Can children use probability to guide their choices under uncertainty?

by

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Author's Declaration

I hereby declare that I am the sole author of this thesis. This is a true copy of the thesis, including any required final revisions, as accepted by my examiners.

I understand that my thesis may be made electronically available to the public.

Abstract

We encounter situations in our everyday lives where we need to make decisions under uncertainty. But what kind of information do we use and what abilities are helpful to us when making decisions under uncertainty? In three experiments (Total N = 180), I examined whether 3- to 7-year-olds could use numerical information (e.g., probability) to judge which of two situations presented with more or less uncertainty. Children were shown two games with different numbers of hiding locations. Using a within-subjects design, they were asked to select the game that would make it either easy or hard for someone else to find a coin that is hidden under one of the locations. Around the age of five, children selected the side with fewer hiding locations when asked to make it easy to find the coin and selected the side with more hiding locations when asked to make it hard to find the coin (Experiment 1). Findings from Experiment 2 suggest that children do this by considering the absolute number of hiding locations, rather than using perceptual cues like surface area (e.g., clutter). In Experiment 3, we simplified our procedure to examine whether younger children could make a similar inference. Findings reveal that even 4-year-olds were selecting the side with fewer hiding locations when asked which ball was easier to find and selecting the side with more hiding locations when asked which ball was harder to find. These results suggest that around age four, children can evaluate probability to make judgements about levels of uncertainty. Moreover, these results highlight that perhaps evaluating the probabilities of outcomes is a helpful tool when confronted with uncertainty.

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Dr. Randy Pausch once said, "Brick walls are there to give us a chance to show how badly we want something." Well, I finally broke through that wall.

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Introduction

Imagine you are playing a game of Hide and Seek. You can hide in the basement where there are more nooks and crannies or in the living room where there are fewer hiding locations. If the goal is to stay undetected, your best option would be to hide in the basement, as the more hiding locations there are, the more uncertain your friend will be when trying to find you. Like playing a game of Hide and Seek, we encounter situations in our everyday lives where we need to make decisions under uncertainty. For example, uncertainty may arise when deciding to take on a new job opportunity or when a student applies to graduate school. To minimize costs, a bakery must decide how many treats to bake each day when faced with uncertain demand. Thus, many situations require us to make judgements based on limited information, with unknown outcomes. But what kind of information do we use and what abilities are helpful to us when making decisions under uncertainty?

Probability can aid our decision-making when faced with uncertainty. When referring to probabilities, we typically define them as the likelihood of an occurrence of a particular outcome (Nutter, 1987). Meanwhile, uncertainty refers to an epistemic state of unsureness involving unknown information (Wakeham, 2015). Often, probability can be used as a quantitative expression of our degree of certainty and uncertainty (Borch, 2015). For example, a baker might examine the sales from the previous day to calculate probability estimates of specific products and use that information to make optimal decisions on the number of treats needed to be baked for the following day. So, even if we are uncertain about future customer demands, we can use probability to predict which products would be the most popular. When faced with uncertainty, it is often the case that decisions must be made under probabilistic conditions. Thus, evaluating the probabilities of different outcomes can help determine the degree of certainty/uncertainty of a specific future outcome.

In three experiments, I explored children's decision-making under uncertainty and whether they can use probability to guide their choices. More specifically, I was interested in whether children could use probability to compare different levels of uncertainty and to select the option that would most likely lead to the desired outcome.

Children may have the pre-requisite abilities to understand the relationship between probability and uncertainty. Past research has shown that infants and non-human animals have been shown to have intuitions about probability (see Denison & Xu, 2019 for a review). For example, Téglás et al. (2007) showed that infants can use probability to make predictions about uncertain future outcomes. In this study, 12-month-old infants were presented with a container that comprised of a single opening. In the container, three identical yellow items and one unique blue item bounced around. During test trials, the contents of the container was occluded so that participants could only see the opening. In the *Probable* outcome, one of the three identical yellow items exited. In the *Improbable* outcome, the unique blue item exited. Infants looked longer at the improbable outcome than the probable outcome, suggesting that they expected one of the yellow items to exit. However, it is possible that infants looked longer at the improbable outcome because they may have a preference to respond to more perceptually salient properties (e.g., ball with a different shape and colour). This interpretation was ruled out in their control experiment, where infants looked longer at an impossible outcome (e.g., seeing a yellow item exit when all yellow items were obstructed by a barrier) versus a possible outcome (e.g., the unique blue item exited).

Other work shows that 12-month-old infants and non-human primates can make choices based on probabilistic information (e.g., Denison & Xu, 2014; Rakoczy et al., 2014). For example, when shown that a single hidden item is sampled from each of two distributions, one with a more favorable ratio of target to non-target items, they choose the hidden item drawn from the more favorable distribution.

Moreover, children's understanding of multiple possibilities might also aid their decision-making under uncertainty. When we are confronted with an uncertain situation, the ability to consider multiple possibilities, the likelihood of each possibility and the relationship between possibilities, can help us formulate logical inferences. For example, when playing a game of Hide and Seek with two hiding locations, failing to find your friend after searching at the first location provides us with confidence that our friend is hiding in the only other location. In this scenario, we can logically reason by disjunctive syllogism: "A or B, Not A, Therefore B." This requires us to represent two possible outcomes and by ruling out one possibility, we can be confident that the other must be true.

Reasoning by disjunctive syllogism has been studied using Call's Cups task (Call, 2004). In such tasks, participants (often children or non-human animals) are presented with two pairs of cups, with the pairs located far apart from each other on a surface and a sticker hidden under one cup in each pair. The experimenter reveals to participants that a cup from one of the pairs is empty. Participants are then given a chance to look under one cup to find a sticker. If they are reasoning by disjunctive syllogism, they should pick the cup that was paired with the empty cup (the certain cup) because the location of that pair's sticker is known, whereas the location of the sticker for the other pair is still uncertain. Indeed, results from Mody and Carey (2016) show that children as young as 3 years select the certain cup around 67% of the time, significantly more than might be expected by chance if one considers chance 33% (choosing randomly among one of three remaining cups) or a stricter bar of choosing the certain cup more than 50% of the time (i.e., choosing randomly between the certain cup and one of the uncertain ones).[1] Nevertheless, children's performance is still limited when compared to adults who select the certain cup 100% of the time.

There is also evidence to suggest that children can mentally represent mutually exclusive events and prepare for uncertain future outcomes. For example, in what is known as the Y-shaped tube task, children are presented with an upside-down Y-shaped tube. A ball is dropped into the tube and participants are asked to catch it. Since there are two openings, participants should place one hand under each tube to catch the ball. Results show that at around age four, children appropriately cover both exits consistently. Younger children do not consistently cover both exits, instead selecting one and covering it (Redshaw & Suddendorf, 2016; Redshaw et al., 2018). Additional experiments and controls suggest that it is not the case that this is due to a physical limitation (with two separate tubes and two balls, younger children cover both exits).

Additionally, children can maintain a mental representation of a sequence of events, providing them with partial information to reason logically about possible outcomes. For example, in a much older study conducted by Sophian & Somerville (1988), they tested four- and six-year-olds' ability to use the observed sequence of events to make inferences about the location of a hidden toy. In this experiment, participants saw four hiding locations where a single toy could be hidden. The experimenter had a "partner toy", and children were told they would leave the partner toy behind with the target toy if they found it during their search. On each trial,

[1] In a re-analysis of this work, questions have been raised about whether 3-year-olds' performance is truly indicative of reasoning through the disjunctive syllogism and if this ability is not convincingly shown until age 4 (Leahy & Carey, 2020). That particular issue is not relevant to the current thesis.

the experimenter searched in two locations – marked to the children. Participants were measured on whether they could correctly identify the two possible locations of where the toy could be. Both four- and six-year-olds correctly reasoned that when the experimenter did not have the partner toy at the end of the test trial, it must be hidden in one of the two searched locations, whereas if the experimenter still had it, it must be in one of the unsearched locations. This shows that children can use partial information from a sequence of events to reason logically about possible outcomes, which can support their reasoning about uncertain outcomes.

So far, I have reviewed evidence suggesting that by around age four, children can reason logically about uncertain events when provided with disambiguating evidence. In the Cups task and the Sophian and Somerville task, children are faced with an uncertain event, but are then given information that allows them to narrow in on the correct choice through exclusion. In Call's Cups task, children were asked to select between a certain outcome versus an uncertain outcome. While this can be considered a comparison of probabilities (1.0 vs. 0.5), this is not a fully probabilistic task because one outcome is guaranteed. The Sodian and Somerville task had a similar structure, where children were asked to identify two out of four possible locations of a hidden toy. Since participants saw where the experimenter searched and the outcome of the search (successful or unsuccessful at finding the toy), they can be certain whether the hidden toy was under one of the two visited locations or vice versa if the experimenter was unsuccessful at finding the toy. Thus, this task does not require children to compare different levels of uncertainty, since it asks children to identify possible or impossible locations (because they could easily infer that the item was not in a particular pair, and they were only asked to identify the pair that could possibly hold the toy).

Together, this literature suggests that when faced with uncertainty, children can reason by exclusion and make predictions about uncertain future outcomes when provided with enough information. However, it is often the case that disambiguating evidence is not available and thus final decisions in many cases must be made under probabilistic conditions.

Here, I examined whether young children could use probability to judge which of two situations presents more or less uncertainty. For example, when tasked with finding an item, do children understand that it will be easier to find the item when there are fewer locations to explore? Can children do this in the absence of disambiguating information, where all options remain uncertain (i.e., probabilistic) at the time of the choice, unlike in the reviewed work on disjunction?

In my prior work, I began investigating children's ability to compare the number of items across sets to make inferences under uncertainty (Lu, 2020; Lu et al., 2021). Three- to six- year-olds were presented with a character who either wanted to find (one within-subjects condition – Finder Condition) or hide (a second within-subjects condition – Hider Condition) a gold coin under one of several hiding locations (e.g., cups). Then we presented them with three trials in each condition that differed in the absolute number of cups to choose from: 2vs4 cups, 3vs6 cups and 5vs10 cups. I manipulated number because beginning around age 2.5, and improving throughout the early years, children can compare sets of discrete items such as 1 versus 3 or 6 versus 9 items to indicate which contains more (even when controlling for variables such as total surface area, that correlate with number (e.g., Cheung & Le Corre, 2018). Moreover, we used a 1:2 ratio for all trials, given that previous research suggests that infants as young as 6 months can perceive and discriminate 1:2 ratios when comparing sets (e.g., Xu & Spelke, 2000).

We examined whether children would select the best option that would lead the character to successfully find or hide a gold coin from another player. Children's responses across conditions statistically diverged at 4.94 years, such that they were reliably choosing the side with fewer cups when asked to help find the coin and the side with more cups when asked to help hide the coin by basically age 5. Given the reviewed literature (e.g., Mody & Carey, 2016; Sophian & Somerville, 1988), we were somewhat surprised that sensitivity to the two conditions did not emerge until age 5. One possible reason for this is that younger children have difficulties choosing between two uncertain tasks, given that those previous tasks had always been disambiguated, such that one option (or set of options in Sophian & Somerville, 1988) always became certain.

However, it is also possible that the instructions for "finding" and "hiding" were too complicated for the youngest children in the sample.[2] Participants were asked to help a character successfully find or hide a hidden item from another player. Thus, success on this task required children to put themselves in the perspective of both the main character and the other player to answer accordingly. Past literature suggests that children start to think about others' state of mind around the age of 4 to 5. So, a rudimentary theory of mind might explain why younger children performed more poorly on our earlier tasks (Astington & Edward, 2010). The procedure for the current experiments was simplified so that participants were not required to make inferences about the state of mind of two different characters.

[2] See Lu et al. (2021) for the procedure of the hiding and finding experiment to see how the current procedure is more straightforward. The hiding and finding procedure required more explanation, particularly for the hiding condition.

Experiment 1

For my Master's thesis, I followed-up on this work in three online experiments. We still asked children to select between two games with different numbers of hiding locations, but we simplified the procedure by asking them to either make it easy or hard for another player to find the coin that is hidden under one of the cups. Thus, if children can appropriately compare the odds between the two sets, they should select the set with more hiding locations when asked to make it hard for someone else to find the hidden item and select the set with fewer hiding locations when asked to make it easy.

In Experiment 1, children were told that they were going to help make a game where another player is going to try to find a coin that would be hidden under one of several hiding locations. They were then told to either make the game easy or hard for the player. After hearing the instructions, children were asked to select between two games that varied in the number of hiding locations for another person to play.

Methods

Participants. Data collection was conducted online via live Zoom calls using screen sharing. Eighty four- to seven-year-olds participated in the study (20 children per age in years, Mage = 6.01 years; 72.14 months; 38 females). Eight additional children were tested but excluded for failing the comprehension question twice (see Procedure). This experiment received ethics clearance through the Research Ethics Board at the University of Waterloo. The sample size, experimental procedures, statistical analyses and exclusion criteria were pre-registered (https://aspredicted.org/blind.php?x=yt3di9).

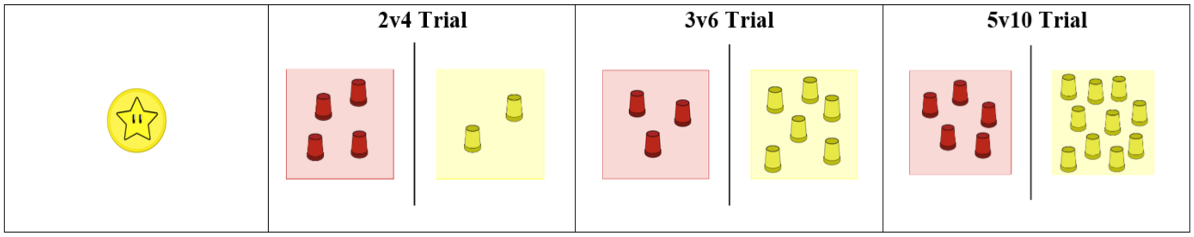
Materials and procedure. Both condition (Easy, Hard) and trial type (2 vs 4 cups, 3 vs 6 cups, 5 vs 10 cups) were tested within-subjects. Trial type order was counterbalanced, and condition order was blocked and counterbalanced (i.e., the easy block appeared first for half the children). Participants were told that they had one gold coin to hide under a cup (see Figure 1 for the procedure). In counterbalanced order, they were told to make the game easy or hard for another player. After being told the task, they were asked the comprehension question: “Do I want it to be easy or hard for another kid to find the gold coin?” For those who answered incorrectly the first time, the task instructions were repeated, and the comprehension question was asked a second time. If children failed the question a second time, the experimenter continued with the task, but these children’s data were excluded from analyses, as planned in the pre-registration.

After the comprehension check, the three trials for that block proceeded and children were asked to select the side they would like to hide their coin in. Then the experimenter proceeded with the next block by saying, “now I want to make it very [easy/hard] for other kids to find the gold coin in my game.” They again went through the comprehension check with the new goal (again, giving children two attempts at it) and children completed the next block of three trials. The visual stimuli in both the Easy and Hard condition were identical and scripts were identical other than the words “easy”/”hard”.

If children are sensitive to odds, they should select the side with fewer cups in the Easy condition, since the odds of finding the gold coin are higher when there are fewer hiding locations. Conversely, they should select the side with more cups in the Hard condition, since the odds of finding the gold coin are lower when there are more hiding locations.

Figure 1.

Sample slides and scripts from Experiment 1



Today you are helping me make games! So you get to choose how to do it. I want to make it very **[*easy* or *hard*]** for other kids to find the gold coin in my game.	I have one gold coin to hide. I could hide it in the red game or the yellow game. Remember I want it to be **[*easy* or *hard*]** for another kid to find my coin. Which game should I hide it in?	I have one gold coin to hide. I could hide it in the red game or the yellow game. Remember I want it to be **[*easy* or *hard*]** for another kid to find my coin. Which game should I hide it in?	I have one gold coin to hide. I could hide it in the red game or the yellow game. Remember I want it to be **[*easy* or *hard*]** for another kid to find my coin. Which game should I hide it in?

Note. The stimuli and script were identical for both conditions; however, in the Easy condition, participants were asked to make it easy and in the Hard condition they were asked to make it hard.

Results

The data for all experiments are openly available via the Open Science Framework and can be accessed at https://osf.io/vhwk3/?view_only=bf36f5062a6749afb3b9ce dfc5cf97a8.

Participants received a score of 1 when they picked the game with more cups and 0 when they picked the game with fewer cups on each trial, regardless of condition. Average responses from the three trials were calculated and the proportion of trials in which participants chose the side with more cups across trials are reported (see Table 1 for means and standard deviations at each age).

We used a GEE (binary logistic, independent correlation matrix) with condition (Easy, Hard) and trial type (2v4, 3v6, 5v10) as within-subjects factors, age in months was mean-centered and entered as a continuous covariate and interactions were included in the model (also see Nyhout & Ganea, 2020; Doan et al., 2021). All statistical analyses were conducted on IBM SPSS Statistics for Windows, Version 28.0, released 2021.

There was a significant main effect of condition, *Wald* X^2 (1) = 41.94, $p < .001$, in that children scored higher in the Hard condition than in the Easy condition. There was also a significant Condition x Age interaction (see Figure 2), *Wald* X^2 (1) = 14.83, $p < .001$. There was no main effect of trial type, *Wald* X^2 (2) = 1.00, $p = .606$.

To further investigate the interaction, we examined the differences in responses in the two conditions for each age group separately. Among the 4-year-olds, there was no significant effect of condition, *Wald* X^2 (1) = 1.32, $p = .250$. In contrast, there was a significant effect of condition for the 5-, 6- and, 7-year-olds, *p*s < .001. The 5- to 7-year-olds were more likely to select the game with more cups in the Hard condition.

To determine the age when children's responses in each condition departed from each other (e.g., more likely to select the game with more hiding locations in the Hard condition versus Easy condition), we examined when the 95% confidence interval for responses in each condition no longer overlapped with each other (also see Lee & Warneken, 2020). This was at age 57.16 months; CI 95% for Easy condition [0.32, 0.56], CI95% for Hard condition [0.57, 0.80]. To determine the age where children's responses in each condition departed from chance, we examined when the 95% confidence intervals no longer overlapped with 0.5. For the Easy condition, children's performance was different by chance at 60.15 months, CI95% [0.29, 0.49]. For the Hard condition, children's performance was different by chance at 53.34 months, CI95% [0.51, 0.78].

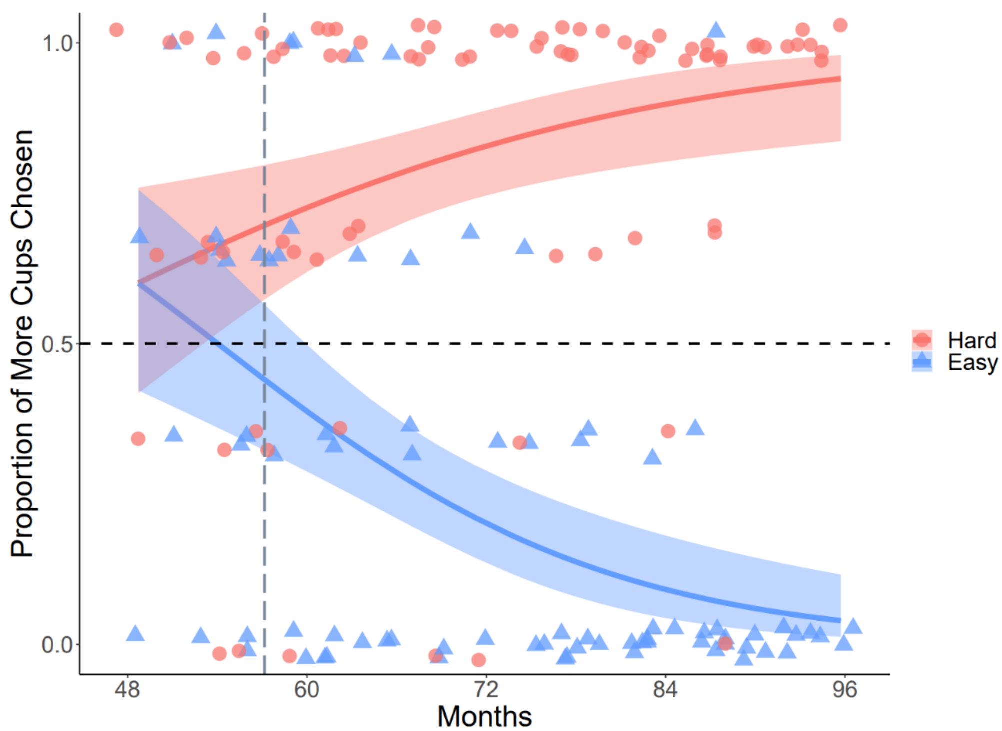
Table 1.

Average score (out of 1) by age in years

Age	Condition	$M_{Average\ Score}$	$SD_{Average\ Score}$
4	Easy	0.48	0.35
	Hard	0.63	0.37
5	Easy	0.32	0.38
	Hard	0.80	0.33
6	Easy	0.12	0.20
	Hard	0.88	0.22
7	Easy	0.07	0.23
	Hard	0.92	0.24

Figure 2.

Proportion of trials in which children chose the game with more cups in Experiment 1



Note. Vertical grey dashed line at 57.16 months represents the age at which participants' responses statistically diverged from each other, across conditions. Dots are jittered for visibility

Discussion

When asked children to select between two games that varied in the number of hiding locations. Around the age of five, children were able to select the best option when asked to either make the game easy or hard for another player. These results suggest that around the age of five, children are able to use probability to compare different levels of uncertainty.

However, with this current set up, we cannot be certain that the children who succeeded on this task do so by truly considering the discrete number of locations. They might be using a heuristic like, “searching is easier when there’s less stuff”. Even if we control for the absolute number of items, it might perceptually seem more difficult to find a hidden item when items are more widely dispersed throughout a room since it occupies more surface area (e.g., more clutter). To determine whether children are considering the number of locations and not just the amount of occluded space, in the next experiment, we controlled for surface area (Experiment 2). Our stimuli were slightly changed so that the cups in the more numerous sets are half the size of those in the less numerous sets.

Experiment 2

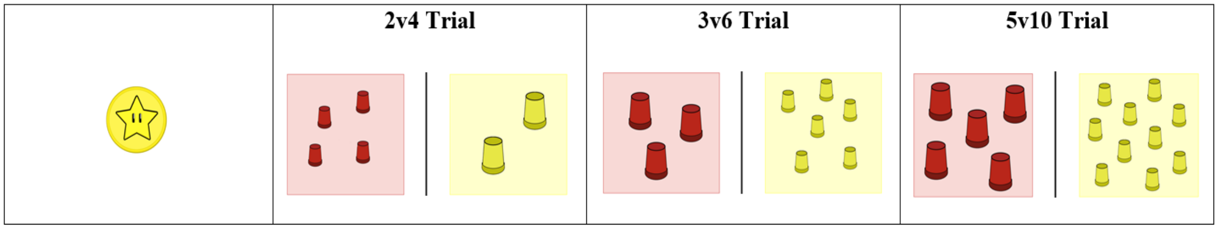
Methods

Participants. Data collection was conducted online via live Zoom calls using screen sharing. Sixty five- to seven-year-olds participated in the study (20 children per age in years, Mage = 6.52 years; 78.28 months; 23 females). Five additional children were tested but excluded for failing the comprehension question twice (see Procedure). In Experiment 1, only the older children correctly selected the game with fewer cups when asked to make the game easy and vice versa. So, we only tested 5- to 7-year-olds in Experiment 2. The sample size, experimental procedures, statistical analyses and exclusion criteria were pre-registered (https://aspredicted.org/2WB_DBP).

Materials and Procedure. The experiment used the exact same design and script as Experiment 1. The only difference was that the cups in the more numerous sets were half the size of those in the less numerous sets (see figure 3 for the procedure). If children consider the discrete number of locations rather surface area, they should select the side with fewer cups in the Easy condition and the side with more cups in the Hard condition.

Figure 3.

Sample slides and script from Experiment 2



Today you are helping me make games! So you get to choose how to do it. I want to make it very **[*easy* or *hard*]** for other kids to find the gold coin in my game.	I have one gold coin to hide. I could hide it in the red game or the yellow game. Remember I want to be **[*easy* or *hard*]** for another kid to find my coin. Which game should I hide it in?	I have one gold coin to hide. I could hide it in the red game or the yellow game. Remember I want it to be **[*easy* or *hard*]** for another kid to find my coin. Which game should I hide it in?	I have one gold coin to hide. I could hide it in the red game or the yellow game. Remember I want it to be **[*easy* or *hard*]** for another kid to find my coin. Which game should I hide it in?

Note. The stimuli and script were identical for both conditions, however, in the Easy condition, participants were asked to make it easy and in the Hard condition, participants were asked to make it hard.

Results

Coding was identical to Experiment 1. Thus, if children reasoned correctly, they should have scored higher in the Hard condition than in the Easy condition (see Table 2 for means and standard deviations at each age).

We used a GEE (binary logistic, independent correlation matrix) with condition (Easy, Hard) and trial type (2v4, 3v6, 5v10) as within-subjects factors, age in months was mean-centered and entered as a continuous covariate and interactions were included in the model.

There was a significant main effect of condition, *Wald* $X^2(1) = 28.67$, $p < .001$, in that children scored higher in the Hard condition than in the Easy condition. There was also a significant Condition x Age interaction (see Figure 4), *Wald* $X^2(1) = 5.38$, $p = .020$. There was no main effect of age or trial type, *p*s > .088.

To further investigate the interaction, we examined the differences in responses in the two conditions for each age group separately. For all age groups, there was a significant effect of condition (*p=.001* for the 5-year-olds; $p < .001$ for the 6- and 7-year-olds). Participants were more likely to select the game with more cups in the Hard condition.

To determine the age when children's responses in each condition departed from each other (e.g., more likely to select the game with more hiding locations in the Hard condition versus Easy condition), we examined when the 95% confidence interval for responses in each condition no longer overlapped with each other. This was at age 62.88 months; CI 95% for Easy condition [0.24, 0.61], CI95% for Hard condition [0.62, 0.90]. To determine the age where children's responses in each condition departed from chance, we examined when the 95% confidence intervals no longer overlapped with 0.5. For the Easy condition, children's performance was different by chance at 66.59 months, CI95% [0.20, 0.49]. For the Hard condition, children's performance was different by chance at 53.34 months, CI95% [0.51, 0.98].

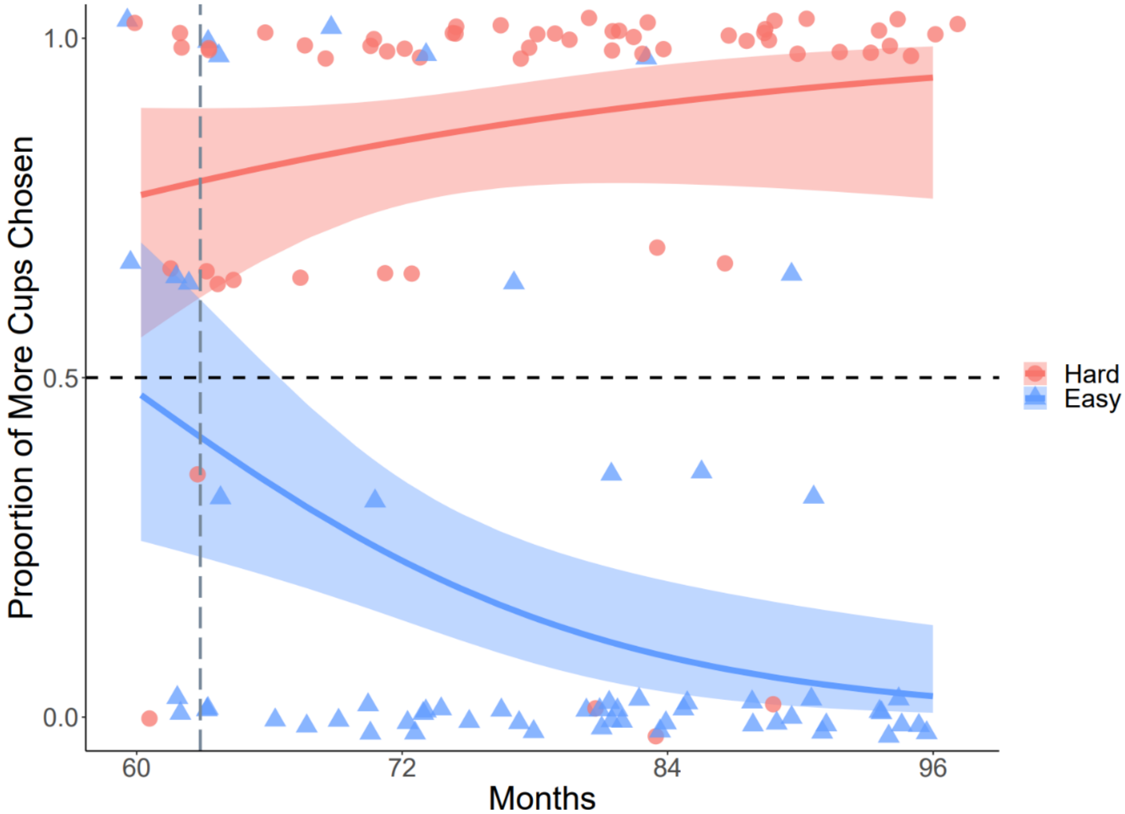
Table 2.

Average Score (out of 1) by age in years

Age	Condition	$M_{Average\ Score}$	$SD_{Average\ Score}$
5	Easy	0.32	0.42
	Hard	0.82	0.28
6	Easy	0.15	0.33
	Hard	0.88	0.31
7	Easy	0.07	0.17
	Hard	0.92	0.24

Figure 4.

Proportion of trials in which children chose the game with more cups in Experiment 2



Note. Vertical grey long dash at 62.88 months represents the age at which participants' responses statistically diverged from each other, across conditions. Dots are jittered for visibility

Discussion

By age 5, children were selecting the side with fewer hiding locations when asked to make it easy to find the coin and vice versa for making it hard. We can also be more confident that children do this by considering the absolute number of hiding locations, rather than using perceptual cues like surface area (e.g., selecting based on more or less clutter).

In the experiments so far, children were not successful until age 5. However, past literature suggests that infants and young children have an intuitive number sense and show some understanding of probability. So, we have reason to believe that younger children could be successful in our tasks. Thus, we ran a follow-up study with 3- and 4-year-olds. We simplified our procedure and made two main changes to our current design. First, we only included two trial types: 1v2 cups and 2v3 cups. The 1v2 cups trial closely mirrors Call's Cups task. Thus, if children as young as three can reason through disjunctive syllogism we would expect our participants to make the correct selection on this trial (Mody & Carey, 2016). Though, it is still unclear as to whether younger children can use numerical information to compare different levels of uncertainty. The inclusion of the 2v3 cups trial will allow us to answer this question. To be successful on this trial, participants will have to compare the chance of uncovering a coin on each side (e.g., 50% versus 33%) and decide between two uncertain events.

Like the other experiments, children were presented with two games side-by-side that varied in the number of hiding locations. This time, however, they were shown two different coloured balls and were told that each ball would be hidden under one of the cups on each side. Once the balls disappeared, children were asked to select the ball that would be easier or harder to find. We thought this might be an even easier design than the previous experiments, as we removed the additional character. If children are reasoning correctly, they should say that the ball from the side with fewer hiding locations is easier to find and that the ball from the side with more hiding locations is harder to find.

Experiment 3

Methods

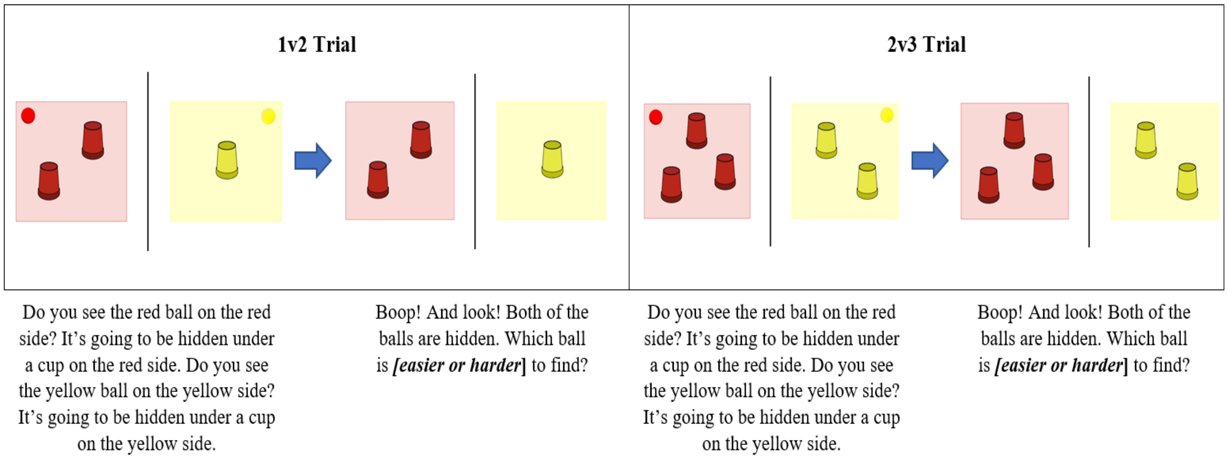
Participants. Data collection was conducted online via live Zoom calls using screen sharing. Forty 3- and 4-year-olds have participated in the study (20 children per age in years, Mage = 3.50 years; 48.66months; 25 females). Four additional children were tested but excluded due to their refusal to answer the test questions, resulting in incomplete data sets. The sample size, experimental procedures, statistical analyses and exclusion criteria were pre-registered (https://aspredicted.org/X9Y_KH9)

Materials and procedure. In this study, participants were shown a red and yellow box side by side that varied in the number of cups superimposed onto the boxes. At the top corner of the two boxes, a ball corresponding to the colour of the boxes was presented. Participants were told that each ball would be hidden under one of the cups on the side corresponding to the colour of the ball. The balls then disappeared. In the Easy condition participants were asked "Which ball is easier to find?" and in the Hard condition, participants were asked "Which ball is harder to find?" See figure 5 for illustrations of the slides and the scripts.

In this experiment, both condition (Easy, Hard) and trial type (1 vs 2 cups and 2 vs 3 cups) were tested within-subjects. Trial type order was counterbalanced, and condition order was blocked and counterbalanced (i.e., the easy block appeared first for half the children). The visual stimuli in both the Easy and Hard condition were identical, and scripts were identical other than the words "easy" and "hard." Participants who refused to answer the test questions on at least one trial were excluded from analyses, as planned in the pre-registration.

Figure 5.

Sample slides and script from Experiment 3.



Note. The stimuli and script were identical for both conditions. In the Easy condition, participants were asked to identify which ball is easier to find, and in the Hard condition, they were asked to identify which ball is harder to find.

Results

Coding was identical to Experiments 1 and 2. Thus, if children reasoned correctly, they should score higher in the Hard condition than in the Easy condition.

We used a GEE (binary logistic, independent correlation matrix) with condition (Easy, Hard) and trial type (1v2, 2v3) as within-subjects factors, age in months was mean-centered and entered as a continuous covariate and interactions were included in the model.

There was a significant main effect of condition, *Wald* $X^2(1) = 16.305$, $p < .001$, in that children scored higher in the Hard condition than in the Easy condition. There was also a significant Condition x Age interaction (see Figure 6), *Wald* $X^2\,(1) = 8.561$, $p = .003$. There was no main effect of age or trial type, *ps* > .478 (see Table 3 for means and standard deviations at each age).

To further investigate the interaction, we examined the differences in responses in the two conditions for each age group separately. Among the 3-year-olds, there was no significant effect of condition, *Wald* X^2 (1) = 2.526, p = .112. In contrast, there was a significant effect of condition for the 4-year-olds, *Wald* X^2 (1) = 14.547, p <.001. The 4-year-olds were more likely to select the game with more cups in the Hard condition.

To determine the age when children's responses in each condition departed from each other (e.g., more likely to select the ball from the set with more hiding locations in the Hard condition versus Easy condition), we examined when the 95% confidence interval for responses in each condition no longer overlapped with each other. This was at age 45.73 months; CI 95% for Easy condition [0.20, 0.44], CI95% for Hard condition [0.45, 0.71]. To determine the age where children's responses in each condition departed from chance, we examined when the 95% confidence intervals no longer overlapped with 0.5. For the Easy condition, children's performance was different by chance at 44.29 months, CI95% [0.22, 0.49]. For the Hard condition, children's performance was different by chance at 47.76 months, CI95% [0.51, 0.75].

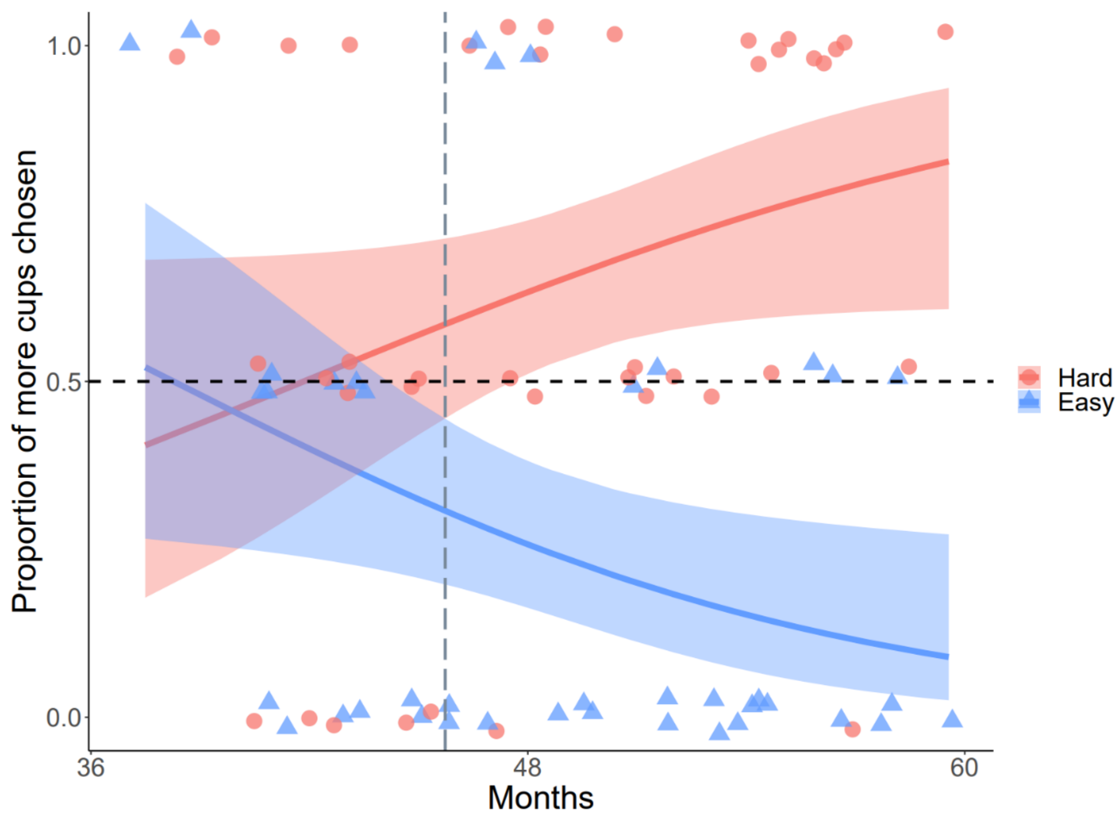
Table 3.

Average score (out of 1) by age in years

Age	Condition	$M_{Average\ Score}$	$SD_{Average\ Score}$
3	Easy	0.35	0.48
	Hard	0.53	0.51
4	Easy	0.18	0.38
	Hard	0.75	0.44

Figure 6.

Proportion of trials in which children chose the ball from the side with more cups.



Note. Vertical grey long dash at 45.73 months represents the age at which participants' responses statistically diverged from each other, across conditions. Dots are jittered for visibility.

Discussion

We found that around the age of 4, children were selecting the side with fewer hiding locations when asked to which ball was easier to find and selecting the side with more hiding locations when asked which ball was harder to find.

Unlike Experiment 1, younger children were successful on this task. The 1v2 cups trial closely mirrors Call's Cups task, thus, it is unsurprising that 4-year-olds made the correct decision on this trial. Interestingly, on average, the 4-year-olds also made the correct decision on the 2v3 trial. Success on this trial provides evidence that they were able to use numerical information to compare different levels of uncertainty. For example, the chances of uncovering a single coin when there are two hiding location is 50% and when there are two hiding locations the chances are 33%. Four-year-olds were able to correctly infer that it is harder to find the hidden coin when the odds are lower and vice versa. Unlike the work cited earlier (Call's Cups task, Y-Shaped Tube task), this task was fully probabilistic in that children had to make predictions about two uncertain future outcomes. Thus, our results suggest that children as young as four are able to use probability to compare different levels of uncertainty.

General Discussion

In three experiments, we investigated whether children could use numerical information to compare different levels of uncertainty based on probability and whether they can use this information to make a decision that would most likely lead to the desired outcome. We presented children with two games that varied in the number of hiding locations to see if they can evaluate the probability of uncovering a hidden item on each set and choose the set that would lead to another player successfully finding the hidden item. We found that around age 5, children were selecting the side with fewer hiding locations when asked to make it easy to find a hidden item and selected the side with more hiding locations when asked to make it hard to find a hidden item (Experiment 1). Moreover, children who succeeded in this task seemed to do so by truly considering the absolute number of hiding locations rather than relying on perceptual cues such as surface area (Experiment 2). Interestingly, we also found that when children were asked to identify which ball was easier or harder to find, around the age of 4, they correctly inferred that the ball from the set with fewer hiding locations is easier to find than the ball from the set with more hiding locations (Experiment 3).

Together these findings provide evidence that around age 4 children can compare different levels of uncertainty between two uncertain events by considering the probabilities of different outcomes. Our findings are consistent with prior research suggesting that children possess an intuitive number sense long before they learn to count. To be successful on our tasks, children must be able to make numerical comparisons, evaluate and compare the probability of success on each of the sets (e.g., 1:2 ratios) and remember the goal of the task (e.g., make it easy or hard to find the hidden item).

Nevertheless, the younger children performed more poorly in Experiment 1. In contrast, the older children in our sample were successful in all our tasks. One explanation of this could involve children's numerical reasoning, including their approximate number system (ANS) acuity, which refers to our basic intuitions about numbers and our ability to make estimations about quantity (e.g., the cardinality of a set of items) without nonverbal representations of numbers (Bonny and Lourenco, 2013). For example, ANS acuity allows us to identify the shortest line to the cashier at a store without counting the exact number of people waiting ahead. ANS acuity is likely implicated in our experiments. Thus, better performance from the older children in our sample could be attributed to improvements in ANS acuity across these ages (e.g., Odic, 2018).

Further, children are learning about quantifier words and their associated concepts at these ages (e.g., Odic et al., 2013), such as *more* and *less*, and better mastery of these concepts could aid the older participants on our tasks. On the other hand though, children might not have struggled with the numerical aspect of our tasks much at all. Even though ANS acuity improves during the ages we tested, none of the comparisons should have been challenging even for 3-year-olds (they were all 1:2 ratios, which are discriminable at 6 months of age). When we simplified our procedure for Experiment 3, children around the age of four correctly inferred that when the probability of finding a hidden item is lower, it will be harder to find and vice versa. Unlike Experiment 1, we slightly modified our design so that children were explicitly asked which ball would be easier or harder to find. The instructions may have been clearer to younger children because, in Experiment 3, there was no mention of another character in the script. Instead, participants' responses required them to only make judgements about their state of mind.

Thus, solving the task in Experiment 3 is not confounded with having an adequate theory of mind.

Moreover, performance on our task is likely associated with improvements in children's general cognitive development. For example, to produce the correct responses in the Easy condition, children must select a smaller number of cups. As revealed in our comparisons to chance, 4-year-olds appear slightly more pulled towards choosing the side with more cups, regardless of condition. Interestingly, in my previous work, we also found a similar trend, in that the younger children (3- and 4-year-olds) were inclined to select the side with more cups, regardless of if they were asked to help find or hide an item. They may be struggling to inhibit a desire to simply select more cups in our task, as significant improvements in executive functioning, including inhibition, are well documented throughout this period (Diamond, 2013). However, children performed better in the Easy condition of Experiment 1 than in the somewhat analogous Finder condition from my previous work (Lu, 2020; Lu et al., 2021). This suggests that inhibiting choosing the larger number of items is perhaps not solely responsible for weaker performance in the Finder than Hider conditions of my Honours thesis at the youngest ages. But, there are many other differences between the two studies, including simplified instructions, online format, and within-subjects design in Experiment 1.

Overall, findings from our studies suggests that children from the age of four can evaluate probabilities and use this information to make judgements about uncertainty. More specifically, they understand that the higher the odds of a specific outcome, the more certain we should be that a particular outcome will occur. Unlike the reviewed work on disjunction (Call's Cup task and Y-Shaped Tube task), children make this inference without clear disambiguating evidence.

Our work also highlights children's sensitivity to uncertainty and their ability to make decisions under uncertainty. Other related literature suggests that preschoolers and non-human primates can adjust their information-seeking behaviour when faced with uncertainty (Marsh & MacDonald, 2012). For example, children search more or are motivated to explore more when faced with uncertainty versus certainty. In a study conducted by Schulz and Bonawitz (2007), they found that 4-year-olds voluntarily spent more time playing with a toy with an ambiguous causal structure. However, under some situations of uncertainty, younger children also experience some confusion about uncertainty. For example, children younger than five have been shown to mistake their guesses for knowledge. When presented with two toys and asked which of the two is hidden in a box, although blinded to the actual hiding of the toy, children younger than five said that they knew which toy was hidden and even specified which toy was hidden under the box. They failed to consider that either of the two toys could be the one hidden and instead, treated their guess as knowledge (Rohwer et al., 2012). Thus, there is still much to uncover about how children understand uncertainty, how they resolve it, and how they make decisions under uncertainty.

Future work could examine whether poorer performance from the younger children in Experiment 1 could be due to a lack of comprehension of the words "easy" and "hard." Results would corroborate whether difficulty among younger children is due to task demands (e.g., understanding the instruction) or if they have difficulties choosing between two uncertain tasks. For example, we can present children with a story about a character who needs to carry a tray of cups. In scenario one, the character is carrying a tray with two cups. In the other scenario, the character is carrying a tray with four cups which is heavier than a tray with two cups. Children would then be asked to select the tray that is easier or harder to carry. If children understand the meaning of “easy” and “hard,” they should say that the heavier tray (with four cups) is harder to carry. Recall that in Mody and Carey’s cups task, 3-year-olds selected the correct cup at above chance levels, so it is possible that the words “easy” and “hard” derailed 3-year-olds in this task, who did not answer the 1vs2 cup trials correctly in my Experiment 3. However, it is unlikely that a lack of comprehension of the words “easy” and “hard” would account for younger children’s incorrect responses. Children as young as three have been shown to use proportional information to make easy and hard judgements. In a study examining children’s understanding of supply and demand, 3-year-olds correctly inferred that it would be easier to obtain treats from a location with a larger supply of treats and with less demand for treats. They were also correct when making judgements about where it would be more difficult to obtain treats based on supply and demand. Nevertheless, future work may be necessary to determine whether task demands weakened younger children’s success in our tasks.

Lastly, one interesting avenue might be to examine children’s inferences about deception, cheating, and lying, based on their intuitions about probability and uncertainty. The current experimental design could provide a potentially fruitful paradigm: By considering the odds of winning or losing in games of chance, such as searching for a target among a number of locations, children might become suspicious of a character who is winning too frequently based on odds and start to wonder whether they had surreptitiously gained knowledge about the hiding locations. Young children can use probability to make a number of other social inferences, including people’s preferences (Diesendruck et al., 2015; Doan et al., 2021; Kushnir et al., 2010; Ma & Xu, 2011) and their emotions (Doan et al., 2018; 2020), suggesting this could be extended to examine other kinds of social inferences involved in cooperation and competition.

In sum, findings from three experiments suggest that around age four, children can use numerical information to make judgements about levels of uncertainty. They understand that it will be easier to find an item when there are fewer locations to search because the probability of uncovering an item is higher and can do so in the absence of disambiguating information. These results highlight that probability is a helpful tool when confronted with uncertainty. Although past work indicates that young children have intuitions about probability that can guide actions under uncertainty, our study suggests that perhaps this ability strengthens with age. Future work will continue to investigate children's ability to make decisions under uncertainty, if task demands explain younger children's inconsistent performance on our tasks, and whether children can use probability and uncertainty to make social inferences about deception.

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