Optimization Models for the Perishable Inventory Routing Problem

by

Ajitesh Rajiv Singla

A thesis
presented to the University of Waterloo
in fulfillment of the
thesis requirement for the degree of
Master of Applied Sciences
in
Management Sciences

Waterloo, Ontario, Canada, 2019

I hereby declare that I am the sole author of this thesis. This is a true copy of the thesis, including any required final revisions, as accepted by my examiners.

I understand that my thesis may be made electronically available to the public.

Abstract

In this thesis, three models for the Perishable Inventory Routing Problem (PIRP) are explored. The first model of the inventory routing problem considers one perishable product and known (deterministic) demands in the context of consignment inventory. The objective of the problem maximizes the profit of the supplier who owns all the inventory until it is sold. The supplier gives a fixed percentage discount on the selling price of the product as it deteriorates. The shelf life and the number of vehicles used in this problem is also fixed. Computational results, comparing the PIRP with the standard Inventory Routing Problem (IRP), are presented. Based on the calculations using the branch-and-cut algorithm, we conclude that adding perishability to the IRP leads to better inventory management as a result of which fresher products are delivered to the customers. The PIRP model is also solved using the Benders Decomposition algorithm. The results indicate that in the above stated problem, the branch-and-cut algorithm yields better results as compared to the Benders Decomposition algorithm.

The second model extends the PIRP to consider uncertain (stochastic) demands (SPIRP). This is done to make the model comparable to a real-life problem. Several demand scenarios are created within a fixed range to capture the uncertainty with respect to the perishable product. Since estimating the actual probability is difficult, a large number of equally probable demand scenarios are assumed. The model was run on the branch-and-cut and the Benders Decomposition algorithm. The results generated were then compared. It was inferred that even when demand scenarios enable easy decomposition of the problem, the Benders algorithm performs worse than the branch-and-cut algorithm.

In the third model, the robust formulation of the SPIRP is proposed to resolve the above-mentioned limitations. Robustness was added to the SPIRP to enable the use of a

small number of scenarios to obtain solutions that are competitive with modeling a large number of scenarios in the case of SPIRP. An innovative way of formulating the robust counterpart of the SPIRP was developed while keeping the probability of each demand scenario uncertain. An algorithm was devised to compare the effectiveness of the robust model to the deterministic and stochastic models.

Computational results compare the average profit values generated by the three models. It is concluded that while the deterministic model captures no uncertainty, the stochastic model with many scenarios accounts for the most demand uncertainty; the robust model, through the use of far fewer scenarios, can account for a significant uncertainty in demand. Another interpretation of the results is that an increased number of robust scenarios has a significant effect on the average profit values of that model.

Acknowledgements

I wish to express my sincere gratitude and appreciation to my supervisors Dr. James Bookbinder and Dr. Joe Naoum-Sawaya, for their support during the course of my thesis. I would like to express my sincere gratefulness to my supervisor Dr. Joe Naoum-Sawaya, for providing me this opportunity to work on this interesting research, and also for his constant support and advice during my research. Moreover, I would like to express my sincere thankfulness to my supervisor Dr. James Bookbinder, for his research guidance and support, and for his continual help in writing of this thesis.

I also would like to express my sincere thanks to Dr. Qi-Ming He and Dr. Hossein Abouee Mehrizi, for their valuable feedback on my thesis that helped me improve. In addition, I am thankful to Ms. Wendy Fleming, and Ms. Lisa Hendel, for their administrative support in my studies.

Finally, I want to express my thanks to all my friends who have always provided me with motivation, moral support, and academic advice throughout my study at the University of Waterloo. They helped make my time here full of amazing memories.

Dedication

I dedicate this thesis to my parents Rajiv Singla and Kshma Singla, and my sister Anandita Singla.

Without you, I am nothing

Table of Contents

List of Tables

List of Figures

Chapter 1

Introduction

A supply chain is the basis of each and every industry and process that happens in the world. Everything, from the food we eat to the clothes we buy, is the result of a well-designed supply chain. Supply chain management is the management of all entities involved in the process of converting raw materials to the finished product. Supply chain is a dynamic process of many interconnected entities and a failure of even one of those entities at any point of the chain will lead to losses. This may be in terms of money or customer service. As stated by Chopra and Meindl (2013), the nodes or entities involved in a supply chain are usually raw-material suppliers, manufacturers, wholesalers/ distributors, retailers, and customers. It involves a constant flow of information, products, and funds among different entities. The flow of a supply chain occurs in both directions. An important thing to note is that each supply chain is different, having a greater number or few interconnected entities. They have different functions at varied times during the course of the supply chain depending on the industry. Even within an industry, the functions performed by the entities could vary from company to company. Furthermore, supply chain nodes are connected by means of transportation that facilitates the pickup and delivery of the product to and from the entities involved.

The objective of any supply chain is to get the product to the consumer at the lowest possible cost for a given level of service. Thus, inter-dependency among all the processes and the people involved is of utmost importance for the smooth flow of the product across the chain. A supply chain can only be profitable if there is a good relation among all the different entities involved in the network, and the overall cost of the supply chain is minimized (or profit is maximized). This dynamism and importance of supply chain management in the world makes it one of the most interesting and ever-growing subjects. That is the main motivation for me to write this thesis in the field of supply chain management.

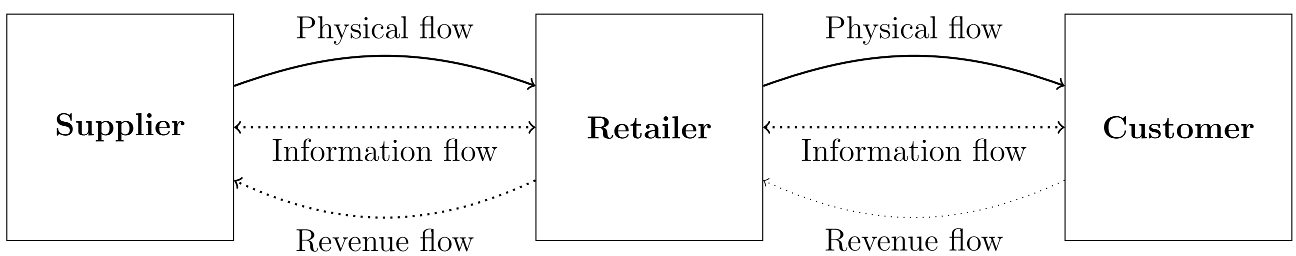
The vast field of supply chain management encompasses several different processes that may be studied while researching the subject. Some of the processes involved in the study of supply chain are production management, by which goods are manufactured in the best way at the lowest possible cost, and inventory management, which requires optimization of stocks at the supplier or at the retailer's end. Transportation, another important process, facilitates the movement of goods among the different nodes of the supply chain. Also significant is warehouse management, which optimizes the storing of goods at a facility such as a distribution centre. These are just some of the processes that exist in a supply chain.

This thesis focuses on two of the processes defined above; inventory management and transportation management. Inventory management is a crucial component when it comes to the overall cost of the supply chain. In the case of excess inventory, a perishable product might degrade and become unsaleable. Whereas a non-perishable product would occupy space in the warehouse and may also be damaged. On the other hand, a lack of inventory would lead to losses and might also negatively impact the customer service level.

Transportation management is necessary for a product to be delivered in an undamaged and timely manner for the customer to get the most value from that product. According to the latest report by Kearney (2019) on the United States Business Logistics Costs, transportation costs have been increasing every year by a large margin. The costs of transportation increased from 622.7 billion dollars in 2009 to 1037.4 billion dollars in 2019. The report also states that transportation currently contributes 5.1% to the Gross Domestic Product (GDP) of the United States, up from 4.3% in 2009. The Total Business Inventory accounts for about 13% and the Inventory Carrying Costs constitute 2.4% of the GDP. These numbers stress the importance of optimizing transportation and inventory for the success of a supply chain.

In this thesis, a simplified version of an actual supply chain with a single point of origin (supplier or warehouse) and a set of stores as destination points is considered. The illustration for the supply chain is shown in Figure 1.1. The decisions to be made in this system are related to inventory (how much and how often to deliver to these stores) and distribution (how to dispatch vehicles to deliver to these stores). A model that combines both decisions and aims at maximizing the profit is the objective of this thesis. Creating such a model is not a trivial task. It requires many underlying assumptions to make the decision model scalable in terms of implementation.

This thesis models the supply chain of an industry that focuses on perishable products. These kind of products add to the complexity of the problem. In a real supply chain, several perishable products that differ in terms of age and shelf life, have stochastic demands. Despite imposing simplifying assumptions, mathematical models are valuable because they provide insights for practitioners and managers. The more realistic the assumptions are, the more reliable the insights will be.



- There is a single supplier
- There are multiple retailers
- Each retailer will have its own customers
- Goods flow from the supplier to the retailers and then are sold to the customers
- The customers pay the retailer for the goods and the retailer pays the supplier from which it got those goods

Figure 1.1: Supply Chain used in this thesis

1.1 Research Motivation

The motivation behind choosing the topic of perishable products comes from the importance of this research in several industries. The present research on perishable products can be used in food retail industries which have very small profit margins; managers involved are looking to optimize their supply chain in a way that increases their profits and customer service level. This research can also be implemented for products like blood, whose supply chain is of utmost significance since it is very demand focused. Optimization here can result in reduced costs, freeing up hospital delivery costs and wastage, and ensuring a safe and stable blood supply.

Integration of the two processes, inventory and transportation, could lead to substantial savings as opposed to the traditional Retailer Managed Inventory (RMI) policy. Under RMI, each store applies its own inventory control system and places its orders independently of other stores with the objective of minimizing long-term inventory costs. In such a system, either transportation costs are completely neglected or a fixed delivery cost is incurred by each store when it is visited.

The RMI policy is simple to implement and abundant in practice but not efficient enough in terms of overall long-term inventory and routing costs. Incorporation of Vendor Managed Inventory (VMI) into the research has helped managers decrease the magnitude of the bullwhip effect and has led to major advancements in the field of supply chain management. Many other relevant works in the literature take into account the synchronization of inventory and transportation. However, those works generally neglect the perishability of products or the randomness of demand from end customers. In this sense, there is a research gap between the real world problems and the problems addressed in literature. This thesis aims to address the existing gap.

1.2 Research Objectives

The following are the objectives of this research.

1. To develop a model that optimizes the inventory and transportation of a supply chain for a perishable product to assist managers in making their decisions

2. To implement an algorithm that solves the model developed and to address the possible shortcomings of the method applied

3. To compare the results of different variations of the problem and provide useful business insights

The main goal of this research is to provide useful insights to supply chain managers on inventory control and distribution of perishable products. The aim is to develop an efficient way of solving this problem. The problem is built through the help of three different models with varying assumptions. For each problem a mathematical model is formulated; the final model is developed to resemble a real supply chain in the best way possible. The model's significance is proved by conducting a comparative analysis with the previous models.

The mathematical models developed in this thesis aim to assist managers in addressing real situations. This thesis addresses a part of a larger problem at a smaller scale. Furthermore, every industry has its own specific set of decisions to make and this thesis cannot make broad generalizations for the industry as a whole. However, it can offer a micro perspective. Similar to other scientific works, this thesis provides some generic models which can be viewed as benchmarks for the inventory and distribution part of any supply chain that deals with perishable products.

1.3 Outline of the thesis

The thesis will start with the standard Inventory Routing Problem (IRP) and Chapter 2 will have the Literature Review. Chapter 3 begins with the deterministic IRP and will describe the problem and the model. This will establish the foundation of the thesis. In Chapter 4 perishability will be instituted in the IRP. This chapter will have the relevant literature of the deterministic perishable IRP, the model, and results. Chapter 5 will introduce randomness to the perishable IRP; the pertinent literature, the model of the two stage stochastic perishable IRP, and results will follow. In Chapter 6, robust optimization will be added to the stochastic perishable IRP. This will be followed by the description and formulation of the robust model. Chapter 7 encompasses the business insights of the problem under consideration, and will present results of the comparison between the models of the deterministic, the stochastic and the robust stochastic perishable IRP. Lastly, Chapter 8 will conclude the thesis.

Chapter 2

Literature Review

The inventory routing problem is a combination of two major components of supply chain management, namely, inventory and transportation. The IRP has been derived from the Vehicle Routing Problem (VRP). This is used to design delivery routes from a depot to a set of a geographically scattered customers with the objective of minimizing the overall routing costs. Each route starts and ends at the depot. The VRP was first introduced 60 years back in 1959 as "The Truck Dispatching Problem". The authors define it as a generalization of the Travelling Salesman Problem (TSP), with constraints on vehicle capacity and on demand at the retailer's end. The problem by Dantzig and Ramser (1959) requires the delivery of a product to all destinations, such that demands are satisfied with minimum routing cost. They use a linear programming approach to solve the problem. Since then, the subject has been extensively studied in combination with several aspects such as inventory management, production scheduling, resource utilization and transshipments.

As described by Desaulniers et al. (2016),the IRP is based on the concept of Vendor Managed Inventory (VMI). VMI is a contrast to the Retailer Managed Inventory (RMI), where the customers are responsible for monitoring their own inventories. Under RMI customers place orders to the supplier, who is responsible for satisfying those orders. The VRP is based on the RMI. In contrast, in a VMI system, the supplier takes over the responsibility of managing the inventory of customers by deciding on replenishment quantities and delivery periods. The goal is a win-win situation for both entities. Customers employ fewer resources for controlling their inventory, and the supplier having to manage several customers at a time is in a better position to manage the routes and replenishment across periods for each of its customers. This minimizes the total costs of production/procurement, inventory, and transportation.

The inventory routing problem was first introduced by Bell et al. (1983). They find a periodic distribution policy where they plan for whom to serve, how much to deliver and the route traversed by the vehicles. This plan is repeated regularly, minimizing the total transportation cost with constraints on the vehicle and the customer capacity being satisfied, with no stockouts at any customer. Although in their model, inventory costs play no role, they deduce that taking a joint decision about inventory and transportation provides a 7% - 10% more efficient decision than when only taking the location of customers into consideration.

Coelho et al. (2013) present an extensive review of the IRP. The review classifies the problem according to two schemes: the first refers to the possible structural variants present in IRPs, whereas the second is related to the availability of information on demand. According to the first scheme, papers can be classified according to seven criteria, namely, time horizon, structure, routing, inventory policy, inventory decisions, fleet composition, and fleet size. The time horizon is categorized as finite or infinite. The number of suppliers and customers may vary, and therefore the structure can be one-to-one when there is only a single supplier serving a sole customer, one-to-many in the case of one supplier and multiple customers, or many-to-many with several suppliers and several customers. Routing can be direct when there is only one customer per route, multiple when there are several customers on the same route, or continuous when there is no central depot.

Inventory policies define preestablished rules to replenish customers. The two most common are the maximum-level (ML) policy and the order-up-to level (OU) policy. Under an ML inventory policy, an upper bound is set on inventory at the retailer and the only constraint is that the quantity delivered should be less than that level. In most cases it is the capacity at the retailer's end. Under an OU policy, both the maximum and minimum levels are specified at the retailer. When the product is delivered, the quantity is such that the inventory is always filled to the maximum defined level. Inventory decisions indicate how the management of the stocks has been modeled. If inventory is allowed to become negative, then back-ordering occurs and the corresponding demand will be served at a later stage; if there are no back orders, then the unsatisfied demand is considered as lost sales. In both cases there is generally a penalty for the stockout. In the deterministic context, where demand of each customer is known beforehand, the inventory can be taken as nonnegative. The fleet composition can be either homogeneous or heterogeneous. The last criterion is the size of the fleet. The number of vehicles available may be fixed at one, fixed at many, or unconstrained.

The second classification refers to the time when information on demand becomes known. If that information is fully available to the decision maker at the beginning of the planning horizon, the problem is then deterministic. If the probability distribution of demand is known, the problem is stochastic, which yields the stochastic inventory-routing problem (SIRP). Dynamic IRPs arise when demand is not fully known in advance, but is gradually revealed over time, as opposed to the full information being available at time zero. In the dynamic IRP case, one can still exploit the statistical distribution in the solution process, yielding a dynamic and stochastic inventory-routing problem (DSIRP).

The basic IRP is NP-hard because it subsumes the classical VRP. Given the complexity of the problem, initially only heuristics were proposed for its solution. With the advancement in technology, the IRP is now being solved using exact algorithms, for problem instances of reasonable size.

The ML policy, as stated, clearly encompasses the OU policy and is more flexible, but also more difficult to solve, given the extra set of decision variables. Archetti et al. (2007) propose the first branch-and-cut algorithm for the IRP with one vehicle. They develop models for both the OU and ML inventory policies. They also modify their model such that there are no constraints on the shipping capacity. Their model is based on the work done by Bertazzi et al. (2002) which uses the OU policy. Archetti et al. (2007) develop an IRP in which a product is to be shipped from a supplier to a set of customers within a time horizon. The supplier has to monitor the level of inventory at the customers and replenish their stock such that the inventory level reaches its maximum allowed. The supplier guarantees that there will be no stockouts at customers. The customers themselves define their maximum inventory level. Therefore, this problem has a vendor-managed, order-up-to-level inventory policy. The demands are known, and the vehicle has a certain capacity. The goal is to determine the quantities shipped in each time period, and to design the routes. Archetti et al. (2007) present a mixed integer linear programming model and solve it by adding some valid inequalities to the LP relaxed model. An exact branch-and-cut algorithm is used to solve all three models.

Solyalı and Süral (2011) suggest a stronger formulation for the model with OU policy given by Archetti et al. (2007), by using the shortest path network representation for the customer inventory replenishment part. Coelho and Laporte (2013) extend the model of Archetti et al. (2007) by considering multiple vehicles. Also, Adulyasak et al. (2014) extend the stronger formulation of Solyalı and Süral (2011) to the case of multiple vehicles. They also solve a model for the Production Routing Problem (PRP) along with IRP. Both the models are solved using a branch-and-cut algorithm.

Another exact approach was the branch-and-price-and-cut algorithm used by Desaulniers et al. (2016). In the most recent work, those authors apply the latest algorithm to solve an IRP with an ML inventory replenishment policy. Route delivery patterns are generated, where such a pattern specifies the quantity delivered to each customer along the corresponding route. Only extreme route-delivery patterns are considered, and their convex combinations are used to generate any other route delivery patterns. In the branch-and-bound algorithm, the lower bounds at each node are computed using a column generation algorithm, where one subproblem for each period is solved as an Elementary Shortest Path Problem with Resource Constraints (ESPPRC). Cutting planes are added dynamically to tighten the linear relaxations. Each of the papers above provides an algorithm to solve the problem under the same assumptions. Those algorithms are tested on the same set of benchmark instances, thus allowing for a clear comparison of the performance of each algorithm.

Chapter 3

Deterministic Inventory Routing Problem

3.1 Introduction

This section discusses the deterministic IRP. This will include the notations and the formulation of the deterministic problem.

3.2 Description

In this variant of the IRP, the time horizon is finite and the structure is one-to-many. Since several stores can be visited on a route in any time period, the routing is multiple. The inventory policy at the retailer's end is ML. Since the problem is deterministic, the demand at each retailer is known over the whole horizon. Thus, the inventory will always be nonnegative. The time horizon is discrete and finite. Finally, the fleet consists of multiple identical capacitated vehicles. The objective is to minimize the inventory cost plus the routing cost. Inventory is held at the depot and also at each retailer, and there is a holding cost associated with that inventory. There is some initial inventory at the retailer as well as at the supplier's end. The cost of routing is dependent on the location of the retailers.

Since it is a VMI system, at the beginning of the time horizon, the supplier has all the information on the demands and inventory at each retailer. In a deterministic case, the routes for each period are decided at the beginning of the time horizon; vehicles will be dispatched to those retailers which require inventory so as to fulfill the demands of the next periods. A route starts at the depot, visits a subset of retailers, and ends at the depot. Each retailer can be visited only once in each time period (no split deliveries). Also, each vehicle can perform only one route per time period although there are no time windows for delivery. The vehicle and the inventory holding capacity at each retailer should never be exceeded in any time period over the course of the problem.

3.3 Mathematical Model

In the literature, the deterministic IRP is described by the MIP formulation containing arc-based decision variables. This makes it is easier to determine the feasibility of the route. The only case of a route-based formulation is in the work of Desaulniers et al. (2016). MIP formulations can differ in the way the models depict the subtour elimination constraints. The constraints can be expressed in two ways, the standard subtour elimination constraints and the Miller-Tucker-Zemlin (MTZ) elimination constraints. The most widely used method is to add standard subtour elimination constraints where the whole family of those constraints is added all at once to the formulation. However, while solving the model, these constraints are removed. They are then introduced when required using the separation algorithm by Padberg and Rinaldi (1991). A more compact way is to use MTZ elimination constraints as given by Desrochers and Laporte (1991). The number of these constraints is proportional to the number of nodes in the model and hence no constraints need to be added later while solving the model.

The mathematical model formulated in this thesis using MTZ constraints is based on Coelho et al. (2012). However, a few changes are made to their model. These include the use of homogeneous vehicles instead of heterogeneous vehicles. The holding cost is included in time period 0 as well.

3.3.1 Notation

The model, defined on an undirected graph $G = (V, A)$ is based on the following notations.

Sets

- Vertex 0 represents the supplier.
- $V' = \{1, ..., m\}$ represents customers.
- $V = V' \cup 0$ represents the vertex set of the graph.
- $A = \{(i, j) : i, j \in V, i \neq j\}$ is the arc set.
- Set of time periods is $T = \{1, ..., p\}$
- $M = T \cup 0$ considers time period 0
- $K = \{1, ..., v\}$ denotes the set of homogeneous vehicles.

Parameters

- h_i denotes the holding cost at the customers as well as at the supplier $(i \in V)$
- The inventory holding capacity at each customer is $C_i\,(i \in V')$
- The length of the planning horizon is p
- At each time period t, the quantity of product made available at the supplier is r^t
- q_i^{kt} denotes the amount of inventory delivered to customer i by vehicle k in time period t
- d_i^t denotes the demand of each customer i at time period t
- Q denotes the capacity of each homogeneous vehicle $k \in \mathrm{K}$
- c_{ij} is the routing cost associated with arc $(i, j) \in A$
- $U_i^{kt}(i \in V')$ is used to avoid subtours by vehicle k in time period t

Decision Variables

- $I_i^t(i \in V)$ defines the inventory levels at the end of period t, at the supplier as well as at each customer i.
- $x_{ij}^{kt} = \begin{cases} 1, & \text{if arc } (i, j) \text{ is traversed by vehicle } k \text{ in time period } t \\ 0, & \text{otherwise} \end{cases}.$
- $y_i^{kt} = \begin{cases} 1, & \text{if vertex } i \text{ is used by vehicle } k \text{ in time period } t \\ 0, & \text{otherwise} \end{cases}.$

3.3.2 Formulation

$$\text{minimize } \left\{\sum_{i\in V}\sum_{t\in M} h_i I_i^t + \sum_{i\in V}\sum_{j\in V}\sum_{k\in K}\sum_{t\in T} c_{ij} x_{ij}^{kt}\right\} \tag{3.1}$$

subject to

$$I_0^t = I_0^{t-1} + r^t - \sum_{i\in V'}\sum_{k\in K} q_i^{kt} \quad t\in T \tag{3.2}$$

$$I_i^t = I_i^{t-1} + \sum_{k\in K} q_i^{kt} - d_i^t \quad i\in V',\ t\in T \tag{3.3}$$

$$I_i^t \geq 0 \quad i\in V,\ t\in T \tag{3.4}$$

$$I_i^t \leq C_i \quad i\in V,\ t\in T \tag{3.5}$$

$$\sum_{k\in K} q_i^{kt} \leq C_i - I_i^{t-1} \quad i\in V',\ t\in T \tag{3.6}$$

$$\sum_{i\in V'} q_i^{kt} \leq Q \quad k\in K,\ t\in T \tag{3.7}$$

$$\sum_{k\in K} q_i^{kt} \leq C_i \sum_{i\in V'}\sum_{k\in K} x_{ij}^{kt} \quad i\in V',\ t\in T \tag{3.8}$$

$$q_i^{kt} \leq C_i y_i^{kt} \quad i\in V',\ k\in K,\ t\in T \tag{3.9}$$

$$\sum_{j\in V} x_{ij}^{kt} = \sum_{j\in V} x_{ji}^{kt} = y_i^{kt} \quad i\in V,\ k\in K,\ t\in T \tag{3.10}$$

$$U_j^{kt} \geq U_i^{kt} + q_j^{kt} + Q x_{ij}^{kt} - Q \quad i,j\in V',\ k\in K,\ t\in T \tag{3.11}$$

$$\sum_{j\in V'} x_{0j}^{kt} \leq 1 \quad k\in K,\ t\in T \tag{3.12}$$

$$\sum_{k\in K} y_i^{kt} \leq 1 \quad i\in V',\ t\in T \tag{3.13}$$

$$q_i^{kt} \geq 0 \quad i\in V',\ k\in K,\ t\in T \tag{3.14}$$

$$x_{ij}^{kt} \in \{0,1\} \quad i,j\in V, i\neq j,\ k\in K,\ t\in T \tag{3.15}$$

$$y_i^{kt} \in \{0,1\} \quad i\in V,\ k\in K,\ t\in T \tag{3.16}$$

$$U_i^{kt} \geq 0 \quad i \in V', \;\; k \in K, \;\; t \in T \tag{3.17}$$

Objective (3.1) minimizes the total cost of inventory and transportation. Constraints (3.2) and (3.3) define the inventory at the end of each period at the supplier and customers respectively. Constraint (3.4) prevents stockouts at the supplier as well as at the customers. Constraint (3.5) requires that the customer's inventory should always be less than or equal to the holding capacity. Since the formulation is based on the ML inventory policy, constraint (3.6) forces the quantity delivered to a customer to fill the inventory to the maximum level. Constraint (3.7) guarantees that the vehicle capacity is maintained. Constraints (3.8) and (3.9) are used to make sure that a customer is serviced by a vehicle only if the vehicle visits that customer. Constraint (3.10) is the degree constraint, stating that if a customer or a supplier is not serviced by a vehicle, then there should be no path to or from that location. Constraint (3.11) is MTZ subtour elimination constraint. Constraint (3.12) makes sure that a vehicle covers at most one route per time period. Constraint (3.13) prevents split deliveries, making sure that each customer is visited at most once per time period. Finally constraints (3.14)-(3.17) impose integrality and non-negativity conditions on the variables.

3.4 Conclusion

This chapter has discussed the standard IRP. However this model cannot be applied to industries that deal with perishable products like food retail and blood industries. To make the IRP model applicable to such kind of products, a component of perishability has to be added to the existing model. This is described in Chapter 4.

Chapter 4

Perishability

4.1 Introduction

This section will introduce the component of perishability to the deterministic IRP. The problem will henceforth be called the deterministic Perishable IRP (PIRP). The present chapter includes a description of the problem followed by the relevant literature for the topic. Then the mathematical model for this problem is described.

4.2 Importance

The management of perishable products forms a major part of the study of Supply Chain Management. The research done on perishability can be applied to several industries. These include the food retail industry which deals with many perishable products with differing lifetimes, and the pharmaceutical industry, where drugs have a fixed expiry date and have no value after expiry. Another industry where this model can be applied is the hospital industry that deals with blood and its components. The management of blood inventory at blood banks and hospitals as well as the transportation of blood during emergencies make it a very crucial perishable product that needs to be protected from wastage.

Looking at the food retail industry, the majority of sales revenue comes from perishable products. The necessity to maintain a high level of consumer confidence in product safety causes the management to be even more challenging. It is difficult to manage the supply chain due to economic trade-offs between the product availability and food supply safety. Many products are discarded at the end of the supply chain by retailers or are not purchased by customers which in turn leads to spoilage. According to O'Byrne (2016), one third of all food produced is lost to spoilage before it can be consumed. This fragility requires the supply chain to move fast. The fresh supply chain is expected to grow by 13.9% by 2020. Thus, it is very important to reduce wastage, increase profits, and improve the customer service level.

The blood supply chain is unlike any other supply chain because blood cannot be manufactured. It is vitally important to health care, and volatile, since blood and its components are perishable. Every unit of blood wastage not only leads to economic losses, but also the effort of the blood donor is dissipated. According to the Canadian Blood Services review, the wastage of blood was down to around 6% in 2017-2018, but there is still scope for improvement. There needs to be better optimization of the inventory and distribution of blood.

4.3 Relevant Literature

In the literature on the IRP, whether for single or multiple products, the shelf life is always assumed to be infinite. This is a major drawback when the product is perishable. Perishable products have a fixed lifetime, and the product will have no value after its shelf life has expired. The major difficulty with the management of perishable products is that the problem size is usually large. This is because any typical retail chain has over 10,000 distinct products with many different shelf lives. Also, a big retail chain would have a large number of geographically dispersed stores with various requirements. Hence, the application of an IRP, where the VMI system is incorporated, is essential for this situation. Overall, these factors make the PIRP complex.

The problem statement for the perishable IRP is the same as for the standard IRP, except that the products have a limited shelf life after which they have no value. In the deterministic case, shortages are not allowed. Moreover, thanks to complete knowledge about the demands, nothing would deteriorate. This implies that the objective of the PIRP remains the same as a standard IRP. Overall profit of the supply chain is maximized, taking account of the inventory holding costs and routing costs.

In the literature, the perishability of products is handled in two ways. The first distinguishes the PIRP and the IRP by defining the maximum quantity delivered to stores. While in the IRP, maximum delivery quantity depends only on the storage capacity and the on-hand inventory at a customer's site, maximum delivery quantity in the PIRP is restricted not only by those two parameters but also by the maximum shelf life of the product. The second way in which perishability can be handled is by maximizing the profit of the supplier. In this case, the objective function has another component of profit which depends on the price at which the product is sold. Constraints are treated in the same manner as in the IRP, with an additional restriction on the product shelf life. As the product deteriorates its price also reduces. This is because an older product cannot be sold at the same price as a new one. The latter approach to handling perishability is used in this thesis.

In the PIRP, delivery frequency plays a big role. Less frequent deliveries reduce the routing costs, but leave behind products with shorter remaining shelf lives until the following delivery. Those items are subject to not only holding cost and deterioration, but will have undesirable freshness from the customers' perspective. Moreover, demand for those products may be adversely affected by their age, perhaps implying a selling price that decreases as the product deteriorates. If deliveries are executed more frequently, freshness of products and consequently customer satisfaction increases. However, higher routing and holding costs are then imposed on the system. Therefore, finding the right trade-off between costs and freshness is crucial.

Hemmelmayr et al. (2009) were the first ones to introduce the concept of VMI to perishable products with known demands. The authors investigate a problem on the delivery of blood products from a blood bank to hospitals using a small fleet of homogeneous uncapacitated vehicles. The objective is to minimize total routing costs during a finite planning horizon; no inventory costs are taken into account. As stockouts may result in loss of life, hospitals actually prefer to have high inventory, even if this results in greater costs. No out-date costs are considered as a deterministic setting is used. Inventory is handled via constraints on minimum and maximum levels with respect to the spoilage period and the product usage.

Hemmelmayr et al. (2009) develop and evaluate two delivery strategies. The first retains the concept of regions and the use of fixed routes, but utilizes integer programming techniques to optimally decide on delivery days. The integer programming-based approach employs a scheme in which the set of hospitals is divided into four regions; hospitals in each of those are served by a single vehicle using a fixed route, which simply skips those hospitals that do not require delivery. Only a two-week period is considered and these delivery decisions are made daily. The concept of “shortcutting” is used in the integer programming model to provide flexibility. It implies that a TSP solution covering all vertices is at hand. This solution is exploited to construct another TSP solution where a subset of vertices is served. In the solution, they simply cut arc (i, j) if either node i or node j is not served in the corresponding period. The remaining partial paths are connected to make a complete TSP solution. The second approach is based on viewing the problem as a Periodic Vehicle Routing Problem (PVRP), with constraints on route length but with none on capacity. They allow each hospital to have a set of visit frequencies and a set of associated (periodic) visit combinations. Visit combinations that lead to an infeasible delivery pattern are deleted. They develop a variable neighbourhood search (VNS) algorithm to solve this variant of the PVRP. The two techniques used by the authors provide considerable transportation savings over the traditional RMI.

Le et al. (2013) propose a mathematical model for the PIRP employing the concept of a feasible route. A feasible route is one that starts from the depot, visits a subset of customers at most once, and then returns to the depot. This differs from the popular notion of a feasible route in the VRP, which is defined there as a route for which the sum of demands of customers on the route is less than vehicle capacity. The approach of Le et al. (2013) is a path-flow formulation; transportation costs as well as constraints depend on the route and not the vehicle. The upper bound on inventory at the retailer is the summation of demands over all time periods following the current period. This does not allow discarding of products. They utilize a column-generation based heuristic algorithm to solve the problem. The pricing problem there can be utilized to solve small to medium instances and forms the basis of a branch-price-and-cut algorithm.

Diabat et al. (2016) address the same problem as Le et al. (2013), minimizing only the transportation costs. They employ a Tabu Search (TS) based heuristic for the Periodic Distribution Inventory Problem. Diabat et al. (2016) construct the feasible visit combinations at the beginning, deciding when each retailer will be visited over the time horizon. They formulate the TS algorithm such that the route is designed first. It is then run in a way that the vehicle-capacity constraint violation is minimized. This requires them to add a penalty cost when that capacity is violated. The results by the TS algorithm out-perform the column-generation based heuristic of Le et al. (2013) in all large instances for shelf life of 2 periods, which the latter authors found more difficult to solve.

Mirzaei and Seifi (2015) formulate a PIRP in which end-customer demand is a decreasing function of inventory age. A portion of demand is considered as lost sales if inventory is not as fresh as it could be. Since the demand is known, the model is deterministic. The model is a Mixed Integer NonLinear Program (MINLP) that is linearized by the authors. The objective is to minimize the total cost of routing, lost sales, and holding inventories. The mathematical model is solved to optimality for small to medium sized problems. The authors develop a hybrid Simulated Annealing (SA) and TS heuristic for solving larger problem instances.

There are a few papers which consider the component of price in the objective function of profit maximization. The price of the product, and in turn the profit, usually decreases as the product gradually deteriorates with age. The present research focuses on profit maximization which is briefly discussed below.

Coelho and Laporte (2014) were one of the first to consider a profit maximization objective for a PIRP. Here inventory holding cost and selling price are age-dependent. The supplier has the choice to deliver fresh or aged products, and each case yields different holding costs and distinct revenues. The objective function maximizes the total sales revenue, minus inventory and routing costs. The sale of products of different ages would decide the revenue. Three different selling-priority policies are investigated: (a) Fresh First policy (FF), (b) Old First policy (OF), (c) Optimized Priority policy (OP). The OP policy lets the model determine which items to sell at any given time period in order to maximize profit. This means that depending on the parameter settings, one may prefer to spoil some items and sell the fresher ones that generate higher revenues.

FF and OF policies sound similar to, but differ from the traditional First in First Out (FIFO) and Last in First Out (LIFO) policies that are common in inventory management. Under a FIFO policy, the first product delivered will be the first to be sold. This coincides with the OF policy when deliveries from suppliers to retailers are of fresh items. However, in this case, where the supplier delivers products of different ages, the sequence of deliveries does not necessarily coincide with the lengths of time the products have spent in inventory. Coelho and Laporte (2014) formulate this IRP for perishables as an MILP, and devise an exact branch-and-cut algorithm for the solution of various models.

Another profit maximization model is given by Alvarez et al. (2018), where a fixed shelf life of the products is considered; sales revenue varies with the quality of those products. Their problem is the same as the one proposed by Coelho and Laporte (2014). Four new reformulations of the original model are proposed. The first variation adds new decision variables with respect to transportation; the new variables relate the quantity delivered to those demands covered by the delivered quantity. In the second reformulation, decision variables do not involve the product age. Instead, age is considered in terms of demands covered in the following periods by the products delivered in the present time period. For the third and fourth formulations, Alvarez et al. (2018) remove the vehicle index from each of the first two formulations respectively. All four formulations are solved using the branch-and-cut algorithm. A hybrid heuristic is also proposed to solve the four variations. Results show that the formulations without the vehicle index (i.e. numbers 3 and 4) provide better and faster solutions as compared to the traditional arc-based formulations.

The Production Routing Problem (PRP), another very common subject where perishability is applied, adds a component of production in the IRP. Yantong et al. (2016) extend the work of Coelho and Laporte (2014) by adding that production component to the inventory and transportation cost in a profit-maximization objective. Inventory capacity at the depot is not considered, but there is a corresponding holding cost there. Product quality is managed throughout the supply chain; the item's selling price changes with each period, as the quality of the product decreases. Selling price is a linear function of product quality, with the price decreasing considerably as the product deteriorates. Yantong et al. (2016) use CPLEX to solve the model on randomly generated instances and different quality degradation rates, concluding that due to the problem complexity, CPLEX is only able to solve small instances. The present research remarks that Li et al. (2016) add inventory capacity at the depot to the model of Yantong et al. (2016) to make it more realistic.

Shaabani and Kamalabadi (2016) develop a PRP for multiple perishable products. Perishability is modeled in the same way as in Le et al. (2013), with the upper bound on inventory being the total demands of the following time periods. Qiu et al. (2019), in the most recent paper on the subject of perishable PRP, treat both the deterioration rates and inventory holding costs as age-dependent. Those authors apply the proposed model to the production and distribution operations of a food company operating in Nanjing, China.

4.4 Description

The deterministic PIRP described in this thesis is based on the model by Coelho and Laporte (2014) with a profit maximization objective. A finite time horizon is assumed with a one-to-many structure. There is one central depot; multiple retailers would be visited in a single trip, but they can be visited only once. Demand is deterministic and is constant across periods. The transportation fleet is homogeneous and is fixed in size.

The model differs from a standard IRP in that it uses the concept of consignment inventory. Consignment inventory can be explained by means of Figure 1.1. The supplier in this case can be the manufacturer that produces the product. The supplier delivers the product to the retailer (retailers) as soon as the item is produced, and hence the supplier holds no inventory. There is thus no holding cost at the depot. The supplier owns the product (which in this case can be called the consignment) until the time it is sold. The retailers keep all the inventory, but the corresponding holding costs at the retailers are charged to the supplier. When the retailer sells the product to the customer, the customer pays for the product, and that money is transferred to the supplier.

This type of inventory policy benefits both entities, as they each share the risks as well as the rewards. The supplier saves the cost of holding inventory at the depot. Also, the supplier benefits from keeping all inventory at the retailers, since they are closer to the customer. The retailer, on the other hand, does not have to pay for keeping the product in inventory. The job of the retailer is to sell the product. Also, the retailers don't have to worry about restocking the product as the supplier, who knows the inventory level at each retailer, will keep the retailer well stocked. This cost-sharing among the supplier and retailers helps to better streamline the supply chain.

The model of the PIRP treats the inventory and transportation costs of a single per-

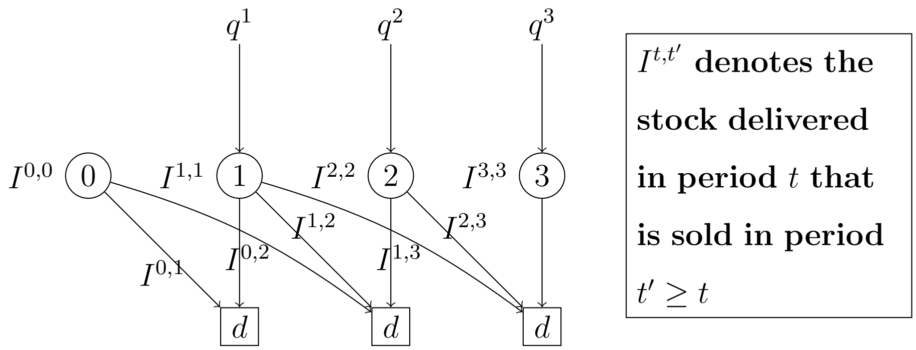


Figure 4.1: Aging process of the inventory at a retailer

Shelf life = 2 periods; time horizon = 3

ishable product with a discount factor. The revenue from that product differs with age, as in Coelho and Laporte (2014). The difference is the way this profit maximization is handled. A fresh product is delivered to the retailers every period. The product deteriorates over time when in inventory. As it deteriorates, the supplier who owns the product has to reduce its selling price (giving a discount) in order to maximize profit. Thus, this model uses the Optimized Priority Policy (OP) described in Coelho and Laporte (2014), where the product is sold in a way that maximizes the system profit, considering all other cost elements. The selling price and discount factor of the fresh product are known and constant. The discount factor is the percentage by which the item's selling price is reduced as the product shelf life decreases with each period it is not sold and kept in inventory. The holding cost is not dependent on the product's age. The shelf life of the product can be any number of periods (one or more), but is constant over the course of the problem.

The product expires after its lifetime and is removed from stock. All inventory is held at the retailer's end based on the holding capacity there. The product's decay affects only the unit price which the supplier receives.

To keep track of decay of the products in stock, another subscript is added to the original inventory variable. I_i^t is replaced by $I_i^{t,t'}$. This represents the usage of the product, where t denotes the time period that the product is delivered to retailer i and t' indicates the period in which the product is used or sold. To illustrate the aging process and how the discount is applied during the planning horizon, the inventory is described in Figure 4.1. There the product shelf-life is taken as 2 periods and a time horizon of 3 is considered for a particular retailer. In the figure, q^t ($t \in (1, 2, 3)$) is the quantity delivered to the retailer in time period t; d denotes the retailer demand in each time period, and is constant across periods.

As an example, we consider a product whose price is \$100 and for which the discount factor is 20 (in %). This means that in any time period, a fresh product will sell at the price of \$100 ($I^{t,t}$ for $t \in (0, 1, 2, 3)$). As the product gets one period older, it is sold at a discount of 20%, hence it is sold at \$80 ($I^{t,t+1}$ for $t \in (0, 1, 2, 3)$). As the shelf-life of the product is 2, if the product is sold after 2 time periods, it will get a further discount of \$20 (20% of the original price), and will finally be sold at \$60 ($I^{t,t+2}$ for $t \in (0, 1, 2, 3)$). After this, the product would be removed from inventory. Thus, the price of the product is constant and the discount rate is also constant (20% of the original price per period) as the product ages.

The objective of the PIRP is to maximize the overall revenue of the supplier. This involves the addition of a term involving sales revenue to the original objective function of the IRP. Thus, the objective function maximizes the profit of the supplier, which is the sales revenue minus the routing and inventory holding costs.

4.5 Mathematical Model

The model here adds the component of perishability to the IRP model described in Chapter 3. This includes modifying the objective function and changing the constraints to include the product aging process in accounting for inventory. Since the model focuses on the concept of consignment inventory, the constraint (3.2) i.e. the inventory balance constraint at the depot, is removed. The rest of the formulation remains the same.

4.5.1 Notation

Only the notations that are changed or added to the earlier model are mentioned.

Parameters

- P denotes the price of the product
- θ denotes the discount factor (%)
- n denotes the lifetime of the product
- I_{i0} denotes the initial inventory at the retailer location i

Sets

- $F = \{0, ..., o\}$ is the set of the time periods starting from the time period 0 to the time periods until which the shelf life of the product is valid $(M + n)$. Here n is the shelf life of the product

Decision Variables

- $I_i^{t,t'}$: Inventory delivered in time period t and used or sold in time period t' ($t' \geq t$) to retailer i

4.5.2 Formulation

$$\text{maximize } \Big\{ \sum_{i \in V'} \sum_{t \in M} \sum_{\tau=t}^{t+n} P(1-\theta(\tau-t)) I_i^{t,\tau} - \sum_{i \in V'} \sum_{t \in M} \sum_{t_1=t-n+1}^{t} \sum_{t_2=t+1}^{t_1+n} h_i(I_i^{t_1,t_2})$$
$$- \sum_{i \in V} \sum_{j \in V} \sum_{k \in K} \sum_{t \in T} c_{ij} x_{ij}^{kt} \Big\} \tag{4.1}$$

subject to

$$d_i^t = \sum_{t_1=0}^{n} I_i^{t-t_1,t} \quad i \in V' \;\; t \in T \tag{4.2}$$

$$\sum_{k \in K} q_i^{kt} = \sum_{t_1=0}^{n} I_i^{t,t+t_1} \quad i \in V' \;\; t \in T \tag{4.3}$$

$$\sum_{t_1=0}^{n} I_i^{0,0+t_1} = I_{i0} \quad i \in V' \tag{4.4}$$

$$\sum_{t_1=t-n+1}^{t} \sum_{t_2=t+1}^{t_1+n} I_i^{t_1,t_2} \leq C_i \quad i \in V' \;\; t \in T \tag{4.5}$$

$$\sum_{k \in K} q_i^{kt} \leq C_i - \sum_{t_1=t-n+1}^{t} \sum_{t_2=t+1}^{t_1+n} I_i^{t_1-1,t_2-1} \quad i \in V' \;\; t \in T \tag{4.6}$$

$$I_i^{t,t_1} = 0 \quad i \in V', \;\; t \in T, \;\; t_1 \in F, \;\; t_1 > p \tag{4.7}$$

$$I_i^{t,t'} \geq 0 \quad i \in V' \;\; t \in T \tag{4.8}$$

$$\sum_{i \in V'} q_i^{kt} \leq Q \quad k \in K \;\; t \in T \tag{4.9}$$

$$\sum_{k \in K} q_i^{kt} \leq C_i \sum_{j \in V'} \sum_{k \in K} x_{ij}^{kt} \quad i \in V' \;\; t \in T \tag{4.10}$$

$$q_i^{kt} \leq C_i y_i^{kt} \quad i \in V' \;\; k \in K \;\; t \in T \tag{4.11}$$

$$\sum_{j \in V} x_{ij}^{kt} = \sum_{j \in V} x_{ji}^{kt} = y_i^{kt} \quad i \in V \;\; k \in K \;\; t \in T \tag{4.12}$$

$$U_j^{kt} \geq U_i^{kt} + q_j^{kt} + Q x_{ij}^{kt} - Q \quad i, j \in V' \;\; k \in K \;\; t \in T \tag{4.13}$$

$$\sum_{j \in V'} x_{0j}^{kt} \leq 1 \quad k \in K \;\; t \in T \tag{4.14}$$

$$\sum_{k \in K} y_i^{kt} \leq 1 \quad i \in V' \;\; t \in T \tag{4.15}$$

$$q_i^{kt} \geq 0 \quad i \in V' \;\; k \in K \;\; t \in T \tag{4.16}$$

$$x_{ij}^{kt} \in \{0, 1\} \quad i, j \in V' \;\; k \in K \;\; t \in T \tag{4.17}$$

$$y_i^{kt} \in \{0, 1\} \quad i \in V \;\; k \in K \;\; t \in T \tag{4.18}$$

$$U_i^{kt} \geq 0 \quad i \in V' \;\; k \in K \;\; t \in T \tag{4.19}$$

The objective function (4.1) maximizes the net profit of the supplier. The first term denotes the revenue, based on the price at which the product is sold by the retailer. The inventory holding costs and the routing costs are subtracted from the price term, and are the same as in the IRP. The standard inventory balance constraints are replaced by constraints (4.2) and (4.3); the inventory balance constraint for the supplier's depot is removed. Constraint (4.2) states that demand can be satisfied only by a product that is fresh or has been in inventory for the previous n time periods, n being the shelf-life of the item. Constraint (4.3) is for the supply, stating that product delivered to the retailer can only be used to satisfy the demand of the current period and the next n periods. Constraint (4.4) is very important to verify that the initial inventory is used (and thus will contribute to the profit of the supplier). Constraint (4.5) maintains the retailer's inventory capacity based on the product age, and makes sure that no stock of an expired product is kept. Constraint (4.6) verifies that the ML inventory policy is maintained even as the product decays. Constraint (4.7) ensures that inventory is zero at the end of the time horizon. Constraint (4.8) guarantees that inventory is nonnegative. Constraints (4.9)-(4.19) for the PIRP are the same as Constraints (3.7)-(3.17) for the IRP.

4.6 Results

This section compares the results of the PIRP model with those of the IRP model.

4.6.1 Instances

The instances used here are derived from the single vehicle IRP instances of Archetti et al. (2007), where single-vehicle algorithms for the IRP were evaluated. Those instances contain three time periods and up to 50 customers, and six time periods and at most 30 customers. They are divided into four classes based on inventory holding cost and planning horizon length: H3, H6, L3, and L6. The H3 and H6 (L3 and L6) classes contain instances with high (low) holding costs, whereas the H3/L3 and H6/L6 classes have three and six time periods, respectively. There are five instances for each combination. All instances involve a single vehicle, multi-vehicle algorithms have been evaluated by simply dividing the original vehicle capacity by the number of desired vehicles. Details regarding these instances can be found at https://www.leandro-coelho.com/instances/inventory-routing/

For the results in this section, the class H3 is used (high cost and 3 time periods). The naming of each instance as given by Archetti et al. (2007) is of the form: "$absjnm$", which is instance j with m retailers. "abs" and "n" in the instance name hold no significance, but we use the same naming convention in accordance with that given by Archetti et al.

(2007). Similarly, the instance numbers go from 1 to 5, and the number of retailers goes from 05 to 50 (in intervals of 5). In total, that makes 150 instances.

For the first test, the models were run on all 150 instances of H3, but that led to many instances timing out. So, in the results shown, only instances with up to 20 retailers (20 instances) are exhibited.

4.6.2 Parameters

The tests were done keeping the number of vehicles (k) as 2, and with the product shelf life of 2 periods. The selling price of the product (P) is taken as 100, and the discount factor (θ) is 20%.

The model for the PIRP has a maximization objective, while the IRP objective is minimization. To make the models comparable, our model for PIRP is made analogous to IRP by taking the discount factor (θ) as 0. In this case then each product is sold at price P, irrespective of when the product is delivered and when it is sold. For the results below, we are thus comparing PIRP with $\theta = 0$ (analogous to IRP) to PIRP (θ ¿ 0).

4.6.3 Computational experiments

Both models are run on the CPLEX solver. The default algorithm of CPLEX is branch-and-cut. The time limit is set to 3 hours.

Comparison between the two models is shown in Table 4.1. It can be seen that, adding the component of perishability to the IRP does not significantly affect the problem. In some cases, PIRP even performs better than the standard IRP. For instance, in abs5n15,

Data Set	IRP				PIRP				Gap
	Obj	BB	Gap(%)	Time(s)	Obj	BB	Gap(%)	Time(s)	Diff
abs1n05	56.44	56.45	0.01	0.28	47.20	47.20	0	0.06	0
abs1n10	187.94	187.96	0.01	967	162.17	162.19	0.01	987	0
abs1n15	245.22	245.24	0.01	233	212.11	212.13	0.01	5662	0
abs1n20	316.42	316.87	0.14	TO	272.55	272.90	0.13	TO	-8
abs2n05	46.18	46.18	0	0.1	36.73	36.73	0	0.1	0
abs2n10	160.49	160.50	0.01	7645	140.15	140.17	0.01	238	0
abs2n15	234.17	234.41	0.1	TO	203.54	203.69	0.07	TO	-43
abs2n20	309.80	309.97	0.05	TO	265.05	265.29	0.09	TO	44
abs3n05	88.62	88.63	0.01	2.45	79.40	79.41	0.01	0.69	0
abs3n10	135.01	135.02	0.01	52.5	113.37	113.38	0.01	82.4	0
abs3n15	250.25	250.28	0.01	2184	213.33	213.35	0.01	447	0
abs3n20	303.82	303.85	0.01	1829	256.44	256.55	0.05	TO	80
abs4n05	51.98	51.98	0.01	0.73	47.20	47.20	0	0.85	0
abs4n10	161.78	161.80	0.01	580	141.75	141.77	0.01	199	0
abs4n15	212.75	213.00	0.12	TO	186.32	186.53	0.12	TO	0
abs4n20	295.63	296.71	0.36	TO	261.95	262.54	0.22	TO	-64
abs5n05	68.85	68.86	0.01	2.45	57.86	57.87	0.01	0.6	0
abs5n10	189.48	189.50	0.01	91.9	165.56	165.57	0.01	24.8	0
abs5n15	201.39	201.69	0.15	TO	179.85	179.86	0.01	7987	-1400
abs5n20	342.72	343.53	0.24	TO	297.50	298.36	0.29	TO	17

Table 4.1: IRP vs PIRP (Vehicles=2; Shelf-life=2)

Obj, BB refer to the Objective Value and Best Lower Bound respectively (both in 1000s); TO refers to Time Out; Gap Diff refers to the difference in Gaps between PIRP and IRP (in %)

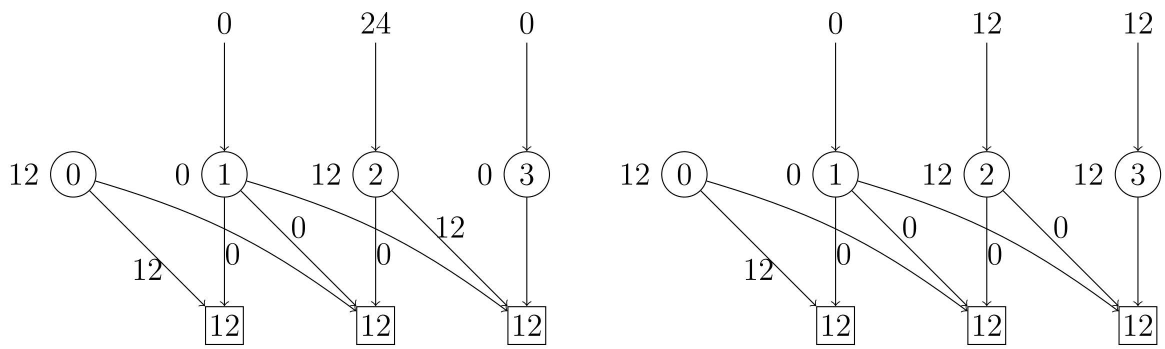


Figure 4.2: Aging process of inventory IRP vs PIRP at retailer 5 of data set abs5n05 (shelf-life = 2; time-periods = 3)

the PIRP solves to optimality whereas IRP does not. In the case of IRP there is no discount factor; hence its objective function value is always greater than that for PIRP.

Comparing two different models does not give us insights on the impact of the component of perishability. How does the declining selling price of the product affect the inventory, routes, and product usage?

For these insights, tests on some of the small instances which solve in reasonable time are done. Four instances with 5 retailers each (abs1n05, abs2n05, abs4n05, abs5n05), and one instance with 10 retailers (abs3n10) are solved. Recall that Figure 4.1 showed the usage of inventory and the quantity delivered at the beginning of each time period, depending on the initial inventory and the retailer's demand.

The aging process of inventory of IRP vs PIRP for retailer 5 in one of the instances (abs2n05) is represented in Figure 4.2. Explanation concerning the remaining retailers in this instance is given in Table 4.2.

In Figure 4.2, the initial inventory at the retailer is 12; demand in each time period is 12. The insights obtained from Figure 4.2 are as follows:

1. For IRP, since it does not matter how long the product is kept in inventory, it is seen that the delivery is done only once in time period 2

2. For PIRP, the product loses value when it is kept in inventory. So, two deliveries are made, in periods 2 and 3 to maximize the profit

3. For both cases, the initial inventory is used first, before any delivery of the product. This is also because constraint (4.4) makes sure that initial inventory never goes to waste

For the data set in consideration (abs2n05), retailer 5 was chosen because it is the only retailer where the initial inventory can satisfy one period's demand. For the other 4 retailers, the initial inventory is double the retailer's demand, and hence can satisfy two periods of demand. This means, for the other retailers (1,2,3,and 4), delivery is required in only a single time period.

Retailer	Demand	In	IRP			PIRP		
		Inv	TP	Vehicle	Quantity	TP	Vehicle	Quantity
1	31	62	2	1	31	3	1	31
2	60	120	2	1	60	3	1	60
3	17	34	3	1	17	3	2	17
4	38	76	3	1	38	3	2	38
5	12	12	2	1	24	2,3	1	12,12

Table 4.2: IRP vs PIRP analysis for abs2n05 (Vehicles = 2; Capacity = 118.5)

Table 4.2 shows the values of the Initial Inventory and Demands for each of the five retailers. Comparison is done between IRP and PIRP as to when the quantity is delivered, which vehicle is used, and how much is delivered. From the table, it is very clear that in this case, since the IRP does not account for perishability, only a single vehicle is used for delivery each period. For the IRP, it does not matter when the product is delivered. For PIRP, the product is delivered only when it is required, and hence in the third time period, two vehicles are used for delivery.

Retailer	Demand	In Inv	IRP TP	IRP Vehicle	IRP Quantity	PIRP TP	PIRP Vehicle	PIRP Quantity
1	65	130	3	1	65	3	2	65
2	35	70	2	2	35	3	2	35
3	58	58	2	1	116	2,3	2,1	58,58
4	24	48	2	2	24	3	2	24
5	11	11	2	2	22	2,3	2	11,11

Table 4.3: IRP vs PIRP analysis for abs1n05 (Vehicles = 2; Capacity = 144.5)

Retailer	Demand	In Inv	IRP TP	IRP Vehicle	IRP Quantity	PIRP TP	PIRP Vehicle	PIRP Quantity
1	53	53	2,3	1	53,53	2,3	2	53,53
2	21	21	2	1	42	2	1	21,21
3	24	24	2,3	1	24,24	2,3	1	24,24
4	67	67	2	2	134	2,3	2	67,67
5	14	28	3	1	14	3	2	14

Table 4.4: IRP vs PIRP analysis for abs4n05 (Vehicles = 2; Capacity = 134)

The results, for instances abs1n05, abs4n05, abs5n05, and abs3n10, are displayed in Tables 4.3-4.6, respectively.

Retailer	Demand	In Inv	IRP			PIRP		
			TP	Vehicle	Quantity	TP	Vehicle	Quantity
1	19	38	3	1	19	3	1	19
2	47	94	3	1	47	3	1	47
3	72	144	3	1	72	3	2	72
4	81	81	2	2	162	2,3	1,2	81,81
5	15	30	3	1	15	3	1	15

Table 4.5: IRP vs PIRP analysis for abs5n05 (Vehicles = 2; Capacity = 175.5)

Retailer	Demand	In Inv	IRP			PIRP		
			TP	Vehicle	Quantity	TP	Vehicle	Quantity
1	63	126	3	1	63	3	2	63
2	22	22	2	1	44	2,3	1	22,22
3	58	116	3	1	58	3	1	58
4	26	52	3	1	26	3	2	26
5	18	18	2,3	1	18,18	2,3	1	18,18
6	99	99	2	1	198	2,3	1	99,99
7	39	78	2	1	39	3	1	39
8	21	21	2	1	42	2,3	1	21,21
9	32	64	3	1	32	3	2	32
10	80	160	3	1	80	3	2	80

Table 4.6: IRP vs PIRP analysis for abs3n10 (Vehicles = 2; Capacity = 343.5)

Results are very similar to what was obtained initially while solving the first instance (Table 4.2). However, there are some differences.

In Table 4.3, since the vehicle capacity is 114.5, and quantity delivered to retailer 3 in time period 2 is 116, the second vehicle needs to be used to deliver the remaining demands during that period. It should be noted that, rather than delivering a quantity of 58 in each of periods 2 and 3 (as in the case of PIRP), a delivery of 116 is made in period 2. This means that the savings in routing costs for retailer 3 is more than the corresponding increase in holding costs.

Another interesting case, for instance abs4n05 is displayed in Table 4.4. Here, even for the IRP, retailers 1 and 3 receive deliveries twice during the time horizon. This leads to a conclusion that additional saving in the inventory holding cost more than compensates for an extra expenditure on transportation cost for these retailers i.e. it is cheaper to deliver twice, than keep the product in inventory for an extra time period.

Only one case of 10 retailers is discussed here. The results, as shown in Table 4.6, are comparable with results for instances having five retailers.

Although only five instances have been gathered in the above-mentioned results, the values for retailer demands, initial inventory, and vehicle capacity change with every instance. Due to this variation, we were able to get the interesting insights of this subsection, as to how introducing perishability to the IRP transforms the way in which the model behaves, and the inventory is handled.

4.7 Benders Decomposition

An exact algorithm based on Benders Decomposition is used to solve the PIRP. Introduced by Benders (1962), this algorithm aims to solve Mixed Integer Programming (MIP) models by distinguishing two different types of variables (continuous and integer). In Benders decomposition, the original problem is partitioned into a master problem and a number of subproblems, which are typically easier to solve than the original problem. By using linear programming duality, all variables that belong to the subproblems are projected out; the master problem contains the remaining variables and an artificial variable that represents a lower bound on the total cost of the subproblems. The resulting model is solved by a cutting plane algorithm in which, at each iteration, the values of the master problem variables are determined first and the subproblems are solved with these variables fixed. If the subproblems are feasible and bounded, an optimality cut is added to the master problem. Otherwise, a feasibility cut is added. An upper bound can be computed from feasible subproblems, and a lower bound is obtained if the master problem is solved to optimality. The process continues until an optimal solution is found or the optimality gap is smaller than a given threshold (Adulyasak et al. (2015)).

When the algorithm was first introduced, Benders (1962) mentioned that the algorithm is fairly successful in problems which have a structure that can be naturally partitioned into two mutually exclusive problems. The problem in this thesis is ideal to solve using Benders Decomposition. The problem can naturally be partitioned into the inventory problem and the transportation problem.

In order that there be a natural partition into the master problem and subproblems, it is required that the two types of variables be mutually exclusive. The integer variables go into the master problem, and the continuous variables go into the subproblem. For that reason, the formulation described in Section 4.5.2 needs to be modified. In the current formulation, the continuous variables are $I_i^{t,t'}$, q_i^{kt}, U_i^{kt}. On the other hand, the integer variables are x_{ij}^{kt} and y_i^{kt}. The goal of the new formulation is to separate the variables that concern inventory from the variables that concern routing. Thus, variables $I_i^{t,t'}$ and q_i^{kt} need to be separated from the variables U_i^{kt}, x_{ij}^{kt} and y_i^{kt}. Also, the variable that is used to avoid subtours, U_i^{kt}, is continuous and needs to be made an integer.

The new formulation that incorporates all the above described changes, and also makes the original formulation more compact, is mentioned below.

4.7.1 Benders Reformulation

The variable y_i^{kt} is removed to render the formulation more compact; the variable x_{ij}^{kt} now incorporates the depot as well.

$$\begin{aligned}
\text{maximize } & \Big\{ \sum_{i\in V'} \sum_{t\in M} \sum_{\tau=t}^{t+n} P(1-\theta(\tau-t)) I_i^{t,\tau} - \sum_{i\in V'} \sum_{t\in M} \sum_{t_1=t-n+1}^{t} \sum_{t_2=t+1}^{t_1+n} h_i(I_i^{t_1,t_2}) \\
& - \sum_{i\in V} \sum_{j\in V} \sum_{k\in K} \sum_{t\in T} c_{ij} x_{ij}^{kt} \Big\} && (4.20)
\end{aligned}$$

subject to $\quad (4.2)-(4.8)$, (4.15), and,

$$q_i^{kt} \leq C_i \sum_{j\in V} x_{ij}^{kt} \quad i\in V' \;\; t\in T, \;\; k\in K \tag{4.21}$$

$$\sum_{k\in K} \sum_{j\in V} x_{ij}^{kt} \leq 1 \quad i\in V' \;\; t\in T \tag{4.22}$$

$$\sum_{j\in V} x_{ij}^{kt} = \sum_{j\in V} x_{ji}^{kt} \qquad i\in V \;\; t\in T \;\; k\in K \tag{4.23}$$

$$U_j^{kt} \geq U_i^{kt} + 1 - M(1-x_{ij}^{kt}) \quad i,j\in V' \;\; t\in T \;\; k\in K \tag{4.24}$$

$$x_{ij}^{kt} \in \{0,1\} \quad i,j\in V \;\; t\in T \;\; k\in K \tag{4.25}$$

$$U_i^{kt} = \{0,1,2,...\} \quad i\in V' \;\; k\in K \;\; t\in T \tag{4.26}$$

The objective function (4.20) remains the same as (4.1). Constraints (4.10)-(4.12), (4.14)-(4.15), and (4.17)-(4.19) are replaced by constraints (4.21)-(4.23) and (4.25). Constraint (4.24) replaces constraint (4.13) as the subtour elimination constraint. It states that, only when there is a route from node i to node j $(i, j \in V')$ will the value of U_i^{kt} get updated. Finally, constraint (4.26) replaces constraint (4.19), where U_i^{kt} is now no longer continuous.

This reformulation ensures that the problem is naturally distinguished into the master problem and subproblems. It also makes it more compact than the original formulation. The variables that correspond to the routing part of the IRP are now all integers. Therefore, they become part of the master problem. Correspondingly, the variables that take care of the inventory part of the IRP are all continuous, and hence are part of the subproblems.

4.7.2 Results

Instances

The instances used here are the same as the ones studied previously. Please refer to the details in Section 4.6.1. Only instances with up to 20 retailers are included in the results.

Parameter values

Values of the parameters employed for results in Table 4.7 below are the same as those mentioned in Section 4.6.2

Computational experiments

The instances are run using CPLEX-based branch-and-cut (BC), and on CPLEX with Benders Decomposition (BD), with the time limit set to 3 hours.

For applying the Benders Decomposition Algorithm, the variables (x_{ij}^{kt}, and U_i^{kt}) that concern the routing went to the Master problem, and the variables concerning inventory ($I_i^{t,t'}$, q_i^{kt}) were put in one subproblem. With CPLEX, there are several ways in which the Benders Decomposition algorithm can be used. Each has a different Benders strategy, giving CPLEX more or less freedom to solve the model. All three strategies (1, 2, and 3) were tested, but strategy 2 (CPLEX works on the annotations provided, and will decompose them further) gave better results and hence was employed to get the results. More information on the Benders strategies can be found on https://www.ibm.com/support/knowledgecenter/SSSA5P_12.8.0/ilog.odms.cplex.help/CPLEX/UsrMan/topics/discr_optim/benders/parameterBenders.html.

Results for implementation of the model with the branch-and-cut (BC) algorithm, in comparison with the Benders decomposition (BD) algorithm, are described in Table 4.7. Those results indicate that the BC algorithm performs better than the BD algorithm for all instances. For small instances, with 5 or 10 retailers, both BC and BD algorithms were able to solve them to optimality, but significantly greater time is taken by the BD algorithm. When the algorithms are run on larger instances, they both time out. Even in those cases, the gaps generated by the BD algorithm are much larger than those generated by BC. The lowest difference is 40%. Apart from that, the differences for the remaining instances are more than 50%, with the number going as high as 93% for two instances. These results for the PIRP clearly imply that the BC dominates the BD algorithm.

4.8 Conclusion

This chapter has discussed the addition of a perishability component to the deterministic IRP. The BD algorithm was also introduced. This was utilized as an alternative to solve the model. However, it did not give the anticipated results.

In terms of formulation, the biggest drawback is that the PIRP model still considers

Data Set	BC				BD				Gap
	Obj	BB	Gap(%)	Time(s)	Obj	BB	Gap(%)	Time(s)	Diff
abs1n05	47.20	47.20	0	0.06	47.20	47.21	0.01	0.13	0
abs1n10	162.17	162.19	0.01	987	162.17	162.19	0.01	4176	0
abs1n15	212.11	212.13	0.01	5662	212.11	212.40	0.14	TO	93
abs1n20	272.55	272.90	0.13	TO	272.11	273.23	0.41	TO	68
abs2n05	36.73	36.73	0	0.1	36.73	36.73	0	0.29	0
abs2n10	140.15	140.17	0.01	238	140.15	140.17	0.01	3381	0
abs2n15	203.54	203.69	0.07	TO	203.47	203.91	0.21	TO	67
abs2n20	265.05	265.29	0.09	TO	264.98	265.46	0.18	TO	50
abs3n05	79.40	79.41	0.01	0.69	79.40	79.41	0.01	3.65	0
abs3n10	113.37	113.38	0.01	82.4	113.37	113.38	0.01	217	0
abs3n15	213.33	213.35	0.01	447	213.33	213.35	0.01	2986	0
abs3n20	256.44	256.55	0.05	TO	256.44	257.02	0.23	TO	78
abs4n05	47.20	47.20	0	0.85	47.20	47.21	0.01	16.3	0
abs4n10	141.75	141.77	0.01	199	141.75	141.77	0.01	8997	0
abs4n15	186.32	186.53	0.12	TO	186.30	186.68	0.2	TO	40
abs4n20	261.95	262.54	0.22	TO	261.50	263.25	0.67	TO	67
abs5n05	57.86	57.87	0.01	0.6	57.86	57.87	0.01	0.85	0
abs5n10	165.56	165.57	0.01	24.8	165.56	165.57	0.01	238	0
abs5n15	179.85	179.86	0.01	7987	179.85	180.11	0.15	TO	93
abs5n20	297.50	298.36	0.29	TO	297.10	298.87	0.59	TO	51

Table 4.7: PIRP: BC vs BD (Vehicles=2; Shelf-life=2)

Obj, BB refer to the Objective Value and Best Lower Bound respectively (both in 1000s); TO refers to Time Out; Gap Diff refers to the difference in Gaps between BC and BD (in %)

demand to be known and constant. Any industry that deals with perishable products, in reality, can never know the demands beforehand. The demand realization is usually different than the predicted demand. Thus, this uncertainty or randomness in demand needs to be captured. Chapter 5 considers stochastic demands, rather than deterministic, for the PIRP.

Chapter 5

Stochastic IRP

5.1 Introduction

This chapter introduces randomness in demand to the perishable IRP. The problem will henceforth be called the stochastic perishable IRP (SPIRP). This chapter will include a description of the problem, followed by the relevant literature for this topic. Then, the mathematical model for this problem is formulated. Results of the solution algorithm for the model are shown next, and then the conclusion and restrictions are noted.

5.2 Importance

Up until this chapter, the demand was always known and hence there was nonnegative inventory at the retailer. The randomness in demand was not considered. When demand is not known with certainty, there can be excess inventory or less inventory when the demand is actually realized at the retailer's end. If demand is lower than predicted, the excess inventory at the retailer will go to waste. With no salvage value, the supplier will face losses. The other case is when the demand is higher than predicted, leading to a stockout. Demand for the product cannot be fulfilled at that time and the extra demand becomes lost sales. This will also lead to losses, since the customer whose demand was not met would most likely go to a rival store/chain to get the product. This lower service level implies a loss in customer satisfaction. Thus, it is very important to consider the case of stochastic demands and make sure that the loss of revenue is minimized by better optimization of the supply chain.

Food wastage is an important issue for supply chain managers to take into consideration. The report by Second Harvest and Value Chain Management International describes the extent of this loss (Nikkel et al. (2019)). The report provides the amount of losses that occurred in Canada in previous years and how each entity of the food retail supply chain contributed to this loss. The statistics show that food losses and wastage during processing, distribution, and retail are 2.25, 0.55, and 1.31 Million Tonnes respectively. The corresponding monetary values of these food losses are 9.78, 2.41, and 5.7 Billion Dollars. These are staggering numbers.

A major cause of food loss is that products are not getting sold at the retail stores. The loss at the supplier is due to inaccurate forecasts of demand. If demand is lower, the stores are left with extra inventory. On the other hand, if the demand is higher, there are penalties and financial losses due to stockout. Thus, to keep the customer satisfied and to avoid penalties, excess inventory is kept at the stores and this leads to wastage of food. Also, to maintain a high customer service level, the stores try to keep the food shelves close to 100% full. This also results in food wastage. This food loss and wastage can be avoided to a large extent if the demand is correctly predicted. Thus, the application of stochastic demands when it comes to perishable products is of utmost importance, as the assumption of deterministic demands could never replicate the real life scenario.

5.3 Relevant Literature

Adding randomness in demand to the IRP is not a trivial task. This is because in any deterministic problem, all demands and the corresponding inventories are known to the vendor/supplier at the beginning of the time horizon. Thus, routing the available vehicles becomes easy when the time horizon is finite and defined. In the case of stochastic demand, the supplier does not have this demand information before making the routing decisions. Because demands can be either greater or lesser than the predicted demands, not all routing decisions can be taken at the beginning. Instead, a two stage process is required. At the start of each time period, the routes are determined, then when the vehicle reaches each retailer, actual demands are realized (can be more or less than previously anticipated). This new inventory is reported to the supplier and then decisions are made again.

Two set of decisions are required. The first pertains to the routing of vehicles, that is which customers should be visited by each. This decision is made regardless of the actual retailer demand. The second set of decisions concerns the inventory and quantity delivered. These decisions can be made once the actual demands are known. The objective of the stochastic version remains the same as that of the deterministic problem, with the addition of the component to account for randomness of demand.

There are two types of stochastic decision processes. This includes a two-stage process and a multi-stage process, as given by Adulyasak et al. (2015). The first stage, i.e. routing, is the same in both processes. The difference is when the actual demand is realized. In the two-stage problem, the demands for the entire planning horizon become known once the first-stage decisions are made. In the multistage problem, the demands for a given stage become known only after the decisions for the previous stage have been made. This thesis discusses only the two-stage stochastic process, and applies it to the perishable IRP.

The concept of uncertainty in demand was first added to the IRP by Federgruen and Zipkin (1984). In their model, only one period is considered; the depot is capacitated, unlike standard IRP models. The objective minimizes routing costs, shortage, and inventory carrying costs. This in turn depends on the demand, the product delivered, and the initial inventory at the depot. Since it is only a one period problem, a two-stage stochastic demand process is followed. Actual demand is realized when the vehicle reaches the retailer. The component of demand uncertainty is considered only in the objective function and not in the constraints. The inventory holding and storage costs are proportional to the inventory left at the end of the time horizon. To solve the model, the problem is decomposed into an Inventory Allocation (IA) problem and the Travelling Salesman Problem (TSP) for each vehicle. The TSPs are solved using Generalized Benders Decomposition. The instances are generated taking the demand as normally distributed. The authors conclude that incorporation of demand uncertainty provides considerable savings when compared to a deterministic VRP. Their model, as discussed above, builds the foundation on which other papers with demand randomness are based.

Jaillet et al. (2002) solve a version of the IRP for the distribution of heating oil. Demands of the customers are not known with certainty but follow a given probability distribution. This problem differs from the standard IRP in the sense that there are satellite centres where the vehicles can fill up before delivering to a customer. Demand at each customer varies daily, so the time period is one day. The objective of the problem is to minimize the annual delivery costs by deciding which customers to serve over each route based on their expected demands. When the realized demand is higher and there is a stockout, a direct delivery is initiated from the depot and a penalty is associated with this emergency delivery. The problem has a time horizon of two weeks and follows a rolling horizon framework developed by Bard et al. (1998). VRP heuristics are used to solve the problem whose solution is approximated as periodic over an annual time horizon.

Geiger and Sevaux (2011) provide an interesting take on the stochastic IRP in order to make the demand similar to any real industry. The problem is solved over a very long time horizon, with only the demand for the first time period known with certainty. The demand over the next several periods is uncertain but is within 10% of the demand approximated initially. Vehicles in their problem can perform several routes in one time period (which is unlike an IRP). Geiger and Sevaux (2011) proposed several policies based on delivery frequencies for each customer. They provide the Pareto front approximation of such policies. The delivery is made when required, thus minimizing the carrying costs. The new policies however, make significant changes in routing costs compared to an OU inventory policy, where the routing costs are minimum but inventory carrying costs are considerable. The authors are able to solve problems with a time horizon of up to 240 periods using VRP heuristics.

Bertazzi et al. (2013) present a stochastic IRP with OU inventory policy. There is a penalty cost associated with any negative inventory and there is no backlogging. The OU inventory policy is the same as the one introduced by Bertazzi et al. (2002) for the deterministic IRP. A Dynamic Programming (DP) model is used to solve the stochastic IRP. A hybrid rollout algorithm enables solution of the DP formulation. The rollout algorithm discussed here requires a cost-to-go approximation for which they solve a Mixed Integer Linear Programming (MILP) Model using BC. They use the MILP model as formulated in Archetti et al. (2007) for the deterministic context, taking demand as its average value in the stochastic case. Bertazzi et al. (2013) conclude that the stochastic problem is significantly more difficult to solve than the problem with no demand uncertainty.

Coelho et al. (2014) propose a dynamic and stochastic IRP. They introduce dynamic demands to an otherwise static literature of IRP. Dynamic demands potentially can change each period and hence decisions on inventory and routing have to be made in every time period. Demand forecasts are added to the model to make the demands less erratic by approximating the future demand. In the case of a stockout, direct deliveries are made to the retailer. In the model, the cost of direct deliveries depends on the distance as well as the volume of goods to be delivered. The use of transshipments also helps in keeping the customer service level high. The model is solved in the context of a rolling horizon framework using heuristics. Coelho and Laporte (2014) conclude that adding the component of dynamic and stochastic demands makes the model much more difficult than the one with deterministic demands. However, adding demand forecasts and transshipments to the new model helps in reducing the solution cost.

Gruler et al. (2018) employ a combination of simulation and metaheuristic to solve the stochastic IRP. The objective is to minimize the routing and inventory costs over a finite time horizon using an ML inventory policy. Since the model follows stochastic demand, there is a penalty cost associated with any excess inventory. In case of a stockout, a direct delivery is made, and the penalty is twice the routing cost from the depot to the retailer. Different replenishment inventory policies and the corresponding routing costs are calculated; the policy with the lowest expected cost is applied in the model. Gruler et al. (2018) use simulation to study several different scenarios and a Variable Neighbourhood Search (VNS) metaheuristic is applied to solve each scenario.

Although it is very important to include demand uncertainty in the perishable IRP, the difficulty of adding stochastic demands makes the model hard to solve. Thus, the literature on this problem is scarce. The first research paper that considered perishable products in a stochastic IRP environment was Federgruen et al. (1986). They extend the model given in Federgruen and Zipkin (1984), adding a fixed product shelf-life to the stochastic IRP. Two types of delivery patterns are employed. The first considers direct deliveries to the retailer. That transforms the problem to an IA problem only, with transportation costs being constant. The second is the standard stochastic IRP problem, with the first stage deciding the inventory allocation and based on that, the routes are determined. The objective minimizes the sum of out-of-date costs until the product perishes, plus the shortage, routing, and inventory holding costs. The old and new products are treated as two different items over the course of the problem. The other delivery strategy is solved by modifying the algorithm used in Federgruen and Zipkin (1984) for multiple products (in this case, fresh and old products). The model requires a little extra time to solve with the addition of the component of perishability.

Hemmelmayr et al. (2010) propose a stochastic IRP model for blood which is an important perishable product. Their model is an extension of Hemmelmayr et al. (2009) where demands are deterministic. To generate several demand realizations, they use external sampling which converts the two-stage stochastic problem to a deterministic problem for each scenario. A single uncapacitated vehicle with a fixed route duration is employed. The demand scenarios are constructed in such a way that the probability of spoilage occurrence is at-most 5%. Since the product being considered is blood, it is crucial that there are no stockouts which is what their model aims to ensure.

Four recourse actions are considered to prevent stockouts. Those include: (1) changing the quantities of planned deliveries (2) introducing out-and-back emergency deliveries (direct deliveries) to hospitals that are likely to experience a shortage based on the inventory at the beginning of the day (3) introducing a single emergency delivery route for each day. The route visits all hospitals that are likely to run out of product on that day (4) introducing emergency deliveries into the regularly planned delivery routes

The cost of each recourse is calculated as the average cost over all0 scenarios. The objective is to minimize delivery costs plus recourse action costs. The LP and the VNS approaches used in Hemmelmayr et al. (2009) are extended to incorporate each scenario for all four recourse actions.

Crama et al. (2018) conducted a recent study on the stochastic PIRP. This work considers a single perishable product, homogeneous vehicles, and a route-based formulation with a fixed route duration. There is a fixed cost for the usage of vehicles and also a variable transportation cost. There is an acquisition cost and the revenue associated with the product, but no inventory holding cost is charged. The objective function is profit maximization including acquisition costs, routing costs, and of course revenue. The product has a fixed shelf life and a FIFO policy is applied (the oldest item is sold first). A predefined Target Service Level (TSL) is also defined.

Four solution approaches are utilized. These include Expected Value (EV) method, deliver Up to Level (UL) method with a high service level, a decomposition scheme, and a decomposition-integration method. The use of expected demand changes the problem into a deterministic one; delivery is done only if inventory is less than the expected demand for that period. The delivery aims to satisfy demands for a fixed number of time periods and once the quantity is decided, the VRP is used to determine the routes. The UL method employs the concept of TSL, where it replaces the EV function with the probability that the satisfied demand is not less than the TSL. The next approach, decomposition, decomposes the UL formulation for each store, with the cost of each considered independently. Finally, the last method integrates the individual costs at each store. Each approach improves on the previous one with respect to the routing cost. The conclusion drawn is that even though routing forms a small part of the IRP, improving the routing cost factors quite a bit into the overall profit of the system.

In the past few years, the topic of *sustainable* IRP has gained prominence. The sustainable IRP includes perishability, but it is usually combined with wastage disposal and transportation emissions. However, a study on the sustainable IRP is out of the scope of this research. Readers are referred to Soysal et al. (2019) for a survey on the latest sustainable IRP literature.

The last publication that is reviewed here is Adulyasak et al. (2015). In this research paper, the authors solve the two-stage and multistage stochastic PRP with multiple vehicles over a discrete and finite time horizon. The PRP, is a generalization of the IRP with the addition of a production component. Those authors use a finite set of scenarios with different demands to incorporate demand uncertainty; the scenarios each have equal probability. The objective minimizes the fixed cost of production plus the routing costs (which are independent of demand). In addition to the preceding costs, the objective function to be minimized includes the penalty cost for lost sales, the production variable cost, and the inventory holding cost. All these costs depend on the particular scenario. Hence, each scenario has a probability and an associated cost which is to be summed over all the scenarios.

In the model of Adulyasak et al. (2015), both the two-stage and the multistage models have the same first stage. The first stage includes scheduling the production and routing. Those enable determination of the production and delivery quantities. Then, depending on the type of the model, when the actual demand is realized, the penalty and the inventory holding costs are calculated. The authors use a Benders decomposition algorithm to partition the original problem into a master problem and a set of sub-problems. The master problem contains all decisions that do not depend on the scenario; there is one sub-problem for each scenario. First the master problem is solved, and then, using the values of the routing variables, each sub-problem is solved. Benders Branch-and-Cut (BBC)

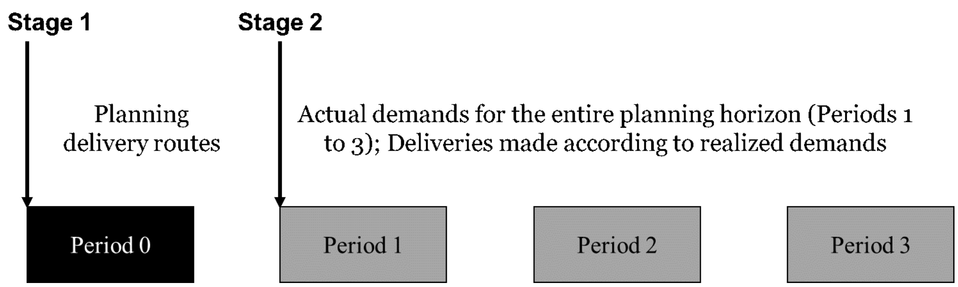


Figure 5.1: Two-Stage Stochastic Demand Process

which is a Benders reformulation in a BC framework, is employed. Scenarios are generated via Monte Carlo simulation, with demands ranging over a specific set of values based on the demands in the data set. The BBC for the two stage problem is improved using cuts. As the instances get larger and the number of scenarios increases, BBC improves on the solution given by BC.

The research paper by Adulyasak et al. (2015) is described at the end because the model employed in this thesis is closely related to their model. The model used here is next described and formulated.

5.4 Description

The stochastic PIRP in the present research is a demand-scenario-based model. The two-stage stochastic decision process used in this model is illustrated in Figure 5.1. In the first stage, decisions on the delivery routes and the allocation of the retailers to the vehicles are made. These decisions can be made regardless of the demand realizations. When the vehicle reaches the particular retailer, the actual demands for the remaining periods in the time horizon are realized. With those actual demand realizations, the second stage decisions regarding inventory and delivery quantity are made. That quantity delivered to the retailer depends on the retailer's holding capacity and demand. Since the quantity is delivered after demand realization, no excess amount will be delivered to any retailer. However, there can be a shortage when not enough stock is available to satisfy demand there.

The two-stage model is an extension of the previous model with deterministic demands; demand is now a random variable whose distribution is assumed to be known. This demand uncertainty is incorporated via a finite number of scenarios, each having a different demand. In a real-life case, there may be an infinite number of scenarios having equal probability. For running this model, a finite sample of those scenarios that account for uncertainty is applied. Naturally, the probability of any scenario would not be known, and since this probability is difficult to find, the probability of each scenario is taken as equal.

The resultant problem is more versatile than the one with known demands. With respect to each demand scenario, when the demand is known, the problem becomes deterministic. This means that Figure 4.1 can be used in the same way, for each demand scenario.

5.5 Mathematical Model

The model and formulation for the SPIRP are based on the two-stage model given in Adulyasak et al. (2015). The objective of the SPIRP remains the same as that of the PIRP, maximizing the profit of the supplier (i.e. maximizing the sales revenue minus the routing and inventory holding costs).

The SPIRP model minimizes the routing cost in the first stage, and maximizes the expected profit in the second stage. The formulation remains the same as that of the PIRP with a subscript for scenario added to the demand, inventory, and the quantity delivered in each period. To account for stochastic demands and to maintain the model feasibility, there are variables to handle shortage and excess inventory. In the objective function the portion representing the price-maximization and holding cost needs to be summed over all scenarios and weighted by the probability of each scenario to get the expected profit. The routing part of the objective and all the routing constraints remain the same as they are independent of the scenario under consideration.

5.5.1 Notation

The notations changed for the SPIRP are mentioned below.

Sets

- $S = \{1, ..., l\}$ represents the set of scenarios

Parameters

- ρ_s denotes the probability of scenario s ($\rho_s = 1/|S|$)
- d_{si}^t is the demand at retailer i in time period t for scenario s
- q_{si}^{kt} stands for the quantity delivered to retailer i by vehicle k in time period t for scenario s
- w_{si}^t represents the shortage at retailer i in time period t for scenario s
- e_{si}^t indicate excess inventory at the retailer i in time period t for scenario s

- I_{si0} denotes the initial inventory at retailer i for scenario s

Decision Variables

- $I_{si}^{t,t'}$ is the inventory delivered in time period t, and used in time period t' ($t' \geq t$), at retailer i for scenario s

Since the initial inventory is independent of the scenario considered and depends only on the retailer, it is assumed that $I_{si0} = I_{i0}$ at each retailer i for any scenario s.

5.5.2 Formulation

$$\text{maximize } \Big\{ \sum_{s \in S} \rho_s \Big(\sum_{i \in V'} \sum_{t \in M} \sum_{\tau = t}^{t+n} P(1 - \theta(\tau - t)) I_{si}^{t,\tau} - \sum_{i \in V'} \sum_{t \in M} \sum_{t_1 = t-n+1}^{t} \sum_{t_2 = t+1}^{t_1+n} h_i(I_{si}^{t_1,t_2}) \Big) - \sum_{i \in V} \sum_{j \in V} \sum_{k \in K} \sum_{t \in T} c_{ij} x_{ij}^{kt} \Big\} \tag{5.1}$$

subject to

$$d_{si}^t = \sum_{t_1=0}^{n} I_{si}^{t-t_1,t} + w_{si}^t \quad s \in S \;\; i \in V' \;\; t \in T \tag{5.2}$$

$$\sum_{k \in K} q_{si}^{kt} = \sum_{t_1=0}^{n} I_{si}^{t,t+t_1} + e_{si}^t \quad s \in S \;\; i \in V' \;\; t \in T \tag{5.3}$$

$$\sum_{t_1=t-n+1}^{t} \sum_{t_2=t+1}^{t_1+n} I_{si}^{t_1,t_2} \leq C_i \quad s \in S \;\; i \in V' \;\; t \in T \tag{5.4}$$

$$I_{si}^{t,t_1} = 0 \quad s \in S, \;\; i \in V', \;\; t \in T, \;\; t_1 \in F, \;\; t_1 > p \tag{5.5}$$

$$I_{si}^{t,t'} \geq 0 \quad i \in V' \;\; t \in T \tag{5.6}$$

$$\sum_{t_1=0}^{n} I_{si}^{0,0+t_1} = I_{si0} \quad s \in S \;\; i \in V' \tag{5.7}$$

$$\sum_{k\in K} q_{si}^{kt} \leq C_i - \sum_{t_1=t-n+1}^{t} \sum_{t_2=t+1}^{t_1+n} I_{si}^{t_1-1,t_2-1} \quad s \in S \;\; i \in V' \;\; t \in T \tag{5.8}$$

$$\sum_{i\in V'} q_{si}^{kt} \leq Q \quad s \in S \;\; k \in K \;\; t \in T \tag{5.9}$$

$$q_{si}^{kt} \leq C_i \sum_{j\in V} x_{ij}^{kt} \quad s \in S \;\; i \in V' \;\; k \in K \;\; t \in T \tag{5.10}$$

$$\sum_{j\in V} x_{ij}^{kt} = \sum_{j\in V} x_{ji}^{kt} \quad i \in V \;\; k \in K \;\; t \in T \tag{5.11}$$

$$U_j^{kt} \geq U_i^{kt} + 1 - M(1 - x_{ij}^{kt}) \quad i, j \in V' \;\; k \in K \;\; t \in T \tag{5.12}$$

$$\sum_{j\in V} x_{ij}^{kt} \leq 1 \quad i \in V' \;\; k \in K \;\; t \in T \tag{5.13}$$

$$q_{si}^{kt} \geq 0 \quad s \in S \;\; i \in V' \;\; k \in K \;\; t \in T \tag{5.14}$$

$$e_{si}^{t} \geq 0 \quad s \in S \;\; i \in V \;\; t \in T \tag{5.15}$$

$$w_{si}^{t} \geq 0 \quad s \in S \;\; i \in V \;\; t \in T \tag{5.16}$$

$$x_{ij}^{kt} \in \{0, 1\} \quad i, j \in V \;\; k \in K \;\; t \in T \tag{5.17}$$

$$U_i^{kt} = \{0, 1, 2, ...\} \quad i \in V' \;\; k \in K \;\; t \in T \tag{5.18}$$

The objective function (5.1) maximizes the expected profit of the supplier minus the routing costs. The terms for supplier revenue and inventory holding costs are dependent on scenario, so the expected profit is the summation of the values over all scenarios weighted by the known probability of each scenario. The term for routing is the same as that in the PIRP. The inventory balance constraints, (5.2) and (5.3), include the variables for excess inventory and shortage of inventory, to take care of stochastic demands for different demand scenarios. Constraints (5.4)-(5.9) are the same as constraints (4.4)-(4.9) with the addition of a subscript for each scenario s. Constraint (5.10) is the same as constraint (4.21) with the addition of subscript s. Constraints (5.11)-(5.13) are the same as constraints (4.22)-(4.24) corresponding to the new formulation. Constraints (5.14)-(5.18) maintain integrality and non-negativity for the variables.

The way this two-stage stochastic model is constructed, variable e_{si}^t should always take the value of zero if the model is feasible. However, the variable w_{si}^t can have positive values when there is a shortage. This happens when the realized demand is only partially met once the vehicle visits the particular retailer. This formulation reduces to that of the PIRP when there is just a single scenario and demand shortage is forced to be zero.

5.6 Results

5.6.1 Instances

The instances here are the same as those discussed in Section 4.6.1.

5.6.2 Parameters

The parameter values for the price of the product and discount are the same as the ones used in Section 4.7.2.

5.6.3 Scenario Generation

In addition, a set of scenarios is required for the stochastic PIRP. Those scenarios are generated via Monte Carlo simulation as in Adulyasak et al. (2015). For the results, 1000 scenarios are randomly generated, with demands varying by 0-40% from the ones given in the data set. Demand varies only in a positive direction, to avoid the case with negative inventory. This is because a negative inventory would render Constraint (5.7) infeasible, and not all the initial inventory will be used. Since the instances are equally likely, the probability of each scenario (ρ_s) is taken as 1/1000 i.e. 0.001. The scenarios are generated in a way that each retailer's demand varies with the same percentage as the scenarios change. This means that if the demand at retailer 1 in scenario 1 is 5 and that at retailer 2 is 10, for the second scenario, the corresponding demands would be $5 * (1 + x)$ and $10 * (1 + x)$. x being the percentage variation in demand.

5.6.4 Computational Experiments

Each instance is run on 1000 scenarios using both the CPLEX- based branch-and-cut and the Benders Decomposition algorithm. The Benders annotations and parameters employed for CPLEX are the same as in Section 4.7.2. The change made for the stochastic case is that each scenario has a separate subproblem. This means that the inventory variables go into 1000 subproblems, one for each demand scenario. The master problem remains the same.

Results obtained by running the stochastic model on branch-and-cut and Benders decomposition are shown in Table 5.1. Comparing the results of BC and BD from Table 5.1, it is clear that even for the stochastic version of the PIRP, the BC is better than the BD for all 20 instances. Even for the smallest instances with only five retailers, BC is able to solve each of those in times less than 100 seconds; the BD algorithm can solve only two of them and requires 7833 and 6568 seconds, respectively. That is an increase of 99% over the time taken by BC to solve the same instances. Now consider the percentage change in the gaps as seen in the last column of Table 5.1. There is a huge difference between the gaps generated by the two algorithms. The minimum difference is 51% for abs1n10, and

Data Set	BC				BD				Gap
	Obj	BB	Gap(%)	Time(s)	Obj	BB	Gap(%)	Time(s)	Diff
abs1n05	58.41	58.47	0.1	5.87	58.41	58.47	0.1	7833	0
abs1n10	200.02	201.12	0.55	TO	194.77	196.94	1.12	TO	51
abs1n15	261.72	262.32	0.23	TO	259.32	263.72	1.7	TO	86
abs1n20	335.76	338.37	0.78	TO	332.00	339.66	2.31	TO	66
abs2n05	46.01	46.05	0.1	5.27	46.01	46.05	0.1	6568	0
abs2n10	171.63	172.95	0.77	TO	169.63	174.50	2.87	TO	73
abs2n15	250.48	251.76	0.51	TO	248.18	252.96	1.92	TO	73
abs2n20	326.77	329.46	0.82	TO	325.26	330.96	1.75	TO	53
abs3n05	96.38	96.48	0.1	53.8	96.38	97.58	1.24	TO	92
abs3n10	140.46	140.78	0.23	TO	139.89	142.11	1.59	TO	86
abs3n15	263.79	264.78	0.37	TO	261.62	266.31	1.79	TO	79
abs3n20	317.39	319.69	0.72	TO	314.73	321.54	2.16	TO	67
abs4n05	57.11	57.17	0.1	20.0	57.11	57.63	0.91	TO	89
abs4n10	173.70	174.65	0.55	TO	171.64	176.02	2.55	TO	78
abs4n15	228.32	229.96	0.72	TO	224.44	231.44	3.12	TO	77
abs4n20	320.23	322.54	0.72	TO	317.06	324.33	2.29	TO	69
abs5n05	71.49	71.56	0.1	122	71.49	71.79	0.42	TO	76
abs5n10	203.80	204.02	0.11	TO	202.47	205.53	1.51	TO	93
abs5n15	219.24	220.72	0.67	TO	217.34	222.04	2.16	TO	69
abs5n20	365.37	368.92	0.97	TO	362.17	370.55	2.31	TO	58

Table 5.1: SPIRP: BC vs BD (Vehicles=2; Shelf-life=2)

Obj, BB refer to the Objective Value and Best Lower Bound respectively (both in 1000s); TO refers to Time Out; Gap Diff refers to the difference in Gaps between BC and BD (in %)

the maximum goes as high as 93% for abs5n10. There is no clear trend as to whether the BD algorithm performs better as the instances get larger.

The conclusion for the results is that BD does not work as well as BC even for the case with stochastic demands, where there is a subproblem for each scenario.

5.7 Conclusion

This chapter has introduced randomness in demand to the PIRP. Adding uncertainty through the use of several demand scenarios makes the model similar to a real life problem. This however comes at the cost of requiring a very large number of scenarios. That is demanding computationally, as seen from the results. It can be overcome by using robust optimization, which can capture the same amount of uncertainty but employing fewer scenarios, and can thus be solved in less time. Chapter 6 considers robust optimization of the SPIRP.

Chapter 6

Robust Stochastic IRP

6.1 Introduction

This chapter formulates the earlier SPIRP in a robust optimization framework. The importance of using robust optimization to solve the problem described in this thesis will be discussed. The notations and the corresponding robust formulation of the SPIRP are introduced.

6.2 Importance

Robust optimization has traditionally been applied to model the case of uncertainty where some infesibility of the problem has to be avoided. When the data are uncertain, the probability of a model having an infeasible solution increases. The usage of robust optimization to solve such a problem decreases this probability to an extent. That comes at a cost of obtaining suboptimal values for the uncertain variables. This suboptimality is preferred to the problem being infeasible.

Bertsimas and Sim (2004) define a "robust problem" as one which is immune to uncertainty. The level of conservativeness defines the likelihood of the problem being infeasible. Making the problem conservative will result in suboptimal solutions most of the time. On the other hand, removing the conservativeness from the uncertain variables will lead to many constraints being infeasible. Thus, a trade-off is required. The importance of robustness lies in the fact that the degree of conservativeness can be controlled. Therefore, depending on how infeasible the problem is, the level of protection can be chosen.

This makes it non-trivial to introduce robustness to any problem. For the problem developed in Chapter 5, many scenarios were needed to make the model account for the demand uncertainty of perishable products. Robust optimization of the model seems like a viable option to solve the difficulty of generating a large number of scenarios. The challenge here is to make sure that the robust model developed in this chapter covers the required uncertainty through the use of a small number of scenarios. This is to be done, keeping in mind that the model should not become too conservative. Thus, a new method of incorporating robustness is applied here. This is different from traditional ways in which robustness has been used. Based on the literature reviewed for this thesis, the present study is the first that aims to examine the use of robust optimization in the context of a scenario-based stochastic IRP for perishable products.

6.3 Relevant Literature

Due to the importance of robust optimization in cases where particular parameters are uncertain, there has been much focus on this subject. Robust Optimization has been applied to many different types of problems and formulations.

In the first study in this case, Soyster (1973) considers linear programming problems with uncertain parameters. The author uses robust optimization to define the feasible region in terms of sets instead of inequalities. The uncertain parameters are equal to their worst-case values in the uncertainty set. This leads to only feasible solutions, no matter what values the parameters finally take. This optimization approach is conservative; optimality is given up to a large extent, in order to ensure robustness.

To overcome this problem of overconservativeness, El Ghaoui and Lebret (1997), and El Ghaoui et al. (1998) ensured that the parameters were feasible with "high probability", rather than the probability of feasibility being 1. During the same time period, the difficulty encountered in the work of Soyster (1973) was being solved by Ben-Tal and Nemirovski (1998, 1999, 2000). All the research papers mentioned solve the issue of overconservativeness by making sure that not all uncertain parameters take their worst-case values simultaneously. Ben-Tal and Nemirovski (2000) in particular use the techniques derived by El Ghaoui and Lebret (1997), El Ghaoui et al. (1998), and Ben-Tal and Nemirovski (1998, 1999) to solve over 90 LP models, in order to prove that their approach improves on that of Soyster (1973). They do so and get nearly-optimal solutions at a much lower cost. The major issue with the above-mentioned papers is that, although they overcome the difficulty of overconservativeness, the resulting model is quadratic as opposed to a linear model in the case of Soyster (1973). This leads to more computational time to solve the model.

Bertsimas and Sim (2004) present a model that takes care of both the above-mentioned restrictions of robust optimization. The new approach retains the advantages of the linear framework of Soyster (1973), along with solving the problem of overconservativeness. A new parameter is introduced to adjust the robustness of the method depending on the conservativeness of the solution. This parameter, called the “budget-of-uncertainty”, is denoted in the paper by Γ_i (where i refers to the constraint). That parameter makes sure that only some of the constraint coefficients change from the original problem, and then the model is deterministically feasible. The parameter (Γ_i) allows flexibility in the sense that, even when the number of coefficients changes more than expected, the constraints are feasible with high probability. This means that the problem remains linear and is not too conservative.

Bertsimas and Sim (2004) illustrate the use of Γ_i by running experiments on three different problems with data uncertainty. Through the results of these experiments, Bertsimas and Sim (2004) prove that as the uncertainty in the problems increases, their method becomes more attractive. This approach to robust optimization is one of the most successful; most of the subsequent research has been based on this method. Their model is known as the “budget-of-uncertainty” model.

Bertsimas and Thiele (2006) were the first to apply robust optimization to the field of inventory management. They use the budget-of-uncertainty approach of Bertsimas and Sim (2004) to solve a stochastic inventory management problem with the distribution of demand unknown. Bertsimas and Thiele (2006) optimize the inventory of a single product at one retailer in two ways, i.e. with and without bounds on the capacity at the retailer. They formulate the robust counterpart of the formulation with and without fixed costs. With no fixed costs, the result is an LP model, and in the case of fixed costs it is an MIP model. The authors conclude that if the parameters of the robust optimization are chosen appropriately, both the feasibility and the performance of the problem at hand can be maintained.

Further work has been done on robust optimization in the field of inventory management; some of these papers are described below. Bienstock and ÖZbay (2008) propose a

Benders decomposition algorithm to find the base stock robust policy for a single echelon inventory problem, where the base stock policy is used to obtain the retailer OU inventory level. See and Sim (2010) consider a single-echelon inventory management problem with nonzero lead times where uncertain demand is characterized by the mean, support, covariance, and directional deviations. These authors formulate the problem as a stochastic optimization model, and approximate it using robust optimization which leads to a non-linear model. See and Sim (2010) conclude that the budget-of-uncertainty model performs less efficiently in their case, when the characteristics of demand are known as a result of forecasting. Even though the resulting model is a non-linear one, they conclude that it can be solved in polynomial time.

Robust optimization was introduced to the subject of IRP by Solyalı et al. (2012), who employ the budget-of-uncertainty model to solve the IRP with uncertain demand. Robustness is applied since the distribution of demand is not known. The unmet demand is backlogged and has a penalty associated with it. When demand is lower than expected, the remaining stock goes into inventory and can be used for satisfying demand for the next time period. There is a fixed cost related to vehicle dispatching. The objective in this case is to minimize the sum of routing, vehicle dispatching, holding, and backlogging costs. Robustness is applied to the model such that even under the worst case scenario of demand, the vehicle capacity is not violated. The nominal formulation and the robust counterpart are solved using branch-and-cut. Solyalı et al. (2012) conclude that the use of robust optimization for the IRP with backlogging protects the problem under uncertainty with only a slightly higher cost than the nominal formulation.

Rahbari et al. (2017) apply robust optimization in the context of the perishable IRP with lost sales. They build on the model developed by Mirzaei and Seifi (2015), and consider multiple products. Demand is a linearly or exponentially decreasing function of the product's age. The authors employ the budget-of-uncertainty model for the robust optimization with two uncertain parameters, namely, the demands and the travel times between nodes. Based on the travel times, the cost of travel between nodes becomes uncertain. The robust model is solved using GAMS. Rahbari et al. (2017) conclude that, for smaller instances, the robust approach is worse than the deterministic model. However, when simulation is performed and large size instances of IRP are considered, the robust model works better than the deterministic one.

6.4 Description

According to Bertsimas et al. (2018), "The strength of a robust optimization lies in choosing the right uncertainty set of possible realizations of the uncertain parameters. If the set is chosen correctly, the robust approach will help in optimizing against the worst-case realizations within the chosen set. Computational experience suggests that with well-chosen sets, robust models yield tractable optimization problems whose solutions perform as well or better than other approaches. With poorly chosen sets, however, robust models may be overly-conservative or computationally intractable".

As can be seen from Chapter 5, SPIRP requires many scenarios to highlight the uncertainty of the data for perishable products. The assumption in the previous chapter was that each scenario is equally probable. This assumption was made because obtaining the true probability of each scenario is difficult. With a large number of scenarios used, taking probabilities as equal is not a bad assumption. However, seldom would this occur in the real world. These limitations of the SPIRP need to be countered to make the problem similar to a real-world case. Robust optimization applied to the SPIRP is a viable alternative to those model limitations.

The works of Solyalı et al. (2012) and Rahbari et al. (2017), both of which apply the budget-of-uncertainty approach to the IRP, had demand as one of the uncertain parameters. This led to uncertainty in the objective function as well as in the constraints of the formulation. Robust optimization was employed to make sure that the error resulting from the worst-case scenario of demand is minimized.

In this thesis, the probabilities in the model of SPIRP are unknown, and the robust optimization approach of Bertsimas and Sim (2004) is used to model that uncertainty. This model will be the robust counterpart of the SPIRP. The motivation behind making the model robust is to be able to overcome the limitations of the preceding stochastic model. Through the use of robust optimization, a small number of scenarios can be used even when the correct probability is not estimated accurately. Secondly, the solutions obtained from solving the robust model are analogous to solving the stochastic model with a larger number of scenarios.

The model described in this chapter is the same as the one for the stochastic PIRP. There is a finite number of demand scenarios, and parameters pertaining to robustness are added to the model. The uncertainty parameter considered here is the probability of each scenario (ρ_s). This means that the component of uncertainty is only in the objective function, not in the constraints.

The goal of any robust formulation is to protect the model against the worst-case scenario of its uncertain parameters. This results in maximizing the deviation of the uncertain parameter from the nominal value, so as to make this parameter take its worst-case value. This also protects the model against parameter values that makes it infeasible. SPIRP has a maximization objective, and to make the problem robust and protect the problem from worst case values of ρ_s, the deviation is minimized in the maximization objective, utilizing a maximin approach.

6.5 Mathematical Model

As mentioned, the Robust SPIRP model is formed by adding the robust parameters as described in Bertsimas and Sim (2004) to the earlier model. That robust counterpart enables protection from the extreme scenarios that lead to undesirable values of profit. The robust formulation is therefore constructed by reformulating the objective function of the SPIRP, with the corresponding addition of robust constraints.

6.5.1 Notations and Robust Formulation

The robust formulation is constructed using five steps. We could have gone directly to the final formulation whose objective function is in equation (6.1). However, five steps enable a presentation of greater clarity in showing how we obtained the robust extension of the SPIRP.

At each step, robust parameters are added to the the SPIRP objective (5.1). The associated notation is now described further.

First Step

For the first step, the nominal objective (5.1) is minimized over the uncertain parameter, ρ_s.

$$\text{maximize } \Big\{ \min_{\rho_s} \Big[\sum_{s \in S} \rho_s \big(\sum_{i \in V'} \sum_{t \in M} \sum_{\tau = t}^{t+n} P(1 - \theta(\tau - t)) I_{si}^{t,\tau} - \sum_{i \in V'} \sum_{t \in M} \sum_{t_1 = t-n+1}^{t} \sum_{t_2 = t+1}^{t_1+n} h_i(I_{si}^{t_1,t_2}) \big) \Big] - \sum_{i \in V} \sum_{j \in V} \sum_{k \in K} \sum_{t \in T} c_{ij} x_{ij}^{kt} \Big\} \tag{6.1}$$

subject to   (5.2) – (5.18)

The objective (6.1) minimizes the effect of extreme scenarios on supplier's profit. Minimization of the objective function over ρ_s will force extreme scenarios to take low probabilities.

Second Step

The notations introduced are:

- $\overline{\rho}_s$ is the nominal value of ρ_s
- $\hat{\rho}_s$ is the maximum deviation from the nominal value

The new formulation is:

$$\begin{aligned}\text{maximize } \Big\{ &\sum_{s\in S}\overline{\rho}_s(\sum_{i\in V'}\sum_{t\in M}\sum_{\tau=t}^{t+n} P(1-\theta(\tau-t))I_{si}^{t,\tau} - \sum_{i\in V'}\sum_{t\in M}\sum_{t_1=t-n+1}^{t}\sum_{t_2=t+1}^{t_1+n} h_i(I_{si}^{t_1,t_2}))- \\ &\min_{\hat{\rho}_s}\Big[\sum_{s\in S}\hat{\rho}_s\Big|(\sum_{i\in V'}\sum_{t\in M}\sum_{\tau=t}^{t+n} P(1-\theta(\tau-t))I_{si}^{t,\tau} - \sum_{i\in V'}\sum_{t\in M}\sum_{t_1=t-n+1}^{t}\sum_{t_2=t+1}^{t_1+n} h_i(I_{si}^{t_1,t_2}))\Big|\Big] \\ &-\sum_{i\in V}\sum_{j\in V}\sum_{k\in K}\sum_{t\in T} c_{ij}x_{ij}^{kt}\Big\} \end{aligned} \tag{6.2}$$

subject to   (5.2) – (5.18),

The objective function (6.2) replaces the original probability of each scenario by the nominal value and the deviation from the nominal value. ρ_s will take values in the range $[\overline{\rho}_s - \hat{\rho}_s, \overline{\rho}_s + \hat{\rho}_s]$. Both the parameters $\overline{\rho}_s$ and $\hat{\rho}_s$ are constants, and the probabilities over all scenarios in the robust formulation will sum to 1. In the objective function, the scenarios which lead to undesirable values for profit are removed by minimizing the impact of those scenarios on the overall profit. The expression is minimized over the deviation $\hat{\rho}_s$ to optimize against the worst-case scenario.

Third Step

Further notations introduced here are:

- Γ is the budget-of-uncertainty parameter
- η_s is the decision variable to decide whether the probability of scenario s goes to its worst-case value or not.

The formulation is:

$$
\begin{aligned}
\text{maximize } \Big\{ & \sum_{s\in S} \overline{\rho}_s \Big(\sum_{i\in V'} \sum_{t\in M} \sum_{\tau=t}^{t+n} P(1-\theta(\tau-t)) I_{si}^{t,\tau} - \sum_{i\in V'} \sum_{t\in M} \sum_{t_1=t-n+1}^{t} \sum_{t_2=t+1}^{t_1+n} h_i(I_{si}^{t_1,t_2}) \Big) \\
& - \min_{\eta_s} \Big[\sum_{s\in S} \hat{\rho}_s \eta_s \Big| \Big(\sum_{i\in V'} \sum_{t\in M} \sum_{\tau=t}^{t+n} P(1-\theta(\tau-t)) I_{si}^{t,\tau} \\
& - \sum_{i\in V'} \sum_{t\in M} \sum_{t_1=t-n+1}^{t} \sum_{t_2=t+1}^{t_1+n} h_i(I_{si}^{t_1,t_2}) \Big) \Big| \Big] - \sum_{i\in V} \sum_{j\in V} \sum_{k\in K} \sum_{t\in T} c_{ij} x_{ij}^{kt} \Big\}
\end{aligned} \tag{6.3}
$$

subject to $(5.2)-(5.18)$, and,

$$\sum_{s\in S} \eta_s \leq \Gamma \tag{6.4}$$

$$0 \leq \eta_s \leq 1 \quad s\in S \tag{6.5}$$

The benefit of making the model robust diminishes if all the values of uncertain parameters are allowed to go to their worst case. In this robust model, if every scenario is allowed to take those values, the objective function value would be extremely low. This would mean that the model is overconservative.

To allow for a degree of control to avoid overconservativess, the budget-of-uncertainty parameter (Γ) is added to the formulation. Also, since there is a finite number of scenarios, a probability should be specified as to how many scenarios go to their worst case, so as to manage the conservativeness of the model. This parameter allows only a fixed number of scenarios to take their worst case values. It is essential to note that Γ is a constant since the parameter of uncertainty (ρ_s) only concerns the objective function.

In the objective function (6.3), a decision concerning whether any scenario goes to its worst case is added for each scenario. The constraint (6.4) specifies that the number of scenarios going to their worst-case values does not exceed the budget-of-uncertainty parameter. Constraint (6.5) defines the bounds on η_s.

Fourth Step

The strong duality theorem, as given by Bertsimas and Sim (2004), is used to obtain the dual of the previous formulation.

The notations used are:

- g is the dual variable for constraint (6.4)
- f_s is the dual variable for constraint (6.5)

The dual of the original formulation is:

$$\text{maximize } \Big\{ \sum_{s\in S} \overline{\rho}_s (\sum_{i\in V'} \sum_{t\in M} \sum_{\tau=t}^{t+n} P(1-\theta(\tau-t)) I_{si}^{t,\tau} - \sum_{i\in V'} \sum_{t\in M} \sum_{t_1=t-n+1}^{t} \sum_{t_2=t+1}^{t_1+n} h_i(I_{si}^{t_1,t_2}))$$
$$+ \max \Big[\sum_{s\in S} f_s + \Gamma g \Big] - \sum_{i\in V} \sum_{j\in V} \sum_{k\in K} \sum_{t\in T} c_{ij} x_{ij}^{kt} \Big\} \tag{6.6}$$

subject to (5.2) − (5.18), (6.4) − (6.5), and,

$$f_s + g \leq -\hat{\rho}_s \Big| (\sum_{i\in V'} \sum_{t\in M} \sum_{\tau=t}^{t+n} P(1-\theta(\tau-t)) I_{si}^{t,\tau}$$
$$- \sum_{i\in V'} \sum_{t\in M} \sum_{t_1=t-n+1}^{t} \sum_{t_2=t+1}^{t_1+n} h_i(I_{si}^{t_1,t_2})) \Big| \quad s \in S \tag{6.7}$$

$$f_s \leq 0 \quad s \in S \tag{6.8}$$

$$g \leq 0 \tag{6.9}$$

The objective function (6.6), and constraints (6.7)-(6.9) give the dual of the minimization portion of (6.3)-(6.5).

Final Robust Formulation

The final formulation combines the two maximization functions in the objective function (6.6), and removes the absolute value function from constraint (6.7) by adding a new parameter v_s.

The final robust formulation is given as:

$$\text{maximize } \Big\{ \sum_{s\in S} \overline{\rho}_s (\sum_{i\in V'} \sum_{t\in M} \sum_{\tau=t}^{t+n} P(1-\theta(\tau-t)) I_{si}^{t,\tau} - \sum_{i\in V'} \sum_{t\in M} \sum_{t_1=t-n+1}^{t} \sum_{t_2=t+1}^{t_1+n} h_i(I_{si}^{t_1,t_2}))$$

$$+\sum_{s\in S} f_s + \Gamma g - \sum_{i\in V}\sum_{j\in V}\sum_{k\in K}\sum_{t\in T} c_{ij}x_{ij}^{kt}\Big\} \tag{6.10}$$

subject to

$$-f_s - g \geq \hat{\rho} v_s \quad \forall s \in S \tag{6.11}$$

$$v_s \geq (\sum_{i\in V'}\sum_{t\in M}\sum_{\tau=t}^{t+n} P(1-\theta(\tau-t))I_{si}^{t,\tau} - \sum_{i\in V'}\sum_{t\in M}\sum_{t_1=t-n+1}^{t}\sum_{t_2=t+1}^{t_1+n} h_i(I_{si}^{t_1,t_2})) \tag{6.12}$$

$$v_s \geq -(\sum_{i\in V'}\sum_{t\in M}\sum_{\tau=t}^{t+n} P(1-\theta(\tau-t))I_{si}^{t,\tau} - \sum_{i\in V'}\sum_{t\in M}\sum_{t_1=t-n+1}^{t}\sum_{t_2=t+1}^{t_1+n} h_i(I_{si}^{t_1,t_2})) \tag{6.13}$$

$$d_{si}^t = \sum_{t_1=0}^{n} I_{si}^{t-t_1,t} + w_{si}^t \quad s\in S \;\; i\in V' \;\; t\in T \tag{6.14}$$

$$\sum_{k\in K} q_{si}^{kt} = \sum_{t_1=0}^{n} I_{si}^{t,t+t_1} + e_{si}^t \quad s\in S \;\; i\in V' \;\; t\in T \tag{6.15}$$

$$\sum_{t_1=t-n+1}^{t}\sum_{t_2=t+1}^{t_1+n} I_{si}^{t_1,t_2} \leq C_i \quad s\in S \;\; i\in V' \;\; t\in T \tag{6.16}$$

$$I_{si}^{t,t_1} = 0 \quad s\in S \;\; i\in V' \;\; t\in T \;\; t_1\in F \;\; t_1 > p \tag{6.17}$$

$$I_{si}^{t,t'} \geq 0 \quad i\in V' \;\; t\in T \tag{6.18}$$

$$\sum_{t_1=0}^{n} I_{si}^{0,0+t_1} = I_{si0} \quad s\in S \;\; i\in V' \tag{6.19}$$

$$\sum_{k\in K} q_{si}^{kt} \leq C_i - \sum_{t_1=t-n+1}^{t}\sum_{t_2=t+1}^{t_1+n} I_{si}^{t_1-1,t_2-1} \quad s\in S \;\; i\in V' \;\; t\in T \tag{6.20}$$

$$\sum_{i\in V'} q_{si}^{kt} \leq Q \quad s\in S \;\; k\in K \;\; t\in T \tag{6.21}$$

$$q_{si}^{kt} \leq C_i \sum_{j\in V} x_{ij}^{kt} \quad s\in S \;\; i\in V' \;\; k\in K \;\; t\in T \tag{6.22}$$

$$\sum_{j\in V} x_{ij}^{kt} = \sum_{j\in V} x_{ji}^{kt} \quad i\in V \;\; k\in K \;\; t\in T \tag{6.23}$$

$$U_j^{kt} \geq U_i^{kt} + 1 - M(1 - x_{ij}^{kt}) \quad i, j \in V' \;\; k \in K \;\; t \in T \tag{6.24}$$

$$\sum_{j \in V} x_{ij}^{kt} \leq 1 \quad i \in V' \;\; k \in K \;\; t \in T \tag{6.25}$$

$$q_{si}^{kt} \geq 0 \quad s \in S \;\; i \in V' \;\; k \in K \;\; t \in T \tag{6.26}$$

$$e_{si}^{t} \geq 0 \quad s \in S \;\; i \in V \;\; t \in T \tag{6.27}$$

$$w_{si}^{t} \geq 0 \quad s \in S \;\; i \in V \;\; t \in T \tag{6.28}$$

$$x_{ij}^{kt} \in \{0, 1\} \quad i, j \in V \;\; k \in K \;\; t \in T \tag{6.29}$$

$$U_i^{kt} = \{0, 1, 2, ...\} \quad i \in V' \;\; k \in K \;\; t \in T \tag{6.30}$$

$$f_s \leq 0 \quad s \in S \tag{6.31}$$

$$g \leq 0 \tag{6.32}$$

The objective function (6.10) is the final objective function obtained from the set of formulations defined above. Constraint (6.7) is replaced by constraints (6.11)-(6.13) and the absolute value is removed. Constraints (6.14)-(6.30) for the Robust Stochastic PIRP are the same as constraints (5.2)-(5.18) for the SPIRP. Constraints (6.31)-(6.32) limit the values that the dual variables can take.

6.6 Conclusion

This chapter has introduced a new way, robust optimization was used to formulate the SPIRP. The benefits of using robust optimization will be quantified next. Chapter 7 will display the results of a comparison between deterministic PIRP, stochastic PIRP, and robust stochastic PIRP.

Chapter 7

Results

7.1 Instances

The instances studied here are based on Archetti et al. (2007). Section 4.6.1 contains more information on those instances.

7.2 Parameter values

Parameters for scenario generation and price are the same as the ones mentioned in Section 5.6.2. For the robust version of the SPIRP, the budget-of-uncertainty parameter (Γ) is taken as 5. For SPIRP (Section 5.6.2), the probability taken was "$1/|S|$"($|S|$ being the number of scenarios in consideration), which means no robustness. To add robustness to the model, a deviation segment needs to be defined for the probability of each scenario of the SPIRP.

Tests were done to to see what probability deviation segment would make the model robust, while not adding too much conservativeness to the model. Probabilities varying from $1/(1.2 * |RS|)$ to $1/(2 * |RS|)$ ($|RS|$ refers to the number of robust scenarios in this case) were employed for the tests. When the value was taken as half the original probability, the objective function value of the robust model came out to be 0. This meant that the model is very robust and is not able to produce any results. A probability of $1/(1.2*|RS|)$ made the model too conservative, as it was too close to the original probability. Finally, the value of $1/(1.5*|RS|)$ was chosen as the appropriate probability that made the model robust, but not too conservative. The value of the deviation ($\hat{\rho}_s$) depends on Γ and $\overline{\rho}_s$. This is done to make sure that the probabilities, when optimized, will always sum up to 1.

7.3 Computational Experiments

The following algorithm is used to generate the result tables:

1. A set of 10,000 demand scenarios is generated from the demand in each instance
2. For each data set, the deterministic model is solved and the routes are generated
3. Similarly, the stochastic model and the robust model are solved, and the routes are generated
4. The time and the corresponding objective values while solving all three of them with the original demands of the data set are recorded
5. The deterministic model is run for all 10,000 demand scenarios, once each for the routes generated in the first run by the deterministic, stochastic, and robust models

6. Min, max, and mean of the objective values over all the scenarios are calculated for each model
7. Also, the last 3 columns of the table, comparing the stochastic with the deterministic, the robust with the deterministic, and the stochastic with the robust, are calculated each time the model is run.

Within the algorithm mentioned above, the results are obtained for the deterministic, the stochastic, and the robust models. The deterministic model is the same as the one generated using the new formulation, used for calculating the final results in Chapter 4 (Section 4.7.2). The stochastic model is the one in Section 5.6, applying the SPIRP formulation on 1000 demand scenarios. The model for the Robust Stochastic case is created with the final robust formulation in Chapter 6. Results are obtained by varying the number of scenarios of the robust model, from 10-100 for each of the 20 instances. The scenarios used for the robust model are from a random subset of the 1000 demand scenarios employed in running the stochastic model.

Here is the insight behind using a subset of the stochastic demand scenarios for the robust model, and varying the demand scenarios for the robust for each instance. This enables a fair comparison between the robust model and the stochastic model. Also, the number of scenarios for the robust varies from 1% to 10% of the number of scenarios taken for the stochastic model. This is done to test the effectiveness of the robust model over a considerably lower number of scenarios than the stochastic.

Results for the 20 instances run on the algorithm mentioned above, with a varying number of robust scenarios (S in each table), are presented in Tables 7.1 - 7.20.

To see the significance of changing the number of robust scenarios on the means of the robust formulation, regression was done. The plot for regression is shown in Figure 7.1.

S	Det					St					Rb					St>	Rb>	St>
	Obj	Time	Min	Max	Mean	Obj	Time	Min	Max	Mean	Obj	Time	Min	Max	Mean	Det	Det	Rb
10	47.20	0.08	47.20	54.72	51.58	58.41	12.63	46.47	69.97	58.29	15.46	0.52	46.55	66.74	56.88	94	94	95
20	47.20	0.08	47.20	54.72	51.58	58.41	12.63	46.47	69.97	58.29	15.72	2.53	46.15	63.84	56.21	94	90	100
30	47.20	0.08	47.20	54.72	51.58	58.41	12.63	46.47	69.97	58.29	15.98	3.07	46.15	63.84	56.21	94	90	100
40	47.20	0.08	47.20	54.72	51.58	58.41	12.63	46.47	69.97	58.29	16.22	2.81	46.15	63.84	56.21	94	90	100
50	47.20	0.08	47.20	54.72	51.58	58.41	12.63	46.47	69.97	58.29	16.27	2.66	46.15	63.28	56.05	94	90	100
60	47.20	0.08	47.20	54.72	51.58	58.41	12.63	46.47	69.97	58.29	16.28	2.63	46.15	63.84	56.21	94	90	100
70	47.20	0.08	47.20	54.72	51.58	58.41	12.63	46.47	69.97	58.29	16.31	3.05	46.15	63.28	56.05	94	90	100
80	47.20	0.08	47.20	54.72	51.58	58.41	12.63	46.47	69.97	58.29	16.22	4.7	46.15	63.28	56.05	94	90	100
90	47.20	0.08	47.20	54.72	51.58	58.41	12.63	46.47	69.97	58.29	16.21	6.55	46.15	63.84	56.21	94	90	100
100	47.20	0.08	47.20	54.72	51.58	58.41	12.63	46.47	69.97	58.29	16.37	4.44	46.15	63.84	56.21	94	90	100

Table 7.1: Results for abs1n05 (Vehicles=2; Shelf-life=2)

Obj, Min, Max, Mean refer to the Objective Value, and the Min, Max, and Mean when run over 10000 scenarios (in 1000s); TO refers to Time Out; Time for the rest is in seconds; $St > Det$, $Rb > Det$, $St > Rb$ refer to instances where Stochastic is greater than Deterministic, Robust is greater than Deterministic, and Stochastic is greater than Robust (in %)

S	Det					St					Rb					St>	Rb>	St>
	Obj	Time	Min	Max	Mean	Obj	Time	Min	Max	Mean	Obj	Time	Min	Max	Mean	Det	Det	Rb
10	162.17	830	162.17	193.83	180.77	200.03	TO	160.29	239.07	199.58	54.87	TO	161.68	200.42	187.94	95	98	87
20	162.17	830	162.17	193.83	180.77	200.03	TO	160.29	239.07	199.58	55.69	TO	161.68	200.42	187.94	95	98	87
30	162.17	830	162.17	193.83	180.77	200.03	TO	160.29	239.07	199.58	56.45	TO	161.68	200.42	187.94	95	98	87
40	162.17	830	162.17	193.83	180.77	200.03	TO	160.29	239.07	199.58	57.19	TO	161.62	202.36	188.86	95	98	87
50	162.17	830	162.17	193.83	180.77	200.03	TO	160.29	239.07	199.58	57.36	TO	161.68	200.42	187.94	95	98	87
60	162.17	830	162.17	193.83	180.77	200.03	TO	160.29	239.07	199.58	57.40	TO	161.68	200.42	187.94	95	98	87
70	162.17	830	162.17	193.83	180.77	200.03	TO	160.29	239.07	199.58	57.37	TO	161.55	207.19	190.73	95	98	87
80	162.17	830	162.17	193.83	180.77	200.03	TO	160.29	239.07	199.58	57.16	TO	161.68	200.42	187.94	95	98	87
90	162.17	830	162.17	193.83	180.77	200.03	TO	160.29	239.07	199.58	57.08	TO	161.60	202.34	188.84	95	98	87
100	162.17	830	162.17	193.83	180.77	200.03	TO	160.29	239.07	199.58	57.50	TO	161.55	207.19	190.73	95	98	87

Table 7.2: Results for abs1n10 (Vehicles=2; Shelf-life=2)

Obj, Min, Max, Mean refer to the Objective Value, and the Min, Max, and Mean when run over 10000 scenarios (in 1000s); TO refers to Time Out; Time for the rest is in seconds; $St > Det$, $Rb > Det$, $St > Rb$ refer to instances where Stochastic is greater than Deterministic, Robust is greater than Deterministic, and Stochastic is greater than Robust (in %)

S	Det					St					Rb					St>	Rb>	St>
	Obj	Time	Min	Max	Mean	Obj	Time	Min	Max	Mean	Obj	Time	Min	Max	Mean	Det	Det	Rb
10	212.11	4641	212.11	252.16	236.13	261.80	TO	210.20	311.71	261.24	72.25	1652	211.74	259.42	244.35	96	98	88
20	212.11	4641	212.11	252.16	236.13	261.80	TO	210.20	311.71	261.24	73.23	TO	211.33	271.55	248.93	96	98	93
30	212.11	4641	212.11	252.16	236.13	261.80	TO	210.20	311.71	261.24	74.16	TO	211.48	275.60	251.53	96	98	88
40	212.11	4641	212.11	252.16	236.13	261.80	TO	210.20	311.71	261.24	75.21	TO	211.46	287.12	254.64	96	98	88
50	212.11	4641	212.11	252.16	236.13	261.80	TO	210.20	311.71	261.24	75.36	TO	211.48	277.99	252.53	96	98	88
60	212.11	4641	212.11	252.16	236.13	261.80	TO	210.20	311.71	261.24	75.44	TO	211.46	287.12	254.64	96	98	88
70	212.11	4641	212.11	252.16	236.13	261.80	TO	210.20	311.71	261.24	75.54	TO	210.63	306.62	260.41	96	97	49
80	212.11	4641	212.11	252.16	236.13	261.80	TO	210.20	311.71	261.24	74.93	TO	210.43	294.16	257.37	96	97	83
90	212.11	4641	212.11	252.16	236.13	261.80	TO	210.20	311.71	261.24	74.96	TO	211.19	281.01	253.25	96	98	91
100	212.11	4641	212.11	252.16	236.13	261.80	TO	210.20	311.71	261.24	75.74	TO	211.46	287.12	254.64	96	98	88

Table 7.3: Results for abs1n15 (Vehicles=2; Shelf-life=2)

Obj, Min, Max, Mean refer to the Objective Value, and the Min, Max, and Mean when run over 10000 scenarios (in 1000s); TO refers to Time Out; Time for the rest is in seconds; $St > Det$, $Rb > Det$, $St > Rb$ refer to instances where Stochastic is greater than Deterministic, Robust is greater than Deterministic, and Stochastic is greater than Robust (in %)

S	Det					St					Rb					St>	Rb>	St>
	Obj	Time	Min	Max	Mean	Obj	Time	Min	Max	Mean	Obj	Time	Min	Max	Mean	Det	Det	Rb
10	272.55	TO	272.55	315.10	299.01	336.03	TO	269.26	401.76	335.30	92.97	TO	272.04	346.65	317.12	95	99	86
20	272.55	TO	272.55	315.10	299.01	336.03	TO	269.26	401.76	335.30	94.21	TO	271.79	350.42	320.59	95	98	85
30	272.55	TO	272.55	315.10	299.01	336.03	TO	269.26	401.76	335.30	95.13	TO	271.52	335.45	315.01	95	98	88
40	272.55	TO	272.55	315.10	299.01	336.03	TO	269.26	401.76	335.30	96.74	TO	271.76	353.54	320.77	95	98	85
50	272.55	TO	272.55	315.10	299.01	336.03	TO	269.26	401.76	335.30	96.91	TO	271.77	338.70	316.72	95	98	85
60	272.55	TO	272.55	315.10	299.01	336.03	TO	269.26	401.76	335.30	97.18	TO	270.69	368.45	329.11	95	97	58
70	272.55	TO	272.55	315.10	299.01	336.03	TO	269.26	401.76	335.30	96.56	TO	269.90	399.30	334.65	95	96	75
80	272.55	TO	272.55	315.10	299.01	336.03	TO	269.26	401.76	335.30	96.50	TO	271.63	335.55	315.12	95	98	86
90	272.55	TO	272.55	315.10	299.01	336.03	TO	269.26	401.76	335.30	96.59	TO	271.73	353.51	320.74	95	98	86
100	272.55	TO	272.55	315.10	299.01	336.03	TO	269.26	401.76	335.30	97.44	TO	270.42	387.78	333.28	95	97	53

Table 7.4: Results for abs1n20 (Vehicles=2; Shelf-life=2)

Obj, Min, Max, Mean refer to the Objective Value, and the Min, Max, and Mean when run over 10000 scenarios (in 1000s); TO refers to Time Out; Time for the rest is in seconds; $St > Det$, $Rb > Det$, $St > Rb$ refer to instances where Stochastic is greater than Deterministic, Robust is greater than Deterministic, and Stochastic is greater than Robust (in %)

S	Det					St					Rb					St>	Rb>	St>
	Obj	Time	Min	Max	Mean	Obj	Time	Min	Max	Mean	Obj	Time	Min	Max	Mean	Det	Det	Rb
10	36.73	0.11	36.73	42.71	40.25	46.00	TO	36.09	55.58	45.90	11.73	3.14	35.96	46.86	43.35	94	94	100
20	36.73	0.11	36.73	42.71	40.25	46.00	TO	36.09	55.58	45.90	11.93	2.91	35.96	46.86	43.35	94	94	100
30	36.73	0.11	36.73	42.71	40.25	46.00	TO	36.09	55.58	45.90	12.14	5.4	35.96	46.86	43.35	94	94	100
40	36.73	0.11	36.73	42.71	40.25	46.00	TO	36.09	55.58	45.90	12.36	3.38	35.96	46.86	43.35	94	94	100
50	36.73	0.11	36.73	42.71	40.25	46.00	TO	36.09	55.58	45.90	12.39	4.94	35.96	46.86	43.35	94	94	100
60	36.73	0.11	36.73	42.71	40.25	46.00	TO	36.09	55.58	45.90	12.40	12.0	35.96	46.86	43.35	94	94	100
70	36.73	0.11	36.73	42.71	40.25	46.00	TO	36.09	55.58	45.90	12.43	9.02	35.96	46.86	43.35	94	94	100
80	36.73	0.11	36.73	42.71	40.25	46.00	TO	36.09	55.58	45.90	12.35	6.88	35.96	46.86	43.35	94	94	100
90	36.73	0.11	36.73	42.71	40.25	46.00	TO	36.09	55.58	45.90	12.35	9.11	35.96	46.86	43.35	94	94	100
100	36.73	0.11	36.73	42.71	40.25	46.00	TO	36.09	55.58	45.90	12.48	7.29	35.96	46.86	43.35	94	94	100

Table 7.5: Results for abs2n05 (Vehicles=2; Shelf-life=2)

Obj, Min, Max, Mean refer to the Objective Value, and the Min, Max, and Mean when run over 10000 scenarios (in 1000s); TO refers to Time Out; Time for the rest is in seconds; $St > Det$, $Rb > Det$, $St > Rb$ refer to instances where Stochastic is greater than Deterministic, Robust is greater than Deterministic, and Stochastic is greater than Robust (in %)

S	Det					St					Rb					St>	Rb>	St>
	Obj	Time	Min	Max	Mean	Obj	Time	Min	Max	Mean	Obj	Time	Min	Max	Mean	Det	Det	Rb
10	140.15	208	140.15	158.10	152.51	171.58	TO	137.64	205.19	171.20	46.60	TO	140.06	158.01	152.66	90	45	88
20	140.15	208	140.15	158.10	152.51	171.58	TO	137.64	205.19	171.20	47.09	TO	139.85	166.81	157.66	90	93	88
30	140.15	208	140.15	158.10	152.51	171.58	TO	137.64	205.19	171.20	47.61	TO	139.85	166.81	157.66	90	93	88
40	140.15	208	140.15	158.10	152.51	171.58	TO	137.64	205.19	171.20	48.13	TO	138.81	192.73	167.20	90	92	85
50	140.15	208	140.15	158.10	152.51	171.58	TO	137.64	205.19	171.20	48.23	TO	138.81	192.73	167.20	90	92	85
60	140.15	208	140.15	158.10	152.51	171.58	TO	137.64	205.19	171.20	48.22	TO	138.88	189.83	166.42	90	92	85
70	140.15	208	140.15	158.10	152.51	171.58	TO	137.64	205.19	171.20	48.27	TO	138.16	201.28	170.17	90	92	62
80	140.15	208	140.15	158.10	152.51	171.58	TO	137.64	205.19	171.20	47.98	TO	138.16	198.93	170.10	90	92	62
90	140.15	208	140.15	158.10	152.51	171.58	TO	137.64	205.19	171.20	48.01	TO	139.78	166.73	157.59	90	93	88
100	140.15	208	140.15	158.10	152.51	171.58	TO	137.64	205.19	171.20	48.47	TO	138.81	193.62	167.21	90	92	85

Table 7.6: Results for abs2n10 (Vehicles=2; Shelf-life=2)

Obj, Min, Max, Mean refer to the Objective Value, and the Min, Max, and Mean when run over 10000 scenarios (in 1000s); TO refers to Time Out; Time for the rest is in seconds; $St > Det$, $Rb > Det$, $St > Rb$ refer to instances where Stochastic is greater than Deterministic, Robust is greater than Deterministic, and Stochastic is greater than Robust (in %)

S	Det					St					Rb					St>	Rb>	St>
	Obj	Time	Min	Max	Mean	Obj	Time	Min	Max	Mean	Obj	Time	Min	Max	Mean	Det	Det	Rb
10	203.54	TO	203.54	235.88	224.86	250.59	TO	201.32	298.07	250.04	68.99	TO	203.36	235.71	226.30	94	64	89
20	203.54	TO	203.54	235.88	224.86	250.59	TO	201.32	298.07	250.04	69.86	TO	203.28	251.42	234.02	94	98	89
30	203.54	TO	203.54	235.88	224.86	250.59	TO	201.32	298.07	250.04	70.59	TO	203.01	257.15	235.19	94	98	89
40	203.54	TO	203.54	235.88	224.86	250.59	TO	201.32	298.07	250.04	71.73	TO	201.63	293.75	249.44	94	95	38
50	203.54	TO	203.54	235.88	224.86	250.59	TO	201.32	298.07	250.04	71.95	TO	202.20	281.94	246.30	94	96	84
60	203.54	TO	203.54	235.88	224.86	250.59	TO	201.32	298.07	250.04	71.97	TO	201.96	286.08	247.35	94	96	70
70	203.54	TO	203.54	235.88	224.86	250.59	TO	201.32	298.07	250.04	72.06	TO	201.86	290.03	247.95	94	96	62
80	203.54	TO	203.54	235.88	224.86	250.59	TO	201.32	298.07	250.04	71.67	TO	202.08	277.36	245.70	94	96	79
90	203.54	TO	203.54	235.88	224.86	250.59	TO	201.32	298.07	250.04	71.59	TO	202.04	277.32	245.66	94	96	79
100	203.54	TO	203.54	235.88	224.86	250.59	TO	201.32	298.07	250.04	72.09	TO	201.67	293.83	249.18	94	95	66

Table 7.7: Results for abs2n15 (Vehicles=2; Shelf-life=2)

Obj, Min, Max, Mean refer to the Objective Value, and the Min, Max, and Mean when run over 10000 scenarios (in 1000s); TO refers to Time Out; Time for the rest is in seconds; $St > Det$, $Rb > Det$, $St > Rb$ refer to instances where Stochastic is greater than Deterministic, Robust is greater than Deterministic, and Stochastic is greater than Robust (in %)

S	Det					St					Rb					St>	Rb>	St>
	Obj	Time	Min	Max	Mean	Obj	Time	Min	Max	Mean	Obj	Time	Min	Max	Mean	Det	Det	Rb
10	265.05	TO	265.05	316.38	294.36	328.04	TO	262.54	392.35	327.32	90.63	TO	264.11	352.69	317.69	96	98	88
20	265.05	TO	265.05	316.38	294.36	328.04	TO	262.54	392.35	327.32	91.92	TO	263.70	364.91	321.85	96	98	83
30	265.05	TO	265.05	316.38	294.36	328.04	TO	262.54	392.35	327.32	93.16	TO	263.68	361.99	321.09	96	98	83
40	265.05	TO	265.05	316.38	294.36	328.04	TO	262.54	392.35	327.32	94.62	TO	263.67	362.94	321.40	96	98	82
50	265.05	TO	265.05	316.38	294.36	328.04	TO	262.54	392.35	327.32	94.61	TO	263.48	381.15	323.64	96	97	88
60	265.05	TO	265.05	316.38	294.36	328.04	TO	262.54	392.35	327.32	94.89	TO	263.62	373.49	323.07	96	97	83
70	265.05	TO	265.05	316.38	294.36	328.04	TO	262.54	392.35	327.32	95.01	TO	263.37	379.67	324.60	96	97	80
80	265.05	TO	265.05	316.38	294.36	328.04	TO	262.54	392.35	327.32	94.56	TO	263.48	369.17	323.22	96	97	73
90	265.05	TO	265.05	316.38	294.36	328.04	TO	262.54	392.35	327.32	94.37	TO	263.60	372.62	322.76	96	97	86
100	265.05	TO	265.05	316.38	294.36	328.04	TO	262.54	392.35	327.32	95.22	TO	262.73	389.18	326.26	96	96	91

Table 7.8: Results for abs2n20 (Vehicles=2; Shelf-life=2)

Obj, Min, Max, Mean refer to the Objective Value, and the Min, Max, and Mean when run over 10000 scenarios (in 1000s); TO refers to Time Out; Time for the rest is in seconds; $St > Det$, $Rb > Det$, $St > Rb$ refer to instances where Stochastic is greater than Deterministic, Robust is greater than Deterministic, and Stochastic is greater than Robust (in %)

S	Det					St					Rb					St>	Rb>	St>
	Obj	Time	Min	Max	Mean	Obj	Time	Min	Max	Mean	Obj	Time	Min	Max	Mean	Det	Det	Rb
10	79.40	0.73	79.40	95.54	89.16	96.38	805	77.70	114.53	96.18	25.56	15.2	79.28	92.24	87.14	88	17	89
20	79.40	0.73	79.40	95.54	89.16	96.38	805	77.70	114.53	96.18	25.86	39.4	79.07	102.41	92.13	88	94	84
30	79.40	0.73	79.40	95.54	89.16	96.38	805	77.70	114.53	96.18	26.18	48.5	79.07	102.41	92.13	88	94	84
40	79.40	0.73	79.40	95.54	89.16	96.38	805	77.70	114.53	96.18	26.51	114	78.82	109.69	94.74	88	94	72
50	79.40	0.73	79.40	95.54	89.16	96.38	805	77.70	114.53	96.18	26.58	65.1	78.82	109.69	94.74	88	94	72
60	79.40	0.73	79.40	95.54	89.16	96.38	805	77.70	114.53	96.18	26.60	72.8	78.82	109.69	94.74	88	94	72
70	79.40	0.73	79.40	95.54	89.16	96.38	805	77.70	114.53	96.18	26.64	79.2	78.82	109.69	94.74	88	94	72
80	79.40	0.73	79.40	95.54	89.16	96.38	805	77.70	114.53	96.18	26.49	101	78.82	109.69	94.74	88	94	72
90	79.40	0.73	79.40	95.54	89.16	96.38	805	77.70	114.53	96.18	26.48	201	78.82	109.69	94.74	88	94	72
100	79.40	0.73	79.40	95.54	89.16	96.38	805	77.70	114.53	96.18	26.71	224	78.82	109.69	94.74	88	94	72

Table 7.9: Results for abs3n05 (Vehicles=2; Shelf-life=2)

Obj, Min, Max, Mean refer to the Objective Value, and the Min, Max, and Mean when run over 10000 scenarios (in 1000s); TO refers to Time Out; Time for the rest is in seconds; $St > Det$, $Rb > Det$, $St > Rb$ refer to instances where Stochastic is greater than Deterministic, Robust is greater than Deterministic, and Stochastic is greater than Robust (in %)

S	Det					St					Rb					St>	Rb>	St>
	Obj	Time	Min	Max	Mean	Obj	Time	Min	Max	Mean	Obj	Time	Min	Max	Mean	Det	Det	Rb
10	113.37	77	113.37	140.16	126.89	140.41	TO	111.46	168.78	140.09	37.68	1116	113.04	150.46	133.96	94	98	90
20	113.37	77	113.37	140.16	126.89	140.41	TO	111.46	168.78	140.09	38.16	2798	113.04	150.46	133.96	94	98	90
30	113.37	77	113.37	140.16	126.89	140.41	TO	111.46	168.78	140.09	38.61	3030	112.71	150.13	135.03	94	96	86
40	113.37	77	113.37	140.16	126.89	140.41	TO	111.46	168.78	140.09	39.19	5413	112.48	149.90	135.43	94	96	83
50	113.37	77	113.37	140.16	126.89	140.41	TO	111.46	168.78	140.09	39.27	TO	112.48	149.90	135.43	94	96	83
60	113.37	77	113.37	140.16	126.89	140.41	TO	111.46	168.78	140.09	39.33	TO	112.48	149.90	135.43	94	96	83
70	113.37	77	113.37	140.16	126.89	140.41	TO	111.46	168.78	140.09	39.39	TO	112.48	149.90	135.43	94	96	83
80	113.37	77	113.37	140.16	126.89	140.41	TO	111.46	168.78	140.09	39.18	TO	112.48	149.90	135.43	94	96	83
90	113.37	77	113.37	140.16	126.89	140.41	TO	111.46	168.78	140.09	39.15	TO	112.48	149.90	135.43	94	96	83
100	113.37	77	113.37	140.16	126.89	140.41	TO	111.46	168.78	140.09	39.51	TO	112.48	149.90	135.43	94	96	83

Table 7.10: Results for abs3n10 (Vehicles=2; Shelf-life=2)

Obj, Min, Max, Mean refer to the Objective Value, and the Min, Max, and Mean when run over 10000 scenarios (in 1000s); TO refers to Time Out; Time for the rest is in seconds; $St > Det$, $Rb > Det$, $St > Rb$ refer to instances where Stochastic is greater than Deterministic, Robust is greater than Deterministic, and Stochastic is greater than Robust (in %)

S	Det					St					Rb					St>	Rb>	St>
	Obj	Time	Min	Max	Mean	Obj	Time	Min	Max	Mean	Obj	Time	Min	Max	Mean	Det	Det	Rb
10	213.33	402	213.33	254.92	237.25	263.92	TO	210.78	316.44	263.35	71.96	TO	212.61	262.17	245.81	96	97	91
20	213.33	402	213.33	254.92	237.25	263.92	TO	210.78	316.44	263.35	73.02	TO	212.05	275.66	252.78	96	97	87
30	213.33	402	213.33	254.92	237.25	263.92	TO	210.78	316.44	263.35	74.07	TO	211.96	275.57	252.98	96	97	85
40	213.33	402	213.33	254.92	237.25	263.92	TO	210.78	316.44	263.35	75.19	TO	211.93	282.23	255.62	96	96	85
50	213.33	402	213.33	254.92	237.25	263.92	TO	210.78	316.44	263.35	75.34	TO	211.93	282.24	255.62	96	96	85
60	213.33	402	213.33	254.92	237.25	263.92	TO	210.78	316.44	263.35	75.44	TO	211.93	282.24	255.62	96	96	85
70	213.33	402	213.33	254.92	237.25	263.92	TO	210.78	316.44	263.35	75.53	TO	211.92	282.23	255.61	96	96	85
80	213.33	402	213.33	254.92	237.25	263.92	TO	210.78	316.44	263.35	75.09	TO	211.93	275.54	252.94	96	96	85
90	213.33	402	213.33	254.92	237.25	263.92	TO	210.78	316.44	263.35	74.98	TO	211.83	282.14	255.52	96	96	87
100	213.33	402	213.33	254.92	237.25	263.92	TO	210.78	316.44	263.35	75.66	TO	211.84	281.75	255.38	96	96	87

Table 7.11: Results for abs3n15 (Vehicles=2; Shelf-life=2)

Obj, Min, Max, Mean refer to the Objective Value, and the Min, Max, and Mean when run over 10000 scenarios (in 1000s); TO refers to Time Out; Time for the rest is in seconds; $St > Det$, $Rb > Det$, $St > Rb$ refer to instances where Stochastic is greater than Deterministic, Robust is greater than Deterministic, and Stochastic is greater than Robust (in %)

S	Det					St					Rb					St>	Rb>	St>
	Obj	Time	Min	Max	Mean	Obj	Time	Min	Max	Mean	Obj	Time	Min	Max	Mean	Det	Det	Rb
10	256.44	TO	256.44	304.80	283.88	318.01	TO	253.40	381.04	317.30	87.62	TO	256.24	324.62	301.76	95	99	80
20	256.44	TO	256.44	304.80	283.88	318.01	TO	253.40	381.04	317.30	88.96	TO	256.24	324.62	301.76	95	99	80
30	256.44	TO	256.44	304.80	283.88	318.01	TO	253.40	381.04	317.30	90.16	TO	256.24	324.62	301.76	95	99	80
40	256.44	TO	256.44	304.80	283.88	318.01	TO	253.40	381.04	317.30	91.34	TO	255.92	343.30	308.64	95	98	80
50	256.44	TO	256.44	304.80	283.88	318.01	TO	253.40	381.04	317.30	91.68	TO	256.05	333.30	305.66	95	99	78
60	256.44	TO	256.44	304.80	283.88	318.01	TO	253.40	381.04	317.30	91.57	TO	255.85	346.80	308.86	95	98	80
70	256.44	TO	256.44	304.80	283.88	318.01	TO	253.40	381.04	317.30	91.83	TO	255.97	333.66	305.77	95	98	80
80	256.44	TO	256.44	304.80	283.88	318.01	TO	253.40	381.04	317.30	91.27	TO	256.00	330.68	304.38	95	98	80
90	256.44	TO	256.44	304.80	283.88	318.01	TO	253.40	381.04	317.30	91.21	TO	256.07	328.96	303.57	95	99	82
100	256.44	TO	256.44	304.80	283.88	318.01	TO	253.40	381.04	317.30	91.68	TO	255.58	368.30	312.42	95	98	84

Table 7.12: Results for abs3n20 (Vehicles=2; Shelf-life=2)

Obj, Min, Max, Mean refer to the Objective Value, and the Min, Max, and Mean when run over 10000 scenarios (in 1000s); TO refers to Time Out; Time for the rest is in seconds; $St > Det$, $Rb > Det$, $St > Rb$ refer to instances where Stochastic is greater than Deterministic, Robust is greater than Deterministic, and Stochastic is greater than Robust (in %)

S	Det					St					Rb					St>	Rb>	St>
	Obj	Time	Min	Max	Mean	Obj	Time	Min	Max	Mean	Obj	Time	Min	Max	Mean	Det	Det	Rb
10	47.20	0.67	47.20	52.32	50.33	57.11	64.4	46.22	67.76	56.98	14.78	25.7	47.18	57.64	52.89	90	98	88
20	47.20	0.67	47.20	52.32	50.33	57.11	64.4	46.22	67.76	56.98	14.93	46.3	47.16	60.48	54.06	90	98	88
30	47.20	0.67	47.20	52.32	50.33	57.11	64.4	46.22	67.76	56.98	15.09	82.6	47.10	61.62	54.55	90	91	87
40	47.20	0.67	47.20	52.32	50.33	57.11	64.4	46.22	67.76	56.98	15.26	154	47.10	61.62	54.55	90	91	87
50	47.20	0.67	47.20	52.32	50.33	57.11	64.4	46.22	67.76	56.98	15.30	196	47.10	61.62	54.55	90	91	87
60	47.20	0.67	47.20	52.32	50.33	57.11	64.4	46.22	67.76	56.98	15.31	217	47.10	61.62	54.55	90	91	87
70	47.20	0.67	47.20	52.32	50.33	57.11	64.4	46.22	67.76	56.98	15.33	205	47.10	61.62	54.55	90	91	87
80	47.20	0.67	47.20	52.32	50.33	57.11	64.4	46.22	67.76	56.98	15.26	140	47.10	61.62	54.55	90	91	87
90	47.20	0.67	47.20	52.32	50.33	57.11	64.4	46.22	67.76	56.98	15.26	221	47.10	61.62	54.55	90	91	87
100	47.20	0.67	47.20	52.32	50.33	57.11	64.4	46.22	67.76	56.98	15.37	253	47.10	61.62	54.55	90	91	87

Table 7.13: Results for abs4n05 (Vehicles=2; Shelf-life=2)

Obj, Min, Max, Mean refer to the Objective Value, and the Min, Max, and Mean when run over 10000 scenarios (in 1000s); TO refers to Time Out; Time for the rest is in seconds; $St > Det$, $Rb > Det$, $St > Rb$ refer to instances where Stochastic is greater than Deterministic, Robust is greater than Deterministic, and Stochastic is greater than Robust (in %)

S	Det					St					Rb					St>	Rb>	St>
	Obj	Time	Min	Max	Mean	Obj	Time	Min	Max	Mean	Obj	Time	Min	Max	Mean	Det	Det	Rb
10	141.75	166	141.75	160.12	154.28	173.59	TO	139.56	206.92	173.21	47.37	TO	141.65	168.01	158.32	91	99	85
20	141.75	166	141.75	160.12	154.28	173.59	TO	139.56	206.92	173.21	47.82	TO	141.50	179.86	162.95	91	98	85
30	141.75	166	141.75	160.12	154.28	173.59	TO	139.56	206.92	173.21	48.38	TO	141.35	173.77	161.92	91	98	84
40	141.75	166	141.75	160.12	154.28	173.59	TO	139.56	206.92	173.21	48.98	TO	141.21	185.63	166.25	91	96	85
50	141.75	166	141.75	160.12	154.28	173.59	TO	139.56	206.92	173.21	49.08	TO	141.21	185.63	166.25	91	96	85
60	141.75	166	141.75	160.12	154.28	173.59	TO	139.56	206.92	173.21	49.14	TO	140.60	192.61	169.75	91	96	69
70	141.75	166	141.75	160.12	154.28	173.59	TO	139.56	206.92	173.21	49.10	TO	141.42	169.48	160.32	91	98	86
80	141.75	166	141.75	160.12	154.28	173.59	TO	139.56	206.92	173.21	48.95	TO	141.21	185.63	166.25	91	96	85
90	141.75	166	141.75	160.12	154.28	173.59	TO	139.56	206.92	173.21	48.91	TO	141.19	185.61	166.23	91	96	85
100	141.75	166	141.75	160.12	154.28	173.59	TO	139.56	206.92	173.21	49.33	TO	140.46	202.92	172.12	91	94	65

Table 7.14: Results for abs4n10 (Vehicles=2; Shelf-life=2)

Obj, Min, Max, Mean refer to the Objective Value, and the Min, Max, and Mean when run over 10000 scenarios (in 1000s); TO refers to Time Out; Time for the rest is in seconds; $St > Det$, $Rb > Det$, $St > Rb$ refer to instances where Stochastic is greater than Deterministic, Robust is greater than Deterministic, and Stochastic is greater than Robust (in %)

S	Det					St					Rb					St>	Rb>	St>
	Obj	Time	Min	Max	Mean	Obj	Time	Min	Max	Mean	Obj	Time	Min	Max	Mean	Det	Det	Rb
10	186.32	TO	186.32	213.78	204.51	228.70	TO	184.18	272.17	228.20	62.98	TO	185.68	234.09	217.17	94	98	86
20	186.32	TO	186.32	213.78	204.51	228.70	TO	184.18	272.17	228.20	63.82	TO	185.56	248.24	221.90	94	98	87
30	186.32	TO	186.32	213.78	204.51	228.70	TO	184.18	272.17	228.20	64.49	TO	185.08	257.23	224.76	94	96	86
40	186.32	TO	186.32	213.78	204.51	228.70	TO	184.18	272.17	228.20	65.54	TO	185.07	261.02	225.99	94	96	77
50	186.32	TO	186.32	213.78	204.51	228.70	TO	184.18	272.17	228.20	65.62	TO	185.03	255.06	224.49	94	96	77
60	186.32	TO	186.32	213.78	204.51	228.70	TO	184.18	272.17	228.20	65.81	TO	185.15	257.50	225.17	94	96	73
70	186.32	TO	186.32	213.78	204.51	228.70	TO	184.18	272.17	228.20	65.59	TO	184.99	261.07	225.22	94	95	78
80	186.32	TO	186.32	213.78	204.51	228.70	TO	184.18	272.17	228.20	65.60	TO	185.19	257.54	225.21	94	96	73
90	186.32	TO	186.32	213.78	204.51	228.70	TO	184.18	272.17	228.20	65.23	TO	184.92	266.68	226.35	94	96	87
100	186.32	TO	186.32	213.78	204.51	228.70	TO	184.18	272.17	228.20	66.09	TO	185.16	251.60	223.64	94	96	73

Table 7.15: Results for abs4n15 (Vehicles=2; Shelf-life=2)

Obj, Min, Max, Mean refer to the Objective Value, and the Min, Max, and Mean when run over 10000 scenarios (in 1000s); TO refers to Time Out; Time for the rest is in seconds; $St > Det$, $Rb > Det$, $St > Rb$ refer to instances where Stochastic is greater than Deterministic, Robust is greater than Deterministic, and Stochastic is greater than Robust (in %)

S	Det					St					Rb					St>	Rb>	St>
	Obj	Time	Min	Max	Mean	Obj	Time	Min	Max	Mean	Obj	Time	Min	Max	Mean	Det	Det	Rb
10	261.91	TO	261.91	288.38	279.76	320.28	TO	258.50	381.07	319.62	87.75	TO	261.18	319.22	298.00	93	98	90
20	261.91	TO	261.91	288.38	279.76	320.28	TO	258.50	381.07	319.62	89.03	TO	260.64	348.38	310.01	93	96	84
30	261.91	TO	261.91	288.38	279.76	320.28	TO	258.50	381.07	319.62	90.32	TO	260.27	362.99	315.18	93	96	74
40	261.91	TO	261.91	288.38	279.76	320.28	TO	258.50	381.07	319.62	91.02	TO	259.35	373.85	317.68	93	94	78
50	261.91	TO	261.91	288.38	279.76	320.28	TO	258.50	381.07	319.62	91.47	TO	259.91	366.29	315.82	93	94	82
60	261.91	TO	261.91	288.38	279.76	320.28	TO	258.50	381.07	319.62	91.65	TO	259.98	373.50	316.99	93	95	82
70	261.91	TO	261.91	288.38	279.76	320.28	TO	258.50	381.07	319.62	91.49	TO	259.85	348.83	310.99	93	94	89
80	261.91	TO	261.91	288.38	279.76	320.28	TO	258.50	381.07	319.62	91.07	TO	259.60	368.43	315.85	93	94	75
90	261.91	TO	261.91	288.38	279.76	320.28	TO	258.50	381.07	319.62	91.05	TO	260.32	349.15	310.44	93	96	84
100	261.91	TO	261.91	288.38	279.76	320.28	TO	258.50	381.07	319.62	91.83	TO	259.78	365.63	315.36	93	94	84

Table 7.16: Results for abs4n20 (Vehicles=2; Shelf-life=2)

Obj, Min, Max, Mean refer to the Objective Value, and the Min, Max, and Mean when run over 10000 scenarios (in 1000s); TO refers to Time Out; Time for the rest is in seconds; $St > Det$, $Rb > Det$, $St > Rb$ refer to instances where Stochastic is greater than Deterministic, Robust is greater than Deterministic, and Stochastic is greater than Robust (in %)

S	Det					St					Rb					St>	Rb>	St>
	Obj	Time	Min	Max	Mean	Obj	Time	Min	Max	Mean	Obj	Time	Min	Max	Mean	Det	Det	Rb
10	57.86	0.63	57.86	70.48	64.36	71.48	614	56.72	85.86	71.32	19.26	8.88	57.85	72.72	66.65	92	98	89
20	57.86	0.63	57.86	70.48	64.36	71.48	614	56.72	85.86	71.32	19.49	45.1	57.85	72.72	66.65	92	98	89
30	57.86	0.63	57.86	70.48	64.36	71.48	614	56.72	85.86	71.32	19.70	127	57.76	72.73	66.97	92	98	86
40	57.86	0.63	57.86	70.48	64.36	71.48	614	56.72	85.86	71.32	19.94	313	57.76	72.73	66.97	92	98	86
50	57.86	0.63	57.86	70.48	64.36	71.48	614	56.72	85.86	71.32	19.98	342	57.76	72.73	66.97	92	98	86
60	57.86	0.63	57.86	70.48	64.36	71.48	614	56.72	85.86	71.32	20.01	537	57.76	72.73	66.97	92	98	86
70	57.86	0.63	57.86	70.48	64.36	71.48	614	56.72	85.86	71.32	20.03	625	57.76	72.73	66.97	92	98	86
80	57.86	0.63	57.86	70.48	64.36	71.48	614	56.72	85.86	71.32	19.94	435	57.76	72.73	66.97	92	98	86
90	57.86	0.63	57.86	70.48	64.36	71.48	614	56.72	85.86	71.32	19.93	440	57.76	72.73	66.97	92	98	86
100	57.86	0.63	57.86	70.48	64.36	71.48	614	56.72	85.86	71.32	20.07	531	57.76	72.73	66.97	92	98	86

Table 7.17: Results for abs5n05 (Vehicles=2; Shelf-life=2)

Obj, Min, Max, Mean refer to the Objective Value, and the Min, Max, and Mean when run over 10000 scenarios (in 1000s); TO refers to Time Out; Time for the rest is in seconds; $St > Det$, $Rb > Det$, $St > Rb$ refer to instances where Stochastic is greater than Deterministic, Robust is greater than Deterministic, and Stochastic is greater than Robust (in %)

S	Det					St					Rb					St>	Rb>	St>
	Obj	Time	Min	Max	Mean	Obj	Time	Min	Max	Mean	Obj	Time	Min	Max	Mean	Det	Det	Rb
10	165.56	22.5	165.56	202.58	186.06	203.79	TO	163.96	243.03	203.35	56.01	321	165.37	202.39	187.76	96	64	90
20	165.56	22.5	165.56	202.58	186.06	203.79	TO	163.96	243.03	203.35	56.67	3770	165.11	209.95	192.43	96	98	90
30	165.56	22.5	165.56	202.58	186.06	203.79	TO	163.96	243.03	203.35	57.41	8292	164.70	230.72	199.87	96	97	87
40	165.56	22.5	165.56	202.58	186.06	203.79	TO	163.96	243.03	203.35	58.25	TO	164.70	230.72	199.87	96	97	87
50	165.56	22.5	165.56	202.58	186.06	203.79	TO	163.96	243.03	203.35	58.38	TO	164.70	230.72	199.87	96	97	87
60	165.56	22.5	165.56	202.58	186.06	203.79	TO	163.96	243.03	203.35	58.44	TO	164.70	230.72	199.87	96	97	87
70	165.56	22.5	165.56	202.58	186.06	203.79	TO	163.96	243.03	203.35	58.52	TO	164.70	230.72	199.87	96	97	87
80	165.56	22.5	165.56	202.58	186.06	203.79	TO	163.96	243.03	203.35	58.21	TO	164.70	230.72	199.87	96	97	87
90	165.56	22.5	165.56	202.58	186.06	203.79	TO	163.96	243.03	203.35	58.18	TO	164.70	230.72	199.87	96	97	87
100	165.56	22.5	165.56	202.58	186.06	203.79	TO	163.96	243.03	203.35	58.67	TO	164.70	230.72	199.87	96	97	87

Table 7.18: Results for abs5n10 (Vehicles=2; Shelf-life=2)

Obj, Min, Max, Mean refer to the Objective Value, and the Min, Max, and Mean when run over 10000 scenarios (in 1000s); TO refers to Time Out; Time for the rest is in seconds; $St > Det$, $Rb > Det$, $St > Rb$ refer to instances where Stochastic is greater than Deterministic, Robust is greater than Deterministic, and Stochastic is greater than Robust (in %)

S	Det					St					Rb					St>	Rb>	St>
	Obj	Time	Min	Max	Mean	Obj	Time	Min	Max	Mean	Obj	Time	Min	Max	Mean	Det	Det	Rb
10	179.85	6844	179.85	191.65	187.08	219.54	TO	177.55	260.66	219.08	60.34	TO	179.53	213.94	202.07	93	95	91
20	179.85	6844	179.85	191.65	187.08	219.54	TO	177.55	260.66	219.08	60.97	TO	178.91	240.04	212.33	93	93	89
30	179.85	6844	179.85	191.65	187.08	219.54	TO	177.55	260.66	219.08	61.71	TO	178.92	240.05	212.34	93	93	89
40	179.85	6844	179.85	191.65	187.08	219.54	TO	177.55	260.66	219.08	62.70	TO	178.09	255.05	218.02	93	93	48
50	179.85	6844	179.85	191.65	187.08	219.54	TO	177.55	260.66	219.08	62.93	TO	178.23	248.69	216.48	93	94	63
60	179.85	6844	179.85	191.65	187.08	219.54	TO	177.55	260.66	219.08	62.67	TO	177.79	260.89	219.31	93	93	17
70	179.85	6844	179.85	191.65	187.08	219.54	TO	177.55	260.66	219.08	62.78	TO	177.81	260.24	219.26	93	93	14
80	179.85	6844	179.85	191.65	187.08	219.54	TO	177.55	260.66	219.08	62.32	TO	177.75	260.36	219.02	93	93	71
90	179.85	6844	179.85	191.65	187.08	219.54	TO	177.55	260.66	219.08	62.28	TO	177.96	254.53	216.87	93	93	87
100	179.85	6844	179.85	191.65	187.08	219.54	TO	177.55	260.66	219.08	63.11	TO	178.06	254.40	217.67	93	93	60

Table 7.19: Results for abs5n15 (Vehicles=2; Shelf-life=2)

Obj, Min, Max, Mean refer to the Objective Value, and the Min, Max, and Mean when run over 10000 scenarios (in 1000s); TO refers to Time Out; Time for the rest is in seconds; $St > Det$, $Rb > Det$, $St > Rb$ refer to instances where Stochastic is greater than Deterministic, Robust is greater than Deterministic, and Stochastic is greater than Robust (in %)

S	Det					St					Rb					St>	Rb>	St>
	Obj	Time	Min	Max	Mean	Obj	Time	Min	Max	Mean	Obj	Time	Min	Max	Mean	Det	Det	Rb
10	297.50	TO	297.50	348.84	329.21	365.77	TO	293.71	436.05	364.97	101.10	TO	297.05	345.98	331.83	94	54	87
20	297.50	TO	297.50	348.84	329.21	365.77	TO	293.71	436.05	364.97	102.31	TO	296.61	375.54	346.01	94	98	87
30	297.50	TO	297.50	348.84	329.21	365.77	TO	293.71	436.05	364.97	103.59	TO	295.53	420.01	360.95	94	97	70
40	297.50	TO	297.50	348.84	329.21	365.77	TO	293.71	436.05	364.97	105.24	TO	295.33	414.88	361.65	94	97	60
50	297.50	TO	297.50	348.84	329.21	365.77	TO	293.71	436.05	364.97	104.94	TO	294.87	401.61	357.77	94	96	68
60	297.50	TO	297.50	348.84	329.21	365.77	TO	293.71	436.05	364.97	105.52	TO	295.48	411.61	360.50	94	97	68
70	297.50	TO	297.50	348.84	329.21	365.77	TO	293.71	436.05	364.97	105.39	TO	295.03	413.27	360.65	94	96	63
80	297.50	TO	297.50	348.84	329.21	365.77	TO	293.71	436.05	364.97	104.58	TO	294.87	412.30	360.29	94	96	76
90	297.50	TO	297.50	348.84	329.21	365.77	TO	293.71	436.05	364.97	104.88	TO	295.18	409.59	359.52	94	97	62
100	297.50	TO	297.50	348.84	329.21	365.77	TO	293.71	436.05	364.97	106.02	TO	295.18	411.98	361.22	94	97	57

Table 7.20: Results for abs5n20 (Vehicles=2; Shelf-life=2)

Obj, Min, Max, Mean refer to the Objective Value, and the Min, Max, and Mean when run over 10000 scenarios (in 1000s); TO refers to Time Out; Time for the rest is in seconds; $St > Det$, $Rb > Det$, $St > Rb$ refer to instances where Stochastic is greater than Deterministic, Robust is greater than Deterministic, and Stochastic is greater than Robust (in %)

7.4 Computational Results

While comparing the values corresponding to the column 'Min' in the tables for the stochastic and the robust models, it is observed that the minimum profit values for the robust are higher than those for the stochastic. This makes it clear that the robust model removes extreme scenarios that affected the overall profit from the stochastic model and hence is better. Also, the maximum profit values for the robust model are less than those for the stochastic model. This implies that the overall variation among values is significantly less for the robust model than those for the stochastic model.

From the values generated in the tables, the column 'Mean' for each of the three models is tabulated. This value refers to the average profit generated for each model over 10,000 scenarios. For each of the 20 instances, it is observed that the mean profit for the deterministic model is the lowest, and the one for the stochastic model is highest. The mean profit for the robust model lies somewhere in between. This is due to the fact that the deterministic model does not capture any uncertainty in demand, the stochastic captures the most uncertainty, and finally the robust captures a significant amount of uncertainty, though not as much as the stochastic one.

The columns, "Stochastic > Deterministic" (column 17) and "Robust > Deterministic" (column 18) from each table convey the same message, as obtained by comparing the average profit values. Averaging over all instances, the value in column 17 is almost 95%, with the lowest being 87% for abs3n05 (Table 7.9). Even for column 18, the average is close to 95%. Thus, the results clearly indicate that the robust model covers a sizable amount of uncertainty when compared to the stochastic model, even though the number of scenarios generated for the robust model is considerably less than the number for the stochastic model.

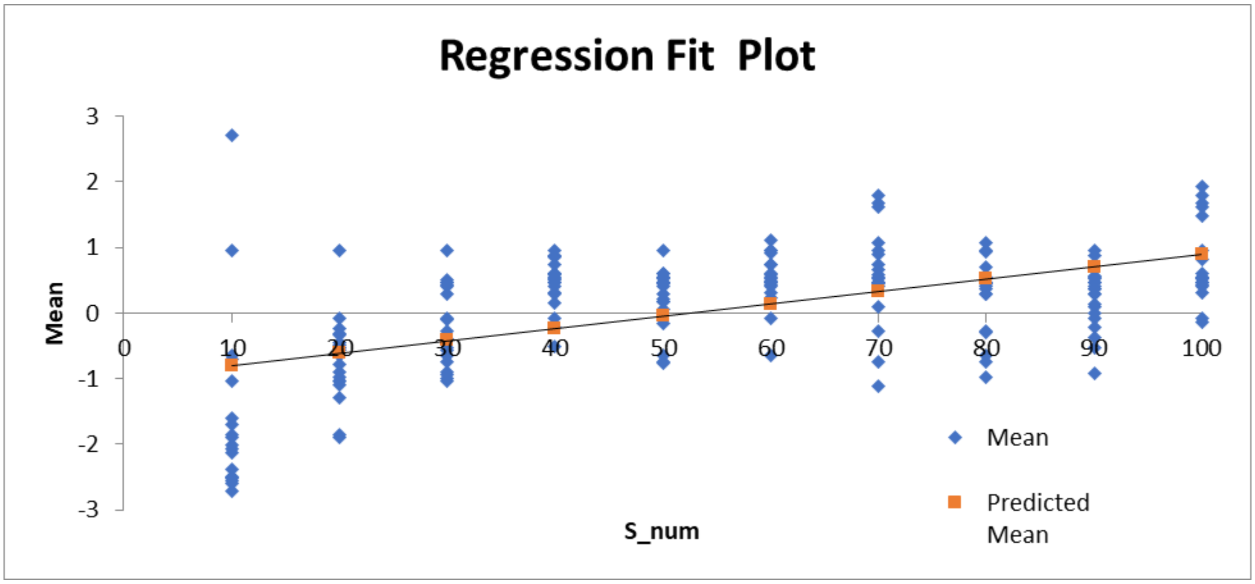


Figure 7.1: Line of Best Fit for Robust Scenario Number vs Robust Mean

S_num is the robust scenario number varying from (10-100). Means are all normalized values of the actual values

The results also project that, as the number of retailers increases, and the problem becomes more difficult, the robust keeps improving. The last column of Table 7.1 shows that the stochastic model is better than the robust for 100% of the scenarios. As the scenarios get more complicated, this percentage decreases to around 80% in Table 7.4, which is an instance with 20 retailers. There is a similar kind of trend for other sets of instances as well. For instances with larger costs (Tables 7.17-7.20), the figures start from 85% and go down to as little as 14%. One such case, in Table 7.19, for scenarios 60 and 70, the stochastic model is better than the robust for only around 15% of the scenarios.

When regression is performed as shown in Figure 7.1, the P-value generated is '1.836E-18 ', which is extremely small, confirming that number of scenarios is actually significant when compared to mean profits for the robust model.

Chapter 8

Conclusion

8.1 Review of the thesis

This thesis has studied the perishable Inventory Routing Problem (IRP) focusing on making decisions for inventory and transportation together. Chapter 2 discussed the literature for the inventory routing problem, and how it involves incorporating the concept of vendor managed inventory into the traditional vehicle routing problem. The several variations of the inventory routing problem, and the algorithms used to solve those variations, along with the assumptions made in the literature, are mentioned.

In Chapter 3, the version of the standard IRP employed in this thesis is stated. The IRP problem discussed here is a one-to-many problem with multiple homogeneous vehicles, and applies the ML inventory policy. The notations and the formulation of the IRP are given. The limitation of the standard IRP, for a product having an infinite shelf life, is mentioned.

Chapter 4 outlines the limitations of standard IRP. It introduces perishability to the

IRP (PIRP). The importance of the IRP with perishable products is discussed in terms of the food retail, and the blood industries. The concept of consignment inventory is introduced in the PIRP framework. The objective of the PIRP is to maximize the profit of the supplier for deterministic demands. The model considers a single product with a fixed shelf life, building on the IRP model presented in Chapter 3. The product perishes with a constant decay factor, and that decay affects the profit of the supplier. The product price is fixed; there is a constant discount that is applied to the price of the product each period that it is in inventory. Once the product shelf-life is reached, the item is discarded from inventory. The formulation with a profit maximization is defined. Results for comparison of the PIRP model with the standard IRP are generated. The results show better usage of inventory at the retailers as the product deteriorates, versus a product that does not deteriorate. However, the PIRP model discussed does not consider uncertainty in demand, which is usually the case in a real-life problem. This limitation is resolved in Chapter 5.

In Chapter 5, the stochastic PIRP is introduced (SPIRP). The demands are no longer known, and the problem is thus complex. Demand uncertainty is added to the PIRP using a finite set of demand scenarios, and the formulation is modified to incorporate uncertainty. The demand scenarios are generated based on the data set in consideration, and are equally probable. Results of running the scenario-based SPIRP on the CPLEX based branch-and-cut, and Benders decomposition algorithms, are generated. Analysis of the results shows that Benders takes more time, and generates worse gaps than the branch-and-cut algorithm. SPIRP resembles a real-life problem, but a huge number of demand scenarios need to be generated to be able to appropriately capture the demand uncertainty. Chapter 6 provides a better and innovative way to handle this limitation of SPIRP.

Chapter 6 applies the concept of robust optimization to the SPIRP. The importance of applying robust optimization to a problem with uncertain parameters is discussed. The limitation of SPIRP is that obtaining the true probability of each scenario is difficult, and hence a large number of equally-likely scenarios have to be generated. This limitation is resolved by making the SPIRP model robust. Applying robustness to the scenario-based model of SPIRP is non-trivial due to the underlying assumption of equiprobable scenarios. The "budget-of-uncertainty" approach is applied to the SPIRP, and the resultant model is robust. The model has been constructed in such a way that the probability of each scenario is uncertain. This method of applying robust optimization to the model is innovative. The purpose of this robust model is to capture the uncertainty in demand using fewer scenarios than the stochastic model while simultaneously making sure that the model is not too conservative in nature, and that not many scenarios assume their worst-case values. This is done by adding an appropriate deviation segment to the probability of the SPIRP scenario, and fixing the robustness parameter that controls the conservativeness of the model. Chapter 7 gives the results that demonstrate the effectiveness of the robust model.

In Chapter 7, the results are obtained using an algorithm that compares the three above-mentioned models. It tests the amount of demand uncertainty that each accounts for. This is done by running each of them on a huge number of demand scenarios, and calculating the average profit values over all the scenarios. The results demonstrate that since the deterministic model accounts for no demand uncertainty it has the lowest average profit values. The stochastic model which accounts for the most demand uncertainty has the highest average profit values. The robust model using a smaller number of scenarios, accounts for much of the demand uncertainty; the average profit values for this model are between the deterministic and stochastic. The results also demonstrate that as the instances get tougher, the robust model gets better. Moreover, increasing the number of scenarios of the robust model has a significant relation to the resultant average profit of that model.

8.2 Future Research

Each of the three models discussed in this thesis impose assumptions on the perishable IRP in a real-life problem. The assumption is that only a single product is taken. VMI is generally applied to the case of several Stock Keeping Unit (SKUs) each managed by the given vendor on behalf of one or more retailers served by that supplier. When there are two or more perishable SKUs, keeping track of the ages of multiple products simultaneously is required. Assuming that can be done, there may be opportunities for coordinated inventory control within the context of the PIRP.

Another area to explore is the possible modeling of the SPIRP problem in a different way. The goal would be that Benders could then better exploit the structure of the problem.

References

Adulyasak, Y., Cordeau, J.-F., and Jans, R. (2015). Benders decomposition for production routing under demand uncertainty. *Operations Research*, 63(4):851–867.

Adulyasak, Y., Cordeau, J.-F., Jans, R., et al. (2014). Formulations and branch-and-cut algorithms for multivehicle production and inventory routing problems. *INFORMS Journal on Computing*, 26(1):103–120.

Alvarez, A., Cordeau, J.-F., Jans, R., Munari, P., and Morabito, R. (2018). *Formulations, Branch-and-Cut and a Hybrid Heuristic Algorithm for an Inventory Routing Problem with Perishable Products*. CIRRELT, Centre interuniversitaire de recherche sur les réseaux d'entreprise .

Archetti, C., Bertazzi, L., Laporte, G., and Speranza, M. G. (2007). A branch-and-cut algorithm for a vendor-managed inventory-routing problem. *Transportation Science*, 41(3):382–391.

Bard, J. F., Huang, L., Jaillet, P., and Dror, M. (1998). A decomposition approach to the inventory routing problem with satellite facilities. *Transportation Science*, 32(2):189–203.

Bell, W. J., Dalberto, L. M., Fisher, M. L., Greenfield, A. J., Jaikumar, R., Kedia, P., Mack, R. G., and Prutzman, P. J. (1983). Improving the distribution of industrial gases with an on-line computerized routing and scheduling optimizer. *Interfaces*, 13(6):4–23.

Ben-Tal, A. and Nemirovski, A. (1998). Robust convex optimization. *Mathematics of Operations Research*, 23(4):769–805.

Ben-Tal, A. and Nemirovski, A. (1999). Robust solutions of uncertain linear programs. *Operations Research Letters*, 25(1):1–13.

Ben-Tal, A. and Nemirovski, A. (2000). Robust solutions of linear programming problems contaminated with uncertain data. *Mathematical Programming*, 88(3):411–424.

Benders, J. F. (1962). Partitioning procedures for solving mixed-variables programming problems. *Numerische Mathematik*, 4(1):238–252.

Bertazzi, L., Bosco, A., Guerriero, F., and Lagana, D. (2013). A stochastic inventory routing problem with stock-out. *Transportation Research Part C: Emerging Technologies*, 27:89–107.

Bertazzi, L., Paletta, G., and Speranza, M. G. (2002). Deterministic order-up-to level policies in an inventory routing problem. *Transportation Science*, 36(1):119–132.

Bertsimas, D., Gupta, V., and Kallus, N. (2018). Data-driven robust optimization. *Mathematical Programming*, 167(2):235–292.

Bertsimas, D. and Sim, M. (2004). The price of robustness. *Operations Research*, 52(1):35–53.

Bertsimas, D. and Thiele, A. (2006). A robust optimization approach to inventory theory. *Operations Research*, 54(1):150–168.

Bienstock, D. and ÖZbay, N. (2008). Computing robust basestock levels. *Discrete Optimization*, 5(2):389–414.

Chopra, S. and Meindl, P. (2013). *Supply chain management: strategy, planning, and operation*. Pearson.

Coelho, L. C., Cordeau, J.-F., and Laporte, G. (2012). Consistency in multi-vehicle inventory-routing. *Transportation Research Part C: Emerging Technologies*, 24:270 – 287.

Coelho, L. C., Cordeau, J.-F., and Laporte, G. (2013). Thirty years of inventory routing. *Transportation Science*, 48(1):1–19.

Coelho, L. C., Cordeau, J.-F., and Laporte, G. (2014). Heuristics for dynamic and stochastic inventory-routing. *Computers & Operations Research*, 52:55–67.

Coelho, L. C. and Laporte, G. (2013). A branch-and-cut algorithm for the multi-product multi-vehicle inventory-routing problem. *International Journal of Production Research*, 51(23-24):7156–7169.

Coelho, L. C. and Laporte, G. (2014). Optimal joint replenishment, delivery and inventory management policies for perishable products. *Computers & Operations Research*, 47:42–52.

Crama, Y., Rezaei, M., Savelsbergh, M., and Woensel, T. V. (2018). Stochastic inventory routing for perishable products. *Transportation Science*, 52(3):526–546.

Dantzig, G. B. and Ramser, J. H. (1959). The truck dispatching problem. *Management Science*, 6(1):80–91.

Desaulniers, G., Rakke, J. G., and Coelho, L. C. (2016). A branch-price-and-cut algorithm for the inventory-routing problem. *Transportation Science*, 50(3).

Desrochers, M. and Laporte, G. (1991). Improvements and extensions to the miller-tucker-zemlin subtour elimination constraints. *Operations Research Letters*, 10(1):27 – 36.

Diabat, A., Abdallah, T., and Le, T. (2016). A hybrid tabu search based heuristic for the periodic distribution inventory problem with perishable goods. *Annals of Operations Research*, 242(2):373–398.

El Ghaoui, L. and Lebret, H. (1997). Robust solutions to least-squares problems with uncertain data. *SIAM Journal on Matrix Analysis and Applications*, 18(4):1035–1064.

El Ghaoui, L., Oustry, F., and Lebret, H. (1998). Robust solutions to uncertain semidefinite programs. *SIAM Journal on Optimization*, 9(1):33–52.

Federgruen, A., Prastacos, G., and Zipkin, P. H. (1986). An allocation and distribution model for perishable products. *Operations Research*, 34(1):75–82.

Federgruen, A. and Zipkin, P. (1984). A combined vehicle routing and inventory allocation problem. *Operations Research*, 32(5):1019–1037.

Geiger, M. J. and Sevaux, M. (2011). Practical inventory routing: A problem definition and an optimization method. *arXiv preprint arXiv:1102.5635*.

Gruler, A., Panadero, J., de Armas, J., Pérez, J. A. M., and Juan, A. A. (2018). A variable neighborhood search simheuristic for the multiperiod inventory routing problem with stochastic demands. *International Transactions in Operational Research*.

Hemmelmayr, V., Doerner, K. F., Hartl, R. F., and Savelsbergh, M. W. (2009). Delivery strategies for blood products supplies. *OR Spectrum*, 31(4):707–725.

Hemmelmayr, V., Doerner, K. F., Hartl, R. F., and Savelsbergh, M. W. (2010). Vendor managed inventory for environments with stochastic product usage. *European Journal of Operational Research*, 202(3):686–695.

Jaillet, P., Bard, J. F., Huang, L., and Dror, M. (2002). Delivery cost approximations for inventory routing problems in a rolling horizon framework. *Transportation Science*, 36(3):292–300.

Kara, I., Laporte, G., and Bektas, T. (2004). A note on the lifted Miller-Tucker-Zemlin subtour elimination constraints for the capacitated vehicle routing problem. *European Journal of Operational Research*.

Kearney, A. (2019). Cscmp's 30th annual state of logistics report. Technical report, CSCMP.

Le, T., Diabat, A., Richard, J.-P., and Yih, Y. (2013). A column generation-based heuristic algorithm for an inventory routing problem with perishable goods. *Optimization Letters*, 7(7):1481–1502.

Li, Y., Chu, F., Yang, Z., and Calvo, R. W. (2016). A production inventory routing planning for perishable food with quality consideration. *IFAC-PapersOnLine*, 49(3):407 – 412. 14th IFAC Symposium on Control in Transportation SystemsCTS 2016.

Mihalopoulos, D. (2009). Expired or lost drugs costing city $1 million. *Chicago Tribune, February*, 3.

Mirzaei, S. and Seifi, A. (2015). Considering lost sale in inventory routing problems for perishable goods. *Computers & Industrial Engineering*, 87:213–227.

Nikkel, L., Gooch, M., Maguire, M., Bucknell, D., LaPlain, D., Dent, B., Whitehead, P., and Felfel, A. (2019). The avoidable crisis of food waste: Roadmap.

O'Byrne, R. (2016). Fresh supply chains - what matters most.

Padberg, M. and Rinaldi, G. (1991). A branch-and-cut algorithm for the resolution of large-scale symmetric traveling salesman problems. *SIAM review*, 33(1):60–100.

Qiu, Y., Qiao, J., and Pardalos, P. M. (2019). Optimal production, replenishment, delivery, routing and inventory management policies for products with perishable inventory. *Omega*, 82:193 – 204.

Rahbari, A., Nasiri, M. M., and Ghorbani, E. (2017). An inventory routing problem for perishable products with demand and transportation time uncertainty: A robust optimization approach. In *Research Gate*.

See, C.-T. and Sim, M. (2010). Robust approximation to multiperiod inventory management. *Operations Research*, 58(3):583–594.

Shaabani, H. and Kamalabadi, I. N. (2016). An efficient population-based simulated annealing algorithm for the multi-product multi-retailer perishable inventory routing problem. *Computers & Industrial Engineering*, 99:189 – 201.

Solyalı, O., Cordeau, J.-F., and Laporte, G. (2012). Robust inventory routing under demand uncertainty. *Transportation Science*, 46(3):327–340.

Solyalı, O. and Süral, H. (2011). A branch-and-cut algorithm using a strong formulation and an a priori tour-based heuristic for an inventory-routing problem. *Transportation Science*, 45(3):335–345.

Soysal, M., Çimen, M., Belbağ, S., and Toğrul, E. (2019). A review on sustainable inventory routing. *Computers & Industrial Engineering*.

Soyster, A. L. (1973). Convex programming with set-inclusive constraints and applications to inexact linear programming. *Operations Research*, 21(5):1154–1157.

Tseng, Y., Yue, W., and Taylor, M. (2005). The role of transportation in logistics chain. *Proceedings of the Eastern Asia Society for Transportation Studies*, 5:1657–1672.

Yantong, L., Feng, C., Zhen, Y., and Calvo, R. W. (2016). A production inventory routing planning for perishable food with quality consideration. *Ifac-Papersonline*, 49(3):407–412.