All the World's a Stage:

Fictionalism, Metaphysics, and Truth

by

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Author's Declaration

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Abstract

Fictionalism has been an appealing position for many philosophers seeking to avoid controversial ontological commitments implicit in certain kinds of discourses, while also trying to account for the usefulness of those discourses. While fictionalists with respect to various domains have made impressive attempts to explain how something can be both fictional and useful, extant fictionalist views retain one problematic commitment: that there are no substantively true assertions made within such domains. My dissertation attempts to develop and defend a semantic anti-realist account of fictionalism which does not share this commitment to error theory. Diagnosing the source of the residual commitment to error theory as arising from a commitment to a particular picture of meaning, I propose that fictionalism can provide an alternative semantics that grants that such assertions can be successfully truth-stating. I begin by describing a general framework for understanding debates between realists and anti-realists in various domains—derived primarily from the work of Michael Dummett and Crispin Wright—according to which realism for a domain roughly boils down to the view that (a) our assertions in that domain, if true, represent a mind-independent reality; and (b) that we can make true assertions in that domain. The fictionalist's aversion to the ontological commitments of realism for particular domains requires a rejection of either (a) or (b). My suggestion is that fictionalists are mistaken in giving up (b), and that the work they have done to bolster their views instead provide us with the tools we need to reject (a). I consider two particular examples to illustrate my case. First, I look at Mary Leng's and Stephen Yablo's respective developments of mathematical fictionalism, both of which make important use of Kendall Walton's theory of make-believe. Contrary to Leng's and Yablo's own views, I present a case for the view that by recognizing mathematics as a sort of make-believe, fictionalism can instead be used to fashion an alternative semantics for mathematical claims, and so provide grounds for rejecting the mathematical realists' commitment to (a). Using lessons derived from the mathematical case, I develop an account of modal fictionalism that similarly outlines modal discourse as a kind of make-believe which provides an alternative, anti-realist semantics for possible worlds discourse. I conclude by moving away from particular instances of fictionalism to consider some potentially controversial consequences of my approach to fictionalism, defending the commitments to alethic and logical pluralism implicit in my view.

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Chapter 1

Story Outline

> The village of Portpatrick, on Scotland's western coast, doesn't so much lie around its harbor as embrace it for dear life. On a map, the harbor resembles the head of a cartoon bunny gnawing its way inland; the main road runs around the muzzle and past one of the ears before curving away from the sea. Nearly all of Portpatrick's establishments—its inns and pubs, its village hall and tennis court—are on this road or just off it. ... In the harbor's two basins—the bunny's rectilinear ears, each of different size—the water can drop from one tide to the next by nearly fourteen feet. (Subramanian 2018)

Subramanian tells us quite a bit about Portpatrick that we can learn in the above passage. From it, we know that many of Portpatrick's businesses can be found on a road which runs up the harbour and past one of its basins. We know that the water level can change dramatically with a change in the tide. If someone were to ask us "Where can I go to get a pint in Portpatrick?" we know we can respond, "Head to the harbour." All of this may seem obvious to many—of course we know these things, the quote tells us these

things. But there is something peculiar in the way that Subramanian describes Portpatrick, something that allows us to know and understand more about Portpatrick than what can be gleaned from simply saying "The main road runs around the harbour" or "The basins are rectilinear."

By inviting us to imagine a cartoon bunny, Subramanian provides us with the means to begin to picture Portpatrick in our heads: to see how it looks on a map, to see the particular curve of the main road, of where the basins are located in relation to the harbour. We are also given a new, more precise, means of answering questions about Portpatrick. When asked where to go for a pint, we can participate in the make-believe ourselves, and respond "Head towards the bunny's nose, you're sure to find somewhere to drink along the way." This is possible by make-believing that Portpatrick is something that it isn't—a cartoon bunny and not just another Scottish coastal village. Through this act of make-believe, we come to potentially understand more about Portpatrick than we could without it.

This act of make-believe is what is peculiar about Subramanian's description of Portpatrick. By creating a fiction for us to participate in, something that is typically understood as not literally speaking true, we can come to learn true things of Portpatrick. Not only do we come to understand true things about Portpatrick, but this knowledge affords us the ability to do things in the real (not cartoon bunny) Portpatrick, such as navigate the village, or find something we are looking for. This is hardly a new means of passing along knowledge—imagining that Italy is a boot, or that Michigan is a mitten, or that a rabbit is going down a rabbit hole when tying our shoes, are all familiar examples of the same sort of use of fiction to facilitate knowledge and understanding. By engaging in make-believe, by participating in a fiction, we can actually sometimes learn more about our world, rather

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than be led astray from it.

This may seem like a mundane fact to many. Something very interesting can arise from this mundane notion, however, when we engage in a classically philosophical endeavour: attempting to determine if the things we talk about in certain discourses, things that we are greatly invested in, are real. Philosophers have long questioned whether things like numbers, normative moral features, and possible worlds, to name a few, are really existing things or not. And if they are not real in the way we thought, then what are they, and why is talking about them so useful to us? Inspired by the notion that we can sometimes use fiction for useful, knowledge producing ends, a view known as fictionalism has seen recent development as an attempt to answer the above question. A fictionalist will typically answer the question posed along these lines: numbers/normative moral features/possible worlds are not real, they are fictitious objects or properties, and the stories we tell with them are fictions, but these fictions are useful to us, in that we can use these fictions to learn something true about our real world, or to accomplish some practical end. And so, from the simple, mundane notion that we can use fiction to learn something real, we can produce a fascinating idea: that perhaps some of the ways of understanding our world that we prize and cherish most are similarly fictitious. But this raises a new philosophical question for us: does this mean that the things we say about these useful, fictitious objects and properties aren't actually true? Are our substantive assertions about, or involving, numbers, moral normativity, or possible worlds, all, strictly speaking, false? Many fictionalists will without hesitation answer yes, that those assertions which involve fictitious entities are not literally speaking true, and so are all false. Many people, including myself, however, will find this answer off-putting or, at the very least, controversial—surely some of the things we say

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about numbers, for instance, are true.

1.1 What I Will Be Arguing

The purpose of my dissertation will be to offer a different answer to the above question. That is, I will offer a way of saying no: although several of our substantive assertions involve or engage in a fiction, we can still affirm that those substantive assertions are true. The key to my suggestion is a reconceptualization of fictionalism as a sort of semantic anti-realism rather than as an error theory. Although my initial focus will be on developing this approach with regards to mathematics, I will show how this approach to fictionalism need not be exclusive to mathematics but can also be conceived of as a general fictionalist position. In writing my dissertation, and outlining how this answer can be given, I will be engaged in the following tasks.

I will begin in the second chapter by introducing the semantic approach to analyzing realist and anti-realist metaphysical positions. By summarizing what is required for a position in a particular metaphysical debate to count as a realist one from this focus on semantics, I identify the key commitments realist positions make with regards to the ontological status of certain entities, and our assertions about them. Identifying these commitments further allows me identify the ways in which anti-realist responses can be pursued, the reasons that motivate anti-realist commitments, and where fictionalism typically fits in this picture. That is, I argue that these realist commitments determine the ways in which anti-realist arguments can proceed, and the forms of anti-realism that they produce. I will then complete my discussion of realism/anti-realism debates with a sum-

mary of the respective advantages and shortcomings of a semantic anti-realist position and fictionalism.

Once I have outlined what is at stake within realism/anti-realism debates and the positions therein, I will move on in the third chapter to develop a semantic anti-realist approach to mathematical fictionalism. I begin this chapter with an overview of a prominent theory within the philosophy of fiction, Kendall Walton's theory of make-believe. After explaining Walton's theory of make-believe, I then illustrate the influence it has had in the development of mathematical fictionalism by providing expositions for Mary Leng's and Stephen Yablo's respective fictionalist arguments. I argue that the the influence Leng and Yablo take from Walton in developing their own fictionalist arguments can be used to provide an alternative, non-representational semantics for mathematical assertions. I will argue that a semantic approach to fictionalism is preferable to the typical error theory approach as it allows one to be committed to mathematical entities being fictitious objects while maintaining that mathematical assertions can be successfully truth-stating, which the error theorist cannot do.

For the fourth chapter, I will go on to show that this semantic anti-realist approach to fictionalism can be generalized. To show this, I will outline a semantic anti-realist modal fictionalism. I begin the fourth chapter by outlining the modal realism of David Lewis, and I show how modal fictionalism was developed as an attempt to preserve possible worlds semantics without the ontological controversy. I then consider the artificiality objection to modal fictionalism, as articulated by Andrea Sauchelli, and respond that by conceptualizing the modal fiction as one that is co-authored, the artificiality of modal fictionalism ceases to be a worry. This reconceptualization opens the door for me to argue that it is possible to

also provide a semantic account of modal fictionalism. I close the chapter by arguing that this semantic anti-realist approach to modal fictionalism has the benefit of not needing a story prefix, unlike the typical approach to modal fictionalism, and so does not face the same challenges that such a prefix poses.

In my fifth chapter, I will defend some of the underlying commitments of my proposed semantic anti-realist account of fictionalism. Specifically, I defend the commitments to alethic and logical pluralism, as well as a commitment to the rejection of a deflationist account of truth. I begin by providing an overview of the problems of mixed inferences, and the challenges that these problems pose for alethic and logical pluralists. I argue that by adopting context-specific variants of alethic and logical pluralism, I am able to defend these commitments. I then argue, counter to Bradely Amour-Garb and James Woodbridge, that a semantic anti-realist approach to fictionalism necessarily entails substantive truth predicates, and so must be committed to a rejection of deflationism.

I conclude my dissertation with Chapter 6, where I will provide general outline of the fictionalist position that I have developed, and what I take its main features to be. I highlight the major strengths of this position, and how it overcomes some long-standing objections to fictionalism.

Chapter 2

Stage Setting

Before giving my own arguments, it is necessary to first provide a general overview of how
I will be approaching realist/anti-realist debates, and of identifying the sorts of commit-
ments that motivate adopting either a realist or anti-realist position. To do so, I begin
by giving an exposition of how these debates can be tied to questions regarding the truth
conditions and semantics of assertions, as first proposed by Michael Dummett, and the un-
derstanding of realism that Dummett's approach produces. This approach to realism/anti-
realism debates will get further refined by an exposition of Crispin Wright's developments
of Dummett's proposal that realism/anti-realism debates can be understood as involving
the question of which properties that a notion of truth can either have or fail to have in
different domains, and the more nuanced explication of the commitments of realism that
Wright offers. With this general approach to analyzing realism/anti-realism debates, and
an understanding of realism in hand, I then go on to give an exposition of the general sorts
of objections that anti-realist positions give in response to realism, and the various sorts

of anti-realist positions that a person can take, depending on which aspect of the realist's commitments is being rejected. I will then offer a brief description of where fictionalism typically falls in these debates. Finally, I discuss Paul Benacerraf's argument that there are two primary desiderata when considering ontological postitions within the metaphysics of mathematics, a homogenous semantics and a workable epistemology, as well as how these two desiderata are seemingly in competition with each other. I close by arguing that the lessons of Benacerraf's argument extend generally, and can help evaluate the strengths and weaknesses of realist and anti-realist positions in other metaphsyical debates.

2.1   Realism

Attempting a neat, tidy, and uncontroversial exposition of realism/anti-realism debates, and the various positions one could take in those debates, is no easy task. Various roadmaps for this intellectual terrain have been proposed, but none have been universally acccepted. Moreover, attempts at defining the words 'realism' and 'anti-realism' seem to often result in some metaphysical positions being placed under the heading of one, despite the insistence of some, often including the proponents of the view in question, that it belongs under the other. Regardless of this difficulty, however, proposing such roadmaps is not a fruitless task. Roadmaps can help to identify key landmarks, which can then allow us to situate the various points on the map in relation to those landmarks, revealing something interesting about those points. In our case, identifying certain philosophical commitments, and situating the various metaphysical positions in relation to those commitments will, despite the controversy, help us to better understand those positions, and reveal their features which

will be of philosophical interest.

Much of this difficulty in developing a roadmap for realism/anti-realism debates has been due to the fact that it has been unclear just what, precisely, is the subject of these debates. The history of philosophy includes many debates between advocates of views styled "realist" and one or more opponents, though the opponents have a wider range of names: phenomenalists, nominalists, behaviourists, constructivists, or idealists, to name a few. To talk of realist/anti-realist debates suggests that these various views have more in common than just that their adversaries are called 'realist.' On the face of it, we may be tempted to say that realism is a commitment to the existence of *entities*—that realism about mental states is committed to mental states existing as entities, and that moral realism is committed to existence of normative moral facts as entities, and so on. However, simply saying, "These debates are about whether or not such-and-such entities exist," is not as useful as it seems at first glance. For one thing, some realism/anti-realism debates don't seem to be questions about entities at all. For instance, realists about the past don't really seem to be advancing a position that the past exists as an 'entity.' The same is true of realists about universals—the defining feature of which, after all, is often taken to be that they are not individuals. As such, debates about the reality of mental states, moral facts, and mathematics, seem to be distinct from debates about the reality of the past or of universals in that the former are debates about the reality of certain classes of entites, while the latter are not. For this reason, it would seem that understanding realism as characteristically committed to the existence of some set of entities would be a non-starter.

Michael Dummett sought to address this gap between question and answer, and to

provide a proper subject for those interested in the questions of realism and anti-realism to argue about. Dummett himself pursued the realism/anti-realism debate in different philosophical topics, most notably about the past and about mathematics.[1] Dummett's understanding of realism is often summarized as simply a commitment to the principle of bivalence for a given domain of discourse. In order to properly understand my argument, however, this summary will not be enough. Understanding *why* Dummett thinks realism can be captured by a commitment to the principle of bivalence will be essential to understand why the semantic anti-realist reconstruction of fictionalism that I am proposing will still conclude that, at least some, fictional discourse is truth-apt. For, Dummett's appeal to the principle of bivalence as an identifying feature of realism is a consequence of his project to recast debates between realists and their opponents as debates about *meaning*, or the proper semantic content of the words in question, and of the *truth conditions* for assertions making use of those words. In particular, Dummett argues that, "since these metaphysical disagreements [between realists and their adversaries] embodied divergent pictures of reality *to which the statements in question related*, it seemed to me apparent that what underlay them were *divergent pictures of the meanings of those statements*" (Dummett 1993, 465).[2]

These divergent pictures of reality, for Dummett, are between those who believe that the reality of the disputed subject is objective in an important, mind-independent way, and those who argue that the subject, more precisely the claims we make about that subject matter, are not mind-independent in the relevant way. The former is best categorized

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[1]See, for example, his "The Reality of the Past" in *Truth and Other Enigmas* (1978) and "What is Mathematics About?" in *The Seas of Language* (1993).

[2]Emphasis added.

as a realist stance, and the latter as an anti-realist stance (Dummett 1993, 465). This divergent picture of reality between realist and anti-realist results in a divergent picture of the truth conditions of the set of assertions in question, with the realist committed to those assertions possessing a definite truth-value that is mind-independent, and the anti-realist who holds that whether assertions about the topic in question are true or not is somehow mind-dependent (though exactly how they argue so varies depending on the variety of anti-realism) (Dummett 1993, 465).

To understand the connection to bivalence, it will help us to consider some domain of discourse that is mind-dependent in some way. Suppose that, for this domain, an assertion is only true if we can definitively demonstrate its truth, such as through mathematical proof or an established procedure of verification. We may now ask ourselves: are we warranted in supposing that every claim in this domain is determinately true or false? For the realist, there is an easy answer—a mind-independent reality will determine the truth or falsity of these assertions, even if we cannot discover it. The anti-realist for this domain, however, cannot warrant bivalence in this way—there can be no such guarantee unless some other reason can be given for supposing that every claim can be definitively verified or falsified, and typically, this will be no easy matter for the domains for which we find anti-realists. It is for this reason that Dummett identifies the commitment to the principle of bivalence as a necessary feature of realism, and those who reject such a commitment as anti-realists. Since the picture of reality as objective and mind-independent would make our assertions about that reality possess a definite truth-value independently of our capacity to recognize it, the principle of bivalence must follow for those sets of assertions. (Dummett 1993, 467). In rejecting the realist's picture of reality, the anti-realist position, for Dummett, "becomes

that of occupying a position that undercuts the ground for accepting bivalence" (Dummett 1993, 467). Thus, the anti-realist can be identified by their rejection of the principle of bivalence.

To help illustrate Dummett's position, we can consider a realist about the subject matter of the past. For Dummett, a realist about the past is someone who holds that the truth or falsity of "Event x happened at past time y," is something that is true or false independently of what human beings think (or are able to discover) about the matter. The meaning of the assertion is such that whether it is true or not is, as we might say, determined by a reality that is independent of us in some important (though perhaps hard to specify) way. This will also be true of mathematical realism—if someone is committed to the position that the semantic content of an assertion such as "$2 + 2 = 4$" is given by some mind-independent reality, and thus the assertion obeys the principle of bivalence, then we should classify them as a mathematical realist (Dummett 1993, 465). In this regard, we can understand the truth-value of these assertions as *epistemically unconstrained*; that is, the truth-values of these assertions, for the realist, will be independent of our epistemic access to them, or our ability to recognize their truth-values. While we may know that $2 + 2 = 4$, there are other mathematical truths that we do not now know and perhaps that we cannot know, but this, for the realist, is independent of the fact that they are nevertheless true. For this dissertation, I will be calling this picture of the meaning of assertions that Dummett attributes to the realist's picture of reality *representational semantics*.

In following this recasting of realism as concerning independent facts rather than entities, we now have a means by which to capture a characteristic feature of realist positions wherever they may be taken. Realists about the past, mental states, universals, or mathe-

matics can all engage in an argument about the proper semantic content of their respective sets of assertions and their truth conditions. More specifically, the realist can now argue that the semantic content of the assertions made in the domain for which they are realist are all representational: that the truth conditions of these assertions is dependent on accurate representation and will obey the principle of bivalence.[3] As such, we can understand Dummett as arguing that this is really what is at issue in many historical debates between realists and opponents, and finds many of the opponents to be rejecting, one way or another, representational semantics.

Crispin Wright, concerned that Dummett's understanding of realism may leave out some important approaches to realism, pushed even further Dummett's view that the question of realism is a question about semantics, and the means by which assertions can be understood as true. Wright argues that realism, as typically pursued, involves a commitment to two claims, one modest and one presumptuous (Wright 1992, 1). The modest claim is simply that there exists a mind-independent reality (Wright 1992, 1). The second, presumptuous, claim is a conjunction of two smaller claims: (a) that our assertions are what I have so far been calling representational; and (b) that some of our assertions are true (Wright 1992, 2).[4] Wright seeks to add nuance to Dummett's approach by arguing

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[3]As noted above, no proposed roadmap for the realism/anti-realism debate is without its detractors. For example, Michael Devitt emphatically objects to this approach to understanding realism/anti-realism debates (Devitt 1984). However, other than simply insisting that metaphysics is not a question about semantics or language, as well as posing several rhetorical questions meant to motivate his position, Devitt provides little, if any, in the way of an actual argument to the contrary of Dummett's position.

[4]In using the word 'assertion,' here and throughtout my dissertation, I follow Wright, which is to say, I use the term as synonomous with a truth-apt locution. This choice is not without controversy, but regardless of how I choose to understand 'assertion,' or what name I choose to give to a truth-apt locution, controversy will follow. I leave this controversy to the side, as addressing it is not necessary to advance my main project with this dissertation.

that Dummett's story, as formulated above, imperfectly captures the true commitments of the realist, and that as a consequence, could neglect to include some forms of realism, most notably, some forms of moral realism (Wright 1992, 9-10). For Wright, Dummett's approach is an attempt to capture the realist's commitment to a kind of *objectivity* that they argue the subject in question commands. The purpose of *Truth and Objectivity* is to parse out this nuance, and what it entails for our understanding of truth.

It is worth pausing here explain that my use of 'representational semantics,' and the notion of *representation* that underlies it, diverges from Wright's own use of 'representation' and 'representational.' When laying out the presumptuous claim, Wright actually characterizes (a) as outling a kind of "fit" between the mind-independent world and our thoughts about it, and not as a commitment to representational semantics, as I did (Wright 1992, 2). The reason for this discrepency between Wright and me regarding (a) is due to Wright's view that there are certain minimal platitudes regarding the nature of truth, and that one of these platitudes is that "to be true is to correspond to the facts" (Wright 1992, 34). For Wright, although the anti-realist will deny that to "correspond to the facts" involves representing a state of affairs which is independent of us and our cognitive activity (e.g. that might obtain independently from our ability to detect it), she can still argue that the discourse is representational, and so "answers to states of affairs which, on at least some proper understanding of the term, are independent of us" (Wright 1992, 4-5). Thus, insofar as any anti-realist is commited to some notion of truth-value, she will nevertheless agree, indeed regard it as a platitude, that the discourse is representational. Since both realists and anti-realists will accept that true assertions represent things as they are, Wright argues that it is a platitude of truth that it is representational.

This leads to a difference between my use of the concept of 'representational' and Wright's. Wright's use of 'representation' includes "representations" of mind-dependent states of affairs, and for Wright, 'to represent' means merely to correspond to the facts in the platitudinous sense that even anti-realists will agree to—for instance, in the sense in which to say that it is true that a joke is funny only if the joke is actually funny. As just shown, this means any anti-realist commited to our ability to provide true assertions will also be commited to the view that assertions in the domain in question are representational, under this usage. What makes someone a 'realist,' for Wright, will be more than simply a commitment to 'representation' when understood this way. Rather, Wright argues that a realist will also be committed to other characteristics that truth might have in different domains of discourse, in addition to the platitudes of truth.[5] Some of the other features that a realist account of truth may possess include *cognitive command*, where disagreement regarding an assertion's truth is always attributable to two agents either having different information or at least one of them making some sort of cognitive error, and *wide cosmological role*, which is the extent of its ability to explain truths in a variety of other domains (Wright 1992, 92-93, 196).

It is here that we can see the nuance that Wright adds to Dummett's semantic approach to analyzing realism/anti-realism debates. Although Wright also pursues an analysis of realisms by way of an analysis of what it means for an assertion to be true, Wright does

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[5]I use the word 'realist' here cautiously, as it is by no means a great stretch to argue that Wright is proposing to recast realist/anti-realist debates as actually debates regarding the level of objectivity a class of assertions can claim, and that words like 'realist' may not be all that useful. I continue to use 'realist' because Wright argues that positions typically understood as realist will be positions that claim a greater level of objectivity for a class of assertions than an anti-realist for that class would claim. Further, I believe that this way of understanding realism is consistent with my proposal that a realist is committed to representational semantics (as understood by me).

not simply reduce the realist's commitments in this regard to a single commitment to successful and accurate reference to a mind-independent state of affairs. Rather, Wright associates various kinds of realisms with positions that argue that the nature of truth is more robust than is reflected in its platitudinal features, and thus reflects a greater claim to objectivity. In this way, there can be gradations of realist commitment, depending on how robust a person argues the notion of truth is for a given domain, and the level of objectivity claimed. Put somewhat crudely, the more robust a person argues truth is for a given domain of discourse, the "more realist" he will be. For example, someone who argues that true assertions in a given domain possess both cognitive command and wide cosmological role will be "more realist" than someone who believes that those assertions only possess cognitive command. This is because an assertion which possesses both features will be able to claim a greater level of objectivity than assertions which can only claim one. Conversely, those with less robust notions of truth are "more anti-realist." And the "most anti-realist" position, as far Wright thinks one can be intelligbly accounted for, will argue that truth will be a notion that satisfies all the platitudes, but has none of the other features that make a notion more objective (Wright 1992, 36). Wright argues that this allows for positions that still reflect strong commitments to objective states of affairs, but not a commitment to a mind-independent subject, to still be counted as realist positions.

How then, does my usage of "representation" differ, and in particular, how is my usage related to Wright's view? Recall that Wright's realist makes the presumptuous claim that, when successful, our assertions "fit" the way things are. The short answer is that I take myself to be following standard philosophical usage by using the term 'representation' to be a term of art that replaces the mysterious notion of 'fit.' It is intended to answer the

question "What could it possibly mean to say that something linguistic 'fits' something non-linguisitic?" by saying "We mean that it accurately represents how things are."[6] So, in discussing this linguistic "fit," I will simply be discussing *representation*, and attempts to outline this "fit" as attempts at outlining a *representational semantics*. As such, my notion of representation, since it is something that is supposed to be part of a full-fledged realist account, includes things like cognitive command and wide cosmological role, and is by no measure platitudinal. Although not the same, exact words that Wright would use to capture the presumptuous claim, there is, I believe, no substantive impact made to the core intuition. This difference between Wright and me is a terminological choice regarding the precision that the concept of 'representation' should have.

In using 'representation' in this way, there should be no worry that any of the strengths that Wright's more nuanced approach to analyzing realist/anti-realist debates offers have been lost. Rather, all of the important respects discussed by Wright in which objectivity can be developed can be recognized precesiely as such, as ways of establishing varying degrees of objectivity. Positions which fall short of proposing a representational semantics, and so in the context of this disseration, fall short of realism, can still propose some kind of semantics which can reflect some measure of objectivity. By proposing non-representational semantics where truth is understood in such ways as to, for example, establish cognitive command or a wide cosmological role, semantic anti-realist positions can come with varying degress of objectivity. In this regard, the insights of Wright can still be accounted for, while remaining true to a Dummettian, mind-independent account of realism.

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[6]Of course, this may merely replace one mystery with another, but I think it's a standard philosophical first step.

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The realism/anti-realism debate, then, for both Dummett and Wright, is intimately tied to the semantics of the discourse in question, and the manner in which a truth-value of true is conferred onto assertions, if it can be so conferred at all. Arguments for realism and anti-realism then become, for Dummett and Wright, arguments about how it is that certain assertions with disputed subjects can be true or false. In particular, we can identify realism with the commitments identified by the presumptuous claim: that is, as commitments to (a) representational semantics; and (b) that some of our assertions are true. It is this paradigm for understanding realism/anti-realism debates, and for understanding realist commitments, that I will be employing for this dissertation.

2.2 Anti-Realisms

With this backdrop for realism/anti-realism debates, as well as the resulting understanding of realism, we can now begin to identify what motivates anti-realist commitments. Using Wright's understanding of realism, we can now define anti-realism as any view that challenges one of the simpler claims conjoined by the realist's presumptuous claim. Although this won't always be tidy and without controversy, it does allow us to identify the significant philosophical commitments that seem to motivate various anti-realist positions. For instance, *semantic anti-realism*, outlined by Dummett and later expanded upon by Wright, finds its roots in philosophical challenges to (a), that our assertions are representational (Dummett 1993, 465; Wright 1992, 3). For the semantic anti-realist, the view that mathematical assertions, for instance, are representational tells the wrong semantic story about those assertions (Wright 1993, 3). Broadly speaking, there are two influential approaches

to making this argument.

One approach to challenging (a), *non-cognitivism*, argues that, despite appearances, some "assertions" are not really assertions at all, as they are incapable of being either true or false (Dummett 1993, 466; Wright 1993, 6). An example of this line of reasoning is traditional *expressivism* in ethics, which typically argues that moral assertions are not really assertions, but are simply the expressing of an attitude held by the utterer (Dummett 1993, 466). The other approach argues that truth is not the exclusive property of the realist, and that what makes an assertion true is not the accurate, face-value representation of facts that are beyond our normal faculties of detection, but something else (Dummett 1993, 468; Wright 1993, 4-5). For instance, *constructivism*, the dominant semantic anti-realist approach in mathematics, argues that truth is not the result of the accurate representation of mind-independent objects, but rather is the result of being able to produce a proof for the sentence asserted, or to be able to provide conclusive verification of it (Dummett 1993, 443-444). With regards to mathematics, constructivism argues that mathematical assertions are not representational, and that mathematical assertions are true if *and only if* they are provable. In other words, the truth of a mathematical assertion such as "2 + 2 = 4" is not the result of accurately representing mind-independent objects that the numerals '2' and '4' may refer to, but instead from the the norms of proof production in mathematical practice.

On the other hand, anti-realists can instead challenge (b) of the presumptuous claim. These sorts of anti-realists, often classified as error theorists, accept that our assertions are representational, or strive to be so, but deny that we ever successfully represent the sorts of facts that would make the assertions of a contested discourse true (Wright 1993,

5). Error theory has been advanced in various domains of philosophical interest, but most prominently in ethics, mathematics, and modal logic. As advanced in ethics, for example, ethical error theory argues that although our assertions about what is right or wrong strive to represent certain normative moral facts, they ultimately fail to do so, since the world is such that it does not include any normative features (Mackie 1977, 48; Wright 1992, 6). As a result, all of our assertions regarding ethical behaviour are false, or at the very least, sincerely made assertions about what is right and wrong are made in error. Similarly, mathematical error theory accepts the representational account of semantics for mathematical assertions, but denies that there are mathematical entities, such as numbers, to be represented, and so concludes that mathematical assertions are, strictly speaking, false, or for the more timid error theorist, incapable of being true (Dummett 1993, 433-434; Field 1980, 7).[7] Regardless of how bold or timid the error theorist in question is, she will generally be committed to the idea that sincere attempts at true assertions will be in error, and that this error can be attributed to the relevant discourse's failure to make reference to the subject matter it purports to be about.

It should be stressed again that this is by no means a definitive means to classify a metaphysical position as necessarily realist or anti-realist, and that I do not intend it to be so. Several positions straddle the lines of distinction given above, and some cannot be so neatly situated within this roadmap. Nevertheless, this roadmap does allow us to hone in on important philosophical commitments that typically motivate the adoption of one

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[7]Of course, some of what we say about numbers and such would come out literally true on such a view—"There are no numbers," for instance. So strict correctness calls for distinguishing vacuous truths from substantial truths in those domains, but consistently observing this nicety quickly becomes quite tedious. Moving forward, I will simply be using 'assertion' rather than 'substantial assertion,' and so on, except in cases where close attention to the distinction is required.

position over others. In particular, we can now focus on the question of whether or not an assertion that purports to refer to some disputed entity or object is capable of being true or not, and why this will be so. Moreover, answering this question will typically reflect the sorts of commitments we have regarding the "reality" of the disputed subject matters.

2.3   The Stakes of these Debates

While what is at stake in any particular realist/anti-realist debate depends on the particulars of the case, insofar as the roadmap presented above describes features common to all such debates, it would not be surprising to find that there are philosophical costs and benefits characteristic of the different positions. Each position seems to have its own philosophical strengths as well as weaknesses, and there seems to be no one position which shares all of the strengths of the others and none of their weaknesses. As such, there will always be costs when pursuing a particular a strength, and many may be motivated to reject a position if they believe the costs of that position to be too great.

One such dilemma involving a trade-off between particular strengths and weaknesses comes up in the realism/anti-realism debate within mathematics, though I will suggest that its lessons apply more generally. In "Mathematical Truth," Paul Benacerraf argued that the choice between realism or semantic anti-realism in mathematics results also in a choice between a straight-forward and homogenous semantics or a workable and uncontroversial epistomology (1983). More specifically, Benacerraf argues that each of these strengths comes at the expense of the other (Benacerraf 1983, 403). To see why, Benacerraf asks us to consider two seemingly similar assertions:

(1)   There are at least three large cities older than New York.

(2)   There are at least three perfect numbers greater than 17.

Benacerraf then asks us if we can correctly represent the logical grammar of both of these assertions as:

(3)   There are at least three FG's that bear R to a.[8]

As Benacerraf notes, it seems fair to say that (1) will have the same logical grammar as (3) (Benacerraf 1983, 405). This means that the truth-value of (1) will be determined if the object picked out by a bears the relation defined by R to at least three objects with the predicates FG. "Thus," Benacarraf argues, "if (1) is true, it is because certain cities stand in certain relation to each other" (Benacerraf 1983, 405). All of this, it seems, is rather uncontroversial.

What is more controversial, however, is if the logical grammar of (2) can be captured by (3). For the mathematical realist, the answer will be yes. This is because the realist will argue that the semantics of both assertions is representational—the semantic relation between a and '17' in (2) will be the same as the semantic relation between a and 'New York' in (1) outlined above. This will mean the logical grammar for (1) and (2) are the same, since the semantics of each assertion can be reflected by (3). In other words, the realist can maintain that there is one homogenous semantic understanding for both (1) and (2), and thus, one theory of truth that can be applied to both assertions (Benacerraf 1983,

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[8]That is, that F and G are the predicates 'large' and 'city' for (1) and 'perfect' and 'number' for (2), a is 'New York' in (1) and '17' in (2), and R is the nature of the relationship between the 3 objects with the predicates FG and a, in this case, 'older than' for (1) and 'greater than' for (2).

408). The mathematical realist approach, however, seems to come at a great epistemological cost. If '17' refers to some real entity, then there must be some epistemological account of our access to such an entity, so as to account for our mathematical knowledge of (2). Unfortunately for the mathematical realist, it does not seem like many of our typical routes for gaining epistemic access, like the ones we would use to gain knowledge of New York, say, offer such an account of how an abstract entity can have a causal relationship with our minds (Benacerraf 1983, 416). As a result, the mathematical realist will have to appeal to controversial or implausible means of epistemic access for mathematical truths, such as some sort of "mathematical intuition" by which mathematical objects can be perceived and so have a causal impact on our minds. So, Benacerraf argues, adopting the mathematical realist position will have the advantage of a straightforward and homogenous semantics for our assertions, but will come at the cost of having to adopt a controversial epistemology (Benacerraf 1983, 416).

If, on the other hand, we were to adopt a semantic anti-realist, most notably, constructivist, approach to (2), the inverse will be true.[9] In pursuing an uncontroversial epistemology, the mathematical constructivist will reject that our mathematical knowledge is the product of some sort of causal connection between abstract objects and our minds. Instead, the constructivist will say that mathematical knowledge is the result of the construction of mathematical proofs (Benacerraf 1983, 416-417). In other words, our

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[9]To be precise, Benacerraf's actual target here is what he calls the "'combinatorial' view of mathematical truth" (Benacerraf 1983, 416). Given the combinatorial view's heavy emphasis on mathamtical truth being discovered by means of proof-production, it would seem the dilemma is just as pressing for the constructivist, and so Benacerraf's argument would extend also to constructivism. Given that mathematical constructivism seems like a more common position in contemporary philosophy of mathematics, I have shifted the focus to constructivism.

knowledge of (2) is not the result of being able to directly recognize that certain objects stand in certain relation to each other, as is the case for (1), but rather of our ability to provide a proof of the sentence. Pursuing mathematical constructivism, however, means that we have to provide a different semantic account of (2) than what was given for (1), since the conditions for the truth of (1) will be different than for (2). That is, although we may accept a representational semantics for (1), we would have to abandon it for (2) since the condition for its truth is not the accurate representation of objects. So, adopting a mathematical constructivist approach means having to adopt semantic pluralism, and by extension, alethic pluralism—a cost that many find controversial, including Benacerraf himself (Benacerraf 1983, 407, 420). Benacerraf concludes that regardless of whether or not we adopt a realist or constructivist approach, when it comes to providing a simple and compelling semantics and epistemology for mathematics, one will always come at the cost of the other (Benacerraf 1983, 410).

Although Benacerraf was focused on the stakes of the realism/anti-realism debate within mathematics, it is not implausible to extend this dilemma more generally to realist and semantic anti-realist positions in other fields of philosophical interest. For example, suppose in evaluating an act of lying to resolve some moral dilemma, someone asserts:

(4)   There are at least three morally acceptable alternatives that are more praiseworthy than lying.

It would seem that the same dilemma of interpretation faced by (2) will also have to be faced when choosing between realist and semantic anti-realist interpretations of (4). It seems as though the moral realist will accept (3) as representative of the logical grammar of (4)

since they will accept a representational semantics for (4). In this regard, the moral realist will have the strength of a homogenous semantic theory. But just as the mathematical realist had to posit some implausible avenue for epistemic access to mathematical objects to account for the truth (2), the moral realist will also face this challenge, having to provide some account for how we perceive normative moral features, and how those features can have a causal impact on our minds—even utilitarians and those with a naturalized realist metaethics will have to address Moore's Open Question Argument (Moore 1993, 62). The moral semantic anti-realist, on the other hand, will be subject to the same controversy that the mathematical constructivist must face—in pursuing an uncontroversial epistemology for knowledge of normative moral features, the moral semantic anti-realist will have to embrace semantic pluralism, and argue that (3) does not capture the logical grammar of (4). And so, just as we had to choose between a homogenous semantics and an uncontroversial epistemology, it looks as though the same dilemma will arise for moral philosophy.

Error theory, on the other hand, would not seem to be subject to the above dilemma. Indeed, one of the characteristic strengths of error theory is its ability to sidestep this dilemma entirely. In accepting that representational semantics are the appropriate semantics for any set of assertions, the error theorist is not committing herself to any drastic semantic revision for any set of assertions, whether global or local to some distinct domain of discourse, and can embrace a homogenous semantic account for all assertions. Thus, the error theorist can avoid the controversy of having to embrace semantic pluralism. Moreover, in simply accepting that we are in error when we believe that some set of assertions are true, the error theorist does not need to provide any account of how controversial entites, abstractions, or states of affairs have a causal connection to our minds. In pursuing the

proverbial desert landscape, an error theorist need not offer any drastic semantic revisions or implausible means of epistemic access to ontologically controversial subject matter.

Despite the ability to avoid controversial semantic or epistemological commitments, error theory is not without its philosophical weaknesses. One of the most common challenges error theory often faces is to provide an account of why something about which all our substantive claims are false can still be useful or important. In other words, if, say, mathematical assertions are false, then why is it that mathematics has been so useful? The same can be asked of most error theorists regarding the domain of discourse for which they are error theorists. How does the modal error theorist account for the utility of modal discourse? Or the historical error theorist for history, or the moral error theorist for moral discourse? It would seem that all of these subjects have served us well before, and continue to do so, and so the error theorist must account for the usefulness of these supposedly error-prone discourses. As a potential response to this challenge, an approach known as fictionalism has been developed, which argues that some of the objects and properties that are the subjects of some discourses are fictional, though they offer us a useful means of interacting with and understanding the world around us (Kalderon 2005a, 3). For instance, mathematical fictionalism understands mathematical entities to be fictional objects, and that mathematics more generally is, in crucial respects, akin to a game of make-believe rather than an actual representation of facts about mind-independent entities (Leng 2010, 9; Yablo 2005, 98). This allows us to continue to make mathematical assertions and explains the nature of their use. Similarly, modal fictionalism argues that the 'possible worlds' often referenced in modal discourse are not real worlds, but rather are fictional objects that modal operators quantify over (Kim 2005, 116). This allows modal

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logic to preserve the usefulness of understanding modal operators as quantifiers over possible worlds without having to be committed to modal realism. I will be giving greater detail on how fictionalism addresses the worry of the usefulness of mathematics and modal discourse in Chapters 2 and 3, respectively.

Another challenge that I believe is fair to pose to error theory is one that has been called *the incredulous stare* in another context.[10] Not a terribly sophisticated challenge, the incredulous stare objection is one that simply points to the incredible controversy, or perceived outlandishness, a view entails, and insists that there must be a better explanation. In the context of fictionalism, and more precisely, mathematical fictionalism, this seems like a relevant challenge to address. As noted in the previous section, the mathematical fictionalist will argue that mathematical assertions cannot be true, and that any sincere attempt at a true mathematical assertion will be in error. Considering that many simple mathematical assertions, such as "$2 + 2 = 4$," seem like exemplars of true assertions, if ever there were any, it seems fair to raise an eyebrow at the fictionalist when they say we are in error when we insist on the truth of such assertions. This issue may be unique to the mathematical fictionalist, but the relevance of this challenge to other domains should not be ruled out, especially if the error theorist ends up denying the truth of what may be widely accepted as paradigmatically true assertions within those domains. For example, if "Murder is wrong," or "I could have worn a different shirt today," are paradigmatically true assertions within moral and modal discourse respectively, then we may wish to glance at moral and modal error theorists with the same incredulity.

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[10]Specifically, the incredulous stare was first thought of as a challenge to modal realism preempted by Lewis, and later pursued by Rosen. I will go over the incredulous stare objection to modal realism in Chapter 3.

2.4 Chapter Conclusion

In closing, the semantic analysis of realist/anti-realist debates advanced by Dummett and further developed by Wright allows us to identify certain characterist commitments of the realist, as well as the characteristics of anti-realist challenges to realism. Realists can be characterized by their commitments to representational semantics and to some of the claims in the disputed domain being true, while anti-realists can be characterized by either their challenge to the realist's commitment to representational semantics or to our ability to make true assertions in the disputed discourse. This roadmap not only allows us to identify these characteristic commitments of the positions a person can take in these debates, but also the characteristic strengths and weakness of those positions. Realists will be able to advance a straightforward semantics that is homogenous with standard approaches to semantics that is presumably applicable in most other domains, which the anti-realist cannot. But such a homogenous semantics comes at the expense of having to embrace a controversial epistemology. Semantic anti-realists will be able to provide an uncontroversial epistemology, but it will come at the expense of having to embrace semantic pluralism. Finally, error theorists can avoid this dilemma, and provide both a homogenous semantics and uncontroversial epistemology. However, doing so comes at the expense of needing to account for the utility of the disputed discourse, as well a controversial immodesty when challenging the truth-values of assertions that are typically understood, and widely accepted, as being paradigmatic examples of true assertions.

Chapter 3

Raising the Curtain

With the preceeding discussion of the philosophical commitments characteristic of realist and anti-realist positions in hand, it is now possible to outline just how fictionalism can be reconceptualized as a semantic anti-realist position. To do so, I will begin with an exposition of Kendall Walton's approach to understanding make-believe and assertions made within the context of a fiction. From there, I outline the influence Walton's theory of make-believe has had on the development of mathematical fictionalism, particularly the mathematical fictionalism of Mary Leng and Stephen Yablo. With this background in place, I will turn to arguing for the main claim of this chapter, namely, that what Yablo and Leng take to be an account of how mathematics can be false and yet remain useful is in fact an alternative semantics for mathematical assertions. Specifically, I argue that by taking inspiration from Walton's theory of make-believe, Leng and Yablo have provided a compelling fictionalist account of mathematics that shares some important features with Dummettian constructivism, though with a different account of the assertability conditions

for mathematical assertions. Finally, I will argue that understanding mathematical fiction-alism as a kind of semantic anti-realism preserves a notion of mathematical assertions as genuine assertions, in which we are capable of making substantively true mathematical assertions, something that error-theoretic approaches to fictionalism cannot do.

3.1 Walton's Theory of Make-Believe

A key topic in the philosophy of fiction has been understanding how it is that assertions made within the context of a work of fiction can be true or not, given that the entities that are the subjects of such assertions do not actually exist. For example, there seems to be a sense in which "Frodo is a Hobbit" is true, while "Frodo is secretly Spider-Man" is false. When we read *The Lord of the Rings*, the former assertion coheres with the story told therein, and so can be thought of as true in a way, while the latter assertion fails to so cohere, and is typically thought false for that reason. But if the real world is such that there is no Frodo, no Spider-Man, and there are no Hobbits, then there doesn't seem to be any means for the former assertion to be true, and the latter assertion, although still false, is false for an entirely different reason than the one mentioned above. Much work in the philosophy of fiction has sought to address this dilemma regarding the truth-values of assertions made within the context of a fiction.

An influential theory which seeks to resolve the above dilemma is Kendall Walton's theory of make-believe. This theory was first advanced in Walton's "Pictures and Make-Believe" (1973), and was further developed in *Mimesis as Make-Believe* (1990) and "Metaphor and Prop Oriented Make-Believe" (2005).

In order to unpack Walton's theory of make-believe, and how there can be true assertions within games of make-believe, it will help to begin with an example of a make-believe game. Suppose there is a group of children playing a game of Cops and Robbers.[1] While playing, they will make a variety of assertions, such as:

> There are three cops guarding the bank.
> The bag of loot is in the safe.
> The getaway car is parked around the block.
> There are several witnesses watching as we make our escape.
> Bang! You're dead!

As Walton argues, these statements are literally false (Walton 1937, 287). The three children standing out front of a garage are not really cops, and the garage is not really a bank. The bag of Monopoly money kept in a lunch box is not literally a bag of loot kept in a safe. A cardboard box down the street from the garage is not a car and the adults sitting on their front porches will not be witnesses to a literal crime. And finally, the children do not possess real guns when they shape their hands into a pointed finger and upturned thumb, and no one really dies when a child points their finger at another and yells "Bang!" Nevertheless, each of the above assertions are what Walton calls *fictionally true* within the context of the children's game of make-believe (Walton 1973, 287). Moreover, there can be assertions that will be fictionally false within the context of the game of make-believe. For example, if "The bag of loot is in the safe" is fictionally true in one game of Cops and Robbers, then "The bag of loot is not in the safe" will be fictionally false in that

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[1]Walton's own example is game of Mud Pies (Walton 1973, 287). I have opted instead to use the example of Cops and Robbers, which I hope will be a more familiar example of a game of make-believe to more people.

game. These fictional truth-values are not necessarily fixed—context informs whether a given assertion will be true or false, and different contexts will confer different fictional truth-values (Walton 1973, 288). So, for instance, in a different game of Cops of Robbers, perhaps "The bag of loot is not in the safe" may be fictionally true.

But this is not all that is interesting about the sorts of assertions made within games of make-believe, nor does it give a complete picture. Walton also draws attention to the role that imagination and real objects play when looking at assertions made in a game of make-believe. Both imagination and real objects play an important role in understanding what Walton calls *imaginarily true* and *make-believedly true* (Walton 1973, 289). When an assertion is fictionally true simply in virtue of the fact that a person or a group of people imagine that it is true, then that assertion is imaginarily true (Walton 1973, 289). For example, in holding up a plastic bag full of Monopoly money and asserting "This is a bag of loot," the child will have uttered an assertion that is imaginarily true, and it will be imaginarily true for all the children who decide to also imagine that the bag of Monopoly money is a bag of loot.

Assertions which are make-believedly true, on the other hand, are distinct from assertions which are imaginarily true in that their truth-value is tied not just to acts of imagination, but also to the real objects and facts about them that are being used to facilitate the game of make-believe (Walton 1973, 289-290). To understand, consider the assertion "The bag of loot is in the safe." Now it may be imaginarily true in the game of Cops and Robbers that a plastic bag of Monopoly money will be the bag of loot, and that a lunch box will be the safe. This alone, however, is not enough to make the assertion true. In order for the assertion to be true, it must actually be the case that the

bag of monopoly money is in the lunch box. The truth-values of assertions which are make-believedly true are thus tied to facts about the real objects which are serving as the vehicles for our imagination (Walton 1973, 298). In this instance, the truth-value of the assertion is tied to facts about where the bag of Monopoly money is actually situated. In this sense, Walton notes, the truth-values of assertions which are make-believedly true are to some degree independent from the players participating in the game (Walton 1973, 292). If, for instance, the bag of Monopoly money wasn't actually in the lunch box, then the assertion would not be true, and moreover, if the bag were actually in the cardboard box down the street, then the assertion "The bag of loot is in the getaway car" would be true. Walton calls the objects that serve as the vehicles for our imagination the *props* of our game of make-believe (Walton 1990, 42).

With this, Walton provides us with an understanding of how it is the assertions listed above can be true. Firstly, there must be a context that the assertions are situated in, in this case, a make-believe game of Cops and Robbers. Secondly, we must have props which serve as vehicles for our imagination, or as occasions for imaginary truths, in our game of make-believe, such as Monopoly money, lunchboxes, and cardboard boxes. Lastly, in some cases, the assertions must reflect actual facts about our props, facts like their actual location in the real world.

Assertions that employ an act of make-believe for their truth-value can be used for a vartiety of reasons, and can serve different ends. All of the assertions listed above, for example, are assertions about some make-believe world. That is, they tell us about the content of the world that we imaginarily inhabit when we play Cops and Robbers. These make-believedly true assertions are ones that Walton calls *content-oriented* as they reveal

something about the content of the make-believe world, and what is true therein (Walton 2005, 65). However, some make-believedly true assertions serve more useful ends other than simply facilitating games of make-believe. Some make-believedly true assertions actually serve to help us better understand the real world and reveal facts about real objects. Walton calls these sorts of make-believedly true assertions *prop-oriented* (Walton 2005, 65). Prop-oriented assertions are assertions making use of make-believe not to understand some make-believe world, but instead to understand the props themselves, and thus, the actual world rather than a make-believe world (Walton 2005, 66).

An example that Walton gives of an assertion that is prop-oriented is "The city of Crotone is in the arch of the boot of Italy" (Walton 2005, 66). Although not engaging in the exact same sort of make-believe as when we play a game of Cops and Robbers, the assertion still heavily relies on an act of make-believe. We begin by imagining that our prop, Italy, is a boot. The truth-value of the assertion is then determined not solely by our act of imagination, but also by facts about the prop, and in particular, the fact of where Crotone is located in Italy. If, for instance, Crotone were actually in the southern-most peninsula of Italy, then the assertion would be false, and instead "Crotone is in the toe of the boot of Italy" would be true. Despite making use of the same criteria to determine the truth-value of assertions made in the context of make-believe, or to classify some of those assertions as warranted, prop-oriented assertions nevertheless serve different ends than content-oriented assertions do. What makes the assertion "The city of Crotone is in the arch of the boot of Italy" distinct from "The bag of loot is in the getaway car" is that the former reveals something interesting about our prop, and is used as a vehicle to understand our prop, rather than as a vehicle to understand or reveal something about

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some fiction, or a make-believe world where Italy is a boot. The former assertion is then prop-oriented, in that it is used a means to assert something about our prop, while the latter is content-oriented, in that it is used as a means to assert something about the content of a make-believe world.

Before moving on, it is necessary to also discuss what Walton calls *principles of generation*, and their role in generating true assertions in games of make-believe. Principles of generation are reflected in conventions, understandings, agreements, or rules in games of make-believe that outline what is to be imagined given certain circumstances (Walton 1990, 38). In Walton's own words, "principles of generation ... constitute conditional prescriptions about what is to be imagined in what circumstances" (Walton 1990, 41). So, in the example given above, when a child holds up a plastic bag full of Monopoly money and asserts "This is a bag of loot," a rule for the game of make-believe will have been established that whenever we see or interact with the Monopoly money, we are to imagine that we see and interact with actual money. In this regard, the imaginarily true assertion "This is a bag of loot" is dependent on a principle of generation, reflected in a rule or convention of the game of make-believe, that mandates that if presented with a bag full of Monopoly money, we are to imagine that it is a bag full of loot. These principles of generation can then be extended to determine which assertions can be make-believedly true. If, for instance, in our game of Cops and Robbers, we are told that every Monopoly dollar is to be imagined as one actual dollar, then that principle of generation can determine whether the assertion "With all this loot we've just stolen, we can buy all the candy in the candy store" will be make-believedly true or false. In particular, the assertion will be make-believedly true if the number of Monopoly dollars corresponds to a number sufficient

to buy all the candy in the candy store.

Principles of generation, in making precriptions about what we are to imagine in our game of make-believe, extend beyond rules which simply identify the imaginary objects that we are to imagine our props to be. They can also include general rules of play for our make-believe, such as "If a cop grabs a robber's shoulder, then the robber is under arrest and goes straight to jail." Although not a rule that identifies a prop as an imaginary object, it is still a rule that is in the business of telling the players what they are to imagine and when, and so is also a principle of generation for our make-believe. In this case, if a player who is a robber is grabbed on the shoulder by a player who is a cop, then the player who is the robber is to imagine that they have now been placed under arrest. Moreover, if we have some prop to serve as our jail for our game, say an empty refrigerator box in the backyard, then the player who is the robber is given directions about what they need to do in the actual world—the player must now make their way to the backyard and sit in the empty refrigerator box. In this way, principles of generation may also be used to make presecriptions about what a player is to do in the actual world, as much as they may produce make-believedly true assertions.

Further, facts of the actual world can also lead to prescriptions about what we are to imagine in our game of make-believe and to give make-believedly true assertions, and so can also contribute principles of generation in their own way. Suppose that in our game of Cops and Robbers, we mean to mimic as best as we can an actual robbery, and so if something could not be the case in an actual bank robbery, then it cannot be the case in our game of Cops and Robbers. In this way, we may say that "rules of the real world," in a manner of speaking, contribute principles of generation. If, for instance, it were a "rule of

the real world" that humans cannot fly nor possess superhuman strength, then we may say that in our game of Cops and Robbers it is a principle of generation that prescribes that all of the players are to imagine that they are normal human beings with no extraordinary abilities. And so, facts and norms of the actual world can also contribute principles of generation to games of make-believe.

This is just one way that the actual world can contribute to a make-believe game's principles of generation, but the actual world's contribution need not be so specific. The manner in which truths or facts of the actual world can contribute to principles of generation can come in a variety of forms, and can serve many different make-believe purposes. For example, we can use the actual world to inform principles of generation in more fantastic acts of make-believe. Suppose that for our game of Cops and Robbers, the children have grown into teenages and have gotten bored with their game, and so to add a bit of excitement, begin to imagine that the bank robbery is now taking place on the Moon. They may then stipulate that any move or action taken in the game of make-believe must be in accordance with what they have learned in their high school physics class—for instance, that there can be no faster than light travel, or that the speed by which the players can run is hindered by the lack of gravity on the Moon. In this way, the actual world and the players' knowledge about it can facilitate make-believe games that depart significantly from the actual world.

This does not mean that principles of generation must be the sorts of claims that are or can be true in the actual world. In fact, principles of generation can also include claims that we know or believe to be false. Instead of pretending that they inhabit a universe that conforms to the actual laws of physics as they've learned them, our players may decide

to adopt a Looney Tunes approach to physics. And so, for their game, they may accept that a robber can plug up the barrel of a cop's gun by sticking their finger in the barrel, thus redirecting the blast to escape from the other end of the barrel and into the cop's face. Ultimately, there are very few restrictions on what can constitute or contribute to a principle of generation—so long as the principle is one that facilitates make-believe and play, then it can be a candidate for a principle of generation. The wide variety of the sorts of make-believe games we can play is a testament to this openness.

Walton's response to the dilemma posed at the opening of this section is to look to games of make-believe and our capacity to engage in pretend and imagination for an answer. For Walton, literally false assertions can nevertheless be *fictionally true* if in certain contexts, *principles of generation* are at work that mandate to us that when presented with certain real objects, we are to engage in a particular act of imagination. These real objects are the *props* of our make-believe, and generate assertions which are *imaginarily true*. Examples of imaginarily true assertions are "That is a bag of loot" when pointing at a bag of Monopoly money, or "That is a safe" when pointing to a lunchbox, while playing Cops and Robbers. Moreover, certain assertions made within the context of a game of make-believe will be fictionally true in virtue not only of our acts of imagination prescribed by principles of generation, but also by virtue of the actual facts of our props, and are thus *make-believedly true*. An example of a make-believedly true assertion would be "The bag of loot is in the safe" when the bag of Monopoly money is actually being held within a lunchbox in our game of Cops and Robbers. Assertions which are make-believedly true can be *content-oriented* when the assertion is being used to assert something about the content of the make-believe world, or can be *prop-oriented* when they are used to assert something about

the actual prop itself, as is the case when we make-believe that Italy is a boot and assert "The city of Crotone is in the arch of the boot of Italy."

This analysis of make-believe, Walton argues, is the foundation for the entirety of enagement with what he calls *the representational arts* (Walton 1990, 11-12). The representational arts, for Walton, include things like paintings, plays, films, and novels (Walton 1990, 11). Even the passage about Portpatrick quoted at the start of Chapter 1 reflects a sort of make-believe game for Walton. In particular, by using the shoreline of Portpatrick as a prop to help us imagine a cartoon bunny, we can then participate in this act make-believe to help us make prop-oriented assertions regarding the location of a pub in Portpatrick, such as "The pub is on the tip of the bunny's nose." That is, the quoted passage about Portpatrick, and its reliance on imagining a cartoon bunny to explain actual features of the actual coastal village, reflects the same sort of make-believe as imagining that Italy is a boot.

3.2 Mathematical Fictionalism

Although not intended as a means by which to understand assertions that are not limited to the representational arts, Walton's theory of make-believe has had an important influence on those attempting to develop anti-realist positions in areas of philosophical inquiry where the question of realism arises. Most notably, Walton's theory of make-believe has influenced the development of mathematical fictionalism, and in particular, the mathematical fictionalism of Mary Leng and Stephen Yablo. Both Leng and Yablo have extended Walton's understanding of how make-believe generates fictional truths to mathematics in

an attempt to explain mathematical practice in the absense of mathematical objects, such as numbers, functions, and sets.

The major motivation of Leng and Yablo for drawing from Walton's theory of make-believe is to respond to a critical question that any mathematical fictionalist must face. The question, simply put, is this: If mathematical objects do not really exist, then how is it that mathematics is so useful? Considering what seems to be the indispensible role that mathematics has in contributing to some of our best and most successful scientific theories, and the incredible things we can achieve as a result of applying mathematics, an answer to this question is deserved. After all, one of the most compelling reasons to accept mathematical realism is the notion that mathematical realism provides an account of the success of mathematics—mathematics is so successful because there are mathematical objects.[2] In an analogous philosophical discussion, Putnam famously said of scientific realism that "realism is the only philosophy that doesn't make the success of science a *miracle*" (Putnam 1975, 73). A similar line of reasoning is also expressed by Quine's argument that the indispensibility of mathematics for successful scientific practice is reason enough to warrant a commitment to mathematical realism (Quine 1983).

The need for an answer to the above question is recognized by both Leng and Yablo. In *Mathematics and Reality*, Leng outlines what she takes to be the primary argument in favour of mathematical realism:

> P1 (Naturalism): We should look to science, and in particular to the statements that are considered best confirmed according to our ordinary scientific standards, to discover what we ought to believe.

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[2]It is worth noting that there are various forms of mathematical realism, and not all realists may actually have an obvious solution to this propblem. For a more detailed discussion of this topic, see Shapiro's article "Mathematics and Realism" (1983).

P2 (Confirmational Holism): The confirmation our theories receive extends to all their statements equally.

P3 (Indispensibility): Statements whose truth would require the existence of mathematical objects are indispensible in formulating our best confirmed scientific theories.

∴

C (Mathematical Realism): We ought to believe that there are mathematical objects. (Leng 2010, 7)

In arguing in favour of mathematical fictionalism, and against mathematical realism, Leng targets the second premise of the above argument. Specifically, she argues the we ought not to accept the second premise as true since many of our best and most successful scientific theories make use of idealizations that we know, or typically believe, to be literally false (Leng 2010, 9). Such idealizations include frictionless planes when designing engines, motors, or roadways, the absence of air resistance when theorizing on the trajectories of projectiles, or that fluids are continuous substances when developing theories about the behaviour of fluids (Leng 2010, 42, 111). Crucially for Leng, though, when we experimentally confirm theories involving such idealizations we do not regard the idealization itself as receiving equal confirmation to the rest of the theory—we do not regard the experiments as confirming the existence of frictionless planes and the like—and so confirmation holism does not seem to be true (Leng 2010, 112). Leng then identifies the major task of her theory to be to give an account of the utility of mathematics, and the utlity of the postulating of mathematical objects, as akin to the utility of idealizations such as frictionless planes, vacuums, and continuous substances. Doing so, Leng argues, shows that despite the indispensibility of mathematics, we need not be any more committed to the existence of mathematical objects than to frictionless planes.

The importance of this task is also acknowledged by Yablo in his article "The Myth of the Seven." There, Yablo also brings attention to the need for an account of the usefulness of mathematics should mathematical objects not exist. Yablo draws specific attention to Hartry Field's argument that mathematical explanations are conservative extensions of nominalistic explanations (Yablo 2005, 91; Field 1980, 11). In other words, although mathematics makes it easier to provide scientific explanations, anything stated or explained using mathematics can be stated or explained nominalistically. Yablo's worry with Field's argument is that it hints at the idea that mathematics *can* be useful, but doesn't explain how mathematics *is* useful (Yablo 2005, 91). Yablo's arguments are intended to address this worry.

Although Leng and Yablo pursue different answers to the question of how mathematics is useful in the absence of really existing mathematical objects, they both take inspiration from Walton's theory of make-believe. In particular, both argue that mathematical practice begins with attempts to reason about real world objects, where those objects and facts about them serve as props to facilitate mathematics as a Waltonian game of make-believe. In brief, both take as a starting point concrete objects and facts about them to serve as props, and make abundant use of the tools of formal logic, which they presumably take to be metaphysically non-committal, to describe principles of generation. From this starting point, Yablo and Leng outline games of make-believe which produce the objects and facts of mathematics.

Leng pursues a top-down approach to explaining the usefulness of mathematics, showing how set theory, and by extension the rest of standard mathematics, can arise out of a Waltonian game of make-believe. Leng argues that non-mathematical objects can serve

as urelements that can be collected into a fictional set, which we will call U (Leng 2010, 177). This act serves as the foundational act of make-believe. From here, the axioms that we've accepted as part of our set theory will serve as principles of generation. So, for example, suppose we are considering non-mathematical objects that can be characterized by some predicate ψ. With non-mathematical objects serving as our props for our set theory, coupled with the sub-set axiom as a principle of generation, those objects can then facilitate our imagining of the subset of the set of urelements U that have the property in question, i.e.: $\{x \in U : \psi(x)\}$ (Leng 2010, 177). With this in place, we can get "pure sets" by, for instance, considering the set of all urelements that have some impossible property (e.g. being non-self-identical) to get the empty set, and from there can use the standard definitions taught in many courses in set theory to define all of standard mathematics using our axioms, that is, our principles of generation. Leng's point is that all of the mathematical objects so generated are part of the game of mathematical make-believe because the original set of urelements is a product of make-believe.

With non-mathematical props acting as urelements of set theory, facts about those objects will make some further assertions made within set theory make-believedly true. The example discussed by Leng is how, depending on the number of fingers on my left and right hand, the assertion "There is a 1–1, onto function f of the set of fingers on my left hand to the set of fingers on my right hand" will be true or false (Leng 2010, 178). Put another way, whether or not there is a 1–1, onto function between the sets R and L, where R is the set of fingers on my right hand and L is the set of fingers on my left hand, will be determined by whether or not our props, the fingers on my left and right hands, make it true—if I have all 5 of my fingers on each of my hands, then it will

be true that there is a 1–1, onto function between the sets of R and L. While this is a very simple example, nevertheless, more complicated situations in which mathematics is applied are no different for Leng. Assuming that all of the other branches of mathematics can be developed in set theory, then, an account of how standard mathematics can be useful without the existence of mathematical objects will have been provided (Leng 2010, 177n). Moreover, an account for the usefulness of mathematics will have been provided by inferring that that mathematical truths reflect, in some way, truths about our props.

For Leng, examples of this sort helps explain the usefulness of mathematics in the absence of actually existing mathematical objects by way of understanding our mathematical assertions as prop-oriented assertions. In particular,the concrete objects in the world serve as props by being the urelements in the imagined initial set of urelements that is the basis of set theory. Although not explicitly stated as such, the axioms of our set theory, which will be expressed using the language of logic, serve as our principles of generation. From here, the assertions we make within the make-believe will be explained, and will help us to explain, and reason about, facts about the objects.

To see more explicitly how this can be useful in the actual world, and to allow us accomplish tasks and make predictions, we can think of The Three Stooges trying to use the above to reason about functions more generally. Suppose Moe wonders whether he could poke all of Curly's eyes using only the fingers of one hand. In order to do be able to predict whether such a task can be accomplished, he needs to be able to demonstrate that he has at least as many fingers on one hand as Curly has eyes. To do this, he begins with the make-believe set of all concrete objects, U, as well as some axioms which serve as principles of generation, such as the Axiom of Subsets, expressed as "For any set S and any

property $P(\mathrm{x})$, $\{x \in S : P(x)\}$." From this, it follows that for any property $P(\mathrm{x})$, the set of concrete objects with that property is a set. Moe, after having identified the properties of being a finger on his hand and of being an eye in Curly's face, can use this game of make-believe to construct two make-believe subsets: R, the set of fingers on his right hand, and E, the set of eyes in Curly's head. Employing some mathematics available via well known methods of defining numbers and arithmetic in set theory using the standard axioms, he is able to conclude that there is an onto function between the set of fingers R to the set of eyes E. Because these are sets of concrete objects, Moe has learned something about what actions are possible in the actual world—namely, he's learned that he can poke all of Curly's eyes with one stab of his right hand. Obviously, this is a homespun example, and not nearly as complex as set theory can get. But this example shows how the availability of sets of concrete objects within Leng's mathematical make-believe provides the material for an explanation of the usefulness of the make-believe to reason and accomplish tasks with actual objects in the actual world and relations.

Yablo, on the other hand, makes use of Walton's theory of make-believe to provide a bottom-up account of the usefulness of mathematics. Yablo argues that mathematics begins with certain existential claims of first-order logic to serve as props that invite us to imagine certain mathematical objects, such as numbers (Yablo 2005, 98, 104). In particular, Yablo imagines a barter society that must quantify over concreta to ensure fair trades (Yablo 2005, 104). They begin with the following definition in first-order logic:

(i) $\exists_0 x F x =_{df} \forall x (F x \rightarrow x \neq x)$.

What this essentially says is that any x that has the property F is not self-identical, or

in other words, that there is no x with F. This provides us with a baseline of nothing, or more accurately, an empty collection of Fs, to begin our counting, as it is a purely logical way of saying that there are exactly zero Fs. In other words, we have something that can now serve as our prop for '0,' at least when talking about Fs. From there, we move on to the following recursive clause:

(ii) $\exists_{n+1}xFx =_{df} \exists y \left(Fy \,\&\, \exists_n x \left(Fx \,\&\, x \neq y\right)\right).$

Simplified, by putting n = 0 on the right, this line tells us how to say that there is an x that has F, but that there are no other F's—in other words, that there is exactly one F. This, then, serves as our prop for '1' when talking about Fs. From here, applying (ii) again with n = 1 on the right, we have a line that says there are two F's but no more, which will be our prop for '2' Fs, similarly we set a prop for '3' by having n = 2, and so on. But this is cumbersome: strictly speaking, these sentences are schematic, which means that we really have a different set of numerical quantifiers defined for each predicate F in the language. By making-believe that there are such things as *numbers*, we can, so to speak, talk about all the predicates of the language at once, and have a far less complex way of counting than what could be done if we simply relied on their first-order logical props.[3]

By using the above first-order quantifications as props to facilitate a make-believe that there are numbers, the barter society now has a means to capture, in a finite language, the notion of an unfair trade—that is, a way of saying that one number is not the same as

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[3]Some people may wonder whether the move to make-believe numbers is what's really needed, and instead propose a move to second-order logic to quantify over the predicates themselves. This move, however, would entail a whole other set of controversial ontological commitments regarding the nature of the predicates. In so far as this is an attempt to outline a story of the development of mathematics that is free from any controversial ontological commitments, I won't be considering this possible move.

another number, that doesn't make use of an infinite conjunction. They do this with the following rule:

(R1)   If $\exists_n x F x$ then $*n =$ the number of Fs$*$, and if $\sim\exists_n x G x$

then $*n \neq$ the numbers of Gs$*$.

In the above conditionals, everything within $**$ gives the identity conditions for the mathematical object we are to imagine given in the antecedent, which is to say, the numbers we are to imagine. With this rule of make-believe in place, or in other words, with this principle of generation in the society's game of make-believe, the barter society can now clearly specify when the claim $(\#x)Rx \neq (\#x)Sx$ is true in their game, allowing them a simple and easy way to assert that the number imagined by one first-order quantification does not equal the imagined number produced by another first-order quantification (Yablo 2010, 104).

Concerned with the inequality of gem-ownership between its members, the barter society decides on pursuing a scheme of gradual gem redistribution, wherein each year sees less disparity in gem ownership than the last between the members of the society. In order to put together a multi-year plan, they need a way of talking about future numbers of gems that will need to be redistributed in the coming years. But to express this goal, they need to be able to talk not only about the number of real objects with some property, but also of the *number of numbers* with a property (like the property of being the number of gems owned by someone). To do so, they propose the following rule:

(R2) $*$If $\exists_n x F x$ then $n = (\#x)Fx*$, and $*$If $\sim\exists_n x G x$ then $n \neq (\#x)Gx*$.

(R2) looks much like (R1), but with a significant difference. This difference is marked with what is now included inside the ** markers—with what is now being treated as objects inside the make-believe game. In particular, the antecdents of (R1) have been moved inside the scope of **, or inside the scope of our game of make-believe. We are now quantifying not only over objects in the real world, but objects within the make-believe, when we state the antecedents of the conditionals. In particular, we treat numbers as objects to be quantified over.

This new rule presents an important shift in the sort of make-believe the society is undertaking. A new act of make-believe has been scaffolded on top of the initial act of make-believe, wherein the numbers that (R1) prescribed us to imagine have now become the props for this further act of make-believe. In proposing and advancing the rules for such instances of scaffolding upon previous acts of make-believe, the society will have to make certain choices about how to best treat the props given the task at hand. Moreover, since the true assertions that can be made within a make-believe that is scaffolded on top of another are used to explain something about our props from the previous make-believe, the assertions will be prop-oriented rather than content-oriented, where the props are the numbers themselves. In this instance, the society now has the means of saying what it is for one number to be less than another number.

From here, the society may begin to wonder whether it is even possible to have an equal distribution of gems. To talk sensibly of such matters, some notion of addition and multiplication, and so of subtraction and division, would be helpful. Departing slightlty from Yablo's story, we can provide a recursive definition for '+' [4]:

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[4]What follows is a generalized account of Yablo's simplification of operations over finite numbers (Yablo

(iii) $\exists_m x F x + \exists_0 x F x = \exists_m x F x.$

This line tells us that (when adding Fs) $m + 0 = m$, with '0' being given to us from (i). Essentially, we are saying that adding 0 to any m leaves us with m. We then move on to:

(iv) $\exists_m x F x + \exists_{n+1} x F x = \exists_{(m+n)+1} x F x.$

This line tells us that for any m, when n + 1, i.e. a non-zero number, is added to it, the result will be equal to $(m + n) + 1$—and we know what $m + n$ is from previous applications of (iv), and we know how to add one more to it from (ii). With this recursive definition, we can now provide an account of addition in purely logical terms. But by using (i) and (ii) as props to facilitate the imagination of numbers, and (R1) as a principle of generation, the society can now add numbers, and not just the number of Fs. That is, facts of addition can now be facts about numbers themselves, and not facts about "the number of Fs" for every possible F.

The society also wants to be able to capture the notion of who counts as an ancestor to settle matters of inheritance. To do so, they propose the following rule:

(R3)   For all $x_1,...,x_n$, *there is a set y such that for all z, $z \in y$ iff

$z = x_1 \lor z = x_2 \lor ... \lor z = x_n$*,

allowing them to capture all of the ancestors of a particular family in a set. In settling issues of inheritance, the society wishes to consider not only ancestors, but also descendants. However, unlike ancestors, there is no fixed number of descendants, since there is always the possibility of adding another descendant. In other words, the number of descendants is

2005, 105)

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indeterminate and indefinitely extensible. Put more formally, there is no x_n to terminate the list of descendants as there is with the list of ancestors, as reflected in (R3). To capture the notion of the descendants of a family, the society proposes a rule that will allow them to include any object into a set so long as it possesses the required predicate:

(R4)   If F is a predicate of concreta, then *there is a set y such that for all z,

$z \in y$ iff Fz*.

This effectively allows the society to propose sets with an infinite number of members, or at least potentially infinite sets. By this stage in the story, it is increasingly difficult to produce homespun examples of how practical concerns motivate each decision the society makes. Instead the motivations are increasingly abstract and, so to speak, intellectually driven. Nevertheless, it will be useful to consider a couple of further steps Yablo includes in the story. Concerned that there might be sets that are not extentions of a predicate that their language gives them the expressive power to describe, they propose the following rule:

(R5)   Whatever x_1, x_2, ... might be, *there is a set containing all and only x_1, x_2, ...*.

In expirementing with what they can do with sets, the society lays out a new proposed rule:

(R6)   *If x_1, x_2, ... are sets, then there is a set containing all and only x_1, x_2, ...*

Much like (R2), the adoption of this rule marks a significant move in the development of the make-believe of the society. Firstly, it is the first rule so far with the potential

to create a paradox that many will be familiar with, the Russell Paradox. In order to avoid this potential paradox, Yablo tells the story as involving a decision by the society that the license to make-believe there are sets and their members will depend on prior liscence to make-believe that there are those members. Or in other words, that there is a cumulative hierarchy of sets. Secondly, as with (R2), what was previously an antecedent outside the scope of the make believe in a prior rule has now been moved to be inside its scope. Specifically, the antecedent of R5 has now been moved to be inside the scope of the make-believe. This marks a similar shift as made in (R2) to a prop-oriented make-believe, with a new act of make-believe being scaffolded upon a prior act. With all of this work done, there will have been enough set theory developed to serve as a foundation for all of standard mathematics, thus providing a fictionalist foundation for mathematics.

Although I have left some details out, this is Yablo's fictionalist account of mathematics. Before moving on though, there are three significant features of Yablo's fictionalism that are important to take note of. Firstly, that acts of make-believe can be scaffolded upon prior acts of make-believe. Secondly, that in so doing, some acts of make-believe can be prop-oriented, where the props can be the make-believe objects produced from earlier acts of make-believe. And lastly, that choices are made throughout the development of the make-believe, and that different choices could have been made, depending on the task at hand. For instance, when dealing with the paradox threatened by (R6), the society could have opted instead to say that all sets must be constructive (in the sense of intuitionistic mathematics), rather than existing within a cumulative hierarchy. Since Yablo is not explicit about which task (R6) was developed for, different choices on how to treat sets could have been made depending on the task that (R6) was meant to facilitate. Thus, the

development of mathematics could have gone differently depending on the choices of the society in question, and the tasks that they want to put mathematics to work for.

Leng explains the usefulness of mathematical assertions as prop-oriented assertions, and the same can be said of Yablo. It is tempting to think that because the props at the beginning of Yablo's game are written in a logical symbolism that his starting point is somehow fundamentally different than Leng's, but that is a misreading. The logical notation here is merely a persipicuous way to express facts about concrete objects, and it is these facts that are the props with which we begin the game. In this regard, we can see that in employing a mathematical make-believe, and making prop-oreiented assertions to reason about our props for that make-believe, we really end up reasoning about facts of the actual world. In particular, Yablo's arguments make clear that we can begin to reason whether the facts of the matter are such that a trade is fair one, or how they can be accomodated in the predictions we must make when planning a more egalitarian redistribution scheme, or whether they establish that a person is an eligible candidate for an inheritance.

With these expositions, we can begin to see how it is that employing Walton's theory of make-believe can help to provide an explanation of the usefulness of mathematics. Both Leng and Yablo appeal to Walton's discussion of prop-oriented make-believe to make sense of how it is that mathematics helps us explain or predict things about our props. For Leng, our props are objects in the real world which we use as urelements to imagine the set of all concreta U, and the axioms of set theory, expressed using the tools of formal logic, serve as our rules of generation in our game of make-believe. For Yablo, our props are certain facts about the world and concreta which we express using logic, which once expressed,

facilitates the imagination of various mathematical objects. In both cases, logical language is used as the means by which to express our principles of generation. In so far as both games allow us to make prop-oriented assertions, these assertions, when true, will reveal something about either objects in the world or facts about such objects, and allow us a means of reasoning, predicting, and accomplishing tasks in the actual world. Moreover, the make-believe can be one where (1) additional acts of make-believe can be scaffolded upon prior acts of make-believe, (2) these acts can be prop-oriented, where the props can be the make-believe objects produced from earlier acts of make-believe, and (3) different choices of how to engage in this scaffolding could have been made, and these choices will have significant ramifications for the development of the mathematical make-believe.

3.3   Mathematical Fictionalism as an Error Theory

With these expositions in place, we can begin to situate mathematical fictionalism within the realism/anti-realism debate in mathematics. More specifically, we can now ask ourselves what sort of anti-realist position Leng and Yablo take mathematical fictionalism to be. As discussed in Chapter 2, there are three main categories of anti-realist positions: semantic anti-realism, expressivism, and error theory. As such, the particular question we will now address is: Is mathematical fictionalism a sort of semantic anti-realism, a kind of expressivism, or is it an error theory?

In her article, "Revolutionary Fictionalism: A Call to Arms," it is clear that Leng is understanding her fictionalism as a kind of error theory (Leng 2005). Although she never uses the phrase 'error theory' to classify her arguments, it is easy to see why her view is

an error theory. In her article, she takes her project to be a defense of "attributing ... systemic error" to mathematicians when making mathematical assertions, and of understanding mathematical discourse, broadly construed, as an "error-prone discourse" (Leng 2005, 277-278). The reason for this error, according to Leng, is simply that the objects that mathematical assertions make reference to do not exist. For Leng, she sees her fictionalism as one that will argue that "the best interpretation of the assertions of mathematical theories is as literally false" (Leng 2005, 280). The reason for this is that mathematical assertions, when taken at face-value, seem to assert the existence of mathematical objects, and fictionalists argue that those objects don't exist. Thus, "the revolutionary fictionalist denies the truth of many claims made ... by mathematicians when doing mathematics" (Leng 2005, 281). Underlying this argument is a commitment to representational semantics for the assertions made within mathematical discourse. Since such representation fails, as the objects that are to be respresented do not exist, all such assertions made are literally speaking false. Thus, any attempt to sincerely make such assertions are made in error. This seems to me to be a textbook example of error theory.

Yablo, on the other, is not quite so explicit about what sort of anti-realism he takes his fictionalism to be. But again, I do not think that it takes any great leaps to classify his arguments as reflecting a commitment to error theory. As noted above, Yablo takes his arguments as building off of those first advanced by Field. Yablo's own understanding of Field's position is one that is committed to mathematical assertions as likely being literally false, and as one that is "on the right track" (Yablo 2005, 91). Yablo's only concern, discussed above, is how mathematics is so useful given that it is likely false. The question Yablo takes his arguments as answering is "how does mathematics manage to be

useful *without being true*" (Yablo 2005, 91).[5]

As with Leng's arguments, Yablo attributes the inability of mathematical assertions to be true to the non-existence of the mathematical objects that a literal interpretation of mathematical assertions seem to be committed to. Yablo associates the commitment that mathematical assertions can be true with a commitment to mathematical realism. In particular, he sees the position as one following a broadly Fregean realist line that the subject of mathematics is "a special sort of logical object" (Yablo 2005, 89). It is in correctly tracking these special sorts of logical objects that mathematical assertions can be true. Since Yablo follows Field's argument that mathematical explantions are only conservative extensions over nominalistic explanations, they do not have any such special sorts of logical objects as their subject. Thus, mathematical explanations cannot be true. Again, it is the commitment to representational semantics for mathematical assertions, and the inability to successfuly represent some putative class of independent objects, that leads to the conclusion that mathematical assertions cannot be true.

3.4   A New Way for Mathematical Fictionalism

I propose that the accounts of mathematical fictionalism offered by both Leng and Yablo can be developed as semantic anti-realist approaches to mathematics, rather than an error theory approaches. More specifically, I will suggest that the Waltonian account for the usefulness of mathematics in the absence of actually existing mathematical objects provides an alternative, non-representational semantics for mathematical assertions. Mathematical

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[5]Emphisis moved from 'does' to 'without being true.'

truth, on this approach, is tied to warrant in making certain assertions depending on the props that serve as the foundation of mathematics as well as the principles of generation that are in play, thus being able to defend an approach to mathematical assertions as capable of being genuinely and substantively true. Essentially, I will be arguing for the following truth condition for mathematical assertions:

(SARF-Math) S is mathematically true iff S is make-believedly true in the game of mathematical make-believe currently in play.

In arguing in favour of a semantic anti-realist approach to fictionalism, it will help to return to Benacerraf's article "Mathematical Truth." Although I analyzed the strengths and weakness of constructivism in light of the arguments made in that article, the actual target of that article's arguments, as I noted above (see footnote 5 of Chaper 2), was a different semantic anti-realist position: *combinatorial* approaches to mathematics. Benacerraf takes the views of David Hilbert as his prime example of a combinatorial approach to mathematical truth (Benacerraf 1983, 406). Benacerraf understands Hilbert's approach as one that sees mathematical truth as equivalent to the provability of the assertion's consistency with first-order arithmetic (Benacerraf 1983, 406). In other words, if a mathematical assertion can be derived from first-order arithmetic without producing a contradiction, or a collection of assertions can be added to first order arithmetic without rendering the resulting system inconsistent, then the assertion(s) is mathematically true. Benacerraf suggests that Hilbert's notion of mathematical truth shares crucial features, or more precisely certain flaws, with a whole range of views about mathematical truth, namely those that see mathematical truth as equivalent to derivability from accepted axioms (Benacerraf

1983, 407). The truth-values of mathematical assertions, for this broader combinatorial approach, are determined by a formal analysis of the syntactic features of those assertions, and by whether or not they can be derived, on that analysis, from the accepted axioms.

As discussed in Chapter 2, Benacerraf sees both realism and combinatorial approaches to mathematics as having their own relative strengths and weaknesses. Despite this, Benacerraf presents what he takes to be a damning objection to combinatorial approaches to mathematics. Benacerraf argues that a satisfactory account of knowledge will be a causal account—in his own words, "for X to know that S is true requires some causal relation between X and the referents of the names, predicates, and quantifiers of S" (Benacerraf 1983, 412). In addition to this commitment to a causal account of knowledge, Benacerraf is also commited to a causal theory of reference, "thus making the link to my saying knowingly that S *doubly* causal" (Benacerraf 1983, 412). Mathematical realism, in adopting a representational semantics for mathematical assertions, accomodates Benacerraf's intuitions about a causal account of knowledge, even if it can't actually provide an account of the causal realtionship between mathematical objects and our ability to make true mathematical assertions (Bencarraf 1983, 414). In pursuing an alternative semantics for mathematical assertions, however, the combinatorial approach to mathematics abandons such causal commitments regarding the nature of truth and knowledge (Benacerraf 1983, 414). This leads Benacerraf to conclude that combinatorial approaches to mathematics are "not only morally but practically deficient" (Benacerraf 1983, 420).

I argue that fictionalist semantics for mathematics can provide an account of how the world, and facts about the objects in it, "plays some causal role" in the production of mathematical knowledge. Simply put, in using objects in the world, and the facts about

them, as the props by which we facilitate the make-believe of mathematical objects, it will be those objects that play the relevant causal role in the production of mathematical knowledge. Although this may not be as direct a causal relation as Benacerraf and others may have in mind, because it is not a causal relation between the knower and the referents of mathematical terms, mathematical knowledge on this account will ultimately be anchored in the props of our make-believe. In this regard, those props will play "some causal role" in the production of mathematical knowledge, even if only a mediated one. At very least, I suggest that this mediated casual role is enough to fend off any charge of being either morally or practially deficient, while also avoiding the controversies of mathematical realism.

Moreover, a semantic anti-realist approach to mathematical fictionalism will also preserve the intuitions of the combinatorial view's approach to mathematical knowledge as intimately tied to the ability to provide and produce proofs. New mathematical truths, on this approach, are not the result of attempts to make observations about the behaviour of abstract objects—in seeking new mathematical truths, we do not seek to construct a telescope to Plato's Heaven. Rather, mathematical truths are proven in accordance to formal rules of derivability of the mathematical make-believe in play. These mathematical truths will involve observations about how the world, furnishing us with our props, informs the game. A semantic anti-realist approach to mathematical fictionalism can thus build up combinatiorial approaches to mathematicas in a way that addresses Benacerraf's intuitions about a causal connection between independent states of affairs and knowers, without having to rely on ontologically suspicious entities.

Another argument in favour of a semantic anti-realist approach to mathematical fic-

tionalism is the ability to accomodate mathematical pluralism that comes with viewing mathematics as akin to games. The view of mathematical practice as akin to participation in a special sort of game is one that can be traced at least as far back as Wittgenstein's concept of *language games*, and many have adopted a similar stance towards mathematical practice since then. This approach was recently defended by Graham Priest in his article "Mathematical pluralism" (2012). There, Priest argues that the simplest means of accomodating the variety of mathematical practices is to view each sort of practice as its own game, with its own set of rules to be followed (Priest 2012, 6). Viewing the variety of mathematical practices, from standard mathematics, Zermelo Frankel Set Theory with the axiom of choice, to category theory, as their own game with their own set of rules to follow preserves the legitimacy of each system of mathematics. Or, to alter the example a bit and to harken back to our discussion of Yablo, we could make different decisions about how to proceed into the infinite.

The ability to accomodate mathematical pluralism is a strength, I argue, because of the simple fact that there is an actual plurality of mathematical practices, from standard mathematical practice to the various approachs to constuctive mathematics. It therefore seems that the burden of argument would not be on the pluralist, but with the monist who wishes to argue that some mathematical practice is the only "true" mathematics.[6] Mathematical fictionalism not only offers accomodation to this plurality, but actually goes some way to explain it. In seeking to develop new or different explanations about some aspect of our real world, or about some quirk of a previous mathematical make-believe, we

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[6]This will hardly be a satisfactory argument for those opposed to mathematical pluralism. Regardless of how much more needs to be done to make the argument a compelling one, I do believe that something like it, at least in broad strokes, is correct.

can always develop new games of make-believe.

The ability to accomodate mathematical pluralism is a strength that is not shared with either realist or constructivist approaches to mathematics. By the arguments of both of those systems of mathematical practice, there will be one, correct manner to practice mathematics. This is largely due to the fact that classical, realist approaches to mathematics insists on the *principle of bivalence*, while constructive approaches make no categorical commitment to such a principle. For the mathematical realist, since true mathematical assertions are ones that accurately represent mind-independent objects, every mathematical assertion will definitevely be either true or false, and thus correct mathematical practice must include the principle of bivalence. On the other hand, in arguing that mathematical truth is tied to an ability to provide a proof, constructive mathematics, and its semantic anti-realist development of mathematics, insists that if no proof can be given for the truth or falsity of a mathematical assertion, then that assertion is neither true nor false. Since constructivists reject mathematical truth as tied to an accurate representation of mind independent objects, they insist that correct mathematical practice need not, and in the absence of some as yet unprovided reason ought not, assert the principle of bivalance.

The semantic anti-realist approach to fictionalism that I am advancing would not have to make such a choice. Rather, by thinking of both classical and constructive mathematics as particular mathematical games, with each free to choose whether to adopt the principle of bivalence as a principle of generation or not, both standard as well as as constructive mathematical practice can preserve their legitimacy. The same will be true for any other approach to mathematical practice, and the axioms or principles adopted therein. A semantic anti-realist approach to fictionalism can thus accomodate mathematical pluralism,

and recongize all systems of mathematical practice as legitimate.

Thus, a semantic anti-realist approach to mathematical fictionalism has various strengths when compared to other anti-realist approaches to mathematics. By tracing the foundations of mathematics to objects in the actual world that serve as the props for games of mathematics, a semantic anti-realist approach to mathematical fictionalism can supplement combinatorial approaches to mathematics to account for a causal connection between the world around us and mathematical knowledge. This building up of the combinatorial approach doesn't go so far, however, so as to produce an intolerance towards the variety of potential avenues for practicing mathematics, as each practice will still be free to choose which axioms and principles to adopt as the principles of generation for their game. The ability to provide an epistemology rooted in, and answerable to, the world around us, as well as to accomodate mathematical pluralism—all while avoiding the controversial ontology and epistemology of realism as well as the immodesty of error theory's conclusion that all mathematical assertions are false—provide a compelling argument in favour of a semantic anti-realist approach to mathematical fictionalism.

3.5   Chapter Conclusion

In summation of this chapter, I have argued for the following truth condition for mathematical assertions:

(SARF-Math) S is mathematically true iff S is make-believedly true in the game of mathematical make-believe currently in play.

In motivating this truth condition for mathematical assertions, I provided a sketch, as well as what I take to be compelling reasons in favour of, a semantic anti-realist approach to mathematical fictionalism. This approach is a reconceptualizing of more typical, error theory developments of mathematical fictionalism which take as their starting point a Waltonian approach to make-believe games. These Waltonian approaches attempt to account for the usefulness of mathematics in the absence of abstract mathematical objects by appeal to objects in the world which serve as props for imaginary truths, along with principles of generation to produce make-believe truths. My suggestion is that this Waltonian approach can do more than simply account for the usefulness of mathematics; it can also furnish us with an alternative, non-representational semantics for mathematical assertions, where mathematical truth simply is the ability to produce assertions that will be warranted given the facts of the props employed, and the prescriptions made by the principles of generation for the mathematical game in play. This approach to mathematical fictionalism has the benefit of not only avoiding the commitment that mathematical assertions are not capable of actually being true, or of being literally false, but of also preserving intuitions about the important role of causation in accounts of knowledge, as well as mathematical truth as closely tied to proof production. Further, it is also a natural fit for those looking to defend the legitimacy of mathematical pluralism, and their respective truths as genuine mathematical truths. With this chapter completed, we can now move on to possible worlds and quantified modal logic.

Chapter 4

The Adaptation

Now that I have developed a semantic anti-realist approach to mathematical fictionalism, I show in this chapter that such an approach to fictionalism is not restricted to mathematics. In particular, the purpose of this chapter is to develop a similar semantic approach to fictionalism for the domain of modal discourse. I begin by providing an exposition of David Lewis's modal realism, and the possible worlds semantics it proposed. I then outline modal fictionalism, which was developed as a means of preserving possible worlds semantics without the controversial ontological commitments of modal realism. I then consider the artificiality objection, which argues that if modal fictionalism were true, the vast majority of humans have never engaged in, and indeed have been incapable of engaging in, modal reasoning. By arguing that the modal fiction is one that is collectively authored, I show why this worry need not concern me. This conclusion that the modal fiction is collectively authored allows me to move forward with my argument that modal discourse, and possible worlds semantics, can be understood as a kind of Waltonian make-believe, allowing me to

argue that modal fictionalism can be reconceptualized as a theory about the semantics of modal assertions. I conclude by arguing that my proposed approach to modal fictionalism has the advantage, over typical forms of modal fictionalism, of not confronting the difficult task of having to propose and explain a story prefix for modal assertions.

4.1 Towards Modal Fictionalism

Before moving forward with my argument, it is first necessary to understand *modal realism*, and the possible worlds semantics for modal claims that serves as its foundation. Modal realism was most prominently developed by David Lewis, in particular in his *On the Plurality of Worlds* (1986). In that work, Lewis gives an exposition of his approach to understanding modal claims—claims that are about the ways that things could or can be, such as "It could have been that I wore my Propagandhi t-shirt today," or "It is possible that I will go to the punk rock show tonight," or "No matter what happens, I will be drinking beer tonight."[1] More importantly, Lewis shows how his approach to modal assertions provides a means for understanding how it is that modal assertions can be true or false. Although many others have since gone on to defend and further develop modal realism, Lewis's exposition of modal realism remains the quintessential articulation of the view.

Lewis's view is that there are, as a matter of fact, many different universes, or "worlds" (Lewis 1986, 2). These worlds each realize different possibilities, and there are as many worlds as there are possibilities, or as Lewis puts it, "absolutely *every* way that a world

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[1]These examples, although used to illustrate Lewis's ideas, are my own examples of modal claims.

could possibly be is a way that some world is" (Lewis 1986, 2).[2] Thus, there are worlds very much like our own world—what Lewis calls "the actual world"—and worlds very much different from it (Lewis 1986, 5). Lewis argues that modal assertions quantify over possible worlds, and that modal operators such as 'possibly,' 'necessary,' 'would,' and 'could,' are all quantifiers over those possible worlds (Lewis 1986, 5). In modal logic, 'necessarily' is represented by $\Box$, while 'possibly' is represented by $\Diamond$. So, a modal claim like:

It is possible that there is an x with property P

is understood to express:

There is at least one possible world where an x has the property P

and is translated in modal logic as:

$$\Diamond(\exists x)Px.$$

On the other hand, a modal claim like:

Necessarily there is an x with property P

is understood to express:

In all possible worlds, there is an x that has the property P

and is translated in modal logic as:

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[2]Lewis's own emphasis.

$$\Box(\exists x)Px.$$

By quantifying over possible worlds, modal assertions are given a reference, and by *correctly* quantifying over those worlds, modal assertions can have a truth-value of true (Lewis 1986, 20). In other words, what makes a modal assertion like "It could have been that I wore my Propagandhi t-shirt today" true is the existence of at least one possible world where I wear my Propagandhi t-shirt today. Thus, for Lewis, it is the successful quantification over real possible worlds that enables modal assertions to have the truth-value of true.

Providing translations of everyday modal claims that make us of the 'necessity' operator is not as straight-forward. To see why, consider a modal assertion like "No matter what happens, I will be drinking beer tonight." It would be a mistake to simply translate this as "In all possible worlds, I will be drinking beer tonight," and to then say that the assertion is true only if in all possible worlds I am drinking beer tonight. This is because surely there are possible worlds where I don't exist, or worse, where I am a teetotaler. It is due to cases like this that Lewis argues that for many of the modal assertions that we make, there is some sort of implicit condition that the modal worlds we are to consider are those that do not significantly depart from our own (Lewis 1986, 7, 21). Lewis calls this relation the "closeness" of possible worlds, where possible worlds that make minimal departures from our actual world are closer than those that make more significant departures (Lewis 1986, 20-21). For example, the possible worlds where I exist and drink beer are closer worlds than those where I don't exist or don't drink beer, since the actual me both exists and drinks beer. For Lewis, accounting for this implicit condition is done by simply restricting

the scope of the modal quantifier (Lewis 1986, 7). Just how we are to formalize and spell out the specifics of this "closeness" relation is a matter of some controversy, but Lewis argues that hisorical background and contextual influences offer general guidance in properly interpreting modal claims (Lewis 1986, 21-22). So, when translating a modal claim like "No matter what happens, I will be drinking beer tonight," we need to translate it more along the lines of "In all possible worlds *where, in certain crucial respects, I exist similarly to how I do now,* I will be drinking beer tonight."[3] In so doing, we restrict the scope of the 'necessarily' qauntifier to the possible worlds where I exist and where the past of my possible selves doesn't stray too far from my own actual history. If in all of those possible worlds I will be drinking beer tonight, then the modal assertion will be true.

The idea of restricting the scope of modal quantifiers along a closeness relation is particularly important for understanding modal reasoning that makes use of subjunctive conditionals, most notably counterfactuals. Subjunctive conditionals are assertions where the antecedent invites us to imagine some hypothetical state of affairs, and then claims a particular consequent would be true in that state of affairs. Counterfactuals, more specifically, deal with subjunctive conditionals that have false antecedents, for example, "Had I not decided to go to the punk rock show tonight, I would be at the arcade." For subjunctive conditionals and counterfactuals, the antecedent informs us how we are to restrict the scope of the modal quantifier, and how to evaluate their truth-value. Specifically, the antecedent tells us that we are to resctrict the scope of the quantifier to be over all of the possible worlds where the antecedent is true. From there, if the closest possible world

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[3]I have done my best to word this quote so as to fend off concerns regarding counterpart theory and determinism. I do so in the hopes of setting these concerns to the side, as they are not relevant to the argument that I am making.

within that scope also has the consequent of the subjunctive conditional or counterfactual as true, then the subjunctive conditional or counterfactual is true. So, "Had I not decided to go to the punk rock show tonight, I would be at the arcade" will be true if after restricting the scope of the modal quantifier to the possible worlds where I decided to not go the punk rock show tonight, the closest of those worlds to the actual world is one where I go to the arcade.[4]

As Lewis himself notes, the proposal that there really exist as many different worlds as there are possibilities is a controversial ontological commitment that is often met with scepticism and, in his own words, an "incredulous stare" (Lewis 1986, 133). Despite the controversy, though, Lewis argues that the commitment to the real existence of possible worlds is warranted because of the simple and serviceable semantics that it provides for modal assertions (Lewis 1986, 3). That is, by making the commitment to the existence of possible worlds, a simple means of understanding what modal assertions are doing, and how it is that they can be true, is made available to us. This, for Lewis, is enough to warrant the ontological commitment to the existence of possible worlds.

In recognition of the usefulness of possible worlds semantics, attempts to preserve this convenient semantics for modal assertions that do not rely on such a controversial ontological commitment have been developed. One such attempt has been the development of a view called *modal fictionalism*. Modal fictionalism was first advanced by Gideon Rosen (1990), and has been defended by others such as Howard Noonan (1994), Seahwa Kim

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[4]This approach to identifying true subjunctive conditionals and counterfactuals actually reflects Stalnaker's version of the semantics of such conditionals, which assumes that there will always be a single distinct closest possible world (Stalnaker 1981). Lewis's own view is more complicated in ways that are not essential to my argument.

(2002; 2005), and Richard Woodward (2011), to name a few. What modal fictionalism generally proposes is that possible words are not really existing worlds, but rather that there is a fiction about possible worlds (Rosen 1990, 330-331; Kim 2005, 116; Woodward 2011, 535). When we engage in modal reasoning, or seek to evaluate the truth-value of modal assertions, we don't quantify over really existing possible worlds, but rather participate in the fiction of possible worlds, and discuss those fictitious possible worlds (Rosen 1990, 330-331). In particular, modal fictionalism is typically developed along the lines of interpreting Lewis's works on possible worlds as outlining a fiction of possible worlds, rather than a genuine account of modal discourse, and indexing the semantic interpretation of modal assertions to the possible worlds semantics outlined in those works. Initially, it was specifically Lewis's *On the Plurality of Worlds* (1986) which was so interpreted (Rosen 1990, 332). However, Noonan, in defending modal fictionalism, shifted focus to Lewis's 1968 article "Counterpart Theory and Quantified Modal Logic," and Rosen, in recognizing the strength of this shift, adopted Noonan's strategy (Noonan 1994, 139; Rosen 1995, 68).[5] Kim and Woodward, however, seem to maintain Lewis's *On the Plurality of Worlds* (1986) as the indexed fiction, although Kim is non-committal as to whether the indexed fiction is to be some other work by Lewis (Kim 2005, 131n; Woodward 2011, 535). In either case, it is the work of David Lewis, and his thesis of a plurality of worlds, which serves as the indexed fiction.

In this regard, evaluating modal assertions would not be very different than evaluating

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[5]Specifically, Noonan is responding to the Brock/Rosen objection, which challanges modal fictionalism to translate a modal assertion such as "Necessarily, there are many possible worlds." Detailing the nature of this objection and the response to it is not necessary for my purposes, but those interested can look to Brock (1993) and Rosen (1993) to understand the objection, and then to Noonan (1994) and Rosen (1995) for the response to it.

an assertion like "In *The Lord of the Rings*, you would have to throw the One Ring into the fires of Mount Doom in order to destroy it"—the assertion is warranted by adhering to the details that are laid out in the fiction. So, a modal assertion such as "It is possible that I will go to the punk rock show tonight" is warranted if, when we look at the fiction of possible worlds, there are possible worlds where I go to the punk rock show tonight, or if the fiction furnishes us with the ability to conclude that such a possible world is part of its fiction. That is, under a typical modal fictionalist framework, the assertion "There is a possible world where I go to the punk rock show tonight" is understood to have an implicit prefix along the lines of "In the fiction of possible worlds..." Rosen calls this prefix the "story prefix" (Rosen 1990, 331). So, the assertion is properly interpreted as "In the fiction of possible worlds, there is a possible world where I go to the punk rock show tonight."

This approach allows us to preserve the usefulness of possible worlds semantics without having to make controversial ontological commitments; that is, we need only be committed to the existence of a fiction about possible worlds, and not to the real existence of possible worlds, to legitimately employ possible worlds semantics. As such, it would seem that modal fictionalism provides a means of doing much of what modal realism can do, but without all of the controversy.

One thing that modal fictionalism seems to give up, however, is a commitment to the possibility that modal claims can actually be substantively true. Though many modal fictionalists do not seem to address the issue directly, some of what they say about the truth-values of modal assertions seems to point in that direction. For instance, Rosen argues that assertions that employ the story prefix, either explicitly or implicitly, are not "intended straighforwardly" (Rosen 1990, 331). Moreover, we may have beliefs and

opinions about assertions that make use of a story prefix without actually believing that such assertions are true outside of the story (Rosen 1990, 331).

When speaking about fictionalism more generally, Rosen understands assertions made in a fictionalist discourse as ultimately "unconcerned, in the end, with representing things as they are," or in other words, of accurately representing facts about the world around us (Rosen 2005, 16). He goes on to identify the key commitment of fictionalism to be that "a *false* claim can be ideally acceptable," and that for assertions made in certain discourses where fictionalism is employed as a means of understanding that discourse, "literal falsity is simply not a defect and literal truth as such is not a virtue" (Rosen 2005, 16).[6] Rosen goes on to note the distinction between assertion and *quasi-assertion*, where the former is meant to express the utterer's belief that what was asserted is true, while the latter merely expresses an acceptance of what was asserted, though not an actual belief that what was asserted is really true (Rosen 2005, 17). For Rosen, when a fictionalist claims that a certain discourse is fictional, they must reject that claims made in that discourse are genuine assertions since they fail to actually and accurately represent, and instead argue that those assertions are actually quasi-assertions (Rosen 2005, 17). To the best of my knowledge, no modal fictionalist has challanged this approach to understanding fictionalism's relation to genuine and substantively true assertions, or made any commitment to the contrary.

All of this leaves an ambiguity with regards to the sort of anti-realism that Rosen, and other modal fictionalists who follow him, take fictionalism to be. Quasi-assertions can be understood in two distinct ways, with each reflecting a distinct metaphysical position. The first possible understanding of 'quasi-assertion' reflects a commitment to expressivism. A

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[6]I have switched the focus of emphasis found in the original text from "ideally" to "false."

quasi-assertion, for a traditional expressivist, is a stament that appears to be an assertion, and so appears to be truth-apt, but is actually used to convey attitudes that are not truth evaluable. The familiar example, already discussed in Chapter 2, would be moral claims as understood by the expressivist. A moral claim such as "Stealing is wrong," despite appearing like a truth-apt assertion, is not actually *asserting* that stealing is wrong, but rather merely conveys that the utterer disapproves of stealing.

The other possible understanding of 'quasi-assertion,' following Joyce McDowell, is one that makes a distinction between truth and *felicity*, and is used to classify assertions which are false but recognized as felicitous (McDowell 1982, 327). To understand, we can observe how truth and felicity need not travel together. Suppose I have tickets to go see the film *Danger: Diabolik* tomorrow night, and when asked when I will be going to see the film, I respond "Sometime in the next few weeks." This assertion, though true, is not very felicitous—it's an answer that is more likely to mislead someone than to help them understand when I intend on seeing the film. It would be both true and much more felicitous for me to respond, "I'm going tomorrow night." Suppose, on the other hand, someone were to ask me how comfortable the seats at the cinema are, and I respond "Sitting on a seat at that cinema is just like sitting on a cloud." This assertion, though false, can still be felicitous—if the seats at the cinema were exceptionally comfortable, then despite the fact that it will always be false that a person could actually sit on a cloud, my assertion will have assisted another to understand something about how comfortable the seats are. One could think of worse comparisons to make a false and less felicitous assertion which attempts to reflect that the seats at a cinema are exceptionally comfortable, such as "Sitting on a seat at that cinema is just like sitting on a bed of nails." If what Rosen has in mind in his use

of 'quasi-assertion' is McDowell's sense, then in accepting that quasi-assertions are genuine assertions that are false, his use of 'quasi-assertion' would reflect a commitment to error theory.

When we observe these distinctive uses of 'quasi-assertion,' it is clear that Rosen is following in McDowell's use of that concept. Rosen does not mean by 'quasi-assertion' an assertion that is not in actuality truth-apt. Rather, as already shown, Rosen takes the assertions in questions to be truth-apt, as being false, but not taking that falsity as reflecting any immediate defect. As such, it would be a mistake to think of Rosen's modal fictionalism as reflective of a commitment to expressivism. Rather, from Rosen's use of 'quasi-assertion,' it is clear that his understanding of fictionalism is one that reflects a commitment to a kind of error theory.

So, it seems, using possible world semantics for modal assertions, and being a fictionalist about possible worlds, would have all modal assertions incapable of being genuinely true. Thus, if we were interested in preserving possible worlds semantics, while maintaining that possible worlds are fictitious, *and that modal assertions can be true*, then we would need to reject representational semantics in favour of a semantic anti-realist approach. But before turning to how such an approach can be developed, it is necessary to first consider what may be a more damning objection to modal fictionalism, in any form. I turn now to that task.

4.2 Addressing the Artificiality Objection

Fictions are the sorts of things that are not typically thought of as having a necessary existence. Rather, fictions are normally thought as being the sorts of things that are created by people. In other words, fictions are often thought of as artificial. The story told by *The Lord of the Rings*, for example, is not seen as something that existed prior to J. R. R. Tolkien's having written it. As a result, prior to Tolkien's writing the story, there would not have been any true assertions about *The Lord of the Rings*, nor things that would be true inside the story, since there would be no determinant to establish the truth or falsity of such assertions. In other words, a person who is told "In *The Lord of the Rings*, you would have to throw the One Ring into the fires of Mount Doom in order to destroy it" prior to the publication of *The Fellowship of the Ring* would have no possible way of determining if the assertion were true or not, since no determinant for the truth of the assertion yet existed. In addition, even after the publication of *The Fellowship of the Ring*, and the establishment of a determinant for the truth-values of assertions about *The Lord of the Rings*, a person must be familiar with the story in order to enable her to be able to effectively evaluate the truth-value of such assertions. The artificial nature of *The Lord of the Rings*, thus, seems to create barriers to being able to make or evaluate true assertions about it or of the story it tells—one must be able to access the fiction, and must have some familiarity with its story.

Some have highlighted the artificial nature of fictions so as to level an objection to modal fictionalism that has come to be known as *the artificiality objection*. This objection has come in various flavours, and defenses have addressed the many ways that the arti-

ficial nature of fictions pose problems for modal fictionalism (Peacock 1999; Nolan 2002; Kim 2005; Woodward 2011). However, there is one particularly strong articulation of the artificiality objection made by Andrea Sauchelli (2013) that warrants a response if modal fictionalism is to be considered a coherent and compelling alternative to modal realism.

In his article, "Modal Fictionalism, Possible Worlds, and Artificiality," Sauchelli argues that because of the artificiality of modal fictionalism, the truth conditions of modal claims are largely inaccessible to the average person (Sauchelli 2013). The problem, briefly put, is that the fiction of possible worlds, like any other fiction, must have a point of creation (Sauchelli 2013, 415). Moreover, familiarity with that fiction is required for any person engaged in modal reasoning to understand the truth conditions for modal claims and to evaluate their truth-values (Sauchelli 2013, 415). As such, the ability to effectively and successfully engage in modal reasoning is contingent on the fiction of possible worlds being accessible to a person engaged in modal reasoning. However, there are many people who existed before the creation of the fiction of possible worlds, and as such, had no determinant available to them to establish the truth-values of modal claims. That is, the truth conditions of modal claims were inaccessible to any person before the genesis of the fiction of possible worlds (Sauchelli 2013, 415). So, we are left with the conclusion that a person engaged in modal reasoning before the creation of the fiction of possible worlds was simply shooting in the dark, and incapable of effectively engaging in modal reasoning.

This is doubly problematic given that the fiction in question is often taken to be made up of the works of Lewis (Sauchelli 2013, 420). As noted above, many prominent modal fictionalists are committed to the idea that it is Lewis's works that serve as the basis for the fiction of possible worlds (Noonan 1994; Rosen 1995; Kim 2005; Woodward 2011). So, when

a person engages in modal reasoning, they must participate in the fictional interpretation of Lewis's works and use the norms outlined therein to know how to correctly make and evaluate modal claims. Most people, however, have not read Lewis's works on possible worlds, and thus are left without access to the truth conditions of modal claims. What follows is a mystery as to how people could or can have any competence in evaluating modal assertions

The artificiality objection, as made by Sauchelli, seems to present the modal fictionalist with a hard pill to swallow. Many people, past and present, have regularly engaged in modal reasoning, and it seems fair to say that at least some have done so with some success. This would suggest that on some basic level, at the very least, the truth conditions of modal claims are available to most people. However, given that the fiction of possible worlds is artificial, modal fictionalists cannot make sense of the successful modal reasoning performed by those unfamiliar with, or existing prior to, Lewis's works on possible worlds. Moreover, it would be unreasonable for the modal fictionalist to assert that the successful modal reasoning of these people is the product of chance, or a lucky shot in the dark. It would seem, then, that modal fictionalism does not offer a successful account of modal reasoning, nor of the truth conditions of modal claims.

4.3   The Possible Worlds Fiction as Collectively Authored

I shall suggest that modal fictionalism can be rearticulated so as to address the artificiality objection. This rearticulation avoids commitment to the view that the indexed fiction of possible worlds is a specific work of fiction by a particular author. Rather, the relevant fiction that serves as the foundation for the truth conditions for modal claims is a collectively authored fiction, one which has been under development for as long as there has been modal reasoning, still is, and will be into the indefinite future. This collectively authored fiction is one that we can all contribute to, and that can be amended or edited by us as we further develop it. This response to the artificiality objection would allow modal fictionalism to account for the accessibility of modal truth conditions, while at the same time maintaining that possible worlds are objects of fiction.

That a fiction can be collectively authored should be uncontroversial, and there are numerous examples of such fictions. For instance, although Len Wein and Bernie Wrightson were the creators of *Swamp Thing*, the fiction of *Swamp Thing* is not restricted to, nor does it end with, the work of Weis and Wrightson (Wein & Wrightson, 1972). Several authors over the decades have contributed to the fiction, with each adding to, editing, and revising the story. As such, the contributions of Alan Moore and Doug Wheeler are just as much a part of the *Swamp Thing* fiction as are those of Weis and Wrightson, and arguably an improvement on it (Moore, 1984; Wheeler, 1989).[7] More controversially, should mathematical fictionalism hold true, then mathematics would also be an example of

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[7]In the case of Moore's contributions, it would be difficult to argue to the contrary.

a collectively authored fiction. Mathematicians can co-author the fiction by constructing and contributing new mathematical proofs or innovations for use in that fiction, and have done so for as long as there has been mathematics.

To help illustrate this response, it may help to look at one of Sauchelli's examples meant to motivate the artificiality objection. In this example, Caesar is preparing his invasion of Gaul, and is engaged in modal reasoning to identify the steps he should take to ensure victory (Sauchelli 2012, 416-417). In particular, we can imagine Caesar engaged in subjunctive or counterfactual reasoning—that is, reasoning about ways that things could be, and what would follow were they the case, in helping him prepare his invasion. For example, he might have thought to himself "If I were to launch my attack at daybreak, then the enemy would not be ready to defend themselves," or "If I weren't to reposition my troops, they would get trampled by the enemy's elephants." If the relevant fiction that grounds the truth conditions of Caesar's modal considerations is a fiction that is totally inaccessible to him, then we could not attribute even a minute portion of Caesar's successful invasion of Gaul to his modal reasoning. On the other hand, if Caesar's advisors were co-authors of the fiction of possible worlds, along with others who came before them and Lewis much later, and Ceaser was participating in that fiction, then the truth conditions of his modal considerations would have been available to him. By rearticulating modal fictionalism to instead propose a collectively authored fiction that informs modal reasoning, modal fictionalism can preserve an account of possible worlds as fictitious objects while still maintaining the accessibility of the truth conditions of modal assertions.

Pursuing this approach to modal fictionalism does raise some important questions that are worth considering and responding to in order to better understand just what I am

proposing. For example, what, precisely, is meant by 'development' of the modal fiction? How does the modal fiction get developed over time? Who counts as a co-author of the modal fiction? Offering comprehensive and complete answers to these questions is daunting, and is indeed too big a task to be fully pursued here, but I do believe that some preliminary answers can be provided.

My primary suggestion is that the modal fiction gets developed as we better develop our understanding in a wide variety of other sorts of studies, such as those that help us to better understand the laws of nature, facts about our actual world and its history, regularities of behaviour, and various fields of mathematics, such as game theory and probability. In this sense, advances in physics, psychology, game theory, historical awareness, and philosophy, or any other contribution to science and the social sciences, can all develop the modal fiction so as to better enable us to imagine different ways things could be, and thus to engage in modal reasoning.

For example, suppose we are imagining what might happen should a large asteroid hit the Earth. We can construct a wide array of subjunctive conditionals that all invite us to imagine such a scenario, such as "If a large asteroid were to hit the Earth, then the Earth would be displaced from its orbit," or "If a large asteroid were to hit the Earth, then a large dust cloud would cover the sun, causing major climate change and famine," or "If a large asteroid were to hit the Earth, then there would be civil strife and massive wars between the survivors of ruined countries," and so on. To help us evaluate the truth-value of these assertions, we can make use of the developments made in studies that are relevant to the their consequents. So, for "If a large asteroid were to hit the Earth, then the Earth would be displaced from its orbit," we can turn to theories of gravity to

help us determine if such a subjunctive conditional is true, or how it could be true. A physicist may tell us that the subjunctive conditional would be true if the asteroid were at least a certain size and travelling at a certain speed, not only helping us to idenitify the truth-value of the conditional, but also what would have to be true of the world that the antecedent invites us to consider. Moreover, the advance of physics from an Aristotelian to Newtonian understanding of gravity better enables us to do this. Likewise, developments in climate and ecological science could help us determine if it would be true that there would be massive climate change and famine should an asteroid hit the Earth, and historians, political scientists, and psychologists can tell us how it could be true that there would be wars between the survivors of ruined countries, if it could be true at all.

Advances in various studies better enables us to imagine possible ways things could be when we propose departures from the actual world, and are particularly useful when we engage in subjunctive or counterfactual reasoning. For since the "nearness" relation between worlds is part of the fiction, it is a development of the fiction whenever we get a better grasp of which worlds are closer to which others, and in particular which are closer to our own than others. Knowing which assertions are more or less likely to be true supposing that some false antecedent is true is crucial to this sort of modal reasoning, so co-authors of the modal fiction would include those who advance or develop our understanding in the various fields that help us to better understand what is true of the actual world, its laws, features, history, regularities, and so on.

To help get a better feel for how modal reasoning is improved by updating the modal fiction, it will help to return to our example of Caeser and his plotting of his invasion of Gaul. As noted above, Caeser might have engaged in a wide array of subjunctive

and counterfactual reasoning in his preparation for his invasion. Some other subjunctive conditionals he might have considered may have been "If I were to make a sacrifice to the god Mars and win his favour, then the likelihood of my success would increase," or "Given the enemy general's brashness and impatience, if I were to send out this small battalion ahead of the others, I would lure him into deploying all of his troops early to meet the battalion, and position my other troops to flank his, thus trapping them." With the evolution of theology over the centuries, the modern general, however, may not pay any concern to winning the favour of Mars, and although some may offer a prayer to Jehova, they will recognize that their time is better spent preparing troops than preparing a sacrifice to win the favour of some deity. In this way, the changes in theologocal thought over history and its impact on the modal fiction could be understood to have rendered the possible world where Mars exists and can offer his favour so distant from our own that it is not worth considering. Likewise, although Caeser may have simply relied on his intuitions regarding the psychology of his enemy and how to effectively manipulate him into falling into a trap, the modern general can make use of the development of strategic tools derived from game theory which were not yet available to Caeser to more effectively set a trap for his enemies. In other words, advances in our understanding of the world allow us to indentify which considerations are relevant and pressing when engaged in modal reasoning. My suggestion is that this progress can be understood as our refining the "story" about which worlds are closer to our own, and so more worthy of attention.

Although this is a departure from modal fictionalism as initially proposed by Rosen and developed by others, where Lewis's works on possible worlds serve as the indexed fiction for modal fictionalism, I do not believe that it is a very large one. Even though

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modal fictionalists would be committed to the notion that Lewis is not the sole author of the fiction grounding modal reasoning, they would still be able to help themselves to all of the innovations his theory provides, and the convenience it affords us. Lewis's works can still serve as a recent and very important contribution to, and revision of, the fiction that grounds modal reasoning—as co-author, Lewis's work serves to further elaborate and develop on the modal fiction, much as Alan Moore advanced the story of *Swamp Thing*.

Sauchelli has a potential response to this approach to modal fictionalism that is worth considering. He argues that if the truth conditions and truth-values of modal claims have, in fact, always been accessible to those engaged in modal reasoning, then no purpose is served by using possible worlds semantics, particularly fictional possible worlds, in understanding modal reasoning (Sauchelli 2012, 419). In his own words, the discourse of fictional possible worlds becomes "gratuitous and theoretically redundant" (Sauchelli 2012, 419). Although Sauchelli is specifically targeting arguments that claim modal truth conditions and truth-values are accessible prior to, and not during, the creation of the possible worlds fiction, it is not difficult to understand how this may still be a problem. Even if the modal fiction is a work in progress, what is gained by the recent addition of possible worlds semantics to it? Why should we continue to edit and amend the fiction after, say, the time of Caesar? If the fiction utilized by Caesar was good enough for him, then it would seem that it is also good enough for us and that we gain nothing by including possible worlds as a central feature of that fiction. So now it would seem that the modal fictionalist can avoid the charge of inaccessible modal truth conditions, but now must face the charge of redundancy.

The response to this worry is, I believe, fairly simple, and I have already hinted at it. By understanding that the fiction that grounds modal reasoning is collectively authored,

we can identify Lewis's works on possible worlds semantics as a contribution to that fiction. But the nature of this contribution is not quite like that of those who make contributions to the sciences, and thus to the modal fiction. Rather, we can think of Lewis, and his proposed possible worlds semantics, as contributing something towards the development of a meta-narrative to the fiction of possible worlds. There are many examples of this in postmodern literature. For example, the novel *If on a winter's night a traveler* is structured as two narratives in alternating chapters. One is about a "you," a reader trying to read a novel called "If on a winter's night a traveler," and the others purport to be the first chapters of the books that you read along the way (though really the story begins anew and changes completely every time the novel-within-the-novel starts up anew). The structural distinction suggested by the alternating chapters is quite porous, and events in each strand influences what happens in the other. The meta-narrative structure allows Calvino to offer insights about the nature of the storytelling enterprise. Arguably, experience with postmodern techniques that lays bare the novelist's tools changes the experience of reading more traditionally structured novels in the future, as one is now positioned to see the machinery being used to generate particular narrative effects.

I want to suggest that Lewis's development of a meta-narrative is a philosophical innovation that brings to light something that was always done when we engaged in modal reasoning, particularly the sort that makes use of subjunctive or counterfactual conditionals. Namely, it makes clear that we make-believe or imagine worlds where the antecedent of subjunctive or counterfactual conditionals are true, and use our knowledge of various aspects of our actual world to help us do so. We then use these worlds to help us reason through various modal assertions, and to establish their truth-values. Moreover, the

development of this meta-narrative allows us the means to critically evaluate certain presuppositions that we are making when engaged in modal reasoning. In discussing the notion of a closest possible world, I noted in a footnote that such a notion is a feature of Stalnaker's account of counterfactuals, but not of Lewis's. The reason Stalnaker includes in his theory of possible worlds the notion of there always being a closest possible world is to serve as a foundation for the law of conditional excluded middle as a feature of modal logic. The establishment of a meta-narrative allows us to evaluate this presupposition, and to critcally reflect on whether or not such a presupposition is indeed warranted by the meta-narrative, and if so, under what conditions. Famously, for example, when considering "If Bizet and Verdi were compatriots...," as the antecedent for a counterfactual, some doubt that the world where Bizet is Italian is any closer than the world where Verdi is French. In reflecting on the meta-narrative for modal fictionalism in this way, we critically reflect on the conditions in which a commitment to the law of conditional excluded middle can be warranted, or whether such a commitment can be warranted at all. In this sense, there is nothing *ad hoc* about the inclusion of possible worlds semantics. Rather, possible worlds semantics provides a meta-narrative that helps us to better understand and reflect on the grander narrative of the modal fiction.

4.4 A Semantic Anti-Realist Modal Fictionalism

With a reconceptualizing of modal fictionalism as collectively authored, I can now outline how a semantic anti-realist approach to fictionalism can take form. In particular, it allows me to show how modal fictionalism can fashion itself after the Waltonian inspired

mathematical fictionalism discussed in the previous chapter. As a result, the same reasons that facilitated the reconceptualizing of Waltonian-inspired mathematical fictionalism as a semantic anti-realist position will also apply to a Waltonian-inspired modal fictionalism. That is, by taking inspiriation from some of the arguments in the previous chapter, I argue for the following truth condition for modal claims:

(SARF-Modal) S is a modal truth iff S is make-believedly true in the modal

make-believe.

I am suggesting that the sort of make-believe that grounds modal fictionalism is one that is built upon of a wide variety of other scientific and social disciplines, in the same manner that in Yablo's telling some mathematical make believes are scaffolded on prior make believes (e.g., treating numbers as props in order to make believe that there are numbers of numbers). The props for the modal fiction are then taken from those disciplines, the result being that the modal make-believe will employ a diverse range of props to help facilitate our imagination. These props can include concrete objects, the facts about them discovered by various disciplines, accepted truths within those disciplines, other fictional objects produced by other useful fictions (such as mathematical objects), and many more. In the same way that some mathematical games found their props in the consequences of some other mathematical game, by scaffolding upon of a variety of scientific disciplines and social sciences, we are provided with the props for a modal make-believe.

The wide applicability of modal reasoning of various sorts—it seems we are able to reason counterfactually about every domain, for instance—means that the modal game has a wide variety of principles of generation. In particular, I suggested above that there

will be two primary sources for principles of generation for such a modal make-believe. The first is established principles, laws, or regularities that have been accepted or established in the vartiety of disciplines which provided us with our props. Principles of generation from this first source can include things like laws of physics, economic regularities, known mechanical processes, the Axiom of Subsets, and many more. The second source for principles of generation will be the modal meta-narrative, and its development of acceptable presuppositions for modal reasoning, which will be expressable in modal logic. Principles of generation of this second sort will determine, depending on how we decide to play the game, whether or not there is always a single, closest possible world where some claim A is true, or when we can ignore certain props or principles of generation of the first sort, should the make-believe require it, such as if we were to wonder what would happen were the laws of physics slightly different. Many of the principles of generation of this second sort may be relatively recent additions to the modal make-believe, and many people may not be as familiar with them. But for those wishing to develop more sophisticated and disciplined modal theories, the meta-narrative will be a necessary source for principles of generation.

To help demonstrate the modal make-believe that I am proposing, we can consider the modal assertion "If a large asteroid were to hit the earth, then life on Earth would end." In playing a make-believe game which has been scaffolded upon a varitey of sources, many props will be available to me to help me imagine a world in which a large asteroid hits the Earth. In addition to concrete objects, such as the Earth, there will also be the facts about it as discovered in a variety of disciplines, such as its size, orbit, climate, the effects of pollutants, its economies, and so on. So, for example, we use the actual Earth and the

facts about the gravitational force between the Earth and the sun that holds the earth in orbit (deliberately simplifying here, of course), as discovered by physics as props to help us imagine the force of gravity on our make-believe Earth. Just which props are most important will be determined by the task at hand—in specifically concerning ourselves with the potentially fatal effects a large asteroid striking the Earth, we prioritize the props that we know have the most relevance to the preservation or termination of life on Earth. The principles of generation will then be the established laws, regularities, processes, etc. of the disciplines from which we receive our props. Continuing with my example, we can use theoretical claims regarding the nature of gravity as principles of generation. From here, we can employ these props and principles of generation to help us conclude that in our make-believe world, the Earth is knocked off it's orbit, thus wiping out all life on it. In addition to making the assertion, "If a large asteroid were to hit the earth, then life on Earth would end," true in our modal make-believe, it would also make it a prop-oriented assertion, in that we will have learned something about our props, namely, what it would take to knock the Earth off its orbit.

By applying Waltonian approaches to make-believe to modal fictionalism, it is possible to reconceptualize modal fictionalism as a semantic anti-realist position. Similarily with my proposed semantic anti-realist shift for mathematical fictionalism, the truth of modal assertions on this approach is tied to warrant in making certain assertions within a game of make-believe. In this case, the truth of assertions will depend on the facts about the props that serve as the foundation of modal reasoning, as well as the principles of generation that are in play. This would allow us to be confident that, despite there being no existing possible worlds to represent and refer to, modal assertions can be genuinely and substantively true.

In favour of this shift, my proposed semantic anti-realist approach to modal fictionalism can evade a whole range of objections that typical fictionalist accounts of modal discourse cannot. As noted above, modal fictionalism is typically argued as proposing an implicit story prefix for modal assertions along the lines of "According to the fictions of possible worlds." The need for this prefix has been the source of some complications for modal fictionalism, with these complications being used to object to modal fictionalism. Gideon Rosen, for instance, has considered and responded to several ways that the implicit story-prefix, particularly as understood as a primitive modal operator, can cause problems for a ficitionalist account of modal discourse (Rosen 1990; 1995). Similarly, Kim considered and responded to complications that the story-prefix causes when engaged in cross-world comparisons (Kim 2002). For the sake of the argument I wish to make, I don't need to delve into the details of these complications. What is important for my argument is simply that the implicit story-prefix that modal fictionalists seek to appeal to has been the cause of complications for modal fictionalism, and that much work has been done in the attempt to resolve those complications.

My semantic anti-realist proposal for modal fictionalism is not subject to this class of complications, as my proposal has no need for employing such a story-prefix. The reason that the story-prefix was employed was as a means of interpreting what is really being asserted when we make modal assertions. That is, that when I assert "It is possible that I will go to the punk rock show tonight," I should be understood as actually asserting "In the fiction of possible worlds, there is a possible world where I go to the punk rock show tonight." The role of this prefix for modal fictionalists is to provide an alternate, anti-realist

translation or *paraphrase* of our every day modal assertions.[8] Rosen, for instance, argues,

> The fictionalist maintains that when he utters, as he might, 'there is a blue swan world' what he really means to assert is ['according to the hypothesis of a plurality of worlds, there is a world W such that at W there are blue swans']. This uncontroversial metafictional thesis is the fictionalist's *paraphrase* of the equally uncontroversial modal claim that there might have been blue swans. (Rosen 1990, 332)[9]

In his later defense of modal fictionalism, he continues this understanding of the paraphrasing nature of the story-prefix,

> Where the modal realist proposes to analyse a modal statement P by means of modal statement about possible worlds, P*, the modal fictionalist proposes the *parasitic paraphrase*: 'According to the hypothesis of a plurality worlds (PW), P*'. (Rosen 1995, 67-68)[10]

This understanding of modal fictionalism as offering an alternate paraphrase or translation is one that underlies several discussions of the relative merits or demerits of modal fictionalism. The reason for this interpretation of modal claims is to address the fictionalist's contention that there are no possible worlds to be represented by modal assertions, and to provide a noncontroversial representational semantics for modal assertions. Brock (1993), Hale (1995), Kim (2002; 2005) and Woodward (2011), as a small sampling, all follow Rosen in this understanind of modal fictionalism.

I am not arguing, however, for an alternative *paraphrase* of modal assertions; I am arguing for an alternative *semantics*. That is, I do not wish to understand some alternate paraphrase of "There is a possible world where I go to the punk rock show tonight." I am

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[8]The modal realist paraphrase of "It is possible that I will go to the punk rock show tonight," being simply "There is a possible world where I go to the punk rock show tonight."

[9]Emphasis added.

[10]Emphasis added.

proposing an understanding of *that assertion*, and *that assertion's truth-conditions.* I am not seeking to preserve a representational semantics for modal assertions, and so do not need to provide a paraphrase of the assertion that idenitfies an alternative representation. I am rejecting representational semantics for modal assertions, and instead proposing an alternative, non-representational semantics for modal claims. By appealing to a Waltonian approach to make-believe, I provide this anti-realist understanding of unparaphrased possible worlds talk and its truth-conditions. This distinction between paraphrased assertions and assertions made in the context of a make-believe is also made by Yablo in "Go Figure: A Path through Fictionalism," and is what makes fictionalism a truly distinct anti-realist position in his mind (2001).[11]

To better understand this distinction, it would help if we again returned to an example of a make-believe game, and the assertions we make while playing it. When I assert "The loot is in the safe" while playing a make-believe game of Cops and Robbers, it would be a mistake to understand what I am really asserting as a paraphrase of something like "In our game of Cops and Robbers, the loot is in the safe." The reason this is so is because it betrays the act of make-believe I am engaged in—I would not be making-believe there is loot in the safe were I really intending all my assertions to be understood as employing a story-prefix. My assertion is not meant to be limited to simply a genuine belief of what is true given some fiction, but also about true facts about the location of real objects, in this case about a lunchbox and its contents. That is, there is what Yablo calls "real-content," or

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[11]What is interesting to me is that Yablo goes on in this paper to discuss Rosen's approach to modal fictionalism as distinct from a theory that provides a paraphrase of modal discourse. For the reasons just given, I think this is a mistake, but regardless, his arguments that make-believe assertions are distinct from mere paraphrases still helps me to make my argument that the story-prefix, which serves as the means by which we paraphrase, is unneeded.

what we've been calling props, that I am attempting to sincerely assert something of (Yablo 2001, 74, 76).[12] In other words, I am relying on the make-believe game to (sincerely) say something about our props; I am not merely discussing a fiction with others, or making comments about it.

The distinction between providing an alternative semantics for an assertion and simply paraphrasing it may be subtle, but it is significant. Since I am arguing in favour of an alternative semantics for possible worlds discourse, and not simply a paraphrase of it that changes the subject of representation, I have no need to offer a story-prefix, or an understanding of one. Rather than offering paraphrases of modal discourse, a semantic-anti-realist approach to modal fictionalism offers a semantics for the assertions *as asserted*. Because there is no need to propose a story-prefix, my proposed semantic anti-realist modal fiction is not subject to any of the complications that it causes. Moreover, as noted above, the initial motivation for modal fictionalism was to preserve, as best as possible, possible worlds discourse without the ontological controversy. Since a semantic anti-realist approach to modal fictionalism looks to the assertion as asserted, it better preserves possible worlds discourse than one that seeks to paraphrase that discourse. Simply put, perhaps rather audaciously, if theft can be considered a virtue of a fictionalist position, then I argue that a semantic anti-realist fictionalism is most virtuous.

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[12]Given the discussion about the nature of assertions while engaged in make-believe that Yablo makes in "Go Figue: A Path through Fictionalism," I am all the more convinced that most of the work of providing a to fictionalist account of the semantics of some assertions has alreay been done by fictionalists inspired by Walton, and that what I offer is really only a small nudge to move fictionalists into this position.

4.5   Chapter Conclusion

The purpose of this chapter has been to show how a semantic anti-realist approach to fictionalism, as developed in the previous chapter, can be applied to provide an alternative fictionalist account of modal assertions. In particular, the purpose of this chapter has been to argue that:

(SARF-Modal) S is a modal truth iff S is make-believedly true in the modal
                  make-believe.

In making this argument, I began by looking at existing modal fictionalist accounts, and their weaknesses at accounting for the accessability of modal truth-conditions due to the artificiality of fictions. I then showed that a Waltonian-inspired approach to fictionalism is well suited to address these concerns, where access is simply a matter of engaging in an act of make-believe, and not a matter of making reference to some specifically indexed fictional story. With a Waltonian-inspired account of modal fictionalism, this made the shift to a semantic anti-realist fictionalist account effortless. This shift is distinct from previous modal fictionalist accounts in its ability to provide an alternative semantics for assertions such as "I might go to the punk rock show tonight," that is non-representational and not simply a paraphrase smuggling in a hidden prefix. This renders the need to provide a story-prefix superfluous, thus enabling my argument to side-step any concerns or complications that such a prefix would cause. If we are motivated to preserve and keep intact, with as little modifcation as we can, the right to understand modal claims in terms of possible worlds without any of the ontological controversy, as well as to address concerns with the artificiality of fictions and the implementation of a story-prefix as a modal operator typical

to modal fictionalism as previously developed, then there is ample motivation to accept my proposal.

Chapter 5

Critical Review

In this chapter, I explore and defend three larger commitments that are made by the approach to fictionalism that I have been advancing. The first two commitments that I address are the commitments to alethic and logical pluralism, which I defend from the shared threat posed by mixed inferences. In response to the problems posed by mixed inferences, I argue that an appeal to context can provide solutions, and outline context-specific versions of alethic and logical pluralism to provide those solutions. After having defended the commitments to alethic and logical pluralism, I then argue that a semantic anti-realist account of fictionalism is committed to rejecting a deflationist account of truth. I do this by arguing against Armour-Garb and Woodbridge who argue that a semantic anti-realist approach to fictionalism can be used to ground a deflationist account of truth-talk. Contrary to Armour-Garb and Woodbridge's arguments, I show how a fictionalist approach to the semantics for some assertions actually necessitates a substantive understanding of truth.

5.1 The Problems of Mixed Inferences

One of the key commitments that my semantic anti-realist approach to fictionalism must make is to *alethic pluralism*, the view that there is no one, single way for an assertion to be true, but rather that there are different kinds of truth for different sorts of assertions—or, as Nikolaj Pedersen has described the view, that "truth is not one, but many" (Pederson 2006, 115). In proposing an alternative to representational semantics for both mathematics and modal logic, with each domain having its own particular semantics informed by its particular game of make-believe, my approach to fictionalism is committed to different kinds of truth for each domain. On this approach, each domain will have its own, distinct truth predicate used to capture the sort of truth that is particular to its domain.[1] Thus, the approach to fictionalism as outlined so far is committed to alethic pluralism.

In making this commitment, my approach to fictionalism faces a challenge familiar to any position committed to alethic pluralism. *The problem of mixed inferences* was first outlined by Christine Tappolet and posed to any position that argued that there is more than one kind of truth. To understand the problem of mixed inferences, it helps to begin with an example of an argument. Tappolet's own example is:

(1) Wet cats are funny.

(2) This cat is wet.

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[1]It is important to note that alethic pluralism can be understood in two distinct ways: pluralism about truth *predicates* and pluralism about truth *properties* (Pederson 2006, 107; 2010, 96). The former is meant to only be a commitment regarding language, while the latter is meant as a more robust metaphysical position. For the purposes of this dissertation, it will be assumed that for each truth predicate discussed, there will be a corresponding truth property, but such an assumption may not always be warranted, and the two may come apart in certain circumstances.

Ergo, this cat is funny (Tappolet 1997, 209).

It should be uncontroversial to conclude that the above argument is deductively valid; if premises (1) and (2) are true, then the conclusion will also be true. However, suppose we are constructivists about humour, and realists about wet cats. We may then say that assertions about humour belong to a *domain* distinct from assertions about wet cats, and that truths about humour will be true in a way that is appropriate to the domain of humour. Truths about wet cats will be true in a different way, namely the way that is characteristic of the realist domain in which cats, wet or otherwise, reside. In accepting constructivism for humour and realism for wet cats, we would be committed to *domain-specific kinds of truth*, and to the view that (1) and (2), if each were true, would have the property of truth particular to their respective domains. The dilemma now arises: If we accept the typical understanding of validity as truth preservation, and as meaning that the truth of the conclusion is inherited from the truth of the premises, then which sort of truth is being preserved in the above argument?

To more explicitly see how the problem of mixed inferences is a challenge for the approach to fictionalism that I have developed, consider how I might respond to a friend who has texted me, asking if there's a chance that I will be at the punk rock show tonight. I might waggishly respond with:

(1) Either it is possible that I will go to the punk rock show tonight, or $2 + 2 \neq 4$.

(2) $2 + 2 = 4$

Ergo, it is possible that I will go to the punk rock show tonight.

Expressed formally:

(1) $\Diamond P \vee \neg M$

(2) M

$\therefore \Diamond P$

According to the position that I have developed so far, $\Diamond P$, when assumed to be true, will be a modal truth. M, on the other hand, will be a mathematical truth. The question my semantic anti-realist fictionalism must face is the same as the one posed for the previous argument: which sort of truth is being preserved by this argument? The conclusion is clearly a modal truth, yet (2) is a mathematical truth. It would seem that we could not call this argument properly valid, since there is no single notion of truth that the argument preserves.[2]

A typical response to this problem has been to propose a minimalist understanding of truth that could be used to capture all true assertions, regardless of which domain they are true in. The essential idea is that the minimalist truth is a core that is shared by all truth properties, though most of them have additional characteristics that distingusih them, so any proposition that is true in any sense is minimally true, but few of them are merely minimally true. This minimalist approach to truth was proposed by Wright (1993), used by JC Beall (2000) as a defense against the problem of mixed inferences, and has been further defended by Pedersen (2006; 2010). As Pedersen has argued, we can always

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[2]The astute reader will no doubt also ask about (1)—each disjunct belongs to its own distinct domain, and possesses a sort of truth specific to that domain. The question naturally arises: which sort of truth is expressed by (1)? This problem of premises that mix together various notions of truth is also pressed by Tappolet as a problem for alethic pluralism. Although I will set this problem aside for now, I will address it later in this chapter.

construct a generalized, or universal, truth predicate out of the disjunction of the truth predicates specific to all of the domains which require a unique truth predicate:

(TG) $(\forall p)(T_G(p) \leftrightarrow T_1(p) \vee ... \vee T_n(p))$ (Pedersen 2006, 106).

To briefly unpack the above principle (TG): for any assertion p, the truth of p can be expressed by a general truth predicate T_G iff at least one of the truth predicates in the disjunctive sequence applies to the assertion, where each truth predicate is a truth predicate specific to a particular domain. In this regard, the alethic pluralist can respond to the problem of mixed inferences by arguing that both premises (1) and (2) can share the truth predicate T_G, and so it is T_G that is preserved by the argument. Thus, by proposing a minimalist, general approach to truth, the alethic pluralist can account for the validity of mixed inferences.

The effectiveness of this response has been questioned, most notably, by Tappolet herself. In a paper where she responds to Beall, she asks, "why should we need the many truth predicates instead of the one that does the inferential job?" (Tappolet 2000, 384). To put the challenge another way, why not simply be committed to a single sort of truth, the sort of truth suggested by the truth predicate T_G? Pedersen has given arguments defending an alethic pluralist commitment to a general truth predicate and for why this commitment should not concern an alethic pluralist (2006; 2010). As I will show, however, regardless of whether or not Pedersen's defense of alethic pluralism is successful, it is a defense that offers no comfort to me and my proposed approach to fictionalism.

To see why an appeal to a minimalist, or general, sort of truth is of no help to me, we must first discuss a second key commitment of my fictionalism. In Chapter 2, I argued

that a strength of the mathematical fictionalism developed therein was its ability to accommodate mathematical pluralism—that is, the mathematical fictionalism that I developed will count both standard and constructive mathematics, to take two examples, as genuine means of practicing mathematics. As is well known, classical logic is the correct logic for standard mathematics, while intuitionistic logic is the correct logic for constructivist mathematics. By commiting myself to mathematical pluralism, and to the view that both standard and constructive mathematics are legitimate mathematical systems producing genuine mathematical truths, it thus seems that I am commited to *logical pluralism*, as I must accept both classical and intuitionistic logic to defend mathematical pluralism.

Moreover, it has been argued by many that alethic pluralism naturally leads to a commitment to a position that has been called *domain-specific logical pluralism*. If different domains of discourse each have their own sorts of truth that reflect the metaphysical commitments of those domains, then there will be different logical systems for those domains. For example, if realism is the appropriate metaphysical position in one domain, while a semantic anti-realism, say constructivism, is what is appropriate for some other domain, then classical logic will be the appropriate logic for the former domain, while intuitionistic logic will be needed for the latter domain. The reason for this is that realism will lead to a metaphysical commitment to the principle of bivalence, and thus be committed to the law of excluded middle as a necessary rule of logic. A constructivist, on the other hand, will not share this metaphysical commitment to the principle of bivalence, and so will not assume that every assertion is necessarily either true or false, and thus, not accept the law of excluded middle as a necessary rule of logic. What follows is that it is legitimate to use classical logic for domains committed to realism, while intuitionistic logic is all that

is available in domains committed to constructivism. In committing myself to an alethic pluralism motivated by the view that different domains reflect different metaphysical commitments, it seems only natural that I would also be committed to logical pluralism. This, coupled with a commitment to mathematical pluralism, seems to leave me with no other option but to endorse logical pluralism.

It is this underlying commitment to logical pluralism that renders Pedersen's defenses of alethic pluralism of no use to me. If, indeed, there is one general sort of truth that can be possessed by all true assertions, regardless of the assertion's domain of origin, then it seems, as Tappolet suggests, that there is one sort of truth that can do all of the inferential work that could ever be needed. A general sort of truth suitable to do the required inferential work seems to indicate a general system of logic that can be used to make inferences regardless of the domain in question. Therefore, the commitment to a minimalized, generalized account of truth would seem to not only work against, but also actively discourage, a commitment to logical pluralism.

To better understand the threat that a minimal truth predicate poses to logical pluralism, we can consider the following dilemma. Suppose you're a constructivist, but that there is a minimal truth which applies to all domains. Suppose further that the logic prescribed by this minimal truth is classical. What are you to make now of a mathematical argument which is classically valid, but is not intuitionistically valid? If you remain committed to logical pluralism, then you would seem to be left with a contradiction—the argument is both valid and not valid, since it can preserve the minimal truth predicate, but not a constructive truth predicate. So, any logical pluralist who accepts a minimalist truth predicate must accept conflicting answers when it comes to the validity of arguments.

That, or settle for logical monism as determined by the preservation of the minimal truth predicate.

The pressure on me to adopt logical monism does not stop there. Much as the mixed inference problem may push the alethic pluralist towards an alethic monism, Rosanna Keefe has identified an analogous problem that pushes the domain-specific logical pluralist towards logical monism (Keefe 2018, 441). As described above, a domain-specific logical pluralist argues that the correct logical system will be determined by domains of discourse. An argument that mixes domains will thus also mix logical systems. We are then faced with a familiar looking question: which system of logic should be used to evaluate such an argument? One potential solution, identified by Keefe, is to suggest that there is a *mixed inference logic*. Such a logic, Keefe argues, suggests that there is a *mixed inference domain*. Since the mixed inference domain, and its logic, is designed specifically to accomodate arguments that mix domains, then it will have to be a domain that can include any assertion from any domain. This further suggests that for any assertion, it will not only belong to the domain of its origin, but also to the mixed inference domain. As a result, an argument whose premises are all, say, modal assertions will not only be an argument made in the modal domain, but also be an argument in the mixed inference domain. What the domain-specific logical pluralist is left with, via the additional commitment to a mixed inference domain, is a general logic that can be used to evaluate arguments regardless of the assertions' domains of origin.

Further complications arise when we consider just which logical system will be the one employed to serve as the general logic. To see how, let's say that when my friend texts me asking if there is a chance that I will be at the punk rock show tonight, I respond with:

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(1) Either it is possible that I will go to the punk rock show tonight, or there is a law against going to punk rock shows.

(2) It is not the case that there is a law against going to punk rock shows.

Ergo, it is possible that I will go to the punk rock show tonight.

Expressed formally:

(1) $\diamond P \vee L$

(2) $\neg L$

$\therefore \diamond P$

Suppose (2) expresses a truth specific to the legal domain, and that relevance logic is the correct logic for making inferences within the legal domain. When I make a modal assertion, the modal make-believe that I employ to make modal inferences accepts disjunctive syllogisms as valid inferences.[3] Relevance logic, however, does not count disjunctive syllogisms as valid inferences. The question we must now ask is whether the generalized logic for the mixed inference domain should count disjunctive syllogisms as valid, or not. The validity of the argument, which is simply a disjunctive syllogism, depends on this decision.

As Keefe argues, there are two options available to the domain-specific logical pluralist. The first would be to only adopt the rules of logic that are common to all of the distinct logical systems. This would suggest a very weak logic (Keefe 2018, 441). The other option, defended by Michael Lynch, would be to count the argument as valid since there is at least

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[3]I do not mean to make any commitment as to whether or not the modal make-believe would actually include disjunctive syllogism. I am merely making the assumption here so as to help illustrate the dilemma.

one logical system that would count it as valid (Keefe 2018, 442). This suggests a strong logic that includes all of the inferential rules of all of the logical systems. Regardless of how we may answer this question, there will have to be a correct answer for the domain-specific logical pluralist, and so there will be one logic that can be used to evaluate arguments regardless of the domains involved. The question then naturally arises: if there is a correct logic applicable to any and all arguments, then why shouldn't that logic be counted as the *real* logic? The logical pluralist, under the duress imposed by mixed inferences, seems to be pressured back into logical monism.

The problem of mixed inferences thus seems especially damning for the semantic anti-realist account to fictionalism I have so far developed. In order for the approach of fiction-alism that I am proposing to be compelling, I must provide an account for the validity of arguments with mixed inferences, but I must do so in a way that preserves both altheic and logical pluralism.

I will argue that instead of adopting domain-specific logical pluralism, I can adopt a position that I call *context-specific logical pluralism.* Simply put, I argue that determining the correct logic to employ is a matter of context, and that the context determines which domain(s) are in play.[4] So, although there may be contexts where only one domain is in

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[4]My appeal to context here should not be confused with Keefe's relativist approach to validity, which also makes an appeal to context. For Keefe, contexts include many more features that can be involved in determining whether an argument is valid or not than which domains are in play (Keefe 2018, 447). For instance, for Keefe the "stakes" of the matter to the discussants may determine whether or not one needs to attend to the vagueness of predicates involved in an argument: for non-life-or-death matters, it may be fair enough to allow that "every coloured object is either red or it is not," but when stakes are high it might be important to recognize that this won't be so for borderline cases. In my account, the role of context is to specify which domains are in play, and it is only the domains that are relevant to whether or not the argument is valid, not pragmatic matters such as whether the issue is high stakes for the discussants. While the matter deserves a fuller discussion elsewhere, my reason for saying so is that

play, there may be other contexts where modal and mathematical domains are in play, and still others where modal and legal domains are in play.

It is worth pausing to consider a terminological point. While I don't know of anyone who has advocated such a context-specific account as a variety of domain-specific logical pluralism, it might seem reasonable to use this label for my view. This would be reasonable if, roughly speaking, any time domains are mixed, a new domain is produced.[5] 'Domains,' in this sense, are treated much like sets, or aggregates—whenever we mix together sets we get a new set, and whenever we mix together aggregates we get a new aggregate. But it is misleading to think of domains in this way. Not all concepts are like *set* or *aggregate*, in that 'mixing' instances of the kind does not always produce a new instance of the same kind. Take for instance persons, or stories—placing two people in a group does not thereby create a new person, and ripping out the pages from one book and gluing them into another does not thereby create a new story.[6] I think that the most natural reading of 'domain' is one that emphasizes its use to identify the *subject matter* with which we are concerned. But, much like the concepts *story* or *person*, not every combination of subject matters gives rise to something else that is naturally called a subject matter. And that's why it's more natural to think of contexts as telling us which domains are in play, but not to think that it thereby specifies a single domain.

So far developed, my proposed context-specific logical pluralism, along with its treat-

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cases of the sort Keefe considers are not really matters of validity, but are better viewed as setting aside borderline cases as "don't cares." That is, *it doesn't mattering that much if sometimes we get something wrong* in low stakes cases, so we are willing to reason in ways *likely* to get things right for efficiency, at the cost of sometimes having true premises lead to false conclusions.

[5]Here, 'mix together' is being used as a general way to refer to at least taking intersections or taking unions, but perhaps also other ways of combining.

[6]My advocacy for post-modern fiction in Chapter 4 notwithstanding.

ment of domains, already goes some way to alleviate the worry outlined above that the resulting logic may be either excessively weak or excessively strong. When considering adopting the intersection of all the logics that apply in any domain, the worry was that we would end up with a logic so weak so as to render almost no argument valid. However, if we only need to take the intersections of the *logics in play in each context*, then this worry is not nearly as pressing. Generally speaking, most contexts won't have such a range of logics so as to make the context-specific logic troublingly weak. And so, the resulting context-specific logic will generally still be a substantive system of logic. As for the worry that the context-specific logic will be excessively strong, l will allay those concerns shortly.

There still remains the issue of whether to adopt the intersection or the union of logics in play in a given context. It is my view that the appropriate response to this question is to start by adopting only the rules of inference as shared by the intersection of the logical systems. The reason for this preference is simply that a stronger logical system will reflect assumptions not shared by all, while the weaker logic will reflect a common ground. To better understand this preference, let's consider arguments more generally. Suppose two friends both see somebody step on a rake, and get whacked square in the forehead with the end of the rake handle. The first friend bursts out laughing, while the second gives a disappointed stare to their friend, and stroking one finger along the next, suggests that it was inappropriate for the first friend to laugh at this poor fellow's misfortune. The first friend, now teary-eyed and holding back his snickering, is not convinced, and argues that there was nothing inappropriate with his laughter. The second friend responds by saying that Jehova is always watching and judging how we treat others, and so we should not laugh at the misfortune of others. The first friend does not share these religious beliefs,

and so the argument continues.

How might the argument in this instance be carried out fruitfully, so as to try to arrive at a conclusion that both disputants can recognize for themselves as correct? It won't be by expecting and insisting that the first friend take on the religious assumptions of his peer. Rather, the natural suggestion would be that they begin with what they both can agree on. There are a few lessons we can draw from this example. Firstly, it may be the case that when working only with what is common ground makes it impossible to arrive at a conclusion, where it *would* be possible were all the the premises of each disputant accepted. For instance, if we were to accept the premises regarding Jehova's existence and dictates, we know that it was wrong to laugh in the above example, but it might be indeterminate whether laughter was right or wrong using only common ground.

Secondly, in the imagined case, our theist friend might feel that he is being treated unfairly, having to give up what seems to him as a thoroughly plausible belief, while the atheist is giving up nothing. But while there is an asymmetry here, it is not an *unfairness*, it is merely an implication of how rational persuasion works—one cannot persuade another by appealing to premises that person does not accept. It is worth noting that it's the artificial simplicity of the example that probably gives rise to the appearance of this asymmetry in the first place. Opting for common ground won't always end in a decision that will favour any one disputant's starting position over another. For example, if we began instead with a messier, more realistic example, such as people debating a difficult political issue, it is likely that both disputants would have to give up beliefs they find highly plausible from their stock of initial premises.

It is my contention that since logic is, in the first instance, a tool of rational persuasion, similar lessons apply and give us reason to prefer intersections over unions. By adopting only the intersection of the logics in play, we are, in essense, adopting the position that is common ground between all those involved. Using the intersection of the logics in play as the context-specific logic ensures that only the rules of inference that are agreed upon by those logics as valid are used. Moreover, assumptions specific to one logical system that are not shared, or are even outright rejected, by other logical systems are not being forced upon the context-specific logic. In pursuing the intersection of the logics present as the context-specific logic, we produce a logic that would be compelling and persuasive to all the logics in play. This seems to me surely the right lesson if the context in question is one in which there are various disputants with commitments to different logics, for instance, in a discussion between classical and intuitionistic mathematicians. My suggestion is that it is reasonable to extend that lesson to all contexts, since in all contexts logic is apt for rational persuasion.

In addressing the question of which sort of truth is preserved across mixed inferences, I argue that I can make a similar appeal to context to propose a position that I call *context-specific alethic pluralism*. This position echoes context-specific logical pluralism by looking to context to determine the correct truth predicate to employ, and that context determines the domain(s) in play. And just like context-specific logical pluralism, although there may be only one domain in play in one context, there may be others where more than one domain is in play. So, with regards to my first example of responding to my friend's question, the context in question, let's call it C1, is one where mathematical and modal domains are in play. In the case of my second example, where the context mixes

the domains of modal and legal domains, I can argue that this intersection produces a new context, C2.

Moreover, I argue I can adopt context-specific alethic pluralism without having to argue that there is one, general truth predicate that can be constructed from all other truth predicates. This is because if games of make-believe can be used to determine the semantics of assertions, which will in turn require their own truth predicates, then there will be no definite collection of truth predicates—in pursuing some useful end, we can always make a new game of make-believe with its own semantics and, thus, always make new truth predicates. Recall that in order to use (TG) to construct a general truth predicate T_G, there needs to be a collection of truth predicates, capturable by a disjunction $T_1(p) \vee ... \vee T_n(p)$. This disjunction presumes that there is a well-defined set of truth predicates, $T_1, ..., T_n$. However, we can never guarentee that the set of truth predicates is well-defined since we can presumably make use of a game of make-believe to construct a new truth predicate when we need one. Thus, we could never construct T_G since there could never be a suitably orderly sequence of truth predicates. To put the point a different way, we may always be able to make use of a spotlight to help us identify contexts and to light the way forward, but we will not have a sun available to us that could cast its light on all things. As such, all we will ever see, are parts of the landscape, but not the whole.

We may consider being a little more sympathetic to Pedersen's attempt to construct a generalized truth predicate. For example, what if instead of capturing T_G in terms of a disjunction, we capture T_G as an existential generalization:

(TG) $T_G(p) \leftrightarrow (\exists w)T_w(p)$.

It is true that we do existentially quantify over inderminate collections, and in doing so sometimes succeed in making true or false claims. For example, the collection of 'blue shirts' may be indeterminate owing to the vagueness of 'is blue,' but we may nevertheless quantify over blue shirts in a way that makes "There is a blue shirt in this room" either true or false. The issue, however, is whether or not existentially quantifying over an indefinite collection can provide a satisfactory *definition* for something. The addition of new make-believes which give rise to new truth predicates might turn claims that were previously false into true ones. And so, if we hope to defend Pederson's basic idea of giving a definition for 'is true,' we must make a decision as to whether our existential quantification is quanitifying over *existing* truth predicates, or *all possible* truth predicates. If we only quantify over already existing truth predicates, we are in precisely the same place as with the disjunction of all truth predicates.[7] On the other hand, if the intention is quantify over all possible truth predicates, then as should be clear from the discussion of modality in the previous chapter, there is room for that "collection" to evolve in different directions depending on how our games of modal and mathematical make-believe evolve. And so, it would seem that existentially quantifying over truth predicates still would not provide us with a satisfactory definition of 'is true.'[8]

This inability to construct a general truth predicate is also why there cannot be a general logic, and why there will only ever be context-specific logics. As already discussed,

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[7]Plausibly presuming, for the technically minded, that there are only finitely many truth predicates in existence.

[8]There are other challenges for such a suggestion. Are there possible make-believes, i.e.: of mathematics, that make some claim P true, while other possibilities make $\neg P$ true? If so, how do we feel about a definition of truth in which both P and $\neg P$ are true? But these would be complex matters to discuss adequately, and I believe the point is insufficiently related the the main argument of this chapter to give them the attention they deserve.

a single, general truth predicate sufficient to perform all of the required inferential work, regardless of the domains in question, suggests a general logic. Since there cannot be a general truth predicate, we are not forced by the Tappolet-inspired reasoning described above to accept that there is a single, general logic that preserves it. Moreover, if a general truth predicate suggests a general logic, then a context-specific truth predicate suggests a context-specific logic. Since the correct logic to use will be prescribed by the context, then there will only ever be context-specific logics.

We may be tempted at this point to argue that we can take inspiration from Pedersen's (TG) to provide a formal construction for the various context-specific truth predicates. For example, it seems that for context C1, we could provide the following:

(TC$_1$) $(\forall p)(T_{C1}(p) \leftrightarrow T_1(p) \vee T_2(p))$,

where T_1 is truth from the modal domain and T_2 is a truth from the mathematical domain. Similarly, for context C2, we could provide:

(TC$_2$) $(\forall p)(T_{C2}(p) \leftrightarrow T_1(p) \vee T_3(p))$,

where T_3 is a truth from the legal domain. Such a formalization of truth predicates for C1 and C2, however, works against my argument that we should only look to the intersection of the logics in play to identify the rules of inference legitimate for those contexts. If the truth predicate T_{C2} is constructed from the disjunction of T_1 and T_3, then the logic we would have to employ for context C2 would have to be the union of modal and relevance logic, since either logic is suggested by the disjunction. Since I think it is a mistake to take the union of the logics in play to determine the rules of inference for contexts that mix

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domains, I reject the formalization for truth predicates for specific contexts as suggested by (TC$_1$) and (TC$_2$).

It may not be obvious why using such disjunctive definitions for the truth predicates for a context gives rise to a stronger logic. The intuition here is just this. The disjunctive definition will mean that, in general, there are more ways for a sentence to be true than there are in either of T_1 or T_3, so there will be a greater number of arguments capable of preserving truth. That is, the number of valid arguments will, if anything, go up rather than down compared to either the logic for T_1 or T_3 considered alone. This would mean that more than simply the intersection of logics would be determining the validity of context-specific arguments. In effect, accepting that context-specific truth can be captured by a disjunction of the truth-predicates present would essentially produce dilemmas where we are forced to either adopt the unions rather than the intersections of logics when mixed by a context, or to accept that an argument can be both valid (in the combined logic) and not valid (in either base logic) *in the same context.* Since, as I've argued throughout this chapter, I believe that neither of these options are very satisfactory, I am left with the conclusion that such disjunctive definitions are of no use when trying to capture context-specific truth. I leave the task of providing an alternative formalization for context-specific truth predicates for a different time. For now, I simply offer as a general suggestion that there is a truth specific to contexts, and that the context determines which domain(s) are in play.

An additional strength to my proposed context-specific alethic pluralism would be its ability to not only preserve valid mixed inferences, but to also provide an account of the truth of premises that mix domains, such as mixed conjunctions and mixed disjunctions.

Examples of such premises are found in premise (1) of the two example arguments above. In the first example, the disjunction mixes the domains of mathematics and modality, while for the second example, the disjunction mixes the modal and legal domains. Tappolet, in arguing against alethic pluralism, asks the alethic pluralist which sort of truth is expressed by premises such as these (Tappolet 2000, 385). By appealing to a context-specific truth, I can provide an answer to this question: in my first argument, (1) is true in a way specific to the context that mixes the domains of mathematics and modality, and in my second argument, (1) is true in a way that is specific to the context that mixes the modal and legal domains.

Some may begin to sweat nervously at the indefinite collection of truth predicates that is generated if my account is correct. The hope motivating my fictionalist approach in the first place was, after all, to restrict the number of metaphysical commitments we would have to make. The appeal of fictionalism was its potential to help us realize this end—to only take seriously a set of basic, uncontroversial things, and then to combine them in creative ways so as to pretend that we have produced new, more complicated things. The commitment to an indefinite collection of truth predicates may seem to betray this initial appeal. To this worry, I respond that the commitment to an indefinite collection of truth predicates is a *semantic* commitment, and not a metaphysical one. As I have been arguing throughout my dissertation, truth is a matter of the semantics of the assertions in question, and only representational semantics carries with it robust metaphysical commitments. In arguing that the mathematical and modal domains employ an alternate, fictionalist semantics, I am arguing that the semantics of those domains is not a matter of metaphysics, but a matter of make-believe. In this way, I don't take the commitment to indefinite number of

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truth predicates to be a controversial commitment, since it does not require controversial ontological commitments. Rather, in light of the arguments that I have made in this section, an indefinite collection of truth predicates is not only consistent with, but also facilitates, restricting our metaphyscial commitments.

5.2   Why I Am Not a Deflationist

I now turn to show why I am not a deflationist, and to argue that a semantic anti-realist approach to fictionalism cannot be used to provide a deflationist account of truth. In doing so, I analyze and argue against the fictionalism advanced by Armour-Grab and Woodbridge (2015). Their fictionalism shares similar inspirations and motivations to the ones I have developed so far. Unlike me, however, they employ their fictionalism to explain the domain of truth-talk, arguing for a deflationist account of truth where truth-talk itself is a sort of make-believe game. Although their fictionalism shares some affinities with the position that I have advanced, and granting that a make-believe account of truth-talk may have certain strenghts in certain contexts, I argue that even if we adopted a fictionalist account of truth-talk, a semantic anti-realist fictionalism could not lead to a deflationist account of truth. Rather, I demonstrate that if we accept a such an account of fictionalism, then truth predicates will be substantive predicates.

Armour-Garb and Woddbridge's fictionalism, outlined in *Pretense and Pathology: Philosophical Fictionalism and its Applications* (2015), takes many of the same inspirations that ground my own fictionalism, and employs them towards similar ends. They too attempt to develop an approach to fictionalism that is not an error theory. And, much like me,

they take inspiration from a Waltonian approach to make-believe to argue that such an approach can be used to unpack the semantics of assertions made in a make-believe game to avoid error theory. Where our projects differ, at least up to this point, is that they employ such an account of fictionalism to explain *truth-talk*.

Truth-talk, simply put, happens whenever we speak of things as true or false. For example, when I make assertions such as, "It is true that $2 + 2 = 4$," or "'It is possible that I will go to the punk rock show tonight' is true," or "Everything Emma says is true," I am engaged in truth-talk. Unsurprinsingly, truth-talk has been a topic that many philosophers have found puzzling, and have sought to explain. Armour-Garb and Woodbridge argue that truth-talk is best explained as kind of make-believe game itself. For Armour-Garb and Woodbridge, 'is true' reflects an illocutionary force of pretense on the part of the asserter—that is, when someone utters "p is true," we should recognize that the use of 'is true' reflects an act of make-believe on the part of the utterer. In this regard, 'is true' is not in the business of representing some property possessed by some claim. In relocating the semantics of 'is true' from one that represents a property of the claim to one that reflects a peculiar illocutionary force of the utterer, there is no opportunity to erronously attribute a property to a claim. And so Armour-Garb and Woodbridge, much like me, use fictionalism to develop an alternative, anti-realist semantics that doesn't lead to an error theory for the discourse in question. Unlike me, however, in outlining a semantic fictionalism for truth-talk, Armour-Garb and Woodbridge commit themeselves to a deflationism about truth, since no substantive property gets expressed by 'is true' under such a semantics for truth-talk.

I believe that a fictionalist account of semantics cannot be used to ground a deflationist

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account of truth, and that Armour-Garb and Woodbridge fail to appreciate the true lessons we can learn about truth-talk that a fictionalist semantics can teach us. By accepting that fictionalism can explain the semantics of various domains of discourse, 'is true' will have to be an ambiguous predicate—there can be no single interpretation of 'is true'—and deflationism cannot accomodate this ambiguity. To see what I mean, consider these two assertions:

(1) '2 + 2 = 4' is true.

(2) 'It is possible that I will go to the punk rock show tonight' is true.

According to a semantic anti-realist fictionalism, the 'is true' found in (1) will express a predicate distinct from 'is true' in (2). This is because (1) expresses a mathematical truth while (2) expresses a modal truth. If we were to formalize the above assertions, we would get something like:

(1*) $T_1(2 + 2 = 4)$

(2*) $T_2(\diamond P)$.

In already having argued for alethic pluralism, this should not be surprising. As I discussed in the previous section, alethic pluralism will necessitate different truth predicates to capture different sorts of truth. Thus, simply reading 'is true' will not be enough for us to know which predicate is being referred to. The predicate to be read by 'is true' will depend on the context and the domains it identifies. Therefore, 'is true' must be an ambiguous predicate if we accept that a semantic anti-realist account of fictionalism explains the semantics of some domains.

A deflationist account of truth could not accomodate the ambiguity of 'is true,' and would have a single predicate to express all instances of it. Crucial to the deflationist account of truth-talk is the commitment that 'is true' is an empty predicate that possesses no significance or meaning. As such, 'is true' always means the same thing for deflationists, namely nothing at all, regardless of context. Because of this, deflationism only ever needs one truth predicate to fully accomodate every instance of 'is true.' Even if a deflationist were to attempt to pursue a pluralism of truth predicates, those truth predicates would all be synonomous in their expression of nothing, and so would all be interchangeable with no loss of significance. To go back to (1*) and (2*), a deflationist would have to allow for the exchange of the truth predicates to get:

(1**) $T_2(2 + 2 = 4)$

(2**) $T_1(\diamond P)$,

and argue that this still fully accomodates the significance of 'is true' in both (1) and (2). Moreover, a deflationist would have to allow that both, and indeed all other, truth predicates be reducible to a single truth predicate that can presumably express nothing more generally, say $T_{whatever}$, and that:

(1***) $T_{whatever}(2 + 2 = 4)$

(2***) $T_{whatever}(\diamond P)$

also expresses the full significance of 'is true' in (1) and (2), respectively.

The same cannot be said of the truth predicates that are suggested by a semantic anti-realist fictionalism. The ascription of T_1 to '2 + 2 = 4,' according to this approach to

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fictionalism, identifies something significant about '2 + 2 = 4,' something that '$\diamond P$' does not possess. In particular, that '2 + 2 = 4' satisfies the semantic conditions for a true assertion in the domain of mathematics. Similarly, T_2 will reveal something significant about (2) not possessed by (1): that (2) satisfies the semantic conditions for truth in the modal domain. For a fictionalist account of the semantics of (1) and (2), T_1 and T_2 each express the satisfation of different semantic conditions. This itself inflates truth predicates in way that demands an ambiguous reading of 'is true,' one where not every instance of 'is true' is synonymous with each other, thus making the position decidedly *not* delfationist.

As our discussion of alethic pluralism in the previous section makes clear, however, domains alone cannot explain all kinds of truth-talk. There will also be truth-talk for contexts that mix various domains. For example, a context that mixes the mathematical and modal domains, C1, will have a truth predicate particular to it, T_{C1} while a context that mixes the modal and legal domains, C2, will have a truth predicate specific to it, T_{C2}. We must also make sense of truth-talk for these contexts. We might be tempted, as indeed I am, to employ a semantic anti-realist approach to fictionalism to explain these instances of truth-talk. That is, we could argue in favour of a make-believe account of truth-talk for contexts that mix domains. The props for these games could be the true assertions of each domain. Identifying the principles of generation will be a more difficult matter, as what determines the choice of principles of generation will depend on each context, and presumably, on the ends we are trying to achieve in those contexts. In contexts that mix two or more fictional domains, say, we may adopt only those principles of generation that all the domains share. Or, in contexts which mix fictional and realist domains, we may adopt the rules of the realist domain, along with the principles of generation of the

fictionalist domain that we would need to help us achieve our ends in that context.

Some principles of generation seem likely for most contexts. For instance, for any context that employs conjunctions that mix domains, A & B will be true in the context exactly if both A and B are; so if A and B are both atomic, then it will be true exactly when each of A and B is true in its home domain. But things will clearly be more complicated with other logical operators, where, for instance, the domains might disagree about the logical behaviour of the operators. For example, it will be more difficult to define the behaviour of material conditionals (if ... then operators) in a context that mixes relevance and classical logic. Providing a satisfactory philosophical account of just which principles of generation help us achieve the ends of contexts may be a matter of some experimentation, but regardless, it could offer a possible explanation of the truth of assertions or arguments made in contexts which mix domains. This may, in part, also explain why it is so difficult to provide easy formalizations of these truth predicates.

Even if we were to adopt such a position, however, this would still not be a deflationist account of truth-talk. This is because in arguing that C1 and C2 employ their own game of make-believe to construct their respective truth predicates, T_{C1} and T_{C2}, those truth predicates will each express the satisfation of different semantic conditions just as T_1 and T_2 do. This contributes to the ambiguity of 'is true,' rather than a reduction to a universal predicate. As already shown, deflationism cannot accomodate this ambiguity. Deflationism must argue that all instances of 'is true' can be disambiguated to the same, empty predicate. This reduction is not available to a semantic anti-realist fictionalism of the truth-talk for contexts that mix domains.

In this section, I have shown that Armour-Garb and Woodbridge are wrong to argue that a semantic anti-realist approach to fictionalism suggests a deflationary account of truth. In accepting that a semantic anti-realist fictionalism can be used to capture the semantics of the discourse of a particular domain, it must be accepted that different domains employ different systems of semantics. Thus, this approach to fictionalism must accept alethic pluralism and the substantive ambiguity of 'is true,' something deflationism simply cannot accomodate. This leaves us with various sorts of truth-talk, depending on the context. It can be conceded that we may be able to provide a fictionalist account of some instances of truth-talk, particularly those instances that deal with contexts that mix domains. But, rather then help us to deflate the 'is true' predicate, this move adds further ambiguity to the predicate, thus digging the position's heels further in the ground in its oppoisition to deflationism. As such, Armour-Garb and Woodbridge cannot appeal to a semantic anti-realist account of fictionalism to argue for a deflationist account of truth. In fact, the exact contrary is true if we accept this approach to fictionalism: a semantic anti-realist account of fictionalism reveals the significance of various sorts of truth predicates.

5.3   Chapter Conclusion

The purpose of this chapter was to defend and develop three of the underlying larger commitments necessitated by a semantic anti-realist account of fictionalism. The first two commitments that I defended were the commitments to alethic and logical pluralism. I developed both of these commitments in light of the problem of mixed inferences, advancing context-specific versions of both, and showing how these versions of alethic and

logical pluralism can address and accomodate mixed inferences. Finally, I defended the commitment to substantive truth predicates and the rejection of deflationism. By arguing against Armour-Garb and Woodbridge, I demonstrated that a semantic anti-realist approach to fictionalism constructs different truth predicates to reflect the satisfaction of different semantic conditions, thus inflating those truth predicates. Moreover, even if we were to accept that some instances of truth-talk are best explained by way of a semantic anti-realist fictionalism, there would still be a plurality of sorts of truth-talk that could be so explained, each with its own make-believe, and thus each would employ different truth predicates to signify the satisfaction of different semantic conditions. In sum, these arguments illustrate that a semantic anti-realist approach to fictionalism must be committed to alethic and logical pluralism, and that truth is a substantive property, but that these commitments seem to be a virtue rather than a liability.

Chapter 6

Curtain Fall

The preceding chapters saw the development and defense of an alternative approach to fictionalism, one that uses the tools and insights of Walton's theory of make-believe to provide an alternative, non-representational semantics for mathematical and modal discourse. This project was motivated by a desire to preserve the core, anti-realist contention of fictionalism, without having to also commit oneself to error theory. In other words, I sought to develop a position that could posit that mathematical objects and possible worlds were fictional creations, but still allows us to speak speak truly about them in a substantive way. I hope to have shown that not only is such an approach to fictionalism coherent and tenable, but also that it can offer compelling and capable anti-realist accounts for both mathematical and modal discourse. Further, in exploring and defending the underlying commitments of this view, intriguing lessons on the nature of truth and its influence on logic can be learned. I take this time to now conclude by providing a brief summary of the fictionalism that I have proposed, its lessons and merits.

6.1 Semantic Anti-Realist Fictionalism

I sought first to establish the basis for proposing a semantic anti-realist approach to fiction-alism by describing an analysis of realist/anti-realist debates and identifying positions in those debates according to their semantic commitments. This analysis is one that follows in the footsteps of Dummett and Wright, and proposes that realist/anti-realist debates for any particular domain are best approached as debates concerning the meaning of some set of assertions, the conditions for their truth, and whether or not we are capable of making true assertions of the sort in question. The realist was identified by the commitment to two claims, one modest and one presumptuous—the modest claim being that that there exists some mind-independent reality, and the presumptuous claim that: (a) our asser-tions about this reality are representational; and (b) that we are sometimes successful in making true assertions. Of particular interest is the realist's presumptuous claim, which allowed us to understand how anti-realist objections could proceed. Anti-realists could target (a) and argue that the semantics of the assertions are not representational, or they could target (b) and claim that our assertions on such matters are false. Anti-realists of the former class include contructivists and expressivists, while error theorists make up the anti-realists of the latter class. As I have shown, fictionalists have typically opted to challenge (b), which, I believe, fails to realize the full potential of many developments in fictionalism. My primary suggestion is that much of the development that fictionalism has seen is better employed as providing a challenge to (a), and as providing an alternative, non-representational semantics. Specifically, I suggest that by following in the footsteps of constructivism to provide such an alternative semantics that is also truth-apt, fictionalism

could furnish a semantic anti-realism capable of accounting for how mathematical claims can be both fictional and substantively true.

To make the case that fictionalism could provide such a semantic anti-realism, and that fictional claims could be genuinely true rather than merely "so to speak" true, I sought to explain how it is that developments of mathematical fictionalism can be used to sketch a semantics for fictional claims. I began by first looking to the arguments of Leng and Yablo, who both use Walton's theory of make-believe to develop a fictionalist account of the usefulness of mathematics. Essentially, Leng and Yablo argue that real, non-mathematical objects and facts about them as expressed in the language of logic serve as props in a sort of make-believe that, when taken with certain principles of generation, also expressed in the language of logic, can be used to generate a make-believe which can be of use to understand and make predictions about the real world. Yablo's fictionalism also illustrates that these fictions can be scaffolded upon each other, so that the fictional objects of one make-believe can be used as props in further make-believe games, and that how we engage in such scafollding is a matter of some choice. These additional observations help explain how mathematics can branch out so as to establish itself as an object of study, and how different mathematical practices can arise. My departure from Leng and Yablo was to not only see their arguments as establishing the plausibility of a fictionalist account of the usefulness of mathematics, but as also outlining an alternative semantics for mathematical assertions. Specifically, I suggested that in capturing mathematical discourse as kind of Waltonian make-believe, I could propose the following truth condition for mathematical assertions:

(SARF-Math) S is mathematically true iff S is make-believedly true in the game of mathematical make-believe currently in play.

As a consequence, an alternative, non-representational semantics for mathematical assertions will have been sketched. Moreover, it will be a semantics that puts the capability of making genuinely true assertions within our reach. Thus, in taking this departure from Leng's and Yablo's arguments, a new semantic anti-realist position seems to reveal itself, one that shares metaphysical territory with constructivism, but is a distinct anti-realism with its own distinct analysis of mathematics.

This semantic anti-realist approach to mathematical fictionalism provides a fictionalist account of the usefulness of mathematics that also allows for mathematical assertions to be genuinely and substantively true. However, that is not all that is on offer when pursuing a semantic anti-realist approach to mathematical fictionalism. Such an approach also goes some way toward providing an account of how the real world and facts about it play, at least in some small part, a causal role in the production of mathematical knowledge, while also focusing the establishment of mathematical truths on their provability. Moreover, a semantic anti-realism approach to mathematical fictionalism also provides some explanation of mathematical pluralism, as well as a legitimation of any established mathematical practice as genuine mathematics. And so, when developed as a semantic anti-realism, mathematical fictionalism can not only shake off its commitment to error theory while preserving the insights of Yablo and Leng, but can also potentially offer a more compelling anti-realist account of mathematics.

To further develop the view, and to show that such an account was not exclusive to

mathematical discourse, I considered how modal fictionalism could be fashioned along these lines. In so doing, I provided an exposition of Lewis's modal realism, and the the usefulness of interpreting modal discourse as discourse about possible worlds. I then showed how modal fictionalism developed as an attempt to preserve possible worlds discourse, without being committed to the actual existence of said worlds, by interpreting Lewis's modal realism as a kind of fiction. Such an approach to modal fictionalism, however, is open to the artificiality objection, which seriously challenges the capacity of everyday ordinary people, past and present, to effectively engage in modal reasoning. The solution, I suggested, is to instead take the modal fiction as a Waltonian make-believe game scaffolded upon a wide-variety of physical and social sciences, so as to provide props and principles of generation to help us imagine possible worlds, what would be true of them, and how far removed from the actual world they are. In so doing, I made it possible to propose the following truth condition for modal assertions:

(SARF-Modal) S is a modal truth iff S is make-believedly true in the modal make-believe.

Much as it did for mathematical fictionalism, this move allows fictionalism to propose a non-representational semantics for modal assertions. And much as this shift in fictionalism allowed mathematical fictionalism to potentially rise above the prospects of its error theory roots, the same is true here. Not only does it provide a fictionalism that preserves the possibility of our capacities to make true modal assertions, but it provides a semantics that more closely resembles the realist semantics it sought to emulate. By not having to propose that modal assertions possess an implicit prefix, a semantic anti-realist modal

fictionalism proposes no paraphrase of possible worlds discourse, which allows it to avoid a whole host of complications that such a paraphrase entails.

With all of this done, I provided not only an account of how fictionalism could be used to provide alternative, non-representational semantics that allowed us the capabiliy to make true assertions. I also provided fictionalist accounts of mathematical and modal discourse that seem to be more capable than their error theoretic counterparts. I take these two examples to establish the viability and philosophical value of a semantic anti-realist version of fictionalism, and indeed as a fairly compelling case that fictionalism is generally going to be a more philosophically compelling view when it is not saddled with the weight of error theory.

In the final substantial chapter I turned from examples to some philosophical questions that confront my view as a general philosophical account of fictionalism. Specifically, I defended its implicit commitments to alethic and logical pluralism, and responded to the general worries those commitments face. In the case of alethic pluralism, the worry was that in defending a plurality of kinds of truth, there could be no accounting for the validity of arguments which mixed assertions that possess different truth predicates. A similar worry was outlined regarding the commitment to logical pluralism—that is, how could such a pluralism provide an account of the validity of arguments which mix different logical systems. My solution was to offer context-specific variants of both alethic and logical pluralism, which allowed me identify context-specific truth predicates and context-specific logical systems for arguments which mix truth predicates and logical systems respectively. Thus, arguments made in contexts which mix truth predicates or logical systems can have their validity explained by appeal to the context-specific truth predicates and logical

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systems in play. Although much more could be done to develop context-specific alethic and logical pluralism as fully robust philosophical positions, I believe that the initial outlines on offer provide some defense to the commitment to pluralities of both truth predicates and logical systems.

In closing, I believe that I have shown that fictionalism can be fruitfully pursued as a metaphysical position that not only seeks explanations without appeal to controversial or embarrassing ontological commitments, but also as one that acknowledges that we can still use fiction to speak truly. Although many have appealed to fictions as a means of explaining how a discourse which fails to produce substantively true assertions can nevertheless be useful, none so far seem to recognize the true reach that an appeal to fiction as providing explanations of the usefulness of certain discourses can have. The semantic anti-realist account of fictionalism that I have offered attempts to explore this reach, and has shown, I hope, how fictionalism's potential to provide useful explanations *follows from* its ability to stand on a useful and truth-apt anti-realist semantics. This approach to fictionalism, I believe, better appreciates and understands the usefulness with which fiction serves us to make sense of, and navigate the world around us.

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