1-\kappa\Delta t \end{bmatrix}_{2\times2} \\
Q_t &= \begin{bmatrix} \sigma_p^2\Delta t & \rho\sigma_p\sigma_\delta\Delta t \\ \rho\sigma_p\sigma_\delta\Delta t & \sigma_\delta^2\Delta t \end{bmatrix}_{2\times2}.
\end{aligned} \tag{3.37}$$

### 3.5.3 One-factor long-term model

The state-space form of this model is the same as the two-factor model. We first estimate the state variables as well as the parameters of the two-factor model. Then, we substitute the estimated values in Equations (3.16) and (3.18) to find the model-implied shadow spot prices and the resulting futures prices, given the constant net convenience yield in Equation (3.17).

## 3.6 Estimation results

### 3.6.1 One-factor mean-reverting model

Table (3.4) shows the calibration results for the one-factor mean-reverting model, using the data of both sub-samples. As noted, each sub-sample covers a different time period and consists of 6 contracts with different maturities, enabling us to evaluate the power of each model to fit the data with shorter versus much longer maturities. Estimation results are significantly different from zero. What stands out in the table is the values of the Root Mean Square Error (RMSE) and Mean Absolute Error (MAE) of log futures prices, constructed using the vector of prediction error in Equation (3.31). These values enable us to decide to what extent the model can explain the data.

For the first copper data set, the RMSE and MAE are around 7 cents per pound and 5 cents per pound of copper, respectively. However, these values for the second sub-

| Period | Aug 1997-Jul 2015 | Nov 2002-Jul 2015 |
|---|---|---|
| Contracts | $F1$, $F6$ , $F11$, $F16$, $F21$, $F24$ | $F1$, $F24$ , $F30$, $F40$, $F50$, $F60$ |
| $\kappa$ | 0.026 (0.001) | 0.006 (0.001) |
| $\mu$ | 2.705 (0.079) | 0.748 (0.069) |
| $\lambda$ | 2.784 (0.026) | 4.664 (0.063) |
| $\sigma^2$ | 0.046 (0.002) | 0.04 (0.001) |
| RMSE | 0.069 | 0.136 |
| MAE | 0.052 | 0.11 |
| Log-likelihood | 9652 | 4548 |
| Number of obs | 937 | 667 |

Table 3.4: *Parameter estimation results for the one-factor mean-reverting model using the weekly futures data. Columns 2 and 3 are associated with the first and second sub-samples, respectively. Standard errors are in prantesis.*

sample are double, reducing the power of this model to explain the longer-term futures prices. This conclusion is consistent with our discussion earlier about the importance of stochastic convenience yield in copper price formation, in particular, for the second sub-sample. Figure (3.3) confirms that the model implied price path cannot track the market spot prices if the maturity of contracts is extended. This issue should be considered when applying the one-factor mean-reverting model to long-term investment projects that would require futures contracts with long maturity dates. However, the first sub-sample would still be interesting for the analysis of such projects due to the highest trading volumes as well as the highest degree of liquidity of the futures contracts with shorter maturities.

The much weaker mean reversion (0.006 versus 0.026) as a result of increasing the maturity dates also makes the second sub-sample less interesting compared to the first one, when using the one-factor mean-reverting model. Schwartz (1997) argues that, in equilibrium, an increase or decrease in prices can be offset by the entry or exit of higher-cost producers from the market. Consequently, prices tend to revert to a long-run equilibrium.[18] The mean-reversion speed, $\kappa$, implies that reducing the distance between the current log

[18] Note that we have not generally observed this process in the data. However, eventually we might see upward pressure on copper prices as more of the resource is used up.

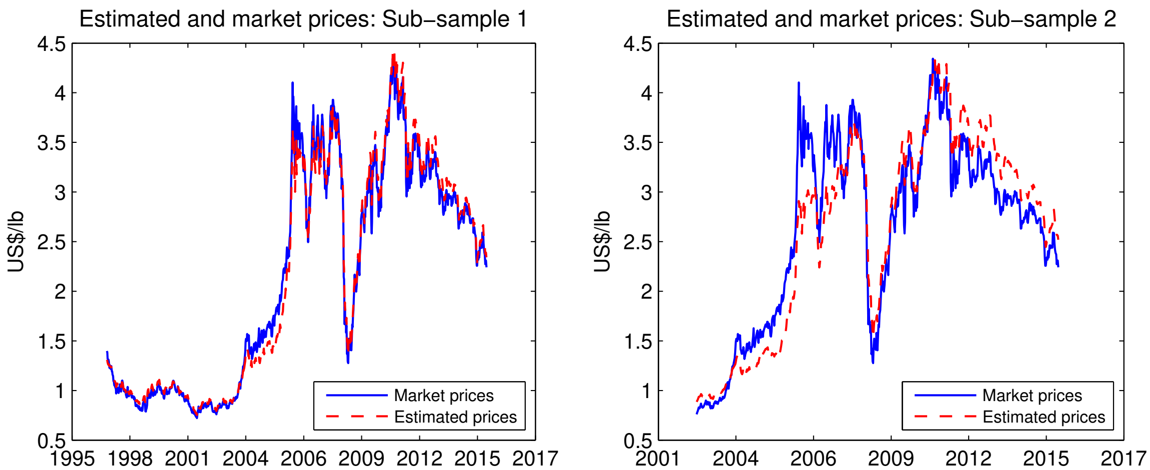


Figure 3.3: *One-factor mean-reverting model implied spot prices versus the market spot prices for copper using the first sub-sample data (the left-hand pannel) and the second sub-sample data (the right-hand panel).*

price and the long-run mean[19] by half takes $ln(2)/\kappa$ years. This implies that the half-life for revering to the long-run mean is 116 years in the second sub-sample. Note that the long-run mean of the price for the second sub-sample is around 0.075,[20] which is unreasonably low. These observations indicate that the one-factor model that is mean reverting in drift fails to forecast the spot prices using much longer maturity contracts, as in the second sub-sample.

It is worth noting that the mean absolute error of using the contracts of the first and second sub-samples are around 8% and 13% of the average log futures prices closest to maturity, respectively. As noted in Schwartz (1997), these percentages indicate some large fluctuations in the prediction error, as shown in the top panel of Figure (3.4). For clarity, the prediction errors are shown for the first, middle, and last contracts of the first sub-sample. The gaps among the prediction errors between 2004 and 2008 could be due to a notable difference between the futures prices and spot prices (and thus high convenience yield) in that period, as shown in the bottom panel of this figure for corresponding futures

[19] i.e., $\hat{\mu} - S$ in Equation (3.4) ignoring volatility and market price of risk.
[20] $exp(\mu - \sigma^2/(2\kappa))$.

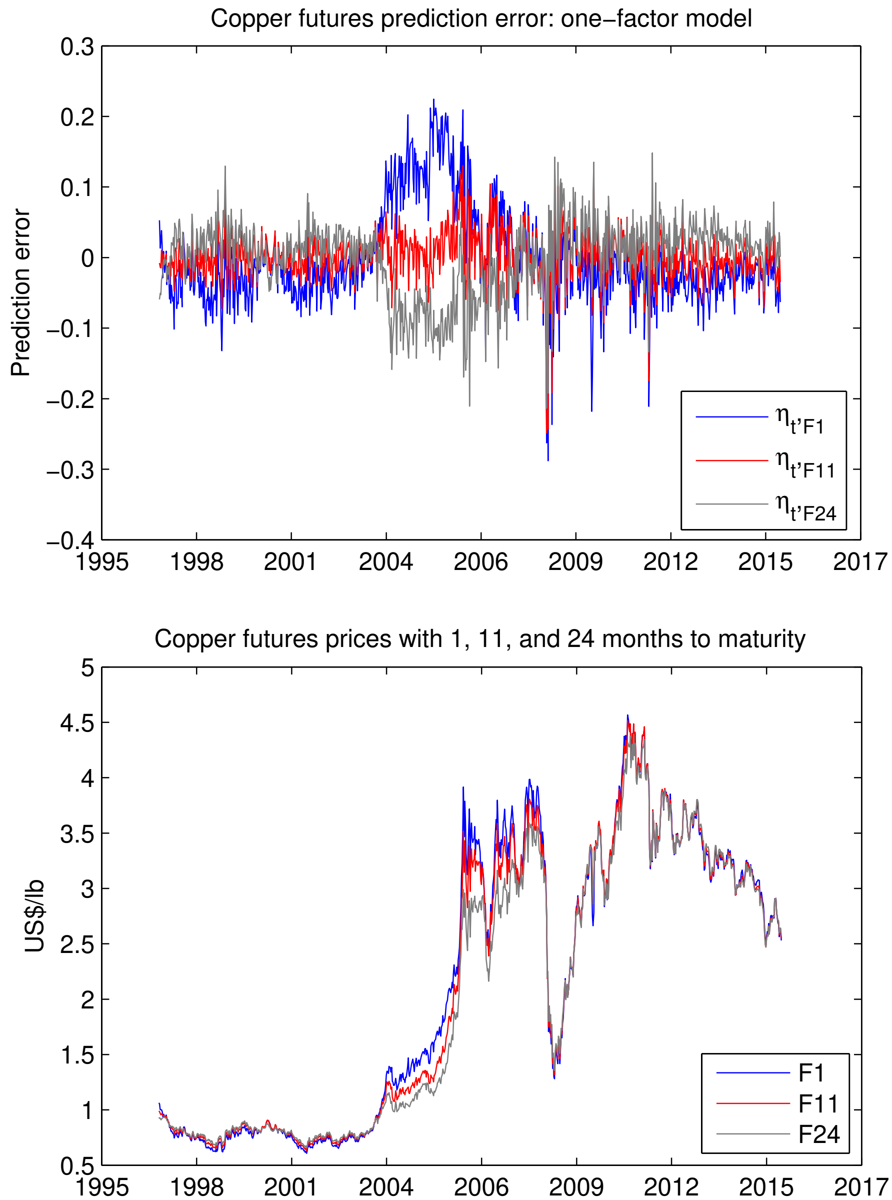


Figure 3.4: *The prediction errors (the top panel) of the one-factor mean-reverting model for three out of six contracts used in estimation and the corresponding futures prices (the bottom panel) in the first sub-sample from Aug 1997 to Jul 2015.*

| Period | Aug 1997-Jul 2015 | Nov 2002-Jul 2015 |
|---|---|---|
| Contracts | $F1,\ F6\ ,\ F11,$ $F16,\ F21,\ F24$ | $F1,\ F24\ ,\ F30,$ $F40,\ F50,\ F60$ |
| $\mu_{TF}$ | 0.303 (0.010) | 0.136 (0.007) |
| $\kappa_{TF}$ | 0.089 (0.027) | 0.178 (0.066) |
| $\alpha$ | 0.010 (0.024) | 0.069 (0.069) |
| $\sigma_p^2$ | 0.071 (0.003) | 0.077 (0.005) |
| $\sigma_\delta^2$ | 0.003 (0.000) | 0.002 (0.000) |
| $\rho$ | 0.555 (0.023) | 0.511 (0.039) |
| $\lambda_{TF}$ | -0.010 (0.021) | 0.007 (0.011) |
| RMSE | 0.039 | 0.047 |
| MAE | 0.028 | 0.033 |
| Log-likelihood | 17,635 | 9071 |
| Number of obs | 937 | 667 |

Table 3.5: *Parameter estimation results for the two-factor model using the weekly futures data. Columns 2 and 3 are associated with the first and second sub-samples, respectively. Standard errors are in parenthesis and $r = 2\%$.*

prices. The prediction errors remain large over that period, implying that the one-factor mean-reverting model cannot explain the impacts of stochastic convenience yield on copper price dynamics.

### 3.6.2 Two-factor model

Table (3.5) shows the estimation results of the two-factor model, using the weekly futures contracts of each sub-sample. The real risk-free interest rate, $r$, is assumed to be 2%. The estimated value of $\rho$ indicates a moderate, positive correlation between the copper spot prices and the net convenience yield. The value of $\kappa_{TF}$ implies that the half-distance between $\delta$ and its long-run mean, $\alpha$, will occur after 8 years for shorter maturities and 4 years for longer-term contracts. This result shows a stronger mean reversion of the net convenience yield with much longer maturities.

We expect that, in equilibrium, the net (marginal) convenience yield goes to zero due

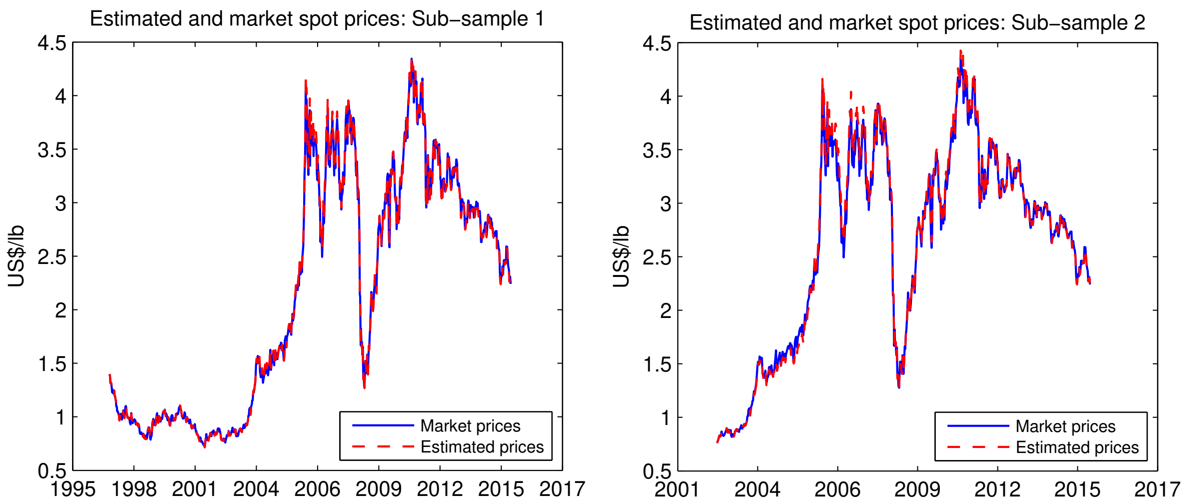


Figure 3.5: *Two-factor model implied spot prices versus the market spot prices for copper using the first sub-sample data (the left-hand panel) and the second sub-sample data (the right-hand panel).*

to the convergence of the marginal benefit of holding an additional unit of inventory and the storage cost of that additional unit. This fact is reflected by the estimated value of $\alpha$, which is close to zero for both sub-samples. Note that since the risk-free rate is a constant parameter in this model, the variations in net convenience yield absorb any fluctuations in the interest rate over time. Consequently, the estimated net convenience yield at each time explains the actual net convenience yield as well as the deviations of the interest rate from 2%. For this reason, the estimate of $\alpha$ for the net convenience yield process is not exactly zero. All estimation results are statistically significant due to low standard deviations.

The root mean square error and the mean absolute error for both sub-samples are low and close to each other. It follows that the two-factor model is able to characterize the dynamics of both short-term and long-term futures contracts, as shown in Figure (3.5). The model prediction errors for the first, middle, and the last contracts of both sub-samples are shown in Figure (3.6). The mean absolute errors of parameter estimations using the data of the shorter-term and the longer-term sub-samples are respectively around 4% and 5% of the average log futures prices closest to maturity, representing some large deviations in the prediction error. There is no significant difference among the prediction errors, in particular

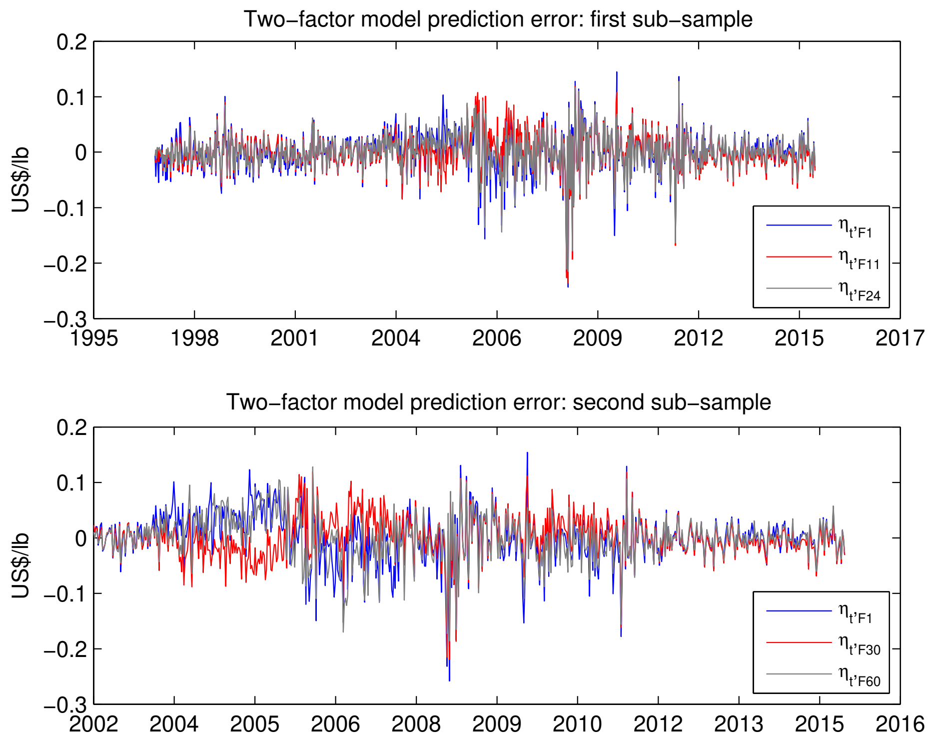


Figure 3.6: *The prediction errors of the two-factor model for three out of six contracts in the first subsample (the top panel) and the second sub-sample (the bottom panel).*

between 2004 and 2008, when there has been strong backwardation in the market, due to including the stochastic convenience yield process. Figure (3.7) plots the model implied state variables for both sub-samples and shows that the estimated net convenience yield has its highest values during that period. Recall that the constant convenience yield has been the main reason for the obvious gaps in the prediction errors of the one-factor mean-reverting over that specific period. Therefore, introducing the stochastic convenience yield has improved the prediction error in this model compared to the one-factor mean-reverting model. The average net convenience yield for the first sub-sample is \$0.028 and for the second sub-sample is \$0.057 per pound of copper.

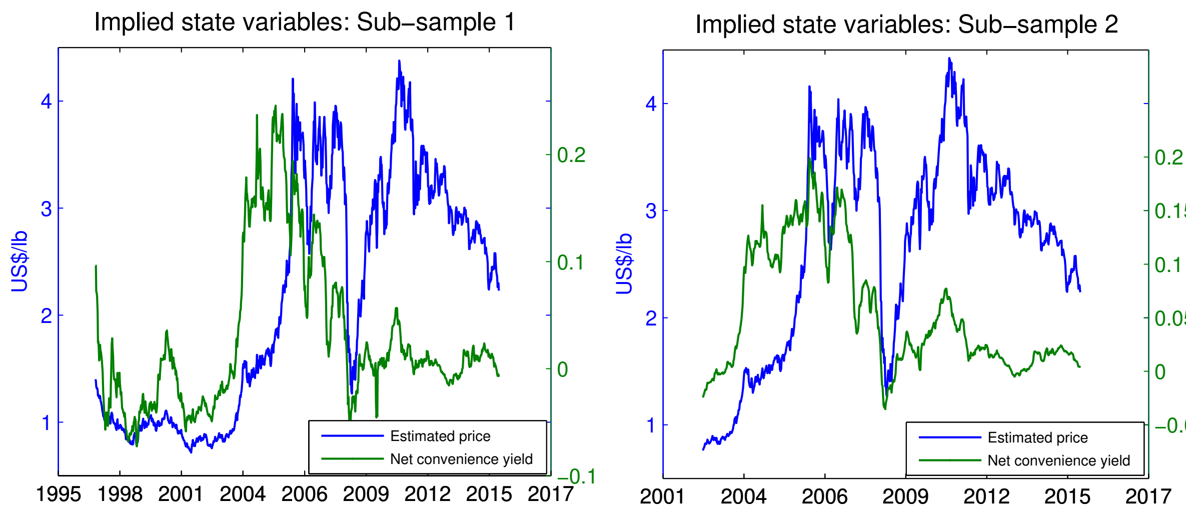


Figure 3.7: *Two-factor model implied state variable using the data of the first subsample (the left-hand panel) and the second sub-sample (the right-hand panel).*

### 3.6.3 One-factor long-term model

Figure (3.8) plots the model implied spot prices, $P(Z, \delta)$, and shadow spot prices, $Z(P, \delta)$, for both sub-samples. The former can be found using Equation (3.16) by writing $P$ in terms of $Z$. To obtain $Z$, we use the vector of each state variable, $P$ and $\delta$, estimated in the two-factor model. In addition, since the shadow spot prices are a mathematical transformation of the two-factor model, the parameters have identical values. Therefore, the constant net convenience yield, $c$ in Equation (3.17), is around 0.054 for the first and 0.016 for the second sub-samples. From this figure, the model implied spot prices have the same values and dynamics as the market spot prices. The estimated shadow spot prices do not have values equal to the market prices, but mimic the dynamics of the spot prices to a good extent. Using the shorter term contracts, we have found a wider range for the shadow spot prices compared to market prices, whereas this range is narrower with longer term futures contracts. In addition, we have observed that the shadow spot prices are almost as volatile as the actual spot prices in both sub-samples.[21]

[21] The standard deviation for the shadow spot prices and market spot prices covering the period of the first sub-sample are 1.076 and 1.099, respectively. These values for the second sub-sample are 0.821 and 0.933, respectively.

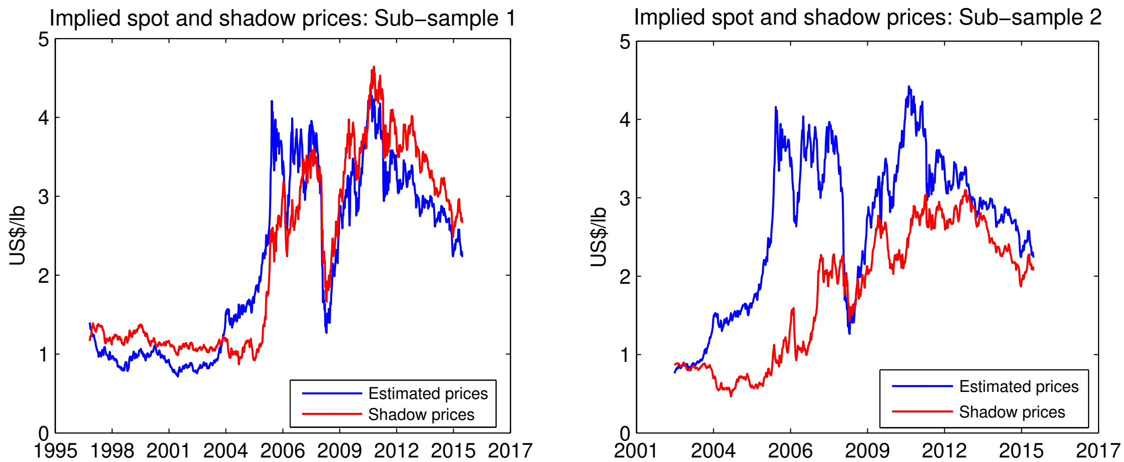


Figure 3.8: *Two-factor model implied state variable using the data of the first subsample (the top panel) and the second sub-sample (the bottom panel).*

## 3.7 Model comparison

This section compares the performance of the one-factor mean-reverting model and the one-factor long-term model relative to the two-factor model, in terms of their ability to explain forward curves. A forward curve shows the futures prices across their maturities at a specific time. The two main shapes of the forward curves are upward sloping and downward sloping. The former implies that the market is in contango, whereas the latter represents backwardation. We will use the forward curve and the term structure of futures prices interchangeably.

In this section, we are interested to learn to what extent each model can explain the different shapes of the copper forward curves. In addition, we investigate the implications of these models for the term structure of futures prices with much longer maturities than the maximum maturity in each sub-sample.[22] This out-of-sample illustration of forward curves is interesting because the main application of these models is in pricing assets contingent on the price of copper with much longer maturities than 2 years and 5 years. We use the

[22]Recall that the maximum maturity date of futures contracts of the first sub-sample is 2 years, and that of the second sub-sample is 5 years.

two-factor model as the benchmark to evaluate the performance of the other two models for two reasons. First, it has a lower RMSE and MAE than the one-factor mean-reverting model and thus more accurately explains the dynamics of market spot and futures prices. Second, the forward curves implied by the one-factor long-term model and the two-factor model should converge as the maturity is extended.

We start our analysis with the term structure of futures prices estimated from the data of the first sub-sample. Figure (3.9) shows all the market futures prices, with two years as the longest maturity. It also illustrates the forward curves implied by each model for contracts up to 15 years, for 4 specific dates. These dates are for illustration only, and the models performance remains the same for the rest of the observations in the sub-sample. In this figure, MR stands for the mean-reverting model, TF refers to the two-factor model, and LT represents the long-term model. It can be seen from this figure that the two factor model can explain the majority of the market futures prices. In contrast, futures prices of the mean-reverting model do not fit the observed futures data and diverge significantly from the two-factor model as the maturity increases. Interestingly, the futures prices implied by the long-term model converge to those of the two-factor model after about 5 years. Note that, for each date, the constant convenience yield for the long-term model is $c = 0.054$. However, the estimated values of spot price, $P$, and instantaneous net convenience yield, $\delta$, as well as the corresponding shadow spot price, $Z$, differ across the two-factor and the long-term models. These values for each panel at $t = 0$ are as follows:

- In panel (a), $P = 0.626$, $\delta = -0.061$, and $Z = 0.891$.
- In panel (b), $P = 1.343$, $\delta = 0.169$, and $Z = 0.893$.
- In panel (c), $P = 3.347$, $\delta = 0.106$, and $Z = 2.749$.
- In panel (d), $P = 3.536$, $\delta = 0.002$, and $Z = 4.093$.

Note that $t = 0$ refers to the less than one month maturity date, which is close to zero if converted to a year. Recall that $P$ and $Z$ are both the spot prices and futures prices at time zero. A comparison between the futures prices of the long-term model and the two-factor model, in particular for maturities before 5 years, reveals that the gap between futures

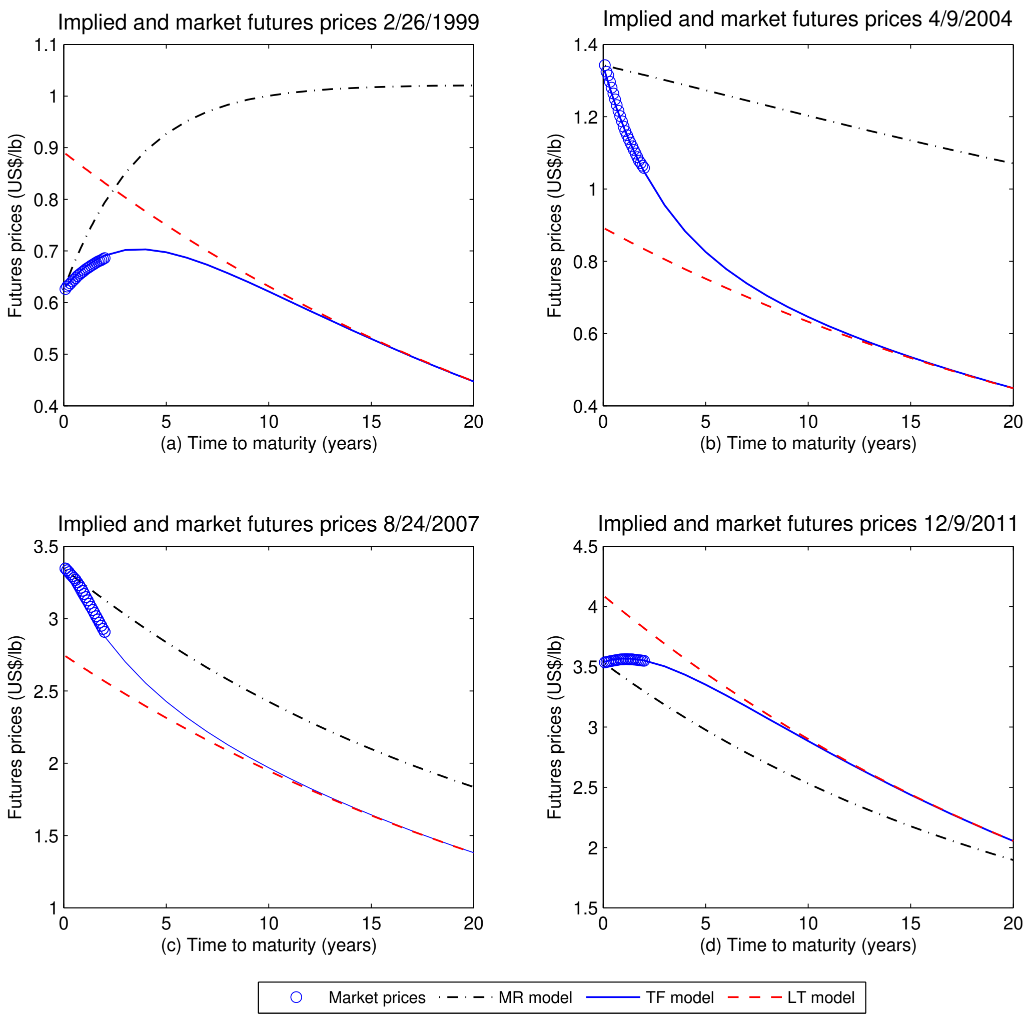


Figure 3.9: *Market and model implied forward curves, at 4 selected dates from the first sub-sample.*

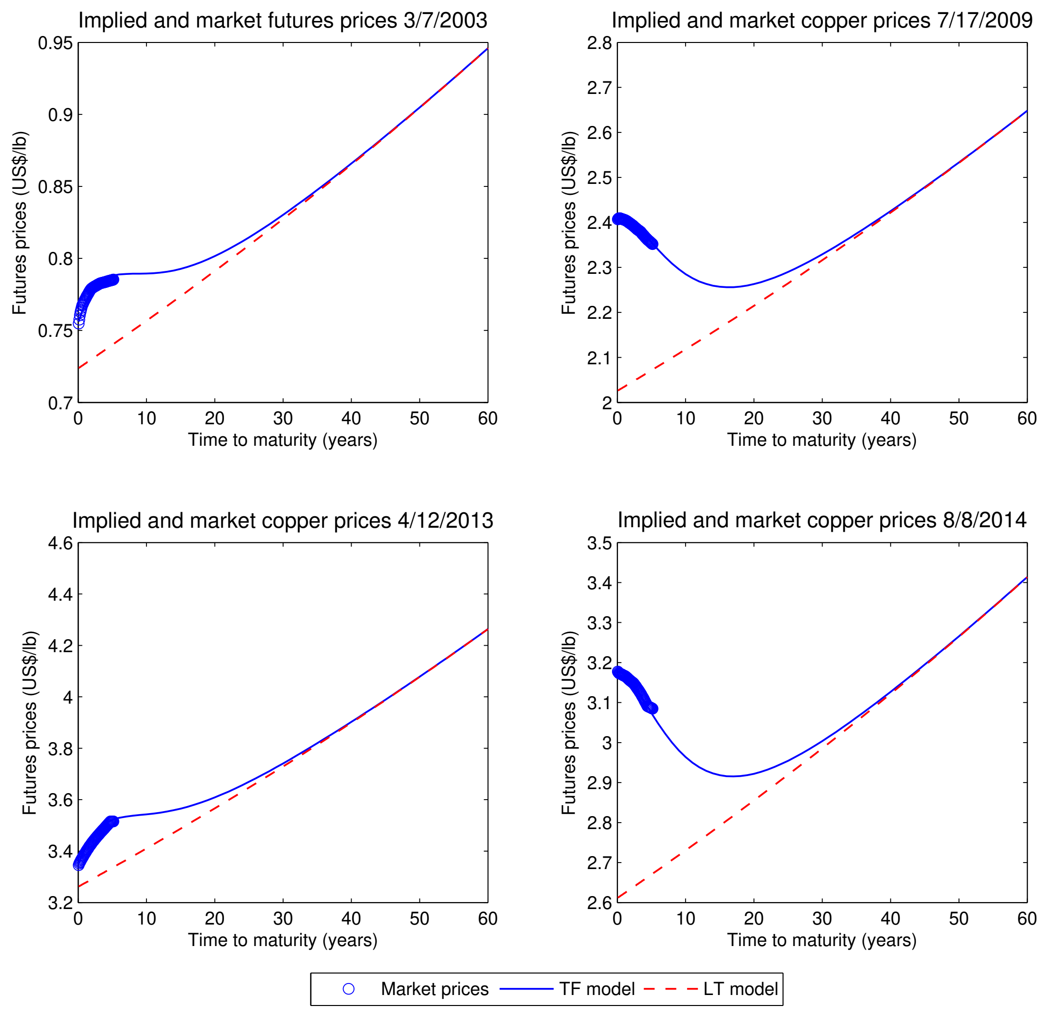


Figure 3.10: *Market and model implied forward curves at, 4 selected dates from the second sub-sample.*

prices of these models gets smaller with maturity and before the two curves converge. In contrast, the gap between the mean-reverting model and the two-factor model increases with maturity, making this model less interesting. Consequently, before the convergence has occurred, the term structure of futures prices of the long-term model can match the two-factor model with a small difference that can be ignored. Specially, Figure (3.10) shows that, using the second sub-sample, the convergence occurs after 15 years. However, the gap is very small before this threshold for the majority of dates, as shown in Appendix (C.1). Since the one-factor mean-reverting model fails to describe the term structure of copper futures prices in the second sub-sample, we have not shown its forward curves in this figure.

## 3.8 Conclusions

This study examines the performance of three stochastic models in terms of their ability to characterize the price of copper derivatives. These models are the one-factor mean-reverting model, two-factor model, and one-factor long-term model. The first model assumes spot prices are mean-reverting in drift. The second model defines two correlated stochastic factors that are spot prices and convenience yield. The third model transforms the two-factor price model into a single factor model. These models are calibrated to copper futures prices using a Kalman filtering approach.

We have found that the mean-reverting model cannot describe the term structure of copper futures prices with long maturities. In contrast, the two-factor and the long-term models are shown to provide a reasonable fit of the term structure of copper futures prices, highlighting the importance of stochastic convenience yield in copper price formation. However, the long-term model is relatively much simpler in its application to asset valuation models and has the same implications as the two-factor model (Schwartz, 1998). We conclude that the long-term model can be applied to long-term investment projects. We also argue that because the first sub-sample has the highest trading volumes and is more liquid, it can still be attractive for calibrating price parameters when the goal is valuing investment projects even with long maturities.

# References

Almansour, A. and Insley, M. (2016), 'The impact of stochastic extraction cost on the value of an exhaustible resource: An application to the Alberta oil sands', *Energy Journal* **37**(2), 61–88.

Amato, J. and Remolona, E. (2003), 'The credit spread puzzle', *BIS Quarterly Review* pp. 51–63.

Ayache, E., Forsyth, P. and Vetzal, K. (2003), 'Valuation of convertible bonds with credit risk', *Journal of Derivatives* **11**, 9–29.

Baumol, W. J. and Oates, W. E. (1988), *The Theory of Environmental Policy*, Cambridge University Press.

Black, F. and Cox, J. C. (1976), 'Valuing corporate securities: Some effects of bond indenture provisions', *The Journal of Finance* **31**(2), 351–367.

Bohm, P. (1981), Deposit-refund systems; theory and applications to environmental, conservation, and consumer policy, Technical report, Resources for the Future, Washington, DC.

Boomhower, J. (2014), 'Drilling like there is no tomorrow: Bankruptcy, insurance, and environmental risk', *Energy Institute at Haas Working Paper* **254**. University of California, Berkeley.

Boyd, J. (2002), Financial responsibility for environmental obligations: Are bonding and assurance rules fulfilling their promise?, *in* 'An introduction to the law and economics of environmental policy: issues in institutional design', Emerald Group Publishing Limited, pp. 417–485.

Brennan, M. J. and Schwartz, E. S. (1985), 'Evaluating natural resource investments', *Journal of Business* **58**(2), 135–157.

Buncic, D. and Moretto, C. (2015), 'Forecasting copper prices with dynamic averaging and selection models', *The North American Journal of Economics and Finance* **33**, 1–38.

Campbell, H. F. and Bond, K. (1997), 'The cost of public funds in Australia', *Economic Record* **73**(220), 22–34.

Chen, S. (2010), 'Modelling the dynamics of commodity prices for investment decisions under uncertainty'. A Ph.D. thesis published by the University of Waterloo.

Chen, Z. and Forsyth, P. A. (2007), 'A semi-Lagrangian approach for natural gas storage valuation and optimal operation', *SIAM Journal on Scientific Computing* **30**(1), 339–368.

Cornwell, L. and Costanza, R. (1994), 'An experimental analysis of the effectiveness of an environmental assurance bonding system on player behavior in a simulated firm', *Ecological Economics* **11**(3), 213–226.

Costanza, R. and Perrings, C. (1990), 'A flexible assurance bonding system for improved environmental management', *Ecological Economics* **2**(1), 57–75.

Dachis, B., Shaffer, B. and Thivierge, V. (2017), 'All's well that ends well: Addressing end-of-life liabilities for oil and gas wells'. C.D. Howe Institute Commentary 492.

Davis, L. W. (2015), 'Policy monitor: Bonding requirements for US natural gas producers', *Review of Environmental Economics and Policy* **9**(1), 128–144.

Dionne, G. and Spaeter, S. (2003), 'Environmental risk and extended liability: The case of green technologies', *Journal of Public Economics* **87**(5), 1025–1060.

Dixit, A. (1989), 'Entry and exit decisions under uncertainty', *Journal of Political Economy* **97**(3), 620–638.

Dixit, A. (1992), 'Investment and hysteresis', *The Journal of Economic Perspectives* **6**(1), 107–132.

Dixit, A. and Pindyck, R. (1994), *Investment under Uncertainty*, Princeton University Press.

Fama, E. F. and French, K. R. (1987), 'Commodity futures prices: Some evidence on forecast power, premiums, and the theory of storage', *The Journal of Business* **60**(1), 55–73.

Farzin, Y. H. (1996), 'Optimal pricing of environmental and natural resource use with stock externalities', *Journal of Public Economics* **62**(1), 31–57.

Ferreira, D., Suslick, S., Farley, J., Costanza, R. and Krivov, S. (2004), 'A decision model for financial assurance instruments in the upstream petroleum sector', *Energy Policy* **32**(10), 1173–1184.

Gerard, D. (2000), 'The law and economics of reclamation bonds', *Resources Policy* **26**(4), 189–197.

Gerard, D. and Wilson, E. J. (2009), 'Environmental bonds and the challenge of long-term carbon sequestration', *Journal of Environmental Management* **90**(2), 1097–1105.

Gibson, R. and Schwartz, E. S. (1990), 'Stochastic convenience yield and the pricing of oil contingent claims', *The Journal of Finance* **45**(3), 959–976.

Grant, J., Dyer, S. and Woynillowicz, D. (2009), *Clearing the Air on Oil Sands Myths*, Pembina Institute.

Hamilton, J. D. (1994), *Time series analysis*, Vol. 2, Princeton university press Princeton, NJ.

Harvey, A. C. (1990), *Forecasting, structural time series models and the Kalman filter*, Cambridge university press.

Igarashi, Y., Andersen, M. A. and Coupal, R. H. (2010), Environmental bonding for land reclamation, *in* '2010 Annual Meeting, July 25-27, 2010, Denver, Colorado', Agricultural and Applied Economics Association.

Insley, M. (2017), 'Resource extraction with a carbon tax and regime switching prices: Exercising your options', *Energy Economics* **67**, 1–16.

Insley, M. and Lei, M. (2007), 'Hedges and trees: incorporating fire risk into optimal decisions in forestry using a no-arbitrage approach', *Journal of Agricultural and Resource Economics* pp. 492–514.

In't Hout, K. (2017), *Numerical Partial Differential Equations in Finance Explained*, Palgrave MacMillan.

Jamshidian, F. and Fein, M. (1990), 'Closed-form solutions for oil futures and European options in the Gibson-Schwartz model: A note', *Merril Lynch Capital Markets* .

Kahn, J. R., Franceschi, D., Curi, A. and Vale, E. (2001), Economic and financial aspects of mine closure, *in* 'Natural Resources Forum', Vol. 25, Wiley Online Library, pp. 265–274.

Kaldor, N. (1939), 'Speculation and economic stability', *The Review of Economic Studies* **7**(1), 1–27.

Kaplow, L. and Shavell, S. (1996), 'Property rules versus liability rules: An economic analysis', *Harvard Law Review* pp. 713–790.

Keohane, N., Van Roy, B. and Zeckhauser, R. (2007), 'Managing the quality of a resource with stock and flow controls', *Journal of Public Economics* **91**(3), 541–569.

Kovalov, P. and Linetsky, V. (2008), 'Valuing convertible bonds with stock price, volatility, interest rate, and default risk'.

Labys, W. C., Lesourd, J. and Badillo, D. (1998), 'The existence of metal price cycles1', *Resources policy* **24**(3), 147–155.

Lappi, P. (2018), 'A model of optimal extraction and rehabilitation'. Working Paper, Presented at the 6th World Congress of Environmental and Resource Economists.

Larson, B. A. (1996), 'Environmental policy based on strict liability: implications of uncertainty and bankruptcy', *Land Economics* **72**(1), 33–43.

Lasheras, F. S., de Cos Juez, F. J., Sánchez, A. S., Krzemień, A. and Fernández, P. R. (2015), 'Forecasting the comex copper spot price by means of neural networks and arima models', *Resources Policy* **45**, 37–43.

Lemphers, N., Dyer, S. and Grant, J. (2010), *Toxic Liability: How Albertans Could end up Paying for Oil Sands Mine Reclamation*, Pembina Institute.

Li, G. and Li, Y. (2015), 'Forecasting copper futures volatility under model uncertainty', *Resources Policy* **46**, 167–176.

Longstaff, F. A. and Schwartz, E. S. (1995), 'A simple approach to valuing risky fixed and floating rate debt', *The Journal of Finance* **50**(3), 789–819.

Madan, D. and Unal, H. (2000), 'A two-factor hazard rate model for pricing risky debt and the term structure of credit spreads', *Journal of Financial and Quantitative analysis* **35**(1), 43–65.

Mason, C. F. (2001), 'Nonrenewable resources with switching costs', *Journal of Environmental Economics and Management* **42**(1), 65–81.

Mastro, M. (2013), *Financial derivative and energy market valuation: Theory and implementation in Matlab*, John Wiley & Sons.

Merolla, A. T. (1998), 'The effect of latent hazards on firm exit in manufacturing industries 37', *International review of law and economics* **18**(1), 13–24.

Merton, R. C. (1974), 'On the pricing of corporate debt: The risk structure of interest rates', *The Journal of finance* **29**(2), 449–470.

Miller, C. G. (2005), *Financial Assurance for Mine Closure and Reclamation: A Study Prepared by for the International Council on Mining and Metals*, International Council on Mining and Metals.

Muehlenbachs, L. (2015), ‘A dynamic model of cleanup: Estimating sunk costs in oil and gas production’, *International Economic Review* **56**(1), 155–185.

Muehlenbachs, L. (2017), ‘80,000 inactive oil wells: A blessing or a curse?’. University of Calgary, School of Public Policy.

Nkuiya, B. (2015), ‘Transboundary pollution game with potential shift in damages’, *Journal of Environmental Economics and Management* **72**, 1–14.

Otto, J. M. (2010), Global trends in mine reclamation and closure regulation, *in* ‘Mining, Society, and a Sustainable World’, Springer, pp. 251–288.

Parente, V., Ferreira, D., dos Santos, E. M. and Luczynski, E. (2006), ‘Offshore decommissioning issues: Deductibility and transferability’, *Energy Policy* **34**(15), 1992–2001.

Peck, P. and Sinding, K. (2009), ‘Financial assurance and mine closure: Stakeholder expectations and effects on operating decisions’, *Resources Policy* **34**(4), 227–233.

Perrings, C. (1989), ‘Environmental bonds and environmental research in innovative activities’, *Ecological Economics* **1**(1), 95–110.

Phillips, C. V. and Zeckhauser, R. J. (1998), ‘Restoring natural resources with destination-driven costs’, *Journal of Environmental Economics and Management* **36**(3), 225–242.

Poulin, R. and Jacques, M. (2007), ‘Mine reclamation bonding and environmental insurance’, *International Journal of Risk Assessment and Management* **7**(5), 589–606.

Roan, P. and Martin, W. (1996), ‘Optimal production and reclamation at a mine site with an ecosystem constraint’, *Journal of Environmental Economics and Management* **30**(2), 186–198.

Routledge, B. R., Seppi, D. J. and Spatt, C. S. (2000), ‘Equilibrium forward curves for commodities’, *The Journal of Finance* **55**(3), 1297–1338.

Sadorsky, P. (2001), ‘Risk factors in stock returns of canadian oil and gas companies’, *Energy economics* **23**(1), 17–28.

Schmitt, A. and Spaeter, S. (2005), ‘Improving the prevention of environmental risks with convertible bonds’, *Journal of Environmental Economics and Management* **50**(3), 637–657.

Schwartz, E. S. (1997), ‘The stochastic behavior of commodity prices: Implications for valuation and hedging’, *The Journal of Finance* **52**(3), 923–973.

Schwartz, E. S. (1998), ‘Valuing long-term commodity assets’, *Financial Management* pp. 57–66.

Shavell, S. (1984), ‘A model of the optimal use of liability and safety regulation’, *The Rand Journal of Economics* **15**(2), 271–280.

Shavell, S. (2005), ‘Minimum asset requirements and compulsory liability insurance as solutions to the judgment-proof problem’, *The Rand Journal of Economics* **36**(1), 63.

Shogren, J. F., Herriges, J. A. and Govindasamy, R. (1993), ‘Limits to environmental bonds’, *Ecological Economics* **8**(2), 109–133.

Slade, M. E. (2001), ‘Valuing managerial flexibility: An application of real-option theory to mining investments’, *Journal of Environmental Economics and Management* **41**(2), 193–233.

Smith, K. (2012), ‘Securitizing environmental risk and the Keystone XL Pipeline’, *The Economists’ Voice* **9**(1), 1–4.

Solow, R. M. (1971), ‘The economist’s approach to pollution and its control’, *Science* **173**(3996), 498–503.

Torsello, L. and Vercelli, A. (1998), Environmental bonds: A critical assessment, *in* ‘Sustainability: Dynamics and Uncertainty’, Springer, pp. 243–255.

Tufano, P. (1998), ‘The determinants of stock price exposure: Financial engineering and the gold mining industry’, *The Journal of Finance* **53**(3), 1015–1052.

Van’t Veld, K. and Shogren, J. F. (2012), ‘Environmental federalism and environmental liability’, *Journal of Environmental Economics and Management* **63**(1), 105–119.

White, B. (2015), 'Do control rights determine the optimal extension of liability to investors? The case of environmental policy for mines', *Journal of Regulatory Economics* **48**(1), 26–52.

White, B., Doole, G. J., Pannell, D. J. and Florec, V. (2012), 'Optimal environmental policy design for mine rehabilitation and pollution with a risk of non-compliance owing to firm insolvency', *Australian Journal of Agricultural and Resource Economics* **56**(2), 280–301.

Working, H. (1949), 'The theory of price of storage', *The American Economic Review* **39**(6), 1254–1262.

Zhou, J. (2014), Beyond liability: An analysis of financial responsibility and care decisions in hazardous waste management facilities. Working Paper, University of Maryland, Department of Agricultural and Resource Economics.

# Appendix A

# Appendix to Chapter 1

## A.1 Boundary Conditions

Boundary conditions at upper and lower bounds of $p$, $r$, $w$, and $t$ are described in this section.

- Evaluation of Equation (1.17) as the commodity price $\boldsymbol{p} \to \mathbf{0}$ implies that

$$0 = \frac{\partial V}{\partial t} + rV + \max_{q,a} \left\{ \pi - q\, \frac{\partial V}{\partial s} + (\phi q - a)\, \frac{\partial V}{\partial w} \right\} \tag{A.1}$$

  No special boundary condition is needed as there is no term involving $p$.

- As the $\boldsymbol{p} \to \boldsymbol{p_{max}}$, we assume $\frac{\partial^2 V}{\partial p^2} \to 0$, which from Equation (1.17) implies:

$$0 = \frac{\partial V}{\partial t} + \kappa(\hat{\mu} - \ln p)p\, \frac{\partial V}{\partial p} + rV + \max_{q,a} \left\{ \pi - q\, \frac{\partial V}{\partial s} + (\phi q - a)\, \frac{\partial V}{\partial w} \right\} \tag{A.2}$$

  The assumption that $V$ is linear in $p$ is common in the literature (In't Hout, 2017).

- As $\boldsymbol{s} \to \mathbf{0}$, the admissible set of $q$ collapses to zero as shown in Equation (1.5). No boundary condition is needed.

- As $\boldsymbol{s \to s_{max}}$, no special boundary conditions is required as Equation (1.17) has outgoing characteristics in the $s$ direction.
- For the boundary $\boldsymbol{w = 0}$, no boundary condition is required as Equation (1.17) has outgoing characteristics in the $w$ direction.
- At the boundary $\boldsymbol{w = \bar{w}}$, Equation (1.6) implies that Equation (1.17) has outgoing or zero characteristics in the $w$ direction. Hence no special boundary condition is needed.
- At $\boldsymbol{(t = T)}$, the obligation to clean up the site from Stages 2 and 3, under the liability rule and the bond in Cases I and II, implies that

$$\begin{aligned} &V(p, s, w, \delta_i, T) = 0 && i = 1, 4 \\ &V(p, s, w, \delta_i, T) = \mathbf{1}_{b=true} C^{tp}(W) - C^f(W) && i = 2, 3. \end{aligned} \tag{A.3}$$

  Note that if the bond is borrowed (Case III), the firm will have to repay the loan at $T$, and thus the second line of the above equation becomes

$$V(p, s, w, \delta_i, T) = -C^f(W) \quad i = 2, 3. \tag{A.4}$$

## A.2 Considering the socially optimal policies

In this paper, we have focused on the effect of an environmental bond on a firm's optimal decisions, but have not specifically addressed whether policies are socially optimal. The objective of the environmental bond is to ensure that firms do not shirk their clean-up obligations. The value of the bond is set to fully cover clean-up costs at any moment in time, so that the government will never be faced with the cost of cleaning up a mining site. In addition, it was assumed that there would be no damages from the waste stock prior to the terminal time, $T$. Clean-up must be undertaken by $T$, which is assumed to be chosen optimally by the government. The Case I bond, which pays interest at the risk-free rate, is used as a benchmark as it gives the same project value and optimal firm decisions as the strict liability policy. Because the Case I bond collateralizes the government and imposes no additional cost on the firm, it represents a socially optimal result. The other two bonds considered impose costs on the firm. Under both Case II and Case III, the profitability of the mine is reduced and hence the project is less likely to be undertaken. Whether these cases represent socially optimal policies depends on the relative size of the cost of the bond to firms versus the costs the government is avoiding by imposing the bonding requirement. This question deserves further investigation, but is beyond the scope of this paper.

The assumption that the waste stock causes no damages prior to the mandatory clean-up date of $T$ is easily relaxed. Damage can be described by a damage function $D(W)$, where $D'(W) > 0$. These damages may reflect contamination of water sources from waste storage, loss of enjoyment of the mine site for recreational or other uses, or potential harm to wildlife of degraded landscapes, to name a few examples.[1] In this case, an additional tool besides the bond is needed for a socially optimal result. In particular a tax per unit could be imposed on the waste stock. This tax would be expected to affect the firm's choices of abatement, production, as well as the timing of opening, mothballing, or abandoning the mine.

## A.3 Decisions to abandon the extraction activities

Consider either the Case I bond or the strict liability rule. At a given level of reserve, the firm's optimal decisions to abandon the mine from the production phase or to mothball the operations depend on the remaining life of the project as well as the quantity of the accumulated waste. Figure (A.1) shows such decisions for $t = 12$, $t = 13$, and $t = 14$ years, when the reserve is full. We have observed that at all decision dates (yearly) before $t = 13$, the firm always plans to mothball first at all levels of waste stock. However, at $t = 13$, when the landfill is almost full, the firm is indifferent between mothballing first and abandoning directly from the production phase. At lower levels of waste, the extraction activities terminate without mothballing, once critical prices are hit. At $t = 14$, critical mothballing prices are lower at all levels of waste, making direct termination of extraction

[1]Syncrude paid a $3 million penalty for the death of 1600 ducks which landed on one of its tailing ponds in Alberta, Canada, in 2008.

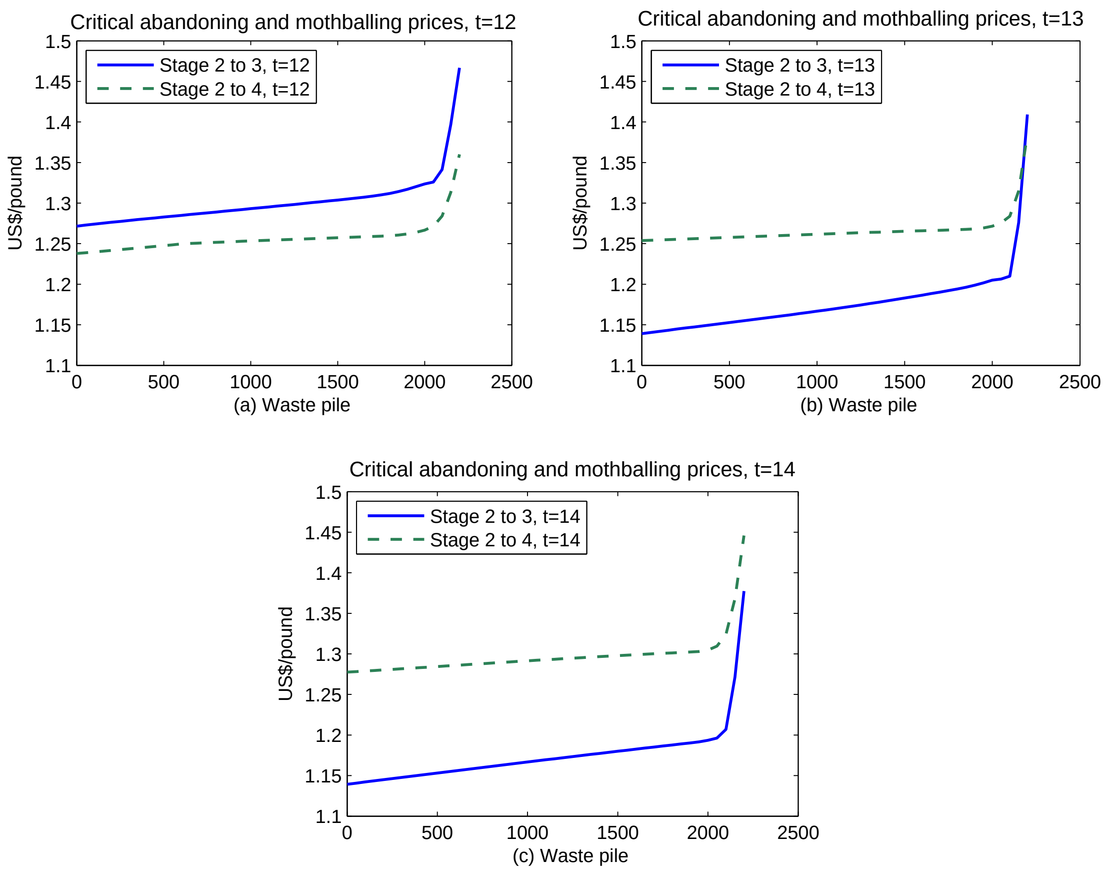


Figure A.1: *Decisions to mothball versus to abandon the extraction activities under the bonding policy (Case I) across the waste stock, for full reserve and for the last three years left in the project.*

activities optimal. As noted, at $T = 15$, the firm is enforced to close the mine from any stages of the project regardless of prices.

# Appendix B

# Appendix to Chapter 2

## B.1 Boundary Conditions

Boundary conditions at upper and lower bounds of $p$, $r$, $w$, and $t$ are described in this section.

- Evaluation of Equation (2.16) as the commodity price $\boldsymbol{p} \rightarrow \mathbf{0}$ implies that

$$0 = \frac{\partial V}{\partial t} + (r + \lambda(\cdot))V + \max_{q,a}\left\{\pi - q\,\frac{\partial V}{\partial s} + (\phi q - a)\,\frac{\partial V}{\partial w}\right\} \tag{B.1}$$

  No special boundary condition is needed as there is no term involving $p$. Note that $\lambda(p \rightarrow 0, w) \rightarrow \infty$.

- As the $\boldsymbol{p} \rightarrow \boldsymbol{p_{max}}$, we assume $\frac{\partial^2 V}{\partial p^2} \rightarrow 0$, which from Equation (2.16) implies:

$$0 = \frac{\partial V}{\partial t} + \kappa(\hat{\mu} - \ln p)p\,\frac{\partial V}{\partial p} + rV + \max_{q,a}\left\{\pi - q\,\frac{\partial V}{\partial s} + (\phi q - a)\,\frac{\partial V}{\partial w}\right\} \tag{B.2}$$

  The assumption that $V$ is linear in $p$ is common in the literature (In't Hout, 2017). Note that $\lambda(p \rightarrow p_{max}, w) \rightarrow 0$.

- As $\boldsymbol{s} \rightarrow \mathbf{0}$, $q$ collapses to zero. No boundary condition is needed.

- As $\boldsymbol{s \to s_{max}}$, no special boundary conditions is required as Equation (2.16) has outgoing characteristics in the $r$ direction.
- For the boundary $\boldsymbol{w = 0}$, no boundary condition is required as Equation (2.16) has outgoing characteristics in the $w$ direction.
- At the boundary $\boldsymbol{w = \bar{w}}$, Equation (2.16) has outgoing or zero characteristics in the $w$ direction. Hence no special boundary condition is needed.
- As $\boldsymbol{\lambda \to \infty}$, both $q$ and $a$ collapse to zero and the project terminates.
- At $\boldsymbol{(t = T)}$, the obligation to clean up the site from Stages 2 and 3, under the liability rule and the bond implies that

$$\begin{aligned} &V(p, s, w, \delta_i, T) = 0 && i = 1, 4 \\ &V(p, s, w, \delta_i, T) = \mathbf{1}_{b=true} C^{tp}(W) - C^f(W) && i = 2, 3. \end{aligned} \tag{B.3}$$

  As noted, the firm receives the restoration benefit under the bond or pays the clean-up cost under the liability if it reaches time $T$.

## B.2 Sensitivity on the parameters of the hazard functions

This section examines the sensitivity of results to the parameters of Equations (2.5) and (2.6). We have increased the values of $k_0$ and $k_2$ compared to the base-case parameter values. This new case is referred to as the higher-risk firm. In what follows, we analyze the higher-risk firm's optimal abatement decisions versus the base case and the solvency case, under both policies.

Figures (B.1) and (B.2) compare the higher-risk firm's optimal abatement rate with the base case and solvency case, at time zero. In Scenario I, as discussed in Section 2.6.2, higher values for $\lambda(\cdot)$ increase the effective discount rate and thus reduces the abatement rate, as future benefits and costs become less important. For a profitable project in Scenario II, at

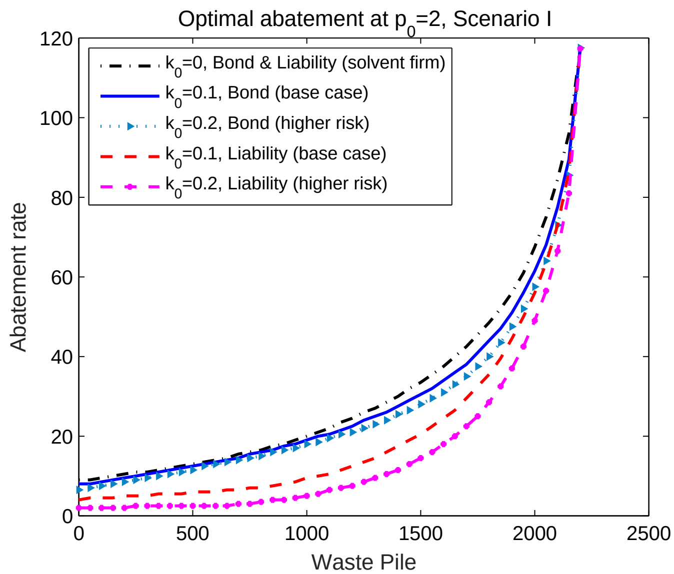


Figure B.1: *Optimal abatement sensitivity to the parameter* $k_0$ *of* $\lambda(p) = k_0/p$*, under both policies at time zero.* $s_0 = 1173$ *million pounds, w: million pounds.*

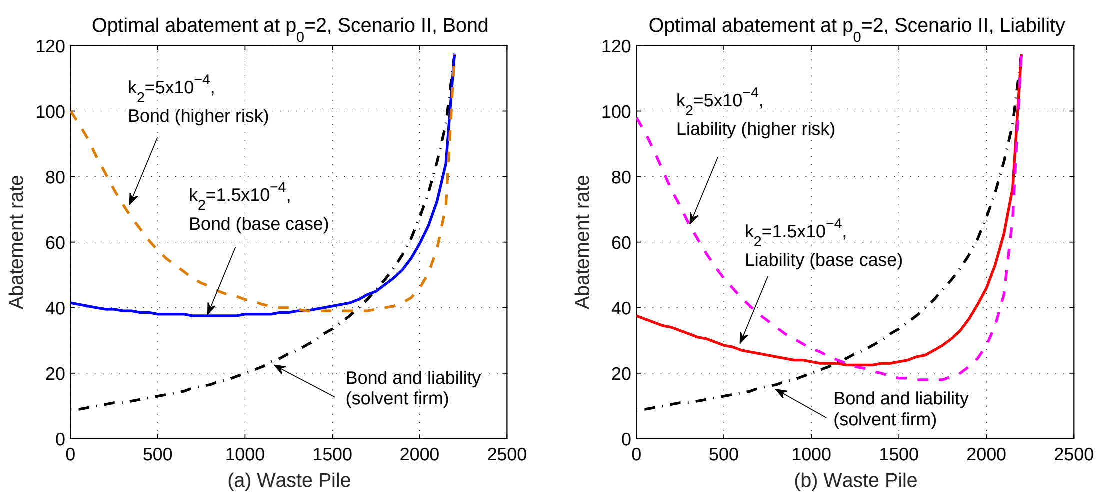


Figure B.2: *Optimal abatement sensitivity to the parameter* $k_2$ *in* $\lambda(p, w) = (k_1 + k_2 w)/p$ *given* $k_1 = 0.1$*, under (a) the bonding policy and (b) the strict liability rule, at time zero.* $s_0 = 1173$ *million pounds, w: million pounds.*

low levels of waste, the higher-risk firm leaves a lower waste for the next period compared to the base case parameters. However, an increase in the level of waste increases the firm's risk of bankruptcy more sharply than in the base case, demotivating the firm to keep abating more, resulting in a sharp decline in the firm's abatement rate, in particular under the liability rule. This behaviour is due to the relatively higher probability of bankruptcy, $\lambda(p, w)$, in Equation (2.15).

## B.3 Sensitivity on the parameter of the clean-up cost function

This section discusses the sensitivities of the firm's optimal abatement decisions under the bond in Scenario II on the scaling parameter of the clean-up cost function, $\beta$ in Table (2.2). In this analysis, the marginal restoration is 2 and 3 times costlier than its original value. To enhance the environmental quality, governments may become more stringent over time in terms of restoration plans, making the clean-up work more expensive. With more expensive restoration plan, the firm's expected bond payment is relatively higher. This higher bond motivates the firm to abate progressively during operations to reduce its annual bond payments as well as its risk of bankruptcy. This result is shown in Figure (B.3).

## B.4 Abatement rates at higher copper prices

Optimal abatement rates under two price levels are shown in Figure (B.4). An increase in copper prices rises the project value and thus the firm exercises a higher abatement rate in both scenarios.

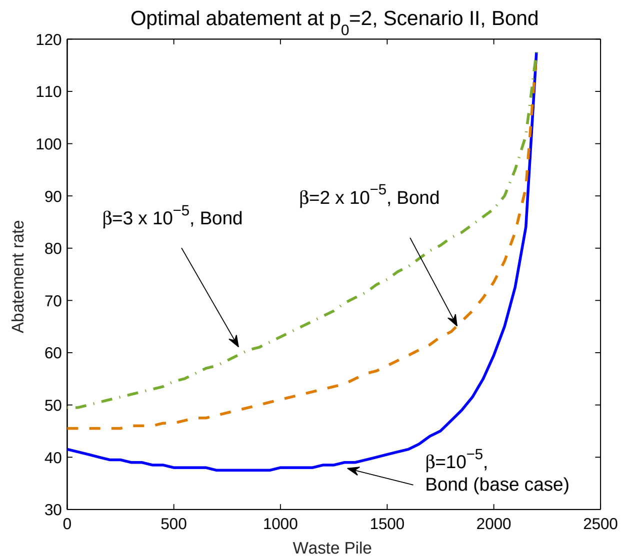


Figure B.3: *Optimal abatement sensitivity to the parameter* $\beta$ *of* $C(w) = \beta w^2$*, at time zero, under the bonding policy and Scenario II, at time zero.* $s_0 = 1173$ *million pounds, w: million pounds.*

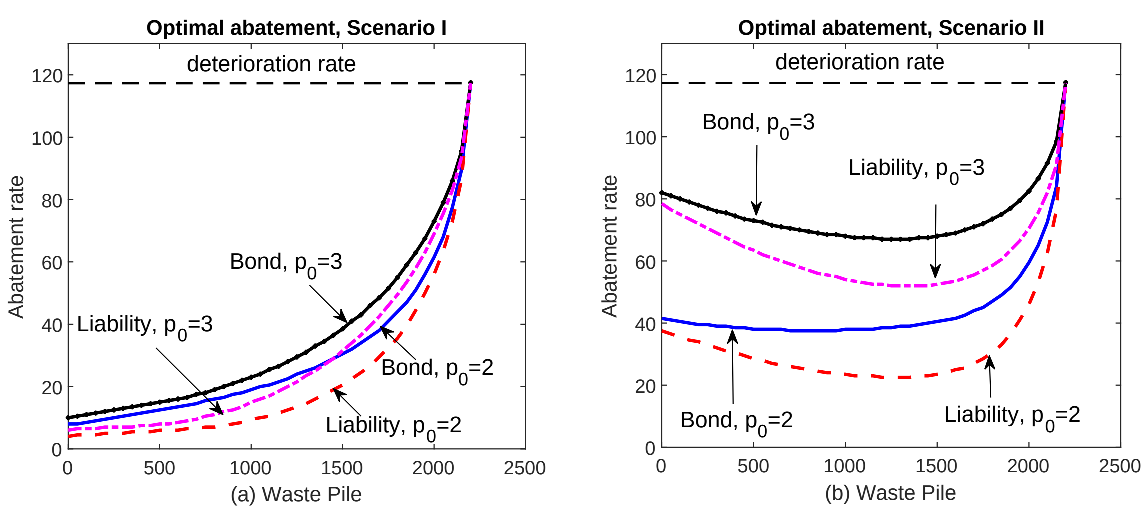


Figure B.4: *Optimal abatement rates for two price levels under the bonding policy and the strict liability rule, at time zero, in (a) Scenario I and (b) Scenario II.* $s_0 = 1173$ *million pounds, w: million pounds.*

# Appendix C

# Appendix to Chapter 3

## C.1 The difference in implied futures prices

Figures (C.1) and (C.2) are the gaps in forward curves shown in Figures (3.9) and (3.10), respectively.

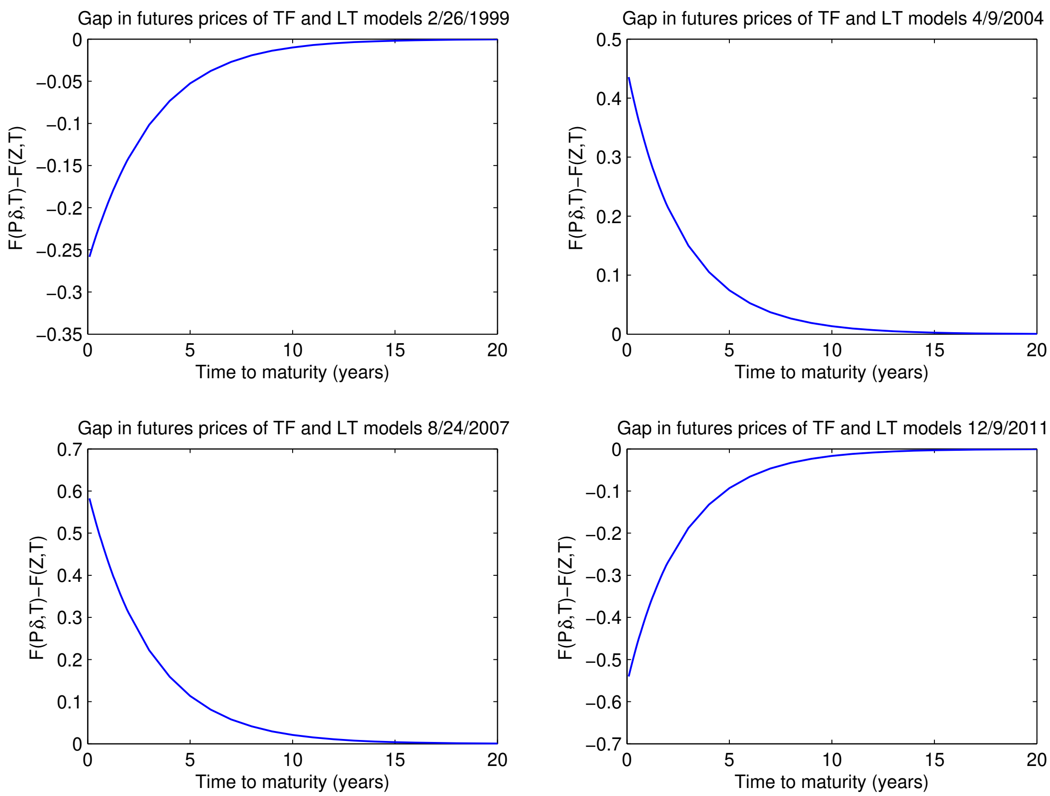


Figure C.1: *The difference between the forward curve implied by the two-factor model and the long-term model, at 4 selected dates from the first sub-sample.*

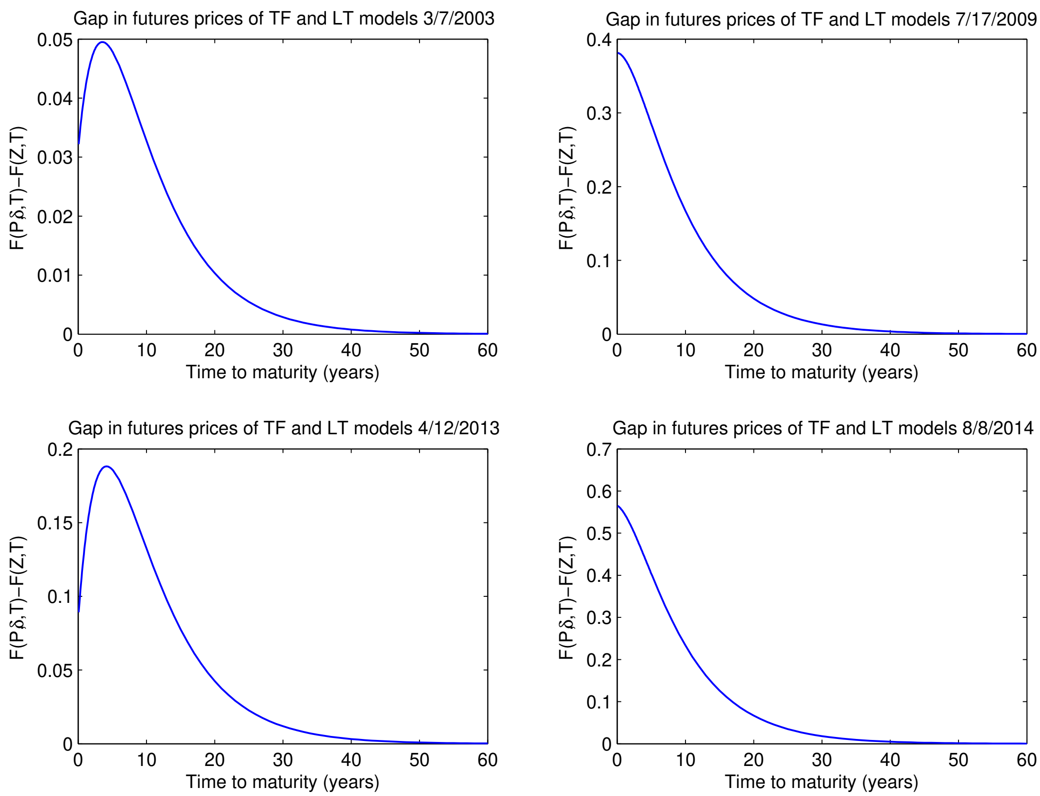


Figure C.2: *The difference between the forward curve implied by the two-factor model and the long-term model, at 4 selected dates from the second sub-sample.*