Staff Scheduling during a Pandemic

by

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Author's Declaration

I hereby declare that I am the sole author of this thesis. This is a true copy of the thesis, including any required final revisions, as accepted by my examiners.

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Abstract

The year 2019 revealed that some of the policies which have shaped the core structure of many organizations in different industries for a long time, could result in an absolute failure in an unprecedented crisis like the COVID-19 pandemic. In the light of such changes, the interaction between the people is a determining factor to limit an outbreak among the staff members of an organization to prevent any disruption in the process of the service/product they provide. Thus, an effective staff scheduling policy can be the clincher to achieve this goal.

In this work, we consider a staff scheduling problem with the goal of minimizing the expected number of staff replacements that happens as a result of getting infected during a pandemic. In this days-off scheduling problem, we discuss a two-stage optimization approach where we first, determine the optimal scheduling patterns for the staff members and next, we will assign them to different resources so that the interaction between the staff members is minimized. In the proposed mathematical formulation for the problem, we consider the characteristics of the disease and the situation of the public health at different stages of the pandemic such as the incubation period, the probability of getting infected on a working day versus a rest-day, and the availability of swab tests. We design a column generation algorithm to solve the optimization model which requires up to 70% less computational power compared to the traditional algorithms that solve the problem when all available patterns are generated. A simulation model is also designed to compare the effectiveness of our suggested policies with the traditional scheduling policies. We examine our findings using data from the Grand River Regional Cancer Centre (GRRCC), which is a comprehensive cancer treatment and research centre located in Kitchener, Ontario. Particularly, we worked closely with the Department of Medical Physics and Radiation Oncology who plans and delivers radiation therapy treatments to cancer patients and treats over 2000 new patients annually. Our results show that depending on the different stages of a pandemic, the proposed staff scheduling policies can lead up to 20% less full-time equivalent staff replacements which have a significant impact on the availability of the centre's resources as well as the patient flow in long-term.

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Dedication

To all people who've been there to support me during the most difficult times in my life.

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Chapter 1

Introduction

The recent COVID-19 pandemic changed the world in many aspects. A wide range of services, industries, and events have been extensively disrupted by the disease. From accommodation and food services to transportation, warehousing, and healthcare, it might take years for them to get back to their pre-pandemic normal. In healthcare for example, dealing with the consequences is even much more challenging. On the one hand, healthcare centres have been under an unprecedented burden of dealing with COVID-19 cases, which has made them allocate most of their resources to confront the pandemic. On the other hand, they need to avoid any disruption in the process of care for the patients who are suffering from other chronic and non-COVID-related diseases. Thus, protecting the health and well-being of staff members is of utmost importance, in order to deliver their services and avoid long wait times for patients. The goal of this work is to develop staff scheduling policies such that the risk of disease transmission to staff members, and the consequent staff shortage during pandemics is minimized.

Mathematical models have played an important role in policy development to address the COVID-19 outbreak [McBryde et al., 2020]. In this work, we provide a mathematical optimization framework for staff scheduling, which takes into account the probability of infection transmission among customers and staff, the effect of interaction between staff members who work in close contact, and the characteristics of the disease such as the incubation period during which customers and staff members can be asymptomatic. The proposed approach consists of two mathematical optimization models that are solved consecutively. The first optimization model determines an optimal days-off schedule that minimizes the expected number of replacements as a result of customer-staff interactions. The second model then uses the output of the first model to assign staff members with a determined schedule to different teams on working days such that the number of interactions between the staff members are minimized. To solve the first model, which is the more computationally challenging step, we develop a column generation (CG) algorithm in which we consider (i) the number of staff members who need to team up for a specific service, and (ii) the minimum and maximum number of days staff members can work during the scheduling time horizon. The second model that we develop would take the optimal output (patterns) of the first model and assign staff members to different rooms so that the total number of "new" interactions between the staff members is minimized. We define an interaction between two staff members as "new" interaction when the pair work on a specific day while they have not worked during the last τ days where τ denotes the incubation period of the disease. Finally, through a sensitivity analysis we consider different scenarios to examine the performance of the proposed scheduling policy in different settings. We also apply our findings, through a discrete-event simulation, in our partner organization, Grand River Regional Cancer Centre (GRRCC) which has teams of therapists who cooperate with each other in different radiation therapy rooms. We observe that the optimal staff scheduling could be highly dependant on the different stages of the pandemic in terms of the probability of getting infected at work on a working day or at home, on a rest day. The simulation results demonstrate that the number of staff replacements because of getting infected would be up to 20 percent less compared to the on-going traditional scheduling policies. Note that the proposed CG algorithm is computationally efficient and performs up to 70% faster compared to the algorithms used to solve the original formulation of the model where all patterns are generated.

Chapter 2

Literature Review

2.1 Staff Scheduling

Staff scheduling problem has been studied extensively in the literature of operations research and management. Basically, the problem is dealing with optimizing the staff schedule with regards to different objectives such as minimizing the number of needed staff, maximizing the utilization, minimizing costs, etc. Usually, the first stage of this problem is to determine the proper number of staff with different skills to meet the service demand. In the second stage, each individual staff member is assigned to shifts so that the required staffing levels at different time periods are met. However, apart from these requirements, different organizational regulations must not be violated. [Ernst et al., 2004] propose a taxonomy of staff scheduling in which they suggest a number of modules starting with the determination of staffing requirements and ending with the specification of the work to be performed, over a time period, by each individual in the workforce. Below, we discuss the most common modules of the staff scheduling problem considered in the literature.

2.1.1 Different Modules of the Staff Scheduling Problem

- Demand Modelling

The goal in this problem is to determine the number of needed staff members for a specific time horizon. In this module, some predicted patterns of incidents are translated into associated duties and then the duty requirements are used to determine the demand for staff. The nature of the demand in this problem is important. In a task-based demand for example, demand is obtained from a list of tasks that needs to be performed. These tasks usually have starting time and duration or a time window within which they must be terminated. The main application of task-based demand is in transportation where the staffing is dealing with crew pairing optimization. To name a few, [Bazargan, 2016, Zeren and Özkol, 2016, de Armas et al., 2017, Deveci and Demirel, 2018] consider staff scheduling with task-based demand.

Another case in demand modelling is when the demand is flexible meaning that there is unknown future demand and prediction/forecasting techniques are needed to find out the future incidents. There are random arrival rates for future services as well as random service times. The aim here is to determine the number of staff required at different times of a day for each day in a time horizon. The application of this case is usually in police services [Thompson and Goodale, 2006, Todovic et al., 2015], call centers [Robbins and Harrison, 2010, Taskiran and Zhang, 2017], airport staffing [Soukour et al., 2013, Zeng et al., 2019], etc. Once the flexible demand pattern has been generated, it is used to determine the shifts and cover the demand. Another approach is that the demand would be used in the second stage of the problem as a constraint on the number of staff working at each time.

- Days-off Scheduling

This module determines how rest days should be interspersed between work days for different working stations. The second phase of such a problem usually deals with the flexible or shift-based demand staffing. We will elaborate on this module in Section 2.3.

- Shift Scheduling

In the shift scheduling problem, we aim to meet the demand by choosing possible shifts from a set of available shifts along with allocating enough staff members to the chosen shifts. Apart from the demand requirements, we also need to consider the organizational rules and regulations such as the breaks, number of consecutive shifts for each staff members, etc. [Volland et al., 2017, El Adoly et al., 2018, Seifi et al., 2021] are examples of recent papers that focus on shift scheduling.

The above-mentioned three modules are the main categories of staff scheduling problems. In fact, all other modules can be transformed to one of these modules with the necessary modifications. In the next section, we will briefly discuss methods and algorithms that are used in the literature to solve different modules of staff scheduling problem.

2.1.2 Solution Methods for Staff Scheduling Problem

Perhaps the most known and simple categorization of solution methods for staff scheduling is proposed by [Bechtold et al., 1991] where they categorize the solutions in two categories of linear programming and construction-based modeling. [Alfares, 2004] also provide a comprehensive review of the employee tour scheduling problem which involves the determination of both work hours of the day and workdays of the week for each employee. In this section, we discuss some mathematical approaches used in the literature to model the problem.

- Linear Programming

The set covering approach proposed by [Dantzig, 1954] is still a very popular method for shift scheduling problem. The advantage of this formulation is that it provides enough flexibility for adding any required constraints. [Bhatnagar et al., 2007] propose a linear programming model to determine the optimal allocation of permanent and contingent workers to all sub-processes in a framework that balances the significant trade-offs for the induction of contingent workers in a complex assembly environment. [Corominas et al., 2010] address a shift scheduling problem by proposing two linear programming models to schedule working time, using working time accounts, at companies in the service industry. In the first model, they plan working hours with working time accounts (WTA), which is defined as the record of the hours of work spent by an employee in any given accounting period that shows 'credit' or 'debit' hours. In the next model, they take into account the need and regularity in the use of WTA working hours. [Dück et al., 2012] address the reactionary delays which usually takes place because of the crews changing aircraft. Based on a stochastic model for delay propagation, they propose an indicator for stability of airline crew and aircraft schedules. As an integrated formulation for the crew and aircraft scheduling problem would be a non-linear stochastic recourse function, they separate it into different linear problems connected by the objective function. In a close work, [Hojati and Patil, 2011] decompose the problem of scheduling of heterogeneous, part-time, service employees with limited availability into two stages of determining good shifts and then assigning the good shifts to employees, and use a set of small integer linear programs to solve each part. [Hochdörffer et al., 2018] address a short-term staff scheduling by using a linear programming based heuristic, which solves the scheduling problem gradually for each rotation round and generates a holistic job rotation schedule for an entire workday.

- Mixed-integer Programming

In many settings, due to various constraints such as shift requirement constraints, the scheduling problem cannot be formulated as a linear program, and thus, other approaches such as MIP is used to model the problem. [Al-Yakoob and Sherali, 2007] study the problem of assigning employees to different stations considering the employees' expressed preferences. Their mixed-integer two-stage approach provides daily schedules for employees for a given time horizon. [Bhulai et al., 2008] introduce a method for shift scheduling in multi-skill call centers. At the first stage, staffing levels are determined, and next, in the second stage, the outcomes are used as input for the staff scheduling problem. Similarly, in their research, [Fırat and Hurkens, 2012] study scheduling complex tasks with an inhomogeneous set of resources. The problem is to assign technicians to tasks with multi-level skill requirements through an MIP model. [Corominas et al., 2012] develop an MILP to solve a planning model which integrates production, human resources, and cash management decisions. They also take into account the consequences that decisions in one area may have on other areas and allowing all these areas to be coordinated. [Shiau et al., 2020] use a scheduling case study of the ground staff in aviation industry that includes three different types of personnel scheduling results: fluctuation-centered, mobility-centered, and project-centered planning. Their work presents an integrated mixed integer programming (MIP) model for determining the manpower requirements and related personnel shift designs for the ground staff at the airline to minimize manpower costs.

- Column Generation

In column generation, large LP models can be solved to optimality without incorporating all variables in the model at once. It establishes a lower bound to the integer programming solution which is guaranteed to be LP optimal. One of the early papers that used CG approach for solving nurse scheduling problems is [Beliën and Demeulemeester, 2008]. [Al-Yakoob and Sherali, 2008] design a column generation model and an effective heuristic to solve the problem of assigning employees to a number of work centres taking into account employees' expressed preferences for specific shifts, off-days, and work centres. In a close work, [Brunner and Edenharter, 2011] present a strategic model to solve the long-term staffing problem of physicians in hospitals using flexible shifts. The objective in their work is to minimize the total number of staff subject to several labor agreements. They formulate the problem as a mixed-integer program and solve it by applying a column generation based heuristic. [He and Qu, 2012] propose a hybrid constraint programming based column generation (CP–CG) approach to model nurse scheduling (rostering) problems. They consider all the complex real-world constraints in several benchmark nurse scheduling problems.

Based on the general classification scheme that we mentioned at the beginning of this section by [Ernst et al., 2004], our work is focusing on "days-off scheduling" module in which we aim to determine how rest days need to be interspersed between the working days. Thus, in the following section, we first discuss the staff scheduling literature during a pandemic and then, we concentrate on the literature of the days-off scheduling problems.

2.2 Staff Scheduling During a Pandemic: COVID-19

The uncertainty in staff scheduling problem is usually studied in terms of workforce demand, availability of resources, and a few papers consider the possible uncertainty around the staff size when examining the medical staff scheduling problem [Chen et al., 2016]. Staff scheduling with a goal of minimizing the risk of infection transmission among staff is a novel emerging aspect that has been the focus of few papers since the start of the COVID-19 pandemic. [Zucchi et al., 2021] propose a mixed integer linear programming (MILP) model that minimizes the total deviation between the amount of weekly contractual hours for each worker and the actual working hours of the personnel in a large pharmaceutical distribution warehouse. Their model meets the contractual working time of the employees, who are divided into mutually exclusive groups to reduce the risk of contagion. Based on the proposed classification scheme proposed by [Ernst et al., 2004], their work is categorized as a shift-based scheduling problem. [Guerriero and Guido, 2021] propose optimization models to address a shift-based staff scheduling during the COVID-19. Their focus is on novel optimization models that take into account demand requirements, employees' personal and family responsibilities, and anti-Covid-19 measures. However, like other works in this new emerging area, they do not consider the characteristics of the disease in their model. Their multi-objective MIP model aims at maximizing employee work on-site as well as maximizing remote work. [Geibinger et al., 2021] present a constraint model that includes the variety of requirements required to ensure day-to-day operations. They introduce an innovative set of grouping constraints to partition the physicians of a children's hospital in Vienna, with the intention to easily isolate a small group in case of an infection. They develop a Constraint Programming model to minimize the interaction between the physicians from different stations in terms of their shift assignments. Similarly, [Güler and Geçici, 2020] address the physician scheduling problem of a hospital in Turkey. They develop a MIP model and embed it into a spreadsheet-based decision support system. In their work, they tend to minimize the deviation of the workload of the physicians between their regular shifts and the shifts in which they are dealing with the COVID-19 patients. To the best of our knowledge, there is no papers in the literature of staff scheduling that considers the risk of losing staff due to infection, characteristics of the disease such as the incubation period and the probability of getting infected at the workplace or at home, as well as pairwise interaction between the staff members.

2.3 Days-off Scheduling

Days-off scheduling has been studied extensively over the past decades. In fact, [Baker, 1976] proposes the days-off scheduling problem by modifying the set covering formulation that was proposed by [Dantzig, 1954]. Years after that, [Chen, 1978] develops a simple solution algorithm for the problem of scheduling workforce for an organization operated seven days a week. For a given number of full-time workers, the objective was to maximize the total number of workers who get two consecutive days off. [Alfares, 1998] develop a two-phase algorithm for cyclic days-off scheduling. The objective in their work is to satisfy daily labor demands with a minimum cost for workers. They study a specific type of days-off scheduling where each work pattern contains five working days and two consecutive off days per week. The problem is modelled as a two-phase optimization framework. First, a simple model was used to calculate the minimum workforce size. Next, they use the minimum number as a constraint in a continuous linear programming (LP) model of the problem. [Azmat and Widmer, 2004] categorize the problem of days-off scheduling into single or multiple shifts depending on the policy which is followed regarding the number and the order of days that the staff members need to work. They consider a set of legal constraints such as holiday arrangements to minimize the defined workforce. [van Veldhoven et al., 2016] study a two-phase decomposition approach to solve the staff scheduling problem. In the first phase, similar to what we will discuss, they create a days-off schedule that determines the working and off days for staff members and in the second phase, they assign shifts to the staff members. Likewise, [Cuevas et al., 2016] propose an MIP model that first determines the workers' days-off schedule and assigns shifts to employees to maximize the demand for on-duty staff across multiple activities. [Shuib and Kamarudin, 2019] propose an integer goal programming model to maximize the over-achievement and minimize under-achievement of the day-off preference schedule for 43 workers in a selected department of the power station for 28 days where workers work in three shifts. Based on the 28-day schedule obtained, the day-off preference's satisfaction of workers increased by 37.21%. In general, most of the days-off scheduling concentrates on two-stage approaches which deal with the shift-scheduling. This work introduces a two-stage approach for the days-off scheduling where we determine the days-off schedule for each staff member in the first stage. In the second stage, the staff members are assigned to available resources to minimize the interaction between them.

Chapter 3

Problem Definition

In many services and industries, a team of staff members need to work together to provide service to the customers. During a pandemic, the interaction between customers and service providers increases the risk of infection transmission between both parties. An outbreak among staff members in many of these services can lead to a potential shortage of staff, especially in healthcare settings where interactions between healthcare staff and patients and also among staff members themselves are inevitable. On each day, in case a staff member is working, we assume that there is a probability of p_w that they get infected from patients. Moreover, the probability of getting infected during a holiday/rest day, is positive and denoted by p_h. Whether p_w is greater than p_h or not, depends on many preventive public health policies and their level of implementation. If social distancing policies are strictly followed for all activities in the society, it may be concluded that the risk of getting infected for a staff member who works on a specific day is higher than that of a colleague who is taking a rest day at home. The reason is that there are likely a larger number of interactions with people on a working day, which increase the risk of disease transmission. On the other hand, the availability of specific proper personal protective equipment in a working setting can decrease the risk of infection compared to that of when the staff member is at home and does not have access to such equipment, and/or do not practice safe distancing policies.

There is also another source of infection which is due to the risk of infection transmission from social interactions among the staff members. To avoid a possible outbreak among the staff members, the best practice is to limit the number of their interactions as much as possible. Once a staff member gets infected, he/she must be replaced with a new staff member. This happens in two occasions: (i) when a staff member shows symptoms of infection, and (ii) when a staff member has an infection-positive test result. An infected person usually becomes symptomatic after the incubation period τ is passed. For example, the median incubation period of COVID-19 is estimated to be at 5 days [Lauer et al., 2020], while the mean incubation period of SARS is about 7 days [Chan-Yeung and Xu, 2003]. Depending on the characteristics of the disease, a transmission may or may not occur during the incubation period. For example, Ebolavirus infected patients do not pass the virus on to others during incubation period [Eichner et al., 2011] while Norovirus can shed during the incubation period [Robilotti et al., 2015]. A recent study by [Johansson et al., 2021] estimates that transmission from COVID-19 infected asymptomatic individuals accounts for more than half of all transmissions. Even the risk for COVID-19 breakthrough and passing infection in fully vaccinated people cannot be eliminated as long as there is continued community transmission of the virus [Centers for Disease Control and Prevention, 2021].

We assume that there is a maximum of m staff members available who need to be assigned to r resources (i.e., rooms, machines, etc.) on a daily basis. Each resource needs t staff members to operate. So, on each day at least $t \times r$ staff members are needed. The scheduling time horizon is n days and each staff member should work at least l and at most u days during the time horizon. There are also test days for staff members and we assume the same-day results are available. This means that at the end of the day, infected staff members are identified and must be replaced with new staff members for the remaining days of the scheduling time horizon. We also assume that at the beginning of the time horizon, there is no infected staff members. Furthermore, we assume that newly replaced staff members are not infected either. To find an optimal staff scheduling, we consider an aggregated objective which aims to optimize the Expected Number of Replacements (EXNR). The objective minimizes the summation of EXNR for all the staff members based on their assigned working patterns. Minimizing EXNR aims to address the shortage of staff due to an outbreak which is an important challenge of many facilities during a pandemic.

3.1 Problem Formulation

In this section, we first discuss how we obtain EXNR. Next, we discuss the optimization models that assign each staff member to a working pattern considering the minimum and maximum number of required working days for each staff member and the required number of staff members for each resource.

3.1.1 Expected Number of Replacements (EXNR)

Consdier a time horizon with n days and h holidays during which none of the staff members work. Thus, there are $c = \theta^{(n-h)}$ working patterns for each staff member where θ represents the number of working status on each day (e.g., full-time, part-time, rest-day).

For example, suppose that we have 2 days with no holidays in between and assume that "1" and "0" indicate working and not working, respectively ($\theta = 2$). In this case, we would have the following 4 patterns:

$$[day1, day2] = [1, 1], [1, 0], [0, 1], [0, 0]$$

Depending on the working patterns employed, the expected number of replacements varies. In the following, we discuss how the expected number of replacements is obtained. Let τ denote the incubation period of the disease which is the time between exposure to the virus and the symptom onset. Also, let x denote a random variable that represents the number of staff replacements as a result of customers-staff infection transmission or infection transmission from external sources. Then, the expected number of replacements with $\theta = 2$ can be obtained as follows:

$$E(x) = \sum_{i=1}^{\lfloor n/\tau \rfloor + 1} i \sum_{(j_1,..,j_i) \in A_i} \pi_{j_1} \pi_{j_2} .. \pi_{j_i} \prod_{k \in B_{j_1,...j_i}} (1 - \pi_k) \tag{3.1}$$

$$A_i := \{(j_1, ...j_i) : \forall r, s = 1, ..., i, \quad r \neq s, \quad |j_r - j_s| > \tau, \quad j_1 < j_2 < .. < j_i\} \tag{3.2}$$

$$B_{j_1,...j_i} := \{1, ..., n\} \backslash (\bigcup_{l=1}^{i} \{j_l, j_l + 1, ...j_l + \tau\}) \tag{3.3}$$

$$\pi_i = \left\{ \begin{array}{ll} p_{wi}, & \text{if } i \text{ is a working day} \\ p_{hi}, & \text{if } i \text{ is a rest day} \end{array} \right\} \tag{3.4}$$

For example, assume that the time horizon is 4 days with the third day as a holiday. The incubation period is 2 days and there is a test day at the beginning and end of time horizon. Then, the expected number of replacements is calculated as follows:

$E(x) = 0 \cdot [(1 - \pi_1)(1 - \pi_2)(1 - \pi_3)(1 - \pi_4)] + 1 \cdot [\pi_1(1 - \pi_4) + (1 - \pi_1)\pi_2 + (1 - \pi_1)(1 - \pi_2)\pi_3 + (1 - \pi_1)(1 - \pi_2)(1 - \pi_3)\pi_4] + 2 \cdot [\pi_1 \pi_4]$ (3.1.1)

As mentioned previously, the total number of patterns that we have in this case is 8. To clarify equation (3.1), consider $[\pi_1(1-\pi_4)+(1-\pi_1)\pi_2+(1-\pi_1)(1-\pi_2)\pi_3+(1-\pi_1)(1-\pi_2)(1-\pi_3)\pi_4]$, which is the second term in (3.1.1). Note that this term covers all outcomes where there is 1 replacement. The infection can occur on any day during the time horizon (day 1 up to and including day 4). Consider the final term of the expression which indicates an outcome that the infection has occurred on day 4. Since we only have one replacement, there should be no infection occurring on days 1, 2 and 3.

Term $(1-\pi_1)\pi_2$ is related to an outcome where the infection occurs on day 2. Again, since there is only one replacement, the infection must not occur on any other days. However, there is a difference here compared to the previously discussed term. The staff member is already infected on day 2, and thus, it is certain (the probability is 1) that no infection occurs on day 3 and 4. The last term in (3.1.1) denotes an outcome (the only one) where two staff members replacements would happen. An infection occurs on day 1 and when the staff member is replaced on day 4, the newly replaced staff members gets infected on the same day. In this way, we enumerate all possible outcomes and calculate the EXNR accordingly.

3.1.2 "*New*" Interactions

The most important and effective policy for minimizing the risk of infection transmission during a pandemic is to limit social interactions. In our problem, assuming that the presence of a possible infected staff member in different teams could increase the risk of an outbreak among them, we define a measure that captures the number of "new" interactions throughout the time horizon for each staff member. We assume that there is a new interaction between a pair of staff members teamed up on day n if they have not been in the same team for the past τ days (i.e, since day $(n-\tau)^+$), where τ is the incubation period. For example, assume there are 5 staff members ($m = 5$) and 2 resources ($r = 2$). Each resource needs 2 staff members to operate on each day ($t = 2$). Table 3.1 shows the working patterns of the staff members for 4 days. Assume that the incubation period is 2 days ($\tau = 2$). Take staff member "A" as an example. This staff member has a new interaction on day 1. However, there is no new interaction for this staff on day 2. The reason is that staff "A" has already worked with staff "B" on day 1. On day 3, however, there is a new interaction for staff "A" as this staff has not worked with staff "E" in the last 2 days (incubation period). On day 4, there is no new interaction for staff "A" because there was an occasion in the last 2 days that staff "A" teamed up with staff "B".

Table 3.1: *An example to illustrate "New" interaction.*

	Day 1	Day 2	Day 3	Day 4
Resource1	A	A	E	A
Resource1	B	B	B	B
Resource2	C	C	A	C
Resource2	D	D	D	D

3.1.3 Optimization Models

In this work, we consider two goals. The first goal is to find out the optimal schedule for the staff members to minimize the EXNR, which is given in (3.1). The second goal is to minimize the interaction between the staff members as much as it is possible. In

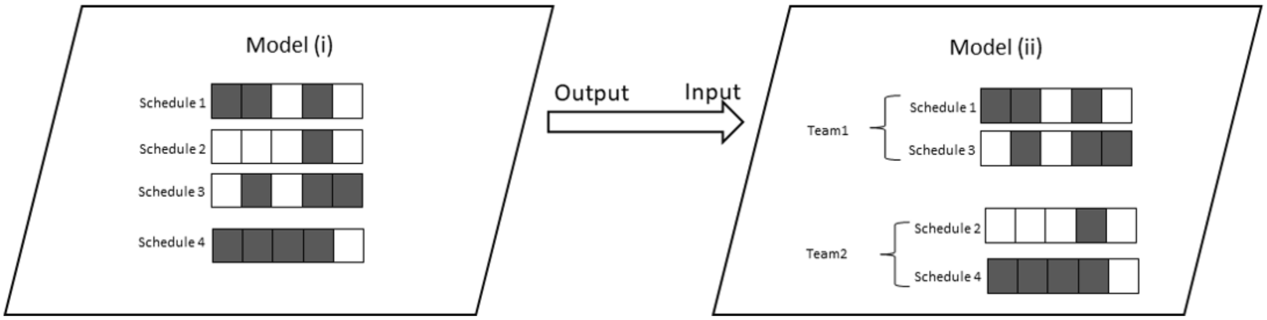

Figure 3.1: *The connection between Model* (i) *output (optimal scheduling) and Model* (ii) *input (optimal teams).*

other words, we also seek to minimize the total number of new interactions between the staff members. To achieve these goals, we design a two-stage optimization framework, as illustrated in Figure 3.1. Specifically, Model (i) assigns staff members to different working patterns to minimize EXNR. Then, Model (i) output will be used as an input for Model (ii) which creates teams of staff members such that the new interactions between the staff members are minimized.

- **Model** (i)

A very straightforward formulation would be to generate all the working patterns, and then determine which pattern should be assigned to each staff member using a mathematical formulation. Below, we introduce the notations used in the mathematical formulations of this model.

Sets

- $N = \{1, 2, .., n\}$: Set of all days

- $M = \{1, 2, .., m\}$: Set of all staff members
- $W = \{1, 2, .., c\}$: Set of all working patterns

Parameters

- n: Number of days during the scheduling time horizon
- m: Number of available staff members
- t: Number of required staff members on each day and for each resource
- r: Number of available resources
- $E(x_w)$: EXNR (cost) of pattern w
- l: Minimum number of days that each staff member is required to work during the time horizon
- u: Maximum number of days that each staff member is required to work during the time horizon.
- J_j: The set of all patterns which have "working" state on day j.

Decision Variables

- μ_{ij}: Binary variable where 1 denotes that staff member i works on day j, and 0 otherwise.
- y_{wi}: Binary variable where 1 denotes that working pattern w is assigned to the staff member i, and 0 otherwise.

Then, the problem can be formulated as follows:

$$Min \sum_{w=1}^{c} \sum_{i=1}^{m} y_{wi} \, E(x_w) \tag{3.5}$$

S.t

$$\sum_{i=1}^{m} \mu_{ij} = t \times r \qquad \forall j \in N \tag{3.6}$$

$$\sum_{w \in \{J_j\}} y_{wi} = \mu_{ij} \qquad \forall i \in M, \forall j \in N \tag{3.7}$$

$$\sum_{w=1}^{c} y_{wi} = 1 \qquad \forall i \in M \tag{3.8}$$

$$l \leq \sum_{j=1}^{n} \mu_{ij} \leq u \qquad \forall i \in M \tag{3.9}$$

$$y_{wi} = \{0, 1\} \qquad \forall w \in W, \forall i \in M \tag{3.10}$$

The objective is to minimize the summation of EXNR for chosen patterns. Constraint (3.6) guarantees that on each day, a total of $t \times r$ staff members are required. Constraints (3.7) and (3.8) ensures that the model assigns one pattern to each staff member and constraints (3.9) determines a lower and upper bound for the number of days that staff members are allowed to work during the time horizon. The challenge with Model (i) is that we need to generate all possible working patterns to calculate the expected number of replacements for each of the patterns. Depending on the number of days (n), working status on each day (θ), and the number of holidays/non-working days (h), there could be thousands of unique patterns (θ^{n-h}) which would necessitate a substantial computational power and memory to solve the model. Thus, in the next section, we propose an alternative formulation for Model (i), which lends itself better to a column generation approach.

- **Alternative Formulation for Model** (i)

In this section, we propose a formulation for Model (i) which is well suited to be solved using column generation. The difference between this formulation and the original one is that here, ϕ_w is a non-negative integer variable denoting the number of working pattern w that is chosen and a_w^j indicates whether day j is a working (1) or a non-working (0) day for pattern w. There are two sets of constraints for this formulation. The first set corresponds to the number of required staff members on each day, and the second set of constraints captures the lower bound and the upper bound of the required number of days to work during the time horizon.

$$Min \sum_{w \in W} \phi_w \, E(x_w) \tag{3.11}$$

S.t

$$\sum_{w \in W} a_w^j \phi_w = t \times r \qquad \forall j \in N \tag{3.12}$$

$$u \geq \sum_{j \in N} a_w^j \geq l \qquad \forall w \in W \tag{3.13}$$

$$\phi_w \geq 0 \quad \& \quad integer \qquad \forall w \in W \tag{3.14}$$

Again, the issue here is that we need to create all working patterns (or at least those within the upper and lower bound of required working days) and calculate the corresponding EXNR for each pattern. However, this formulation is suitable to be solved through the column generation approach where we can avoid generating all possible patterns.

3.1.4 Column Generation Approach

The general idea of CG is that many linear programs are fairly sizable to consider all the variables. In such problems, usually, most of the variables will be non-basic and would have a value of zero in the optimal solution. Thus, only a subset of variables need to be considered in theory when solving the problem. CG leverages this idea to generate only the variables which have the potential to improve the objective function. This approach consists of two problems. The Restricted Master Problem (RMP) which is the original problem with only a subset of variables (patterns in our case) and the sub-problem (SP) which is a new problem to identify a new variable that potentially improves the objective of RMP. In what follows, we demonstrate how our model can be solved using the column generation approach.

- **Restricted Master Problem (RMP)**

The restricted master problem starts off with a small number of patterns to solve the LP relaxation restricted to these variables. Let W' denote the small set of feasible patterns that starts off the algorithm. Then, an LP relaxation of the original problem can be written as,

$$Min \sum_{w \in W'} \phi_w \, E(x_w) \tag{3.15}$$

S.t

$$\sum_{w \in W'} a_w^j \phi_w = t \times r \qquad \forall j \in N \tag{3.16}$$

$$\phi_w \geq 0 \qquad \forall w \in W' \tag{3.17}$$

where ϕ_w is a non-negative, real-valued variable.

- **Sub-problem (SP)**

Generating columns is addressed in the sub-problem where we aim to find a pattern with the lowest reduced cost. The sub-problem is defined as follows:

$$Min \quad E(x_{new}) - \sum_{j \in N} y_j a^j_{new} \tag{3.18}$$

S.t

$$l \leq \sum_{j \in N} a^j_{new} \leq u \tag{3.19}$$

$$a^j_{new} \in \{0, 1\} \qquad \forall j \in N \tag{3.20}$$

The objective is to create a column (pattern) with the minimum reduced cost. The objective in the sub-problem has a non-linear term, $E(x_{new})$, which is the expected number of replacements for the new generated pattern. Decision variable a^j_{new} in this problem is a binary variable. Based on the definition given in (3.1), the objective of the SP contains a product of binary variables $(a^1_{new}, a^2_{new}, .., a^n_{new})$. We can potentially linearize the product of binary variables by adding new variables and constraints. However, the number of variables and constraints of the problem exponentially increase. Note that using the sub-problem, we aim to find a pattern that improves the objective function of the RMP model. Thus, we do not really need to solve the sub-problem to global optimality, and therefore, this non-linear objective function can be easily solved by non-linear approaches. In our numerical examples, we used the GEKKO Optimization Suite [Beal et al., 2018] which specializes in dynamic optimization problems for mixed-integer, nonlinear, and differential algebraic equations (DAE) problems. In each iteration, a new column is created and added to the LP relaxed master problem. These iterations continue to a point where the optimal objective value of the sub-problem becomes greater than or equal to zero, which is the stopping criteria for the algorithm.

- **IP Formulation for Model** (ii)

This model uses the output of Model (i) to assign each working schedule (staff member) to a room in a way that the total number of new interactions is minimized.

Sets

- $N = \{1, 2, .., n\}$: Set of all days
- $M = \{1, 2, .., t \times r\}$: Set of all chosen patterns
- $R = \{1, 2, .., r\}$: Set of all resources

Parameters

- n: Number of days during the scheduling time horizon
- t: Number of required staff members on each day and for each resource
- r: Number of available resources
- a_i^j: The working state of the chosen pattern for staff i on day j
- τ: Incubation period of the disease

Decision Variables

- μ_{ij}^k: Binary variable where 1 denotes that staff member i works on day j with resource k, and 0 otherwise.
- $S_{ii'j}^k$: Binary variable where 1 denotes that staff members i and i' work with resource k on day j, and 0 otherwise.

- $Q^j_{ii'}$: Binary variable where 1 denotes that there has been a new interaction between staff members i and i' on day j, and 0 otherwise.

$$Min \sum_{i \in M} \sum_{i' \in M/i} \sum_{j \in N} Q^j_{ii'} \tag{3.21}$$

S.t.

$$\mu^k_{ij} a^j_i + \mu^k_{i'j} a^j_{i'} \geq 2S^{jk}_{ii'} \quad \forall i \in M, \forall j \in N, \forall k \in R, \forall i' \in \{M \mid i' > i\} \tag{3.22}$$

$$\mu^k_{ij} a^j_i + \mu^k_{i'j} a^j_{i'} \leq S^{jk}_{ii'} + 1 \quad \forall i \in M, \forall j \in N, \forall k \in R, \forall i' \in \{M \mid i' > i\} \tag{3.23}$$

$$\sum_{k=1}^{r} S^{jk}_{ii'} - \sum_{k=1}^{r} \sum_{z=max(1,j-\tau)}^{j-1} S^{zk}_{ii'} \geq \tau Q_{ii'j} - \tau \quad \forall i \in M, \forall j \in N, \forall i' \in \{M \mid i' > i\} \tag{3.24}$$

$$\sum_{k=1}^{r} S^{k}_{ii'j} - \sum_{k=1}^{r} \sum_{z=max(1,j-\tau)}^{j-1} S^{zk}_{ii'} \leq \tau Q_{ii'j} \quad \forall i \in M, \forall j \in N, \forall i' \in \{M \mid i' > i\} \tag{3.25}$$

$$\mu^k_{ij}, Q^j_{ii'}, S^{jk}_{ii'} = \{0, 1\} \quad \forall i, i' \in M, \forall j \in N, \forall k \in R \tag{3.26}$$

The objective of this model is to minimize the total number of new interactions between each pair of staff members. Constraints (3.22) and (3.23) deals with the pair-wise interaction between two staff members. Thus, $S^{jk}_{ii'}$ equals to 1 if and only if the pair works together on a same day and on the same resource. Constraints (3.24) and (3.25) address the new interaction between a pair on each day. There are 2 conditions for a new interaction. First, a pair of staff members must work together on day j, and second, they must not have worked together since day $(j - \tau)^+$. In this way, we can calculate the pair-wise new interaction between all staff members and find out what resources the staff members are assigned to. Since there are usually limited number of staff members and resources and the size of the problem is not often too large, a branch and bound algorithm can solve this problem in a reliable time.

Chapter 4

Case Study: Grand River Regional Cancer Centre (GRRCC)

In this section, we examine the efficiency of the the proposed staff scheduling models using data from the Grand River Regional Cancer Centre (GRRCC). It is a comprehensive cancer treatment and research centre located in Kitchener, Ontario, and the sole provider of radiation treatment in the Waterloo-Wellington Region in Ontario, serving a population of over 775,000 people. We particularly worked closely with the Department of Medical Physics and Radiation Oncology who plans and delivers radiation therapy treatments to cancer patients and treats over 2000 new patients annually.

The partner organization has a growing concern about the possibility of facing a shortage of resources during the pandemic, particularly with an increase in the number of infected staff. Any interruption and delay in the care process for the radiation therapy patients can be highly detrimental towards the efficiency of their treatment. Thus, the scheduling of radiation therapy staff has become an increasingly challenging task during the COVID-19 pandemic to avoid an outbreak among their staff members and the consequent shortage of staff members. The radiation therapy process starts with patients undergoing a Computed Tomography (CT) simulation to determine their clinical condition. The full dose of radiation is usually divided into a number of smaller doses called fractions. The patients get their pre-planned fractions in consecutive days until the treatment process terminates. Radiation therapists are responsible for positioning the patient and for delivering the radiation dose. So, on most occasions, interaction between therapists and patients is inevitable and therefore, infected patients, whether they are symptomatic or asymptomatic, may transmit infection to the therapists. Because of the importance of the continuity of treatment for these patients, even in some cases, symptomatic patients need to get their required radiation dose according to the physicians' decision. Since COVID-19 can remain asymptomatic for several days and there is a high probability of disease transmission during this time, it is extremely hard to identify infected patients and normal protocols cannot be applied. It is also impossible to test all patients for the virus mostly because the number of tests required exceeds the capacity of hospitals and labs, especially since cancer patients must visit the hospital every day for several weeks. Moreover, an infected therapist may transmit the infection to his/her colleague with a higher probability and this can trigger an outbreak among the therapists and paralyze the whole care process.

Since the start of the pandemic, GRRCC implemented different preventive staff scheduling policies such as full isolation policy where teams of therapists do not interact with each other. There are four radiation rooms, and each room needs a team of three radiation therapists to operate. A 5-day schedule is repeated for two weeks and then modified on a biweekly basis (10 working days), and there are a total of 16 radiation therapists. Their full isolation policy is maintained through four specific therapists who are assigned to each room for a specific scheduling time horizon. Figure 4.1 shows the current scheduling policy of four therapists which is the same for each room. There is one therapist who works 0.6 FTE (full-time equivalent) and three other works 0.8 FTE.

4.1 Numerical Examples

In this section, we first discuss the results of the proposed models based on the parameters given in Table 4.1. Next, we will perform a sensitivity analysis to find out the behaviour of the model in different scenarios. All our experiments have been solved using the discussed algorithm and the ILOG CPLEX software and carried out on a Core i7-7700K with 4.2 GHz and 32 GB RAM.

Table 4.1: *Models inputs and default values*

Input	**Default Values**
incubation period (τ)	5 days
Scheduling time horizon (n)	14 days (with Sat and Sun as holidays)
Maximum number of therapists (m)	16 therapists
Number of needed therapists in each team (t)	3 therapists
Minimum and Maximum needed days of working (l, u)	6 and 8 days
Number of teams/rooms (r)	4 rooms
Probability of getting infected on a working day (p_w)	0.05, 0.1
Probability of getting infected on a rest day (p_h)	0.1,0.05

We discuss the results under two different scenarios, namely, $p_w > p_h$ and $p_w < p_h$. As mentioned before, each of these scenarios could be possible at any stage of the pandemic. At the early stages, where the PPEs are limited and are not yet designed to be the most efficient, being present in a hospital and interacting with numerous patients would probably be riskier than staying at home especially in case of lockdowns. In contrast, at some points during the pandemic, staying at home and interacting with other people while there are no high-quality PPEs, could be more hazardous than being in a safer working environment. We explain the inputs of the model and their default values in Table 4.1.

4.1.1 Integer Feasible Solution

In order to find an integer feasible solution for Model (i), in each iteration of the column generation algorithm, we store the unique final optimal solution (patterns) of LP relaxed master problem to a set S. Set S will finally contain small number of patterns that finding the integral optimal solution from this set is just a matter of enumeration. Table 4.2 shows the integrality gap between our suggested algorithm versus the optimal solution of the LP relaxed column generation as a lower bound of the problem.

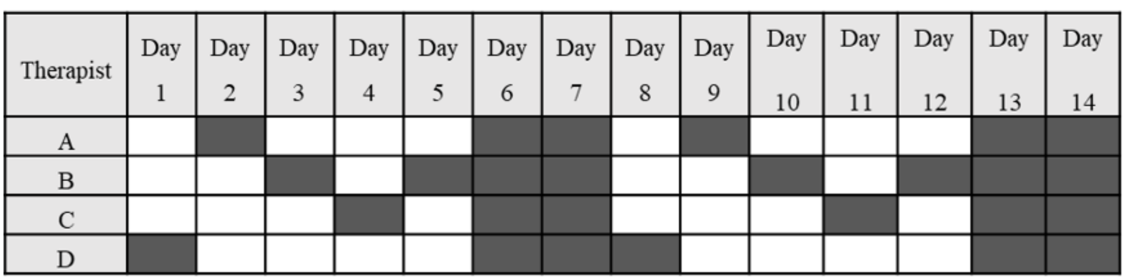

Figure 4.1: *Current scheduling of therapists at each room of GRRCC (Dark: Not working/ White: working).*

Table 4.2: *Integrality gap between the proposed algorithm and the LP relaxation optimal solution*

$t \times r$	m	Gap%	$t \times r$	m	Gap%
2	2	0	6	9	1.4
2	3	0.7	6	12	1.5
2	4	0.65	12	15	0.95
2	5	0.62	12	18	1.01
3	4	1.2	15	20	1.32
3	5	1.1	15	25	1.78
4	5	0.94	20	24	1.03
4	6	0.98	20	25	0.96

4.1.2 The Risk of Getting Infected on a Working Day is Higher

In this section we assume that $p_w > p_h$. We numerically observe that the minimization of the new interactions, considered as the objective of Model (*ii*), leads to the full isolation policy. This means that 4 therapists are assigned to each room with no interaction between the therapists of different rooms during the time horizon. The schedule for each therapist is shown in Figure 4.2. As indicated, there are more consecutive rest days in general compared to the current scheduling of the clinic. The reason mostly lies in the fact that the risk of getting infected on a working day is higher, and therefore, there is a higher probability that a new therapist who replaces an infected therapist, gets infected on a rest day, which has a lower probability of getting infected. Thus, the expected total number of replacements would be approximately 4% (13.2 vs 13.7) less than the current schedule.

Figure 4.2: *Optimal Schedule for each room when there is a higher chance to get infected at the hospital (Dark:Not working/ White: working).*

4.1.3 The Risk of Getting Infected on a Rest Day is Higher

In this section, we assume that $p_w < p_h$. In this setting, the model tries to create more gaps between working and rest days vis a vis the incubation period so that a possible new infection takes place on a working day, which has a lower risk. Note that in this scenario as well as the scenario discussed in Section 4.1.2, we made assumptions and set the inputs in a way that they would be similar to the current setting. For example, we consider that all available therapists need to work and $m = 16$. Nevertheless, in this scenario, the expected number of replacements is about 5% lower than the expected number of replacements based on the current schedule.

In both scenarios (i.e., $p_w > p_h$ and $p_w < p_h$), we have sufficient resources to maintain a full isolation policy. However, the shortage of staff members is an important challenge faced by frontline health and social care workers during COVID-19 pandemic [Nyashanu et al., 2020]. An interesting scenario is that there are not sufficient therapists to maintain a full isolation policy and some of the therapists need to switch teams during the time horizon.

- **Insufficient Staff for Full Isolation**

Figure 4.3: *Optimal schedule for each room when there is a higher chance to get infected at home (rest day) (Dark:Not working/ White: working).*

In this scenario, some of the staff members (therapists) must work with different teammates during the time horizon. Assume that we have 2 rooms, and each room needs 3 staff members to operate on each day, the maximum number of available therapists is 7 and no therapists is allowed to work for the full time-horizon ($u = n - 1$). In this scenario some therapists need to switch between the teams inevitably. As shown in Figure 4.4, therapist

Days	1	2	3	4	5	6	7	8	9	10	11	12	13	14
Room1	A	A	A	A	C			A	A	A	A	A		
	C	C	C	C	F			C	C	F	C	C		
	F	F	F	F	G			F	G	G	F	F		
Room2	B	B	B	B	B			B	B	B	D	B		
	D	D	D	D	D			E	D	D	E	D		
	E	E	E	E	E			G	E	E	G	G		

Figure 4.4: *A scenario where full isolation policy cannot be maintained* ($p_w < p_h$).

G is switching between teams. Again here, since the probability of getting infected on a working day is higher than a rest day, therapist G works in consecutive days. Moreover, his starting day is delayed as much as possible so that in case of an infection, the new therapist who replaces G, would be in less danger of getting infected. On the other hand, in Figure 4.5, we can see that since the probability of getting infected on a working day is higher, the extra therapists who switch between the teams start their schedule as early as possible. In the next section, we perform a sensitivity analysis to examine how the

Days	1	2	3	4	5	6	7	8	9	10	11	12	13	14
Room1	C	A	A	A	A			A	A	A	A	A		
	F	C	C	F	C			C	C	C	C	C		
	G	F	G	G	F			F	F	F	F	F		
Room2	B	B	B	B	D			B	B	B	B	B		
	D	E	D	D	E			D	D	D	D	D		
	E	G	E	E	G			G	E	E	E	E		

Figure 4.5: *A scenario where full isolation policy cannot be maintained* ($p_w > p_h$).

proposed models behave in different circumstances.

4.1.4 Sensitivity Analysis

- p_w and p_h

As illustrated in Figure 4.6, the model shows higher sensitivity to the change of p_h when p_w is fixed as it has a steeper slope. This means that as the probability of getting infected on a rest day increases, the current scheduling policy gets more and more risky for the operations of the clinic. On the other hand, higher probability of getting infected on a working day does not necessarily leads to wider gap between the two policies.

In extreme situation where the infectivity is dramatically higher in the working place, the gap between the two policies starts to shrink. Such situations may happen in case

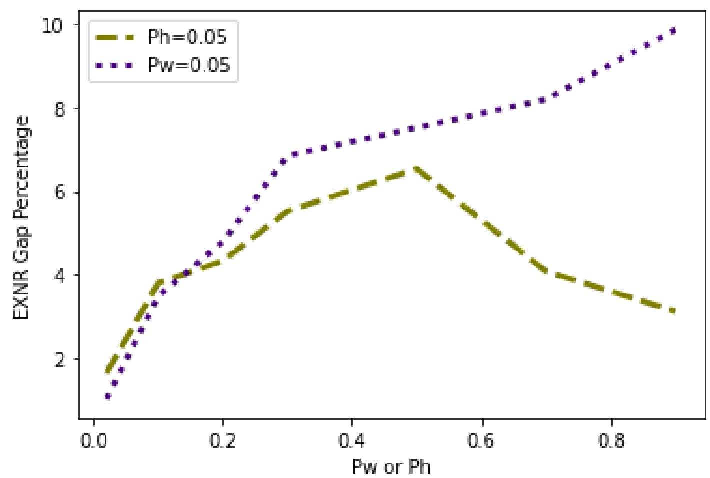

Figure 4.6: *The EXNR gap between the optimal and current schedules for different probabilities.*

of an outbreak in the clinic, or a variant of the virus with very high infectivity rate that makes PPEs less effective.

- Incubation Period (τ)

As it is shown in Figure 4.7, the widest gap between the optimal solution and the current schedule would take place when the incubation period is less than 6 days. The underlying reason of the decrease in the gap from $\tau = 7$ is that when the incubation period increases, the effect of scheduling policy becomes negligible and finally the gap converges to zero ($\tau = n$). Interestingly, the highest difference between the scheduling policies happens when τ is between 4 and 6 days which is the most documented incubation period for COVID-19 virus [Lauer et al., 2020, McAloon et al., 2020, Tan et al., 2020].

Changes of τ has a direct impact on the number of “new” interactions resulted from Model (ii) as illustrated in Figure 4.8. Intuitively, it makes sense that when the incubation

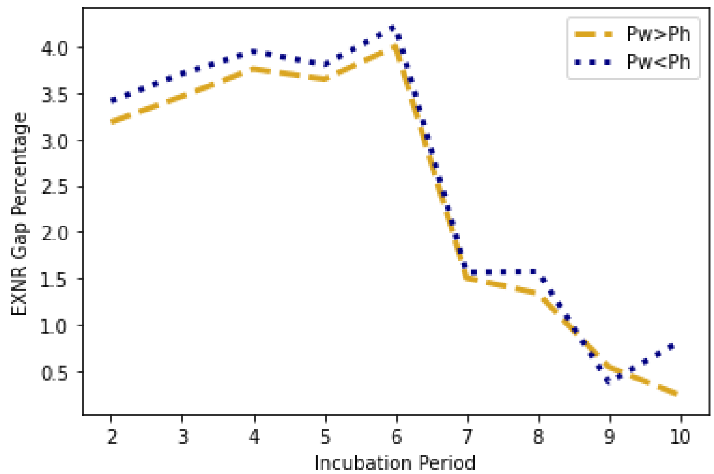

Figure 4.7: *The EXNR gap between the optimal and current schedules for different incubation periods.*

period of a contagious disease is long, there are less means of preventing an outbreak at the workplace. However, regular testing for the staff members can be helpful to limit the spread of the disease.

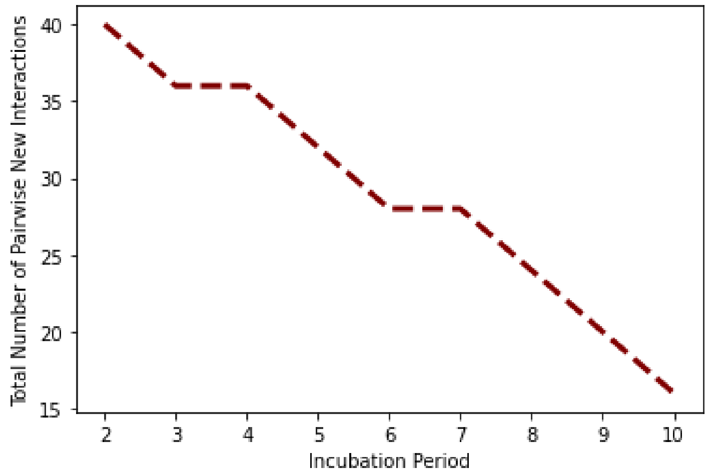

Figure 4.8: *Number of new interactions for different incubation periods.*

- l and u

The minimum and maximum number of days to work is closely related to the number of available therapists. For example, it is not possible that in case of $m = 4$ and $t = 3$, all therapists work full-time. Here, all other parameters fixed, we change the value of l and u to evaluate the effects of these two parameters on the EXNR while the feasibility is maintained. As shown in Figures 4.9 and 4.10, in case of inflexibility for the lower and upper bounds of needed working days for each therapist (l and u), the optimal policy might change. For example, when it is decided that all therapists need to work for at least 8 days and the risk of getting infected is higher on a rest day, the optimal scheduling is to schedule all therapists to work either 8 or 9 days to have lower EXNR.

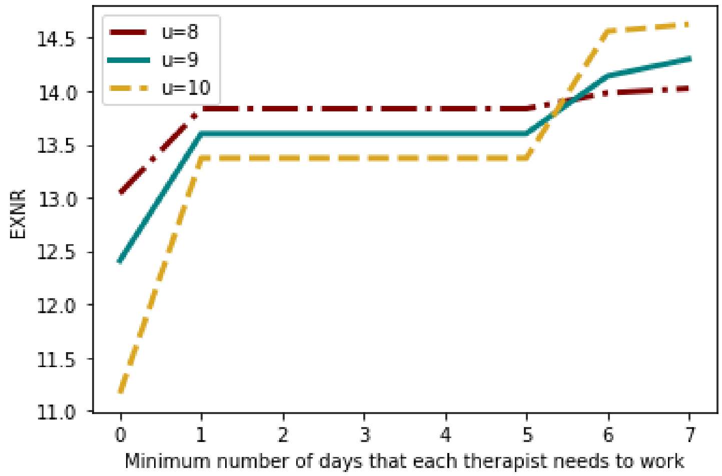

Figure 4.9: *EXNR for different values of u and l ($p_w > p_h$)* .

4.2 Performance of the Models

The main advantage of the propose CG algorithm is that there is no need to create all available patterns to find the optimal solution. This hugely helps in terms of memory usage and accordingly, the computation time. Depending on the value of θ, there would be situations where it is impossible to generate all available patterns (θ^{n-h}) because of the memory limitations. Therefore, the supposedly simple "original formulation", is impossible to be solved. Table 4.3 provides the average computation times (in seconds) required to solve the proposed CG algorithm and the original formulation. It also provides the standard deviation of the computation time in parenthesis. In fact, the CG algorithm computation time in the table consists of the time to solve both Models (i) and (ii). For the original formulation, we consider the time to generate all patterns, the time to calculate the cost of each pattern, as well as the time to solve the model. To be consistent with the assumptions provided in previous sections, we assume that there are only two different working status on each day ($\theta = 2$). Note that when θ increases, the number

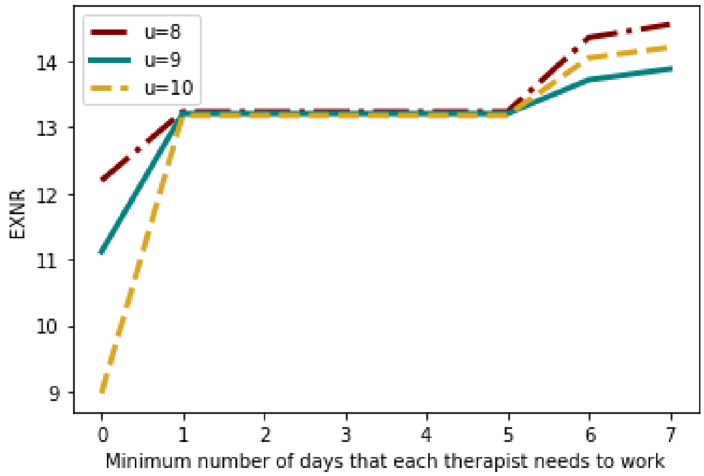

Figure 4.10: *EXNR for different values of u and l ($p_w < p_h$)* .

of possible patterns increases dramatically, and thus, the CG algorithm outperforms the Original formulation in even more instances. As expected, for both CG algorithm and the branch and bound algorithm used to solve the original formulation, the larger the number of working days, the larger the computation time needed to find the optimal solution. However, as shown in Figure 4.11, the marginal increase in computation time for the branch and bound algorithm that solves the original model is greater than the computation time of the CG algorithm. When the number of working days increases, the computation time of the original formulation increases significantly because of high computation power required to generate and calculate the cost of all possible patterns while the CG algorithm solves the model with limited number of patterns.

Table 4.3: *Computation times comparison between the models (standard deviation is given in parenthesis)*

$n-h$	Time	
	CG algorithm	Original formulation
10	14.5 (1.7)	7.1 (2.1)
15	21.3 (3.2)	14.6 (2.9)
20	39.65(7.3)	42.3 (8.6)
25	48.5 (8.2)	75.7 (13.2)
30	54.75(12.1)	112(17.2)
35	85.3 (11.9)	252 (23.6)

4.3 Simulation

In this section, we develop a discrete-event simulation (DES) to compare the output of the proposed models with some other possible scheduling policies. In the simulation model, we consider all assumptions that we made for the optimization models and add the impact of interaction such that an infected therapist, on each day, might pass the infection to the uninfected teammates. Let $\hat{p}_\nu$ denote the probability of getting infected from teammates if there are ν infected therapists in the team on a specific day. Similar to the previous sections, we assume that an infected therapist would be replaced with a new therapist after τ days, or on a test day at the end of the scheduling time horizon (day 14). In the following, we explain the scheduling policies that we consider in the simulation.

- Policy 1: current scheduling policy

The current scheduling of the clinic is illustrated in Figure 4.1 where in each room, 3 therapists work 0.8 FTE and 1 therapist works 0.6 FTE. Under this policy, a full isolation is followed where 4 therapists are assigned to each of 4 available rooms ($m = 16$).

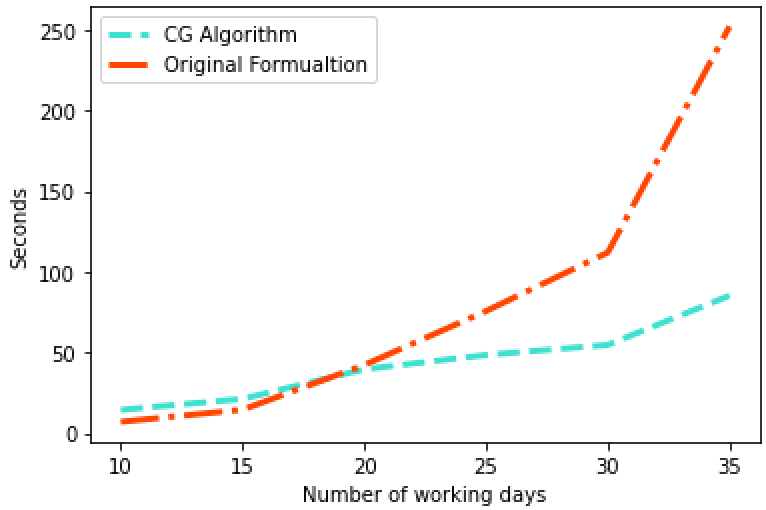

Figure 4.11: *Computation time comparison*

- Policy 2: interaction between the members of different rooms (optimization model's outcome)

Under this policy, illustrated in Figures 4.4 and 4.5, we assume that there are 7 therapists for each two rooms ($m = 14$). One therapist (0.6 FTE) switches between the teams and other 3 therapists (0.9 FTE) remain in the same room and have no interaction between 3 therapists of the other room.

- Policy 3: full-isolation (optimization model's outcome)

This scenario is the closest policy to the current scheduling in terms of the FTE of the therapists and the total number of therapists. The optimal scheduling policy of this scenario is discussed in Sections 4.1.2 and 4.1.3.

- Policy 4: lower FTE for each therapist (optimization model's outcome)

In this scenario, we assume that we have sufficient flexibility to assign 6 therapists to each room with no interaction between the teams ($m = 24$). Furthermore, no therapists is allowed to work more than 2 consecutive days. Figure 4.12 illustrates the schedule of the 6 therapists in each room where 3 therapists work 0.6 FTE and 3 therapists work 0.4 FTE.

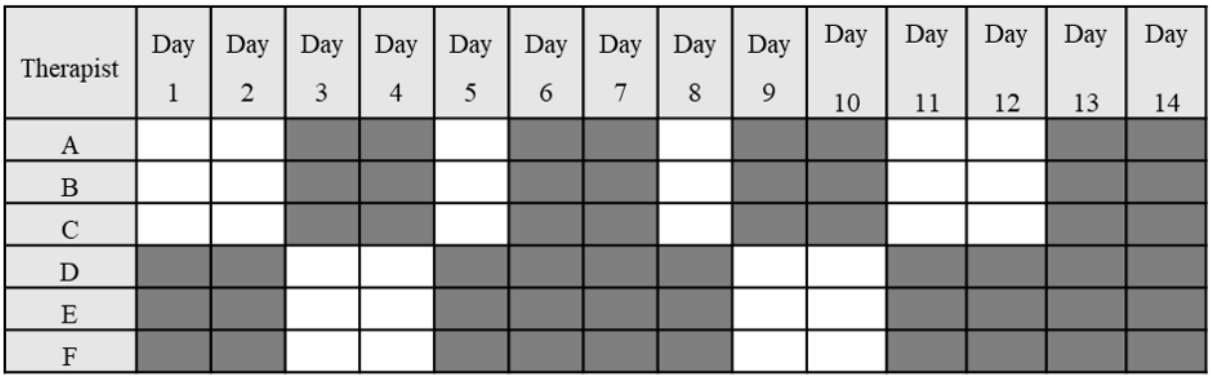

Figure 4.12: *A 2-2 scheduling policy*

Table 4.4 shows the parameters and their default values that are used in the simulation model.

4.3.1 Simulation Results

We present the results of the simulation under two scenarios, namely, $p_w > p_h$ and $p_w < p_h$. We run the simulation for 1000 times and we report two measures. The first measure is the average total number of replacements, which is the objective of Model (i) discussed in Section 3.1.3. The second measure is the average FTE number of replacements which is the average number of replacements per each scheduling pattern times the FTE of the pattern. The second measure provides a good estimation for the decision makers to know how each scheduling policy might affect their available resources in terms of FTE. As shown in

Table 4.4: *Simulation parameters*

Parameter	**Default Values**
incubation period (τ)	5 days
Scheduling time horizon (n)	60 days with testing day at the end every 2 weeks
Probability of getting infected on a working day (p_w)	0.05, 0.1
Probability of getting infected on a rest day (p_h)	0.1,0.05
Probability of getting infected if there is 1 infected therapist in the team ($\hat{p}_1$)	0.7
Probability of getting infected if there are 2 infected therapists in the team ($\hat{p}_2$)	0.9

Figures 4.13 and 4.14, in most instances, there is not much variation between the patterns with the same FTE. There are only differences in part (a) of Figures 4.13 and 4.14 where some patterns have less average of replacements compared to the other patterns with the same FTE. The reason is mainly because of the effect of interaction where the switching therapists probably passed the infection from one team to the other team members. The question here is that to what extent this policy can cause replacements compared to the other 3 scenarios where we have full isolation and no inter-teams connection.

As shown in Figure 4.15, Policy 2 has the lowest total number of replacements and it's partly because we have less number of therapists compared to the other scenarios. In fact, although the probability of getting infected through interaction is very high, the effect is not significant enough to cause an outbreak among the staff members. In terms of FTE though, Policy 4 has the least number of replacements. It's because of the high turnover of the therapists in this scenario and less working days. However, we cannot neglect that

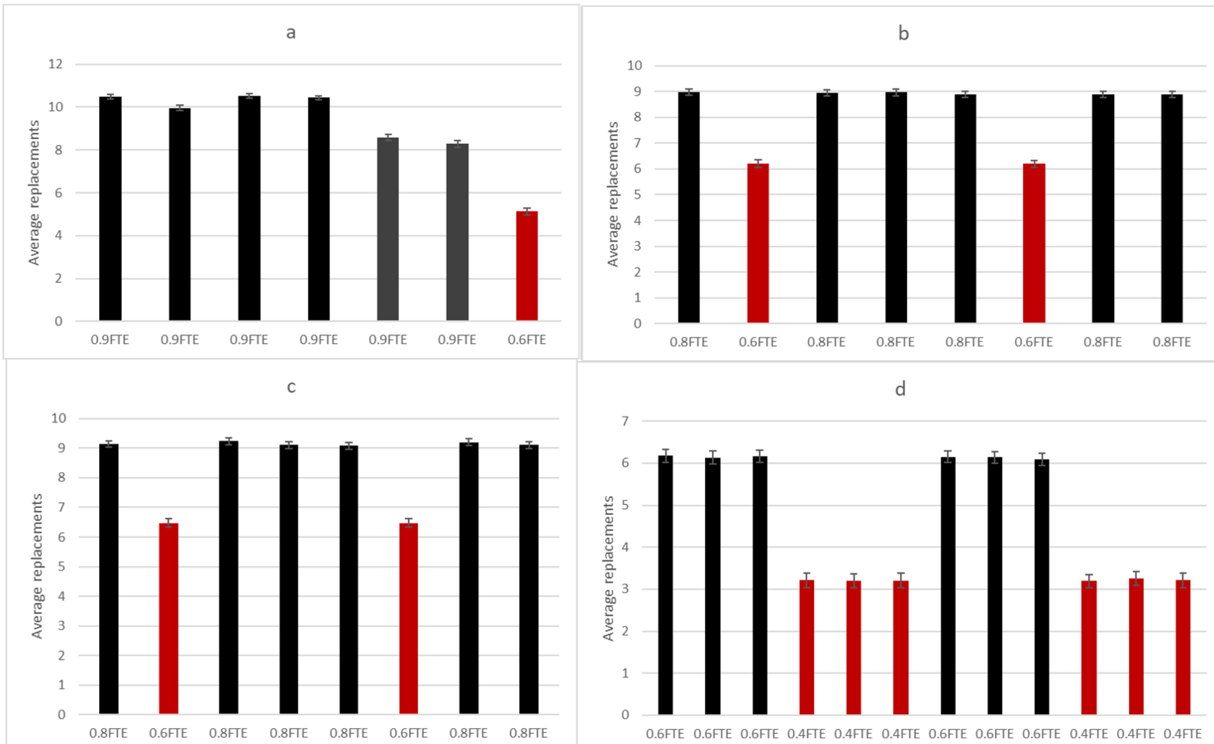

Figure 4.13: *Average replacements per each working pattern ($p_w > p_h$) for a: Policy 2, b: Policy 3, c: Policy 1, d: Policy 4*

in most occasions, and specially during an unprecedented workload during a pandemic, it is not possible to recruit and use large number of staff members.

The gap between Policy 1 and Policy 4 is around 21%. This means that in situations where there is a lower risk of getting infected at work place, in long term there are 21% less FTE replacements if a policy with more therapists and lower FTE per therapist is followed. In case of low flexibility of the total number of therapists, Policy 3 leads to around 8% less FTE replacements. When $p_w < p_h$ (Figure 4.16), the full isolation policy (Policy 3) is the best policy in terms of FTE replacements. Compared to the current policy (Policy 1), there is about 5% less FTE replacements when the order of the working days is changed

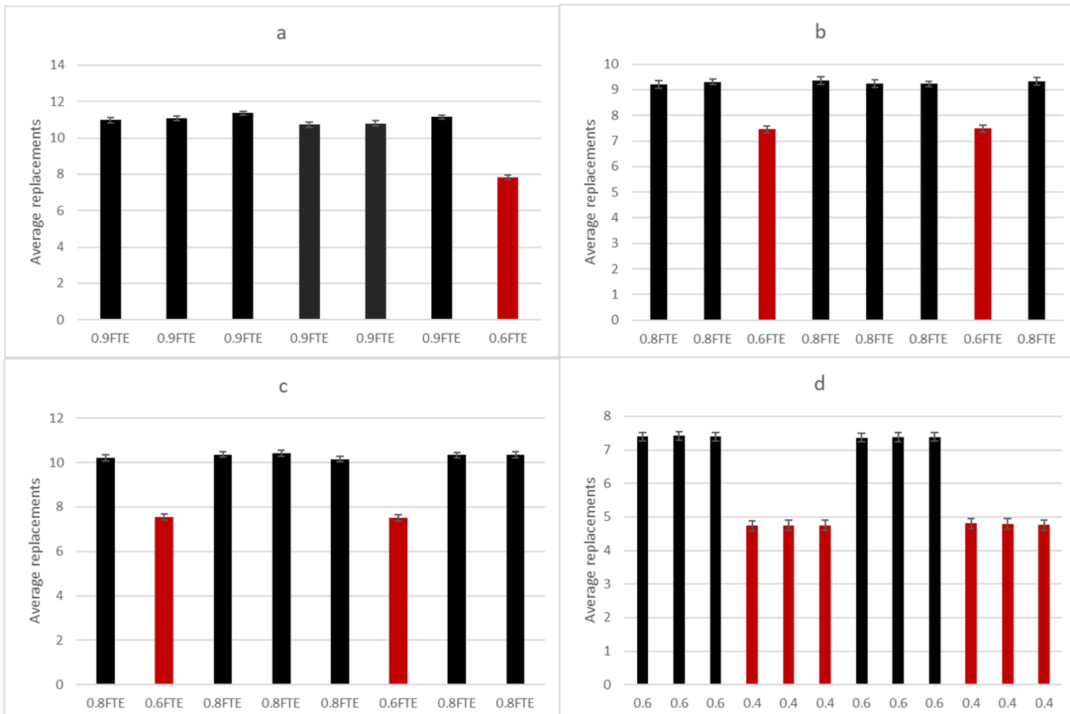

Figure 4.14: *Average replacements per each working pattern ($p_w < p_h$) for a: Policy 2, b: Policy 3, c: Policy 1, d: Policy 4*

according to what our model suggests.

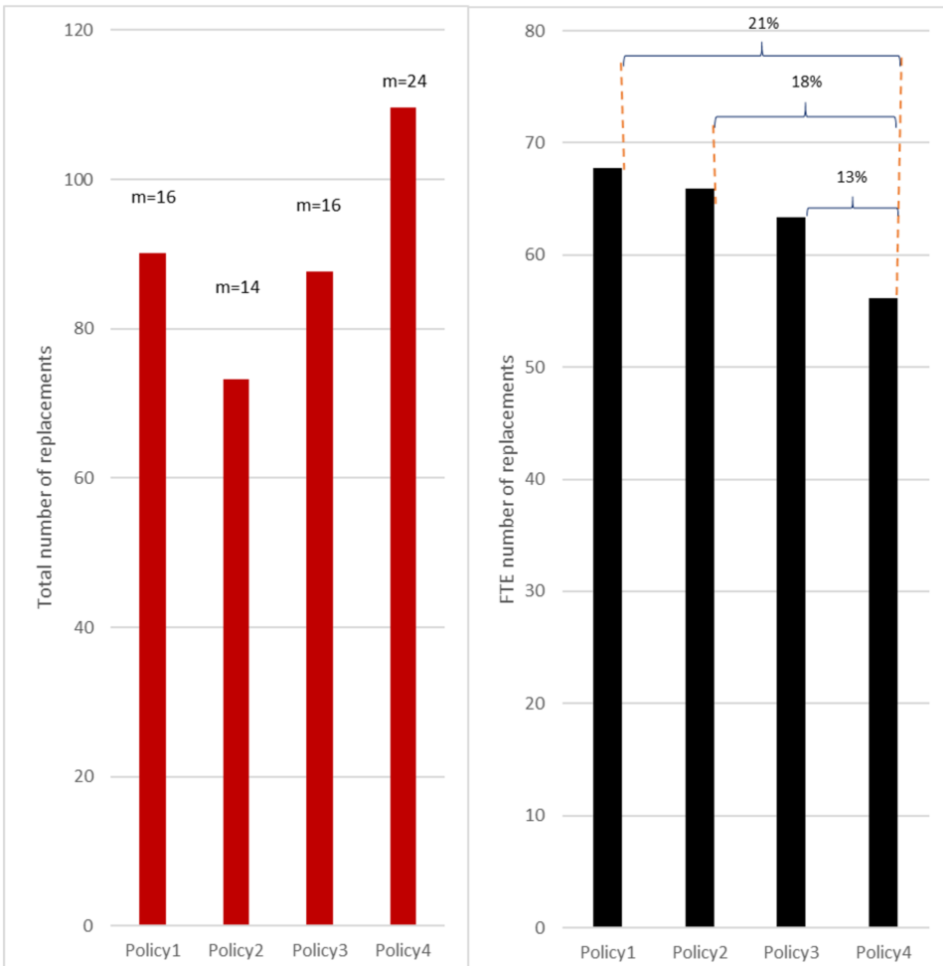

Figure 4.15: *Comparison between the policies ($p_w > p_h$) (left: Total number of replacements, Right: FTE number of replacements)*

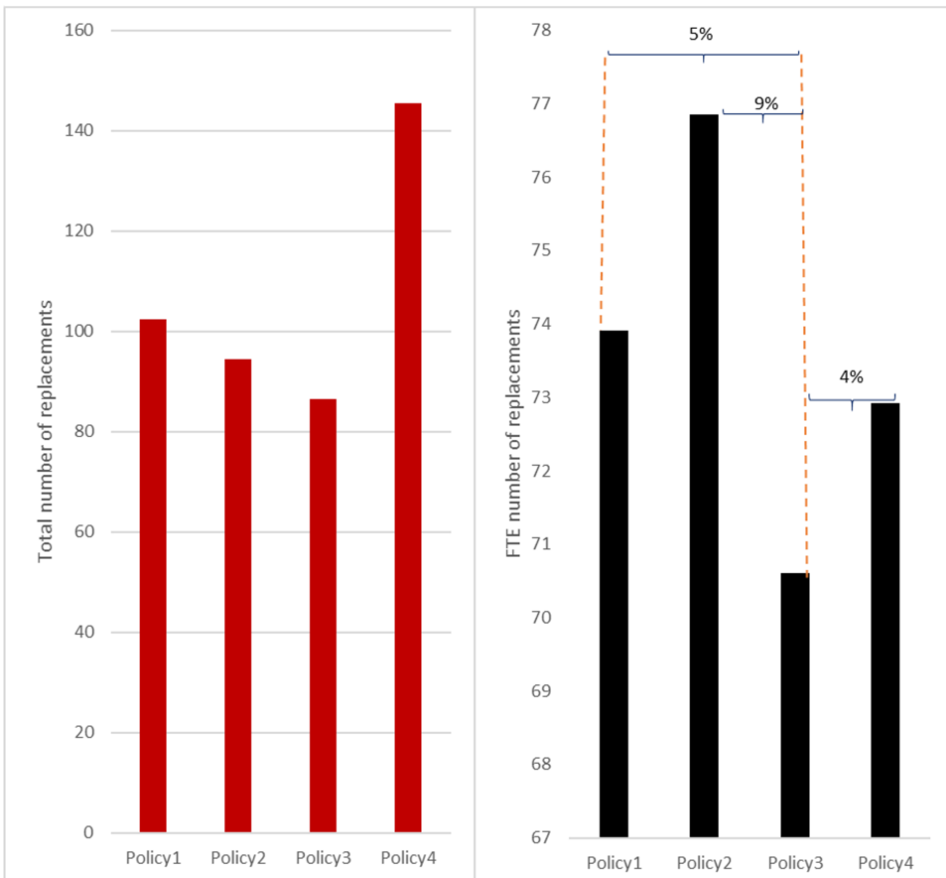

Figure 4.16: *Comparison between the policies ($p_w < p_h$) (left: Total number of replacements, Right: FTE number of replacements)*

Chapter 5

Discussion

In this work, for the first time, we introduced a mathematical framework for staff scheduling during a pandemic with focusing on the characteristics and nature of the disease causing the pandemic. As [Mascha et al., 2020] discussed, there is a high chance that staffing along with epidemiological considerations reduce the healthcare workers shortage. Amid the low rates of vaccination in developing countries, as well as the uncertainty around the efficacy of available vaccines against COVID-19 and its new variants in developed world, the importance of an optimal scheduling policy to minimize the risk of staff shortage in undeniable. We showed that the optimal staff scheduling policies could be different depending on the characteristics of the disease such as the incubation period and the public health policies at different stages of the pandemic.

At early stages of the pandemic where the nature of the disease is unknown and the availability of PPEs is limited, there is probably a higher risk for staff members to get infected from the workplace on their working days ($p_w > p_h$). In this situation, our findings show that the staff members should be scheduled in a way that in case of getting infected, they become symptomatic (and replaced with a new staff member) on a rest day where there is a lower chance of getting infected for the new staff member. On the other hand, in situations like mid-2021 in COVID-19 pandemic, it seems that there is a higher probability for a healthcare staff member to get infected on a rest-day in the absence of social distancing, rather than getting infected in a secured, well monitored workplace. The probable reason behind it is that step by step, the knowledge about the characteristics of the disease improves and vaccines are developed which leads to relaxed lockdown policies while there are uncertainties about the vaccines' efficacy on the new variants.

In contrast to the previous situation, our findings from Model (i) suggest that with respect to the holidays and the incubation period, the rest days should be interspersed in consecutive days so that there is a less probability to get infected on a working day for new replaced therapists. Thus, the optimal scheduling policy is highly dependant on the parameters and characteristics of the disease at each time and it is essential that the models be solved in reliable time according to the needs of each organization.

To address this matter, we designed a Column Generation algorithm which is able to solve the proposed models without generating all working patterns.More importantly, in terms of memory usage, it is much more efficient since generating all possible working patterns will become impossible when the number of working days during the scheduling time horizon increases.

On the other hand, Model (ii) is designed to assign the staff members to different teams so that the interaction between them is minimized. This will help to avoid outbreaks as much as it is possible especially in cases where there are not enough resources (staff members) for a full isolation policy and some staff members must switch between the teams inevitably.

Our results showed that for our partner organization, we can reduce the Expected Number of Replacements (EXNR) by up to 20% in some occasions compared to the current scheduling policy.Our optimization and simulation models can be used as managerial tools to implement, examine and compare different scenarios in different settings. For example, in our case, if there is no flexibility for the minimum number of working days for each therapist, when $p_w > p_h$, it is better that each therapist works between 7 and 8 days rather than 7 and 9 or 10 days. In another example, in case there is a higher risk to get infected at work, it is better to follow a scheduling policy where more people are assigned to each room. This leads to less FTE for each therapist and more rest days. However, it should be carefully analyzed by the decision makers to find out if it is possible to hire more staff members to maintain a 21% less FTE replacements. Even in the lack of flexibility in terms of the total number of staff members, just by re-ordering the days that the therapists work, our results suggest that there could be up to 8% less FTE replacements.

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