Local measurements in Relativistic Quantum Information: localization and signaling

Τοπικές μετρήσεις στη Σχετικιστική Κβαντική Πληροφορία: εντοπισμός και μετάδοση σήματος

by

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Author's Declaration

This thesis consists of material all of which I authored or co-authored: see Statement of Contributions included in the thesis. This is a true copy of the thesis, including any required final revisions, as accepted by my examiners.

I understand that my thesis may be made electronically available to the public.

Statement of contributions

Maria-Eftychia Papageorgiou was the sole author of this thesis, although large portions of this thesis are direct adaptations of collaborative publications/preprints.

The introduction and second chapter are based on these two works in collaboration with Prof. Doreen Fraser:

1. *Eliminating the 'impossible': recent progress on local measurement theory for quantum field theory*, M. Papageorgiou and D. Fraser, http://philsci-archive.pitt.edu/22322/.
2. *Note on episodes in the history of modeling measurements in local spacetime regions using QFT*, D. Fraser and M. Papageorgiou, to appear in EPJ H Special Issue: History for Physics: Contextualizing modern developments in the foundations of quantum theory, http://philsci-archive.pitt.edu/22742/

The third chapter is partially based on the first preprint listed above and mostly based on published work in collaboration with Prof. Eduardo Martín-Martínez and Dr. José de Ramón Rivera:

1. *Relativistic causality in particle detector models: Faster-than-light signaling and impossible measurements*, J. de Ramón, M. Papageorgiou and E. Martín-Martínez, Phys. Rev. D 103, 085002 (2021).
2. *Causality and signaling in noncompact detector-field interactions*, J. de Ramón, M. Papageorgiou and E. Martín-Martínez, Phys. Rev. D 108, 045015 (2023).

Finally, the last chapter is based on unpublished work with Dr. Jason Pye and Prof. Achim Kempf (section 4.4 and part of section 4.1.1) and with Dr. José de Ramón Rivera and with Prof. Charis Anastopoulos (sections 4.1-4.3):

1. *Particle-field duality in QFT measurements*, M. Papageorgiou, J. de Ramón and C. Anastopoulos, https://arxiv.org/abs/2308.14718

Abstract

In this thesis, we study some foundational aspects of detector models in quantum field theory (QFT) related to signaling and localization, and we analyze certain frictions with relativistic causality. We characterize the spatiotemporal information that can be extracted from the field using various detector models in different regimes and we define a signaling estimator, based on quantum metrology, that can be used to quantify how much signaling can be transmitted reliably through the quantum field. We analyze 'impossible measurements' scenarios in which the microcausality condition in QFT is not sufficient for blocking superluminal signaling between multiple detectors coupled to the field.

Further, since QFT does not admit a straightforward particle or field ontology, we ask: what do detectors detect? We answer this question by interpreting the detector's response in different regimes, for single-particle wavepacket states or coherent states of the field. In the weak coupling regime, we demonstrate in detail how detector models can be used to save particle-like phenomenology, related to the phenomenon of resonance and 'time-of-arrival'. In the strong coupling regime, we demonstrate how a continuous pointer variable can get correlated with smeared field time-averages. Finally, adapting the formalism of the quantum Brownian motion, we develop an improved field-detector interaction model that is exactly solvable and can be used to characterize the weak, strong and intermediate regime. Apart from an improved description of field measurements and resonance, this models clearly demonstrates the modulation of particle-field duality by a single tunable parameter (the coupling strength), which is a novel feature that is in principle experimentally accessible.

Extended abstract in Greek

Η Κβαντική Θεωρία Πεδίου ενοποιεί τη κβαντική θεωρία με την (ειδική) θεωρία της σχετικότητας και είναι η περισσότερο πειραματικά επιβεβαιωμένη θεωρία εως σήμερα. Αυτή η ενοποίηση κβαντικής θεωρίας και σχετικότητας αφορά τους νόμους της δυναμικής εξέλιξης, ενώ μέχρι σήμερα δεν υπάρχει πλήρης και κοινώς αποδεκτή θεωρία μέτρησης (π.χ. αναγωγή της κβαντικής κατάστασης) για τη Κβαντική Θεωρία Πεδίου που να μην παραβιάζει τις αρχές της θεωρίας της σχετικότητας. Τα μοντέλα μέτρησης είναι χρήσιμα εργαλεία για την περιγραφή των τοπικών μετρήσεων στην Κβαντική Θεωρία Πεδίου, μέσω της τοπικής σύζευξης ενός κβαντομηχανικού συστήματος μέτρησης (ανιχνευτή) στο κβαντικό πεδίο.

Σε αυτήν την διατριβή μελετάμε θεμελιώδεις πτυχές των μοντέλων μέτρησης στην Κβαντική Θεωρία Πεδίου, σε σχέση με τον σωματιδιακό εντοπισμό και την αιτιακή μετάδοση πληροφορίας. Αναλύουμε τις ασυμβατότητες που προκύπτουν μεταξύ των προβλέψεων των μοντέλων και συγκεκριμένων εννοιών σχετικιστικής αιτιότητας π.χ. την απαίτηση για μη υπερφωτεινή μετάδοση σήματος. Χαρακτηρίζουμε την τοπική πληροφορία που μπορεί να εξαχθεί από το κβαντικό πεδίο μέσω μιας σειράς μοντέλων μέτρησης, για ασθενείς και ισχυρές αλληλεπιδράσεις μεταξύ του ανιχνευτή και του κβαντικού πεδίου, χρησιμοποιώντας διαταρακτικές και μη διαταρακτικές μεθόδους αντίστοιχα. Με βάση τη κβαντική μετρολογία, ορίζουμε έναν νέο εκτιμητή που ποσοτικοποιεί την πληροφορία που λαμβάνει ο ανιχνευτής, τον οποίο χρησιμοποιούμε για να αναλύσουμε ποιοτικά και ποσοτικά την πληροφορία που μπορεί να μεταδοθεί αξιόπιστα μέσω του κβαντικού πεδίου. Επίσης, μελετάμε περιπτώσεις 'αδύνατων μετρήσεων' [1] στις οποίες το αξίωμα της μικροαιτιότητας στην Κβαντική Θεωρία Πεδίου δεν επαρκεί για τον αποκλεισμό της υπερφωτεινής μετάδοσης πληροφορίας μεταξύ απομακρυσμένων ανιχνευτών που είναι συζευγμένοι με το κβαντικό πεδίο.

Από την σκοπιά της Κβαντικής Θεωρίας Πεδίου όπως χρησιμοποιείται στην φυσική υψηλών ενεργειών, τα κβαντικά πεδία περιγράφουν τη δυναμική και τις συμμετρίες μίας σειράς σωματιδίων. 'Ομως υπάρχουν θεωρήματα τα οποία αναδεικνύουν ότι η Κβαντική Θεωρία Πεδίου δεν επιδέχεται ερμηνεία μόνο με βάση την έννοια του σωματιδίου, ή μόνο με βάση την έννοια του πεδίου. Επανεξετάζουμε τον κυματοσωματιδιακό δυισμό στην Κβαντική Θεωρία Πεδίου με βάση την απόκριση των ανιχνευτών σε διάφορα μοντέλα μέτρησης, και συγκρίνουμε τις προβλέψεις διαφορετικών μοντέλων για κυματοπακέτα ή συνεκτικές καταστάσεις του πεδίου. Στην περίπτωση ασθενούς σύζευξης πεδίου-ανιχνευτή, ανακτούμε σωματιδιακή φαινομενολογία που σχετίζεται με το φαινόμενο του συντονισμού και του 'χρόνου άφιξης' του κυματοπακέτου. Στην περίπτωση ισχυρής σύζευξης πεδίου-ανιχνευτή, δείχνουμε πώς μια συνεχής μεταβλητή του ανιχνευτή συσχετίζεται με τις πεδιακές μέσες τιμές. Τέλος, με βάση επιλύσημα μοντέλα, προτείνουμε ένα βελτιωμένο μοντέλο μέτρησης το οποίο μπορεί να περιγράψει τις περιπτώσεις ασθενούς, ισχυρής, αλλά και ενδιάμεσης σύζευξης. Εκτός από τη βελτιωμένη περιγραφή της τοπικής μέτρησης πεδίων και σωματιδίων, αυτό το μοντέλο αναδεικνύει ότι ο σωματιδιακός δυισμός ρυθμίζεται από μία μόνο παράμετρο, την ισχύ της σύζευξης μεταξύ πεδίου και ανιχνευτή. Αυτή είναι μία νέα πτυχή των τοπικών μετρήσεων μέσω μοντέλων μέτρησης στην Κβαντική Θεωρία Πεδίου, η οποία μπορεί να διερευνηθεί και πειραματικά σε πειραματικές διατάξεις στις οποίες η ισχύ σύζευξης δεν είναι δεδομένη, αλλά μεταβαλόμενη (π.χ. οπτομηχανική σε κοιλότητα). Συνολικά, η έρευνα αυτή συμβάλλει στην καλύτερη κατανόηση και εφαρμογή των μοντέλων μέτρησης, με σκοπό τη συνεπή εφαρμογή τους στον τομέα της κβαντικής οπτικής και της σχετικιστικής κβαντικής πληροφορίας.

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As you set out for Ithaka hope
your road is a long one, full of
adventure, full of discovery.

Ithaka by C.P Cavafy
Transl. by E. Keeley

This is the end of a very long road that was, indeed, full of adventure and discovery. The many lessons I learned on the way, I owe to the people I interacted with; people who have been generously sharing their wisdom and support.

During this journey, I experienced many setbacks that I would not have overcome without Achim Kempf's generous and unconditional support and mentorship. His deep insights in Physics, as well as his teaching skills and clarity, were essential for my understanding of Physics at the postgraduate level when I first moved to Canada to pursue a rather fast-paced Masters program. As a supervisor, he inspires the students to investigate and to identify their own research interests, providing as much freedom as support; a golden ratio that is very hard to achieve. This stance I interpret as a big faith in people's ability to succeed in their own terms and to achieve their full capacity, when provided with the conditions to do so; like different plants in a garden, each of them requiring a different amount of water and sun. When I decided to pursue a dual PhD between UWaterloo and the University of Patras in Greece, he was always eager to help with the (endless!) bureaucratic procedure. I consider myself very lucky for having received this kind of support, and this is why Achim Kempf represents the supervisor that I would like to become, if I ever get the chance to be one.

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Dedication

To my grandmothers, Eftychia and Anatoli.

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Chapter 1

Introduction

This thesis is concerned with two aspects of local quantum measurements in relativistic spacetime, that are implemented using detector models in Quantum Field Theory (QFT):

- **Localization**: interpreting the spatiotemporal information that can be extracted from detection events that are localized in space and time. Addressing questions like the following: Which field observable is being probed in a particular detection set-up? How much does this depend on the state of the field, the characteristics of the detector and the detector-field coupling? In which cases does the particle-like phenomenology of the detector's response arise, and in which cases can we measure local field averages? Or, overall, what do detectors detect?

- **Signaling**: characterizing the signaling possibilities between probes that are locally coupled to a quantum field, and identifying possible frictions with relativistic causality. Addressing questions like the following: What constraints does relativity impose on the possible quantum operations? Are the causal relations that are encoded in the structure of relativistic spacetime, and are reflected in the kinematics and dynamics of the quantum field, sufficient for preventing the probes from signaling superluminally? Which notion of relativistic causality in QFT can guaranty the causality of the detectors' response?

Detector models are a simple and useful tool for exploring many issues relevant to information and measurements in QFT [2]. The main example of such models is the Unruh-DeWitt particle detector [3, 4], originally employed in the discussion of the Unruh effect, and extensively used in the field of relativistic quantum information.

Particle detector models, as their name suggests, were originally introduced to tackle the challenges associated with the notion of particle in relativistic QFT, that is, to deal with the complications that the naive notion of particle introduces in scenarios involving non-inertial observers or curved spacetimes.

Many known arguments suggest that QFT does not admit a particle ontology [5, 6, 7] and that detector models can be used to save particle phenomenology [6, 8]. In addition, there is controversy around field ontology [9]. We are agnostic about the issue of QFT ontology and we make no claim that particle detector models are fundamental. Rather, we view them as useful tools for extracting particle or field phenomenology in certain regimes. The inapplicability of the particle concept in relativity has motivated the pragmatic response "a particle is whatever a particle detector detects" [10]. But since no (particle or field) ontology is straightforwardly applicable in QFT, it is still worth asking (in a sense the inverse) question "what do detectors detect?". For example, it is interesting to analyze relevant cases in which the detector's response is insensitive to the particle content (in the Fock space sense) of the quantum field, and establish cases in which the detector is correlated with field (rather than particle) quantities, shedding new light to the particle-field duality.

When more than one detection events are considered, one can analyze causality in detector models in QFT. In particular, in this thesis (and the corresponding joint work [11, 12]) we analyze the signaling relations between detectors. The analysis of causality in this context is seemingly independent of answering what a detector detects. One can analyze causality by means of joint probabilities of abstract measurement outcomes. These outcomes correspond to induced positive operator-valued measures (POVM) elements (first defined in [13] and further developed in [14]) which do not admit an obvious interpretation. In this thesis (and the corresponding joint work [15]) we offer an interpretation of the induced POVMs by means of the field or particle aspect of the field in certain detection set-ups, using a variety of models that capture the weak and/or strong coupling regime. Conceptually, this can be helpful also for analyzing the reliable transmission of information between detectors.

Overall, it is curious that the particle notion seems to be particularly problematic in relativity, both at the level of ontology (what the theory is about) and phenomenology (particle detection). As we will explain, QFT models that reproduce a clean particle phenomenology introduce friction with relativistic causality. We start by reviewing the main no-go result against particle ontology in the next subsection, to then dive into the details of particle phenomenology in QFT measurements (chapter 4).

No particles in relativistic quantum theory

Here I include a brief exposition of a no-go theorem by Malament [5] against the particle concept in relativistic quantum theory (following my previous exposition in [16]). Then, I make some remarks that are relevant for the content of this thesis. The theorem states that any candidate of a relativistic quantum theory will include the following elements: a Hilbert space $\mathcal{H}$, the rays of which represent the pure states of the system, a strongly continuous unitary representation $U(\alpha)$ in $\mathcal{H}$ of the translation group α in Minkowski space $\mathcal{M}$, $\alpha \to U(\alpha)$. Considering the foliation of Minkowski spacetime by a family of spacelike hypersurfaces Σ_t in a fixed frame, we also require an assignment of a projection operator P_Δ on $\mathcal{H}$ for every open bounded spatial subregions $\Delta \subset \Sigma$, that is $\Delta \to P_\Delta$. These elements, put together, comprise what is called a localization structure $(\mathcal{M}, \mathcal{H}, \Delta \to P_\Delta, \alpha \to U(\alpha))$ and amounts to specifying states, dynamics and measurement tools, along with the symmetries of the underlying spacetime that we wish to represent in the Hilbert space. Note that the theorem is only concerned with translations, and not boosts. The premises and the conclusion of the theorem are the following (see figure 1.1):

P1 Translation Covariance: for all vectors α and spatial sets Δ in $\mathcal{M}$ it holds that $P_{\Delta+\alpha} = U(\alpha) P_\Delta U(-\alpha)$.

P2 Energy Condition: for all future directed unit timelike vectors in $\mathcal{M}$, $\hat{U}(t\alpha) = \exp(-it\hat{H}(\alpha))$. The spectrum of the self-adjoint Hamitonian operator $\hat{H}$ is bounded from below.

P3 Localizability: for Δ_1,$\Delta_2 \subset \Sigma_t$ disjoint spatial sets in the same hyperplane Σ_t, it holds that $P_{\Delta_1} P_{\Delta_2} = P_{\Delta_2} P_{\Delta_1} = 0$.

P4 Locality: if $\Delta_1 \subset \Sigma_{t_1}$,$\Delta_2 \subset \Sigma_{t_2}$ for $t_1 \neq t_2$ and such that the two spatial intervals are spacelike separated, it holds that $[P_{\Delta_1}, P_{\Delta_2}] = 0$.

Conclusion: $P_\Delta = 0 \ \ \forall \Delta$.

The theorem states that the localization structure $(\mathcal{M}, \mathcal{H}, \Delta \to P_\Delta, \alpha \to U(\alpha))$ that satisfies the premises P1-P4 has trivial projectors. This means that given a state ρ of the system, the probabilities that correspond to measurements over *any* region Δ will vanish, i.e., $\text{Prob}(\Delta) = \text{tr}\,(P_\Delta) = 0, \forall \rho, \Delta$ (independently of the size of Δ). Some remarks are in order:

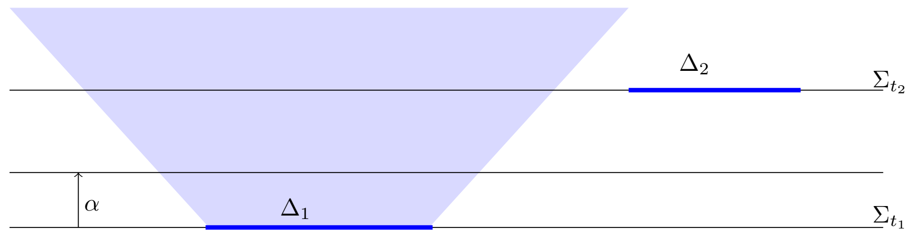

Figure 1.1: By Σ_t we denote a set of spacelike hypersurfaces that foliate Minkowski spacetime. Δ_1, Δ_2 are two spacelike separated spatial subregions (Δ_2 does not intersect the forward lightcone of Δ_1) and α is a timelike vector.

- **The role of the locality axiom**: The locality axiom P4 is obviously stronger than P3. The two together comprise what is also known as the microcausality condition. P4 is the only relativistic premise of the theorem, and it is linked to the following statement: If one performs a non-selective measurement in region Δ_1, then the statistic of a measurement in region Δ_2 should be unaffected. Namely, microcausality implies that

$$\mathrm{tr}\left(\rho P_{\Delta_2}\right) = \mathrm{tr}\left(\rho' P_{\Delta_2}\right) \tag{1.1}$$

where

$$\rho' = P_{\Delta_1}\rho P_{\Delta_1} + (\mathbf{1} - P_{\Delta_1})\rho(\mathbf{1} - P_{\Delta_1}). \tag{1.2}$$

It is easy to check that microcausality implies statistical independence in this sense [1], by plugging (1.2) into (1.1). The relation between microcausality and statistical independence (e.g. no-signaling) will be discussed in the next chapters, in a QFT context.

- **The role of symmetries**: The theorem assumes only spacetime translations and not Lorentz boosts, so in this sense the full spacetime symmetry of Minkowski cannot be blamed for the result. This is important for clarifying that the conclusion follows due to the microcausality (and some dynamical assumptions) and it is not an issue of Lorentz covariance. It is often assumed that the friction comes from the fact that the non-selective state update (1.2) is not Lorentz covariant, but this is an independent issue. A covariant state

[1] The inverse is trickier. In [17] one finds a general proof that microcausality is necessary and sufficient for statistical independence of bipartite spacelike separated generalised non-selective operations.

update rule does not necessarily address the frictions that come from assuming microcausality (after all, microcausality is an independent axiom [18]).

- **The role of dynamics**: P2 assumes that the Hamiltonian is bounded from below. This is a dynamical assumption that is also typically blamed for the result of the non-existence of a position operator in relativity. Indeed, the proof of the theorem is making use of a non-trivial Lemma [2] which is about how the local projectors are 'moved around' by the unitaries (see figure 1.1). This lemma is exactly what links P3 with P4 (which refers to local projectors at different hyperplanes). We will not elaborate on this more, but the moral is that the representation of dynamics can play an important role in restricting the measurement theory.

- **Local entities vs local measurements**: The moral drawn from Malament's theorem is typically that there is no position operator in relativistic set-ups and, as a result, the quantum mechanical wave function of a relativistic particle cannot be defined in Minkowski spacetime. This is because we could parameterize the points of each spatial hypersurface in figure 1.1 by $\boldsymbol{x}$ and define the local projectors as PVMs of a position operator with spectrum $\boldsymbol{x}$ as

$$P_\Delta = \int_\Delta \mathrm{d}\boldsymbol{x}\, |\boldsymbol{x}\rangle \langle \boldsymbol{x}| \,. \tag{1.3}$$

  Indeed in Quantum Mechanics (QM) only the position operator has local outcomes in terms of its spectrum, and as a result P_Δ can only correspond to the proposition 'the particle is found in region Δ'. One could argue that the result is in fact more general than that (see discussion in [19]) and it concerns the possibility of local measurement records in general, not necessarily attached to position measurements. Of course, in a full QFT set up the local projectors would have to be linked with possible values of field-related quantities (e.g. field amplitudes). Also, some of the premises, e.g., P3 would not hold since the local QFT projectors are not orthogonal in spacelike separation. But this alternative way of viewing Malament's theorem highlights that one's commitment to ontology (e.g. particle ontology) is necessarily linked to the measurement theory that can or cannot be formulated 'on top' of a given theory, as well as to the consequences that it might have for the measurement problem [19, 20].

[2] Let $U(t) = e^{itH}$ be a family of strongly continuous one-parameter group of unitary operators, where the spectrum of the generator H is bounded from below, and P_1, P_2 two projector operators such that (i) $P_1 P_2 = 0$, and (ii) there is $\epsilon > 0$ such that $[P_1, U(-t)P_2U(t)] = 0 \;\; \forall t \in (-\epsilon, \epsilon)$. Then it follows that $P_1 U(-t) P_2 U(t) = 0 \;\; \forall t$.

Relativistic causality in QFT

Many discussions of causality in QFT focus on the axiom of microcausaltiy, namely that spacelike separated field operators commute. Nevertheless, the operational meaning of the microcausality condition is not clear unless there is a clearly formulated measurement theory [3][18, 22]. Further, there are other assumptions, e.g. about the dynamics, that play an important role. In [23], Earman and Valente argue that a dynamical axiom (the local time-slice property) is needed in order to enforce relativistic causality. They distinguish two aspects of relativistic causality that are relevant to our discussion: no superluminal signaling (i.e., by performing local operations on quantum fields) and no superluminal propagation of quantum fields.

Assume an association of local algebras $\mathcal{A}(O)$ to bounded regions of spacetime O. Then microcausality states that if two regions O_1 and O_2 are causally disjoint, then the elements of $\mathcal{A}(O_1)$ commute with the elements of $\mathcal{A}(O_2)$. The local time-slice property states that if $O_1 \subset O_2$ and O_1 contains a Cauchy surface for O_2, then $\mathcal{A}(O_1) = \mathcal{A}(O_2)$. Microcausality is a kinematical axiom that imposes an independence or separability requirement. In contrast, the axiom of the time-slice property concerns dynamics. Positing an axiom that imposes a dynamical constraint can exclude spacelike dependencies between expectation values in one region and unitary operations performed in a spacelike separated region by enforcing the requirement that fields cannot propagate faster than the speed of light. Intuitively, if the fields cannot propagate faster than the speed of light, then the effects of local operations on the fields should not be able to propagate faster than the speed of light either.

Earman and Valente [23] argue that this intuition about needing a dynamical axiom like the time-slice property to exclude superluminal signaling is supported by considering classical field theories. In classical relativistic field theories, the prohibition on superluminal field propagation is typically enforced by the field equations. More specifically, the field equations are a system of symmetric, quasi-linear, hyperbolic partial differential equations that are associated with a set of causal cones that typically[4] do not permit superluminal propagation of the field [24]. Determinism keeps the fields propagating within the causal cones. Consider the initial value problem for a system of field equations. The specification of 'initial' data on

[3]The intuition that microcausality should suffice for causality considerations comes from scattering theory (because it enforces the cluster decomposition property) [21] but beyond asymptotic scattering it does not have an a priori operational meaning for *local* measurements.

[4]In atypical cases the causal cones of the hyperbolic partial differential equations could differ from the null cones of the spacetime, which would in principle permit superluminal signaling [24, 23].

a closed subset S of points in Cauchy surface Σ picks out a unique solution of the field equations in the future and past domains of dependence of S, $D(S)$.[5] Note that determinism is a fact about what the initial state and dynamical laws entail about future states, not an epistemic matter of what we can know or predict.

To appreciate the role of microcausality, one needs to introduce a notion of operation. QFT by definition is a theory of local 'observables' that reside and dynamically evolve in spacetime. In QFT it is far from obvious how to move from local observables to local operations, and in fact the standard axioms of measurement theory (Born rule, Lüders' rule) are not part of the standard axiomatic formulations of QFT. Nevertheless, algebraic QFT has traditionally been given an operational interpretation in terms of local measurement operations [25] that are in principle implementable in a local laboratory, but this is a schematic interpretation that is not obvious how to apply. Indeed, in a recently proposed framework for measurement theory in algebraic QFT proposed by Fewster and Verch (FV framework)[26] this interpretation is abandoned, as we argued in [27, 28].

An argument by Sorkin [1] that we will examine carefully in the next chapter establishes that the straightforward extension of non-relativistic measurement theory to QFT is not possible due to frictions with relativistic causality. As we will argue, the infeasibility of idealized measurements in QFT necessitates the formulation of measurement frameworks and models, in which some kind of probe is locally coupled to the quantum field. Whether the causality conditions of the underlying theory are respected or not, depends on the locality properties of the dynamical interaction between the probe and the system. Indeed, as we emphasized in [27] the fact that the FV framework does not reproduce the 'impossible measurements' (see [29]) is thanks to the local time-slice-property. In this thesis we will focus on analyzing frictions with relativistic causality that are introduced by considering non-relativistic models in QFT, tracking the role of microcausality as well as the dynamical assumptions about the field-detector couplings.

Detector models in QFT

Detector models are defined through specifying the details of the dynamical interaction of a detector system with the quantum field. When the detector system is modeled as a non-relativistic quantum mechanical system, the model is not guaranteed to comply with the notions of relativistic causality of the underlying QFT.

[5] The domain of dependence $D(S)$ of S is the set of points p such that every inextendible causal curve through p meets S [23].

Different aspects of the detector response depend on different field quantities. A field theory can be equivalently described through its correlation functions. For example, leading order in perturbation theory, the excitation probability of a single two-level system weakly coupled to the field is a functional of the field two-point function (Wightman function) $W(\mathsf{x},\mathsf{x}') = \langle\hat{\Phi}(\mathsf{x})\hat{\Phi}(\mathsf{x}')\rangle$. This functional depends on the characteristics of the coupling and the detector system [30]. It is well known that the Wightman function leaks outside the lightcones, but this does not imply friction with causality. The Wightman function contains information about the dynamics, but it is not a dynamical object, it is a correlator, and it is known that generic states in QFT (not only the vacuum) exhibit spacelike correlations [31]. The fact that this leakage of the Wightman function outside the lightcones does not lead to causality issues at the level of detector models has been analyzed in the context of the Fermi two-atom problem [32, 33].

The Wightman function can be decomposed into its symmetric and antisymmetric parts, which depend on the anti-commutator and the commutator respectively:

$$W(\mathsf{x},\mathsf{x}') = \frac{1}{2}\langle\{\hat{\Phi}(\mathsf{x}),\hat{\Phi}(\mathsf{x}')\}\rangle + \frac{i}{2}\langle[\hat{\Phi}(\mathsf{x}),\hat{\Phi}(\mathsf{x}')]\rangle. \tag{1.4}$$

It is known in the particle detector literature that, roughly, the anti-commutator part is related to correlations (that can be extracted from the vacuum even in spacelike separation [34, 35]) and the commutator part is related to signaling. The interplay between the two term has been studied in the context of entanglement harvesting, to ensure that the detectors do not get correlated through communication [36].

As we will analyze in detail, signaling between detectors is governed by the (time-ordered) commutator [37, 12], i.e., the retarded Green's function that is supported in the future lightcone of the source. As a result, there is no 'leakage' of signaling outside the lightcone unless the source itself is not compact. In chapter 3 we will investigate non-trivial dependence of the signaling term on the internal states and frequencies of the detectors. Other features of the detector's response depend on other Green's functions of the field [38], all of which can be expressed in terms of the Wightman function [39]). As it becomes more explicit in histories-based approaches, the joint/conditional probabilities (or the signaling possibilities) of an arbitrary number of detectors depend on higher-order correlators [40]. One of the morals that we will draw from the analysis of the Sorkin-type problem is that when more than two detectors are involved the fact that the commutator vanishes in spacelike separation (microcausality) is not sufficient for blocking superluminal signaling.

Finally, in this thesis, we will also consider non-perturbative methods and solvable models. In these cases, the analysis of causality changes, and it depends on the method we use to solve the dynamics of the combined field-detector system. For Unruh-DeWitt-type detector models, we will demonstrate non-perturbative arguments that rely on the causal factorization of detector-field interactions (in the interaction picture). In solvable models like the Quantum Brownian Motion (QBM) the dynamics is solved in the Heisenberg picture, which introduces non-trivial 'mixing' of the degrees of freedom of the detector system with the environment. We will not analyze the causal behavior of the QBM model (even though potential causality issues have been observed in the literature [41]) but we will draw attention to some related features. Overall, it remains an open challenge to formulate notions of causality that are beyond the scattering-like treatment of detector models in QFT and beyond perturbation theory [6] (e.g. for cases in which the causal factorization of detector-field interactions does not hold).

The content and the structure of this thesis

Chapter 2 can be viewed as an extension of the introduction, as it is based on review material that we presented in [27, 28]. First, we identify some episodes in the early history of QFT that are relevant for the formulation of local QFT measurements. This is valuable for tracking the role that the scattering paradigm played in these developments [42], and for revisiting the arguments that went into the initial debates around quantum field measurability [43]. Then, we will explicitly present the 'impossible measurements' arguments [1, 44] as a no-go result with the logical form of a reductio argument and investigate the consequences for measurement in QFT. This is useful for classifying all possible responses to 'impossible measurements', and for setting up the ground for analyzing 'impossible measurements' that are induced by detector-field interactions in the next chapter (based on our analysis in [11]).

In chapter 3 we focus on causality and signaling in detector models in QFT. We define a signaling estimator based on quantum metrology that captures non-trivial dependencies of signaling on the internal states and the internal frequencies of the detector systems. This estimator can also be used to characterize causality for non-compact detector-field interactions, leading order in perturbation theory [12]. Non-perturbatively, we prove the causal factorization of compactly supported and causally orderable detector-field interactions. We demonstrate that causal factorization is sufficient for blocking superluminal signaling and retrocausation in bipartite scenarios, but it is not enough for blocking the Sorkin-type problem.

[6] It is scattering-like in the sense that the detected is measured after the interaction is off (not necessarily in the asymptotic future).

In chapter 4 we address the question "what do detectors detect?". For some detector systems, the question can be phrased as "when and where does a detector 'click', and why?". We demonstrate that the answer highly depends on the physical interaction that one chooses for probing the field's state locally, for particle-like states of the field that are defined by means of local creation operators [45]. Typically, fixed-particle states of the field (e.g. wavepackets) are defined by means of the global momentum-space creation and annihilation operators. Accordingly, the particle-like phenomenology relies on the phenomenon of resonance, i.e., the transition between the detector's energy levels ('click') that happens when, roughly, the energy of the wavepacket matches the detector's energy gap. We study resonance in detail, for the Unruh-DeWitt model as well as for a solvable model that we propose adapting the formalism of the quantum Brownian motion (QBM). In contrast to standard perturbation techniques, this model also recovers the relativistic Breit-Wigner resonant behavior in the weak coupling regime. For detector models that work in the strong coupling regime, we show that the detector's pointer variable is correlated with time-extended smeared field amplitudes, and it is insensitive to the particle content of the field's state. Finally, we demonstrate that the solvable model based on QBM confirms the association of field and particle properties in the strong and weak coupling regimes, respectively, but it can also describe the intermediate regime, in which the field-particle characteristics 'merge'. The modulation of particle-field duality by a single tunable parameter (the coupling strength) is a novel feature that is, in principle, experimentally accessible.

We summarize and conclude in chapter 5. Note that most of the calculations that go into the results presented in this thesis are in appendices in an attempt to maintain continuity. Of course, the appendices are referenced frequently in the main text. In the beginning of each chapter, there is a small introduction to the chapter and summary of the main results.

Chapter 2

Local measurements in QFT

This chapter starts with a historical flashback to the 1930's, when QM was already well established and von Neumann was publishing the famous axiomatic formulation [46], which initiated quantum measurement theory as we know it today. At the same time, QFT was in the making and people like Bohr, Rosenfeld, Landau and Peierls were arguing about quantum field measurability, and the very concept of a quantum field [43, 47]. The overlap between these two research fields (quantum measurement theory and quantum field theory) has not been significant in the decades to follow, mostly because QFT got established as a theory about scattering [42]. The no-go argument by Sorkin (1993) [1] is precisely about the incompatibility between standard measurement theory and QFT. Today, three decades after Sorkin's argument and nine decades after the initial debates, we are witnessing a convergence of the two topics and the development of a variety of approaches to local measurement theory for QFT [26, 14, 40, 48].

In this chapter, we briefly revisit the initial debates about quantum field measurability. Roughly, Bohr and Rosenfeld were the first to argue that the mathematical idealisation of 'field at a point' is not physically meaningful and that instead one can measure field averages over *extended* spacetime regions by coupling suitable test bodies to the quantum field. Then, we present Sorkin's argument about 'impossible measurements' in the form of a no-go result, to analyse the relationship between all recent responses (including the one that we presented in [11], see section 3.3). In the end of this chapter we motivate the introduction of detector models in QFT, as a useful tool for modeling local measurements in QFT, before we dive into certain aspects of detector models in the next chapters.

2.1 Some historical episodes

Bohr and Rosenfeld's 1933 [43] and 1950 [49] papers are two of only five references in Sorkin 1993 [1]. Sorkin mentions them as "one of the few attempts I know of to design concrete models of field measurements". We will first consider the historical context of Bohr and Rosenfeld's papers, and then discuss Sorkin's suggested use of Bohr and Rosenfeld's proposal.

The debate about quantum field measurability in the 1930's was centered around the uncertainty principle or, more generally, Bohr's complementarity. First, Heisenberg attempted to extend the uncertainty principle to a relativistic set-up to argue that the limitations on quantum field measurements are exactly analogous to the ones in non-relativistic quantum mechanics [50]. Landau and Peierls [47] argued that the limitations on quantum measurement are more severe in QFT than in quantum mechanics, challenging the physical basis of the theory. Bohr and Rosenfeld [43] responded to their argument, challenging their assumption of electrically charged pointlike particles as test bodies. Instead, they argue that one must consider spatially extended charged test bodies, whose atomistic structure can be ignored and whose charge density can be adjusted. As a result, the physical predictions of the theory would correspond to field averages over extended spacetime regions, and would not rely on the idealisation of 'field at a point'. By controlling the macroscopic charge density of the macroscopic test body one can control the effect of local field fluctuations and, envisioning a suitable compensation mechanism, the spacetime averages of field amplitudes over bounded regions can be measured in principle (up to the limitations that follow from the field commutation relations). The emphasis on the macroscopic aspect of the test body that is 'measuring' the quantum field is in line with Bohr's views about quantum measurement.

The debate between Bohr & Rosenfeld and Landau & Peierls about quantum field measurement has been characterized as the "small war of Copenhagen" [51]. This debate framed much of the discussion about the role of complementarity, the correspondence principle, and the relation of a (not yet settled) mathematical formalism for measurable quantities. From 1936 to 1946, with the gradual development and establishment of S-matrix theory by Heisenberg, there is a gradual shift from epistemological to more pragmatic arguments in the spirit of the S-matrix program [42]. After the renormalization of QED at the end of the 1940's, Bohr and Rosenfeld write the second paper on the measurability of QFT [49], where they reviewed the proposal for measuring field averages over an extended region, and they also propose an idealized arrangement for measuring charge-current densities over the boundary.

The proposed arrangement involves a distribution of test bodies over the boundary of a region for measuring the flux. They consider "the effect of the charge-current density appearing as a consequence of actual or virtual electron pair production by the field action of the displacement of the test bodies during the measuring process" and determine that "these effects, which are inseparably connected with the measurements, do not in any way limit the possibilities of testing the theory" [49]. In this second paper they do not put as much emphasis on the macroscopic aspect of the test bodies, since for accurate flux measurements the atomistic structure of the test bodies might come into play [52, p.411]. The issue of microscopic versus macroscopic, as well as quantum versus classical treatment of the measuring apparatus, continued to be relevant in the debate surrounding quantum measurement. In the 1960's, Rosenfeld and collaborators (Daneri, Loinger, Prosperi, see [53]) worked out an account of macroscopic quantum apparatus based on thermodynamical arguments ('irreversibility' of measurement records etc.), arguing against some of the philosophical consequences that followed from von Neumann's (and later Wigner's) account of measurements, such as interpretations of the state 'collapse' [54].

Sorkin suggests that Bohr and Rosenfeld's proposal for measuring smeared-field amplitudes might provide a testing ground for Sorkin's claim that there are ideal measurement scenarios in QFT in which superluminal signaling is predicted to occur. Sorkin suggests that the set-up in Bohr and Rosenfeld (1933) could be used to model ideal measurement in his proposed scenario if the 'classical' treatment of the apparatus were replaced by a quantum one "in order to learn how close they come to actually fulfilling the requirements for an ideal measurement" [1]. He elaborates that "specifically, one can ask whether they actually measure the field averages they claim to, and whether the probabilities of the different possible outcomes are those predicted by the quantum formalism (with special reference to the use of the projection postulate after the first measurement, since its effect could only be seen in a full quantum treatment)." Sorkin conjectures that the quantum version of Bohr and Rosenfeld's model for ideal measurements on fields in local regions will exhibit superluminal signaling when applied to Sorkin-type scenarios.

Essentially, Sorkin is calling for Bohr and Rosenfeld's proposed measurements of field averages to be modified to fit into the framework of von Neumann measurements, which is commonly used today. This is indeed very natural (and in section 4.1 we present how local field averages can get correlated with a continuous pointer variable that is locally coupled to the quantum field), nevertheless, it should be noted that Bohr and Rosenfeld's view of quantum measurement is very different in spirit than von Neumann's, in which another *microscopic* system is coupled to the quantum system that is to be measured. Moreover, Bohr and Rosenfeld intentionally avoided adopting von Neumann's framework for measurement, which laid the groundwork for what is today known as quantum measurement theory [46]. Both works were published in the early 1930's. Rosenfeld had studied what he later called the 'Neumanistics' ([54], p.30), and he later clarified that Bohr's goal had never been to provide a measurement theory in von Neumann's style [54]. Nevertheless, Sorkin's comments on the Bohr and Rosenfeld papers raise the following questions, that are partially addressed in the next two chapters: Does a microscopic probe that is dynamically coupled to the field actually measure field averages, and with what accuracy? What are the constrains imposed on the probe by the field commutation relations? Do we get impossible measurements by suitably applying the state update rule on probes that are locally coupled to the field?

2.2 Impossible measurements

As Sorkin's 1993 paper "Impossible measurements on quantum fields" [1] illustrates, the natural generalization of the non-relativistic measurement scheme to relativistic quantum theory fails because it entails superluminal signaling. Sorkin uses a minimal theoretical framework for relativistic quantum theory to construct examples of impossible measurements. He assumes the basic elements of ideal measurement theory for quantum mechanics, including Lüders' rule for state update for non-selective measurements. Measurement theory is adapted to Minkowski spacetime by making the natural assumption that causal order defines a partial temporal order. The microcausality principle that operators associated with spacelike separated regions commute is also imposed. When the system is not being measured, the Heisenberg picture representation for the dynamics is used. Sorkin produces examples of measurement scenarios that comply with all of these requirements, yet the expectation values for a measurement confined to one bounded region depend on the details of a measurement that is carried out in a spacelike separated bounded region. This conclusion is clearly unacceptable because it violates the prohibition on superluminal signaling or information transfer that is typically understood to be a hallmark of relativistic theories.

As we shall explain, Sorkin-type impossible measurement scenarios yield a no-go result that takes the form of *reductio ad absurdum* argument. Different approaches to formulating a measurement theory for QFT can be classified according to how they respond to this reductio argument.

2.2.1 The reductio argument

The 'impossible measurement' scenarios presented by Sorkin [1] and Borsten, Jubb, and Kells [55] are a type of no-go result. No-go results (such as Bell's theorem, or Malament's theorem) have played an important role in foundations of quantum theory because they identify a set of assumptions that cannot all be true. There is an essential difference between no-go theorems and no-go results, like the one we present here. No-go theorems can also take the form of a reduction argument, where the conclusion is provable based on mathematically-stated premises, i.e., a deductive argument. The 'impossible measurements' reductio argument is a more informal argument, i.e., the conclusion is established by producing examples that fulfill all the premises and they lead to the unacceptable conclusion. This is a significant difference, and one that makes no-go theorems much more powerful, but for our purposes what is important is that the informal 'impossible measurement' reductio argument serves the heuristic functions of motivating and guiding the formulation of a measurement theory for QFT.

Following the presentation of 'impossible measurement' examples in Borsten, Jubb, and Kells [55], here is the reductio argument:

P1 Local degrees of freedom: An observable A_k is associated with a region of Minkowski spacetime O_k by restriction of the field Φ to O_k [1]. We denote this as $A_k \in \mathcal{A}(O_k)$.

P2 Dynamics: When measurements are not being performed, use the Heisenberg picture representation (i.e., time-dependence is carried by the observables).

P3 Ideal measurement theory for relativistic quantum theory:

(a) <u>Detection assumptions</u>:

(i) eigenstate-eigenvalue link: the measurement outcomes are the eigenvalues of the self-adjoint operator corresponding to the observable [55]

(ii) Born rule: In a state ρ, the probability of an outcome n that corresponds to a projector E_n is given by $\text{Prob}(n) = \text{tr}(\rho E_n)$.

(b) <u>Preparation assumption</u>:

The state at time t' after a non-selective measurement is determined by applying Lüders' rule (for non-selective measurement) to the state at time t prior to the measurement.

Lüders' rule for non-selective measurement for arbitrary self-adjoint observables:

By the spectral theorem, $A = \int_{-\infty}^{\infty} \lambda dE(\lambda)$ where $E(\cdot)$ maps Borel subsets $B \subseteq \mathbb{R}$ to projectors on $\mathcal{H}$. For a set of mutually disjoint Borel sets $\mathcal{B} = \{B_n\}_{n \in \mathcal{I}}$ that covers $\mathbb{R}$ (with $\mathcal{I}$ some countable indexing set), each B_n represents a possible bin for a measurement outcome. The corresponding projectors (not necessarily rank-1) $E_n := E(B_n)$ resolve the identity $\sum_{n \in \mathcal{I}} E_n = \mathbb{1}_{\mathcal{H}}$. Lüders' rule for non-selective measurement for arbitrary self-adjoint observables:

$$\rho(t) \to \rho(t') := \mathcal{E}_{A,\mathcal{B}}(\rho(t)) = \sum_n E_n \rho(t) E_n \tag{2.1}$$

(c) Relativistic temporal ordering:

Define the temporal ordering relation $O_j \prec O_k$ iff some point of O_j causally precedes some point of O_k. Take the transitive closure of $\prec$. Regions must be chosen such that this extended $\prec$ is a partial order (i.e., cannot have both $O_j \prec O_k$ and $O_k \prec O_j$).

P4 Microcausality: If O_j and O_k are spacelike separated, then $[A_j, A_k] = 0$ for all $A_j \in \mathcal{A}(O_j)$, $A_k \in \mathcal{A}(O_k)$.

C Conclusion: There are bounded, spacelike separated regions O_1 and O_3 for which the expectation values of a measurement confined to O_3 depends on which unitary operation is performed in O_1.

That the conclusion of the argument follows from the premises is established by the examples set out in the following two subsections. Premise P3 sets out the assumptions of ideal measurement theory for relativistic quantum theory. Parts (a) the detection assumption and (b) the preparation assumption are carried directly over from NRQM. This is a generalization of Lüders' rule for non-selective measurement for discrete observables: For a compact self-adjoint observable A, $A = \sum_n \lambda_n E_n$, where λ_n are distinct eigenvalues and E_n are associated projectors onto associated eigenspaces that resolve the identity. A selective measurement is conditioned on obtaining the outcome λ_n. A non-selective measurement is not conditioned on obtaining any particular outcome. Lüders' rule for non-selective measurement for discrete observables: $\rho(t) \to \rho(t') = \sum_n E_n \rho(t) E_n$. As we will see, the notion of finite resolution of the measurement will play an important role in the analysis of impossible measurements.

The relativistic ingredient of P3 is (c), which specifies a temporal ordering relation for regions in Minkowski spacetime. Ordering spacetime regions is much more involved than ordering spacetime points. This is because for two spacetime points p,q we say that $\mathsf{p} \prec \mathsf{q}$ simply if a future-directed timelike curve connects $\mathbf{p}$ with $\mathbf{q}$ (then p causally precedes q). P3 makes use of this to define a partial order between regions, by declaring that a region that includes p causally precedes a region that includes q. P4 Microcausality is an uncontroversial assumption within QFT that is also known as Local Commutativity or Einstein Causality. As Sorkin [1] notes, $\prec$ may be extended to some non-unique linear order. P4 Microcausality ensures that different choices of linear order do not affect the expectation values for any sequence of projective measurements associated with the set of regions.

The conclusion is that there are bounded, spacelike separated regions O_1 and O_3 for which the expectation values of a measurement confined to O_3 depends on which unitary operation is performed in O_1. A further argument can be made that this conclusion is unacceptable because it allows for superluminal signaling. This further argument relies on additional assumptions, including assumptions of ideal measurement theory. As a consequence of the detection assumptions P3(a), the probabilities for measurement outcomes in O_3 are dependent on which measurements are carried out in spacelike separated region O_1. If we assume that parties can make multiple measurements on identically prepared systems to build up statistics following Borsten et al. [55], then in principle an observer in O_3 could determine whether a measurement was carried out in spacelike separated region O_1. This violates the prohibition on superluminal signaling or information transfer which is typically understood to be a hallmark of relativistic theories.

2.2.2 Examples that establish the reductio argument

Sorkin offers two versions of his no-go result, a QFT version and a QM version with qubits on Minkowski spacetime. We will begin by reviewing the QFT version and then we will argue that the QM version is not so compelling. Consider O_1 and O_3, two bounded spacelike separated regions of Minkowski spacetime, and a unitary element of the local algebra $\mathcal{A}(O_1)$ that is characterized by a parameter λ, i.e., $U_\lambda \in \mathcal{A}(O_1)$. This can be thought of as a local unitary 'intervention' that will transform the state of the field $|\psi_0\rangle \to U_\lambda |\psi_0\rangle := |\psi_1\rangle$. Independent of the interpretation of this 'local kick', prohibition of superluminal signaling entails that expectation values of observables outside the causal future of O_1 should not depend on the value of λ. In this case of two spacetime regions, this is guaranteed by microcausality, which

Figure 2.1: Region O_2 is a thickened hypersurface between O_1 and O_3.

imposes $[U_\lambda, C] = 0\ \forall\lambda$ and for all $C \in \mathcal{A}(O_3)$. As a result of microcausality

$$\begin{aligned} \langle\psi_1|C|\psi_1\rangle &= \langle\psi_0|U_\lambda^*\, C\, U_\lambda|\psi_0\rangle \\ &= \langle\psi_0|CU_\lambda^* U_\lambda|\psi_0\rangle \\ &= \langle\psi_0|C|\psi_0\rangle. \end{aligned} \tag{2.2}$$

This expectation value is independent of λ, and so the value of λ cannot be used to signal to spacelike separated regions.

The situation changes dramatically if one considers a third region O_2 'between' O_1 and O_3 that is partially in the causal future of O_1 and partially in the causal past of O_3. Roughly speaking, this third region can 'link' the first two in counterintuitive ways. The region O_2 is chosen by Sorkin to be a thickened hypersurface that lies in the chronological future of O_1 and in the chronological past of O_3 (see Figure 2.1). Associated with O_2 is a non-selective measurement of the projector $P_2 = |\psi_2\rangle\langle\psi_2|$. Applying the non-selective Lüder's rule to the state $|\psi_1\rangle = U_\lambda|\psi_0\rangle$, it is easy to see that the expectation values of C is

$$\langle C\rangle = \langle U_\lambda^* P_2 C P_2 U_\lambda\rangle_0 + \langle U_\lambda^*(\mathbb{1} - P_2)C(\mathbb{1} - P_2)U_\lambda\rangle_0, \tag{2.3}$$

where we denote with $\langle ...\rangle_0$ the expectation value over the state $|\psi_0\rangle$. This expression is equal to $\text{prob}(P_2 = 1)\text{Exp}(C, P_2 = 1) + \text{prob}(P_2 = 0)\text{Exp}(C, P_2 = 0)$ and will generally depend on λ. Sorkin is choosing a particular state $|\psi_2\rangle$ to be a superposition of the vacuum and an one-particle state to demonstrate the $\lambda-$ dependence, but the details of the derivation are not important. Simply one has to notice that, in general, the $\lambda-$dependence on the r.h.s. of (2.3) will not drop out (as it did in (2.2)) since U_λ is guaranteed to commute with C but *not* with $P_2 \in \mathcal{A}_2(O_2)$, because O_1 and O_2

are not spacelike separated. This λ-dependence instantiates the conclusion of the no-go result, since it allows for superluminal signaling between the spacelike separated regions O_1 and O_3 (bacause, in principle, a signal can be encoded in the value of λ).

To fully appreciate the no-go result, it is important to analyze the role of region O_2 in terms of the premises that are laid down in the previous section. Local Commutativity (P4) provides the ground for thinking of regions O_1 and O_3 as 'separate' or statistically independent in a *bipartite* scenario. By invoking a third 'intervention region' O_2 we open up the possibility of signaling between O_1 and O_3 (Conclusion). This is because the non-selective measurement that is associated with region O_2 updates or 'prepares' the state over which the expectation value of C is evaluated, in accordance with the standard rules set out in P3. More explicitly, the preparation assumption (b) (Lüders' rule) is used for the measurement over O_2, while the detection assumptions (a) go into the evaluation of the expectation value of C.

A temporal ordering relation $t < t'$ is needed to apply Lüders' rule. Premise (c) defines a relativistic temporal ordering relation, which reflects the causal structure of Minkowski spacetime, i.e., $O_i \prec O_j$ if O_j is partially in the causal future of O_i. Before the transitive closure is taken, this ordering relation does not apply for regions O_1 and O_3, that is, they are not 'comparable' and it does not hold that $O_1 \prec O_3$. Based on this ordering relation we can only claim that $O_1 \prec O_2$ and $O_2 \prec O_3$. Once we take the transitive closure to obtain a partial order, $O_1 \prec O_2$ and $O_2 \prec O_3$ implies $O_1 \prec O_3$ and we can apply the measurement rules accordingly. Then, the influence, or signaling, between the regions O_1 and O_3 is 'mediated' by region O_2, and it was made possible through taking the transitive closure (as Sorkin points out in [1]). Perhaps the involvement of a third region would partially demystify the conclusion, but from a local perspective of the observers that one could associate with O_1 and O_3, the *non-selective* measurements over O_2 should be irrelevant. Thus, one of the problems posed by this example is the consistent description of multi-partite measurements (involving more than two parties) in relativistic spacetimes. For an arbitrary number of measurements, one would have to extend the partial order to a total order (which always exists, but it is not unique).

Sorkin also offers a 'baby' QM version of the no-go result. In this case, there are two qubits that one can think of as embedded over regions O_1 and O_3 in Minkowski spacetime. The two qubits are initially in an entangled state, and the first one can potentially be flipped by a local unitary operation (analogue to the local unitary over region O_1) before a global projector is applied to the total system (analogue to the non-selective measurement over O_2). Evidently, the expectation values of observables of the second qubit (analogue to O_3) will generally depend on whether

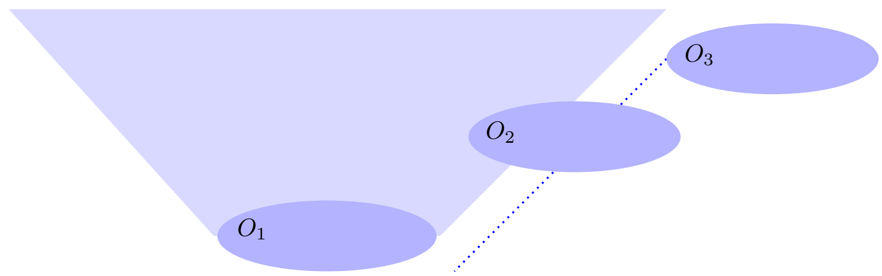

Figure 2.2: Region O_2 partially invading the future lightcone of O_1 and the past lightcone of O_3.

the first qubit was flipped or not before the global operation. This is not surprising because the global projection presupposes some notion of global access to the total system. Sorkin suggests that this example is "in a sense ... all we need, since one would expect to be able to embed it in any quantum field theory which is sufficiently general to be realistic" [1].

While it is true that NRQM should somehow be related to QFT, it does not follow that the QM example is sufficient for Sorkin's purposes. Precisely how NRQM relates to QFT is a non-trivial and somewhat controversial matter. It is not obvious which features of Sorkin's QM example should be expected to carry over to QFT. The value of Sorkin's quantum field theoretic example is that it clearly demonstrates which set of assumptions adapted from NRQM cannot be transferred to QFT. Furthermore, there are disanalogies between the two examples that seem relevant. In the case of the two qubits there is no third 'disjoint' party. The 'third' system is simply the total system. Of course, operations over the total system are by definition global. In the QFT example, there is a non-trivial third party O_2, seemingly 'disjoint' from O_1 and O_3. Nevertheless, that third party is responsible for an operation which, loosely speaking, would also 'connect' O_1 and O_3. Another disanalogy is that the initial state of the QM system must be entangled over the qubits, while there are no restrictions on the initial state in the QFT example. This is more obvious in an example given by Borsten et al. that is presented in the next section, which uses a factorized state.

At first glance, Sorkin's QFT example that illustrates his no-go result seems to be very particular to the choice of O_2 to be a thickened hypersurface (Figure 2.1). It is definitely bothersome, but not really surprising, that such *global* operations, like the one over region O_2, can cause signaling between the two spacelike separated parties.

The global projector represents an operation that presupposes some notion of global access to the total system. Sorkin recognizes this shortcoming of his example, but insists that there is still a genuine problem for QFT: "in a way it is no surprise that a measurement such as of $[A_2]$, which occupies an entire hypersurface, should entail a physical non-locality; but surprising or not, the implications seem far from trivial...What then remains of the apparatus of states and observables, on which the interpretation of quantum mechanics is traditionally based?"

Unfortunately, the problem raised by Sorkin cannot be easily dismissed by simply excluding global operations. Borsten et al [55] supply examples that establish that the problem persists for general *bounded* regions O_2 that partially invade the future lightcone of O_1 and the past lightcone of O_3, i.e., $J^+(O_1) \bigcap O_2 \neq \emptyset$ and $J^-(O_3) \bigcap O_2 \neq \emptyset$[1] (Figure 2.2). As we shall discuss in the next section, Borsten et al. posit a general condition on allowed local operators that guarantees no-signaling for non-selective measurement.

Some examples of seemingly innocent locally implementable operations that lead to 'impossible measurements' are given in [55] (and also [56]). For finite-dimensional Hilbert spaces, it is particularly interesting for quantum information purposes to analyse the causal behaviour of operations that correspond to measuring observables of the type $\hat{A} \otimes \mathbb{1} + \mathbb{1} \otimes \hat{B}$ versus $\hat{A} \otimes \hat{B}$ on the tensor product of two local subsystems $\mathcal{H}_1 \otimes \mathcal{H}_2$. In [55] it was shown that the latter can be problematic, despite the expectation that such a 'factorized' operation should be locally implementable in the Hilbert space sense (by means of local operations and classical communication (LOCC)).

2.3 Eliminating the 'impossible'

Sorkin-type impossible measurement scenarios clearly illustrate the moral that microcausality (P4) is not by itself sufficient to rule out superluminal signaling in relativistic quantum theories. There are three general strategies for responding to a reductio argument: (1) rejecting one (or more) of the premises, (2) adding a premise that blocks the derivation of the conclusion, or (3) arguing that the conclusion is only apparently unacceptable and can actually be tolerated. We will investigate the former two approaches.

[1] The causal future/past $J^{+/-}(x)$ of a spacetime point x is the set of all points reached from x by smooth future-directed causal curves. For a spacetime region O we write $J^{\pm}(O) = \bigcup_{x \in O} J^{\pm}(x)$ [26].

2.3.1 Ad hoc approach

The most straightforward response is an ad hoc one: target P1 and P3, which taken together entail that the measurable observables include all A_k that can be obtained by restricting the field Φ to any region O. An ad hoc resolution of the reductio can be obtained by simply excluding any observable that can lead to superluminal signaling. Sorkin proposes (but does not endorse) restricting the regions to which observables may be assigned. For example, imposing the restriction that measurable observables may only be defined on regions that are strictly causally ordered (i.e., for regions O_j and O_k, *all* $x \in O_j$ causally precede *all* $y \in O_k$ or vice versa). As Sorkin notes, it is difficult to imagine how the possibility of performing a measurement operation could depend on spacetime in this way (see also [57]). There are presumably not 'spacetime police' to ensure that laboratory measurements are only carried out when they are strictly causally ordered.

Borsten et al. [55] propose a different ad hoc resolution of the reductio that imposes a restriction directly on the observables rather than the associated regions (see also [56, 58]). They argue that the following condition rules out superluminal signaling by non-selective measurements in Sorkin-type scenarios:

$$\text{An operator } A_2 \in \mathcal{A}(O_2) \text{ with resolution } \mathcal{B} \text{ will not enable signaling iff} \\ [\mathcal{E}_{A_2,\mathcal{B}}(A_3), A_1] = 0, \text{ as an operator equation, for all } A_{1,3} \in \mathcal{A}(O_{1,3}), \tag{2.4}$$

where $\mathcal{E}_{A_2,\mathcal{B}}$ is the (dual) operation that is defined in (2.1) [2]. One way to understand this condition is the following: for measurements of A_1 and A_3 to be statistically independent in a scenario where an operation is performed over region O_2, it is not sufficient that A_1 commutes with A_3, but it should commute with the outcome of the dual non-selective map on A_3 (and a given measurement resolution). Again, the logic is that this condition is imposed for the purpose of excluding superluminal signaling. The condition can be enforced by 'banning' observables A_2 that do not satisfy it, or else bringing in some notion of coarse-graining that entails a measurement resolution that is large enough for the criterion to be met [3]. Both options are ad hoc, as long as they are demanded only to avoid superluminal signaling, and would have to be further motivated on physical grounds. The ad hoc approach is agnostic regarding

[2]Comparing with (2.1), the operation in (2.4) is the dual operation, since it acts on the observables and not the states. See subsection 3.3.2 for a concrete example.

[3]In [59] the measurement resolution is introduced by considering Gaussian measurements. It is pointed out that, in particular examples, the allowed accuracy of a Gaussian measurement is determined by all future experiments. Some mechanism would have to constrain future experiments accordingly.

the details of how such a measurement would be implemented (model-independent) but it provides a consistency check for a given model.

2.3.2 The detector models approach

Detector models in QFT are introduced to model the coupling of non-relativistic quantum mechanical detector systems to relativistic quantum fields. The relativistic quantum fields are not 'directly' measurable, but can only be measured indirectly through their dynamical coupling to a *controllable* detector system, or probe[4]. This von Neumann-like approach involves modeling the measurement as a dynamical process, where the quantum field and the probe are suitably coupled for a finite (or possibly infinite) time, the measurement duration. After the coupling has been switched off (or becomes negligible), the probe can be directly measured, and quantum measurement theory is applied to the detector system. The outcomes can be translated into statements about the quantum field, at least in principle. In this sense, the quantum field is measured by the detector system. If the detector-field coupling is local, then the detector can be thought of as a local probe that is locally measuring the quantum field.

The requirement that the detector system is controllable is more naturally fulfilled if the detector is chosen to be a non-relativistic system. This means that it is well-described by non-relativistic quantum mechanics and the measurement theory that comes with it. Crucially, we can consider projective measurements over the detector system with the usual Lüders state update rule and probability assignments that correspond to each possible outcome. This is an advantage because the notion of a measurement outcome that is associated to a finite-rank projector is typically not available in QFT, as an implication of the Reeh-Schlieder theorem and arguments that local algebras are generically Type III von Neumann algebras, which by definition do not contain finite-rank projectors [60, 26, 61]. It is not clear that generalizing from projectors to POVMs addresses this problem because the spectral theorem no longer holds, and therefore cannot be appealed to as support for the interpretation of the probabilities as probabilities of measurement outcomes [62]. Even though it can be a relief that the usual notion of a measurement outcome can be maintained

[4]Often the terms 'detector' and 'probe' are used interchangeably, especially if it is not clear from the context whether we are modeling a macroscopic or microscopic detector coupled to the field. A microscopic quantum mechanical system (like a spin or an atom) is commonly called a 'probe' of the field, while this term is not used for explicitly macroscopic detector systems (like a superconducting qubit, see below).

through the introduction of a detector system, the association of detector outcomes with induced field observables is far from straightforward. Partial answers to the question 'What do detectors detect?' have been given [63, 64, 13], but a systematic account is still missing.

Originally particle detector models were introduced to extract particle phenomenology in QFT (in curved spacetimes) related to the Unruh and Hawking effects [3, 65]. Since quantum field theories do not permit a particle ontology [66, 7], this motivated the operational approach that 'a particle is what a particle detector detects' advocated by Davies and others [3, 67, 7]. The Unruh-DeWitt detector model has become a paradigm example in the field of Relativistic Quantum Information (RQI). RQI was born out of the need to merge quantum information theory with relativity theory, using tools from QFT [68, 2]. RQI describes quantum communication through quantum fields (e.g. [69]) and the entanglement structure of QFT by locally coupling multiple detectors to the quantum field (e.g.[35, 70]). In the realm of quantum information, the notion of operations performed in local regions that is informally used in the application of quantum mechanics becomes central. As we argue in [28], RQI-related research has revitalized the topic of formulating local measurements for QFT.

2.3.3 Other approaches

Fewster and Verch recently proposed a measurement framework (FV framework) for algebraic quantum field theory (AQFT) [26], in which they adopt a 'top down' approach that aims to treat measurements in general and quantum field systems in general. Both the quantum field system and the measurement probe are modeled using AQFT. The initial motivation for this approach was to provide a framework in which the localization properties of observables of Unruh-DeWitt detectors could be studied [71]. Subsequently, their framework was used to addresses the 'impossible measurements' problem [29]. The strategy involves rejecting many of the premises of the 'impossible measurements' reductio argument as well as adding as premises axioms from AQFT. In particular, a new measurement theory for AQFT is formulated to replace much of premise P3. In the axiomatic context of AQFT, it is recognized that microcausality by itself is insufficient to rule out superluminal signaling for reasons unrelated to impossible measurements (see [17, 23] and the discussion in the introduction); additional dynamical axioms or assumptions are needed, like the axiom of local primitive causality. An important goal of this approach is the principled one of determining which physical principles are needed to consistently represent relativistic quantum systems and the measurements performed on them.

As we have stressed, the detector models approach is pragmatic in spirit. The adoption of different approaches means that the detector models approach and the FV framework prioritize different goals. A central goal of the detector models framework is to construct models that adequately describe realistic detectors, including detectors that can actually be built in a lab. In contrast, the FV framework has the primary goal of supplying a framework for measurement in QFT that is generally applicable. Many pragmatic choices are made in the course of constructing a model for a particular detector, including the use of NRQM to model the detector, the smearing functions and field-detector couplings, and the acceptance of FAPP arguments ruling out impossible measurements. In contrast, the FV framework focuses on formulating a fully relativistic measurement theory based on the general physical principles of AQFT in which impossible measurement scenarios cannot arise at all. There is a lot of interesting work to be done in (search of) the overlap of the two approaches (see e.g. [72]), as well as interesting interpretational questions (see e.g. [73]).

Finally, it is important to point out that Sorkin's motivation for formulating the 'impossible measurements' issue is to advocate for the sum-over-histories approach to quantum theory. As he puts it in the abstract of [1], "It is argued that this problem leaves the Hilbert space formulation of quantum field theory with no definite measurement theory, removing whatever advantages it may have seemed to possess vis a vis the sum-over-histories approach, and reinforcing the view that a sum-over-histories framework is the most promising one for quantum gravity." To the best of our knowledge, one cannot find a complete response to the Sorkin problem in the histories literature, even though the problem is clearly articulated in older [74] and more recent [75, 76] literature. Here we will simply expose Sorkin's arguments in favor of histories-based approaches with some comments.

Sorkin presents the following dilemma: one can either further restrict the allowed-measurement regions and the corresponding ordering relation, or else select the allowed observables on "some more ad hoc basis." In his words, the problem is "foreshadowed by our need to take a transitive closure in defining $\prec$" (see premise P3c) and as a result one could "further restrict the allowed measurement regions O_j in such a manner that the transitive closure we took in defining $\prec$ would be redundant. For example, we could require that for each pair of regions O_j, O_k all pairs of points $x \in O_j$ and $y \in O_k$ be related in the same way" [1, p.9]. Of course, this would block the Sorkin problem by excluding the configuration of regions in figures 2.1 and 2.2. This further restriction of $\prec$ would imply that one can only consider temporal supports that consist of spacetime regions that are pairwise related like two 'thickened' spacetime points (fully spacelike, fully timelike, or fully lightlike), blocking the possibility that a region partially invades the forward lightcone of another region. This is a global restriction, perhaps an 'all-at-once' constraint as in [77], that is hard to reconcile with the local perspective. As Sorkin puts it "it is difficult to see how the ability to perform a measurement in a given region—or the effect of that measurement on future probabilities—could be sensitive to whether some other measurement was located totally to its past, or only partly to its past and partly spacelike to it" [1]. It is also noteworthy that the initial definition of $\prec$ already excludes cases that might be of physical interest, such as overlapping regions, which were considered by Bohr and Rosenfeld in [43], and regions that intersect the causal past of each other, which are relevant for the study of possible spacetime embeddings of general process matrices [78]. In general, by restricting $\prec$ one might exclude physically interesting cases. Finally, regarding the second possibility of restricting the allowed observables, Sorkin points out that the inability of two coupled subsystems to signal through the measurement of an observable that is additive suggests that one could still allow integrals of spatially smeared observables over a spatial subset of a hypersurface (see [76]). However, *spacetime* smeared fields do not possess this additive character due to the time-extension (see, e.g., [79]).

A histories-based formalism that explicitly treats QFT measurement through the introduction of coarse-grained pointer variables is the Quantum Temporal Probabilities (QTP) formalism [80, 40]. Joint probabilities of the pointer variables are defined by means of unequal-time correlation functions, and the consistency condition is satisfied for a certain degree of coarse-graining (see also [81]). A connection of this formalism to the closed-time-path (CTP) integral was recently established in [40]. In general, as is also pointed out in [75], it is not obvious in general how to establish standard causality conditions in the path-integral formalism ('in-out' formalism) beyond scattering theory (see discussion in [42]). The CTP formalism (Schwinger-Keldysh or 'in- in' formalism [82, 83]) is better suited for analysing the causal behaviour of *local* QFT measurement, thanks to its emphasis to real-time causal evolution [84]. Also, the QTP program demonstrates how the CTP formalism provides the 'right' correlation functions that go into the definition of joint probability distributions over outcomes of coarse-grained pointer variables that are locally coupled to the field [40]. Time is also treated as a random variable (in analogy to stochastic processes) and time-of-arrival problems can be described accordingly. It is work in progress to evaluate the causal behaviour in bipartite scenarios and in multi-partite Sorkin-type set-ups using this framework, and to fully analyse how the possibilities of signaling are encoded in the CTP correlation functions.

Chapter 3

Detector models in QFT: frictions with relativistic causality

In this chapter, we review the zoo of Unruh-DeWitt-type detector models in QFT, paying careful attention to the derived or ad hoc elements that are introduced by construction of the model. This is helpful for analyzing the signaling properties of each variant, and identifying sources of friction with relativistic causality. We start by analyzing signaling in general bipartite scenarios, defining a signaling estimator that is inspired by quantum metrology. Then we analyze tripartite scenarios, to see how 'impossible measurements' can arise in detector-induced measurements. As we will argue, the advantage of the detector approach is that such a causality violation can be quantified in terms of the relevant scales, and can be handled by specifying the regime of validity of the model on a case-by-case basis.

3.1 Constructing detector models

Like any other model, detector models are an addition to the underlying theory and, as a result, they are not a priori guaranteed to comply with its premises. Detector models raise a major concern when the underlying theory is relativistic QFT: Are the predictions of the non-relativistic model respectful of relativistic causality? This is a justified concern, especially because the detector is chosen to be a non-relativistic quantum-mechanical system and, as such, alien to Minkowski spacetime. From this perspective, the non-relativistic quantum-mechanical nature of the detector seems like a serious drawback. On the other hand, thanks to its non-relativistic nature, the detector system is localizable in the usual quantum-mechanical terms. First-quantized non-relativistic systems admit a position representation, which implies that their states will be representable, and localizable, by means of their spatial wavefunction. Such a representation is known not to be available for relativistic systems [5]. Relativistic quantum fields are localized in a different sense: they are operator-valued objects that are locally defined over space and time (e.g., in AQFT by associating algebras of observables to bounded spacetime regions [25]). Since the field (relativistic) and the detector system (non-relativistic) enjoy very different notions of localization, it is first important to clarify the sense in which they can be locally coupled. The fact that issues of localization are relevant for detector models in QFT is more explicit when one is not only considering the detector's internal degrees of freedom, but also the detector's position (of center-of-mass, see e.g. [65, 85, 86, 87]).

There is a zoo of detector models in the literature. The simplest version of the Unruh-DeWitt (UDW) model involves a scalar quantum field coupled to a non-relativistic quantum system (e.g. a two-level system, a harmonic oscillator, or an atom). There have been attempts to extend the model beyond the scalar field, e.g., to spinor fields [88], but this complication is not relevant for our purposes. Also, for simplicity, we will only refer to the case of linear coupling between the detector and the field, even though more complicated couplings, e.g., quadratic, have been investigated in the literature [88, 89, 90]. A careful treatment of the modeling of light-matter interaction with UDW-type detectors beyond the scalar approximation can be found in [91].

Perhaps the most well-known detector model that has been considered in this literature is the pointilke UDW detector model, in which it is assumed that the detector is coupled to the field over a timelike trajectory. This is prescribed by the interaction Hamiltonian that generates translations with respect to the proper time τ associated to the detector's trajectory. In the interaction picture, this Hamiltonian is given by

$$\hat{H}_{\text{int}} = \lambda \chi(\tau) \hat{\mu}(\tau) \otimes \hat{\phi}(\mathsf{x}(\tau)). \tag{3.1}$$

Here λ is the coupling strength, $\chi(\tau)$ is the switching function, which is usually assumed to be integrable, and $\mathsf{x}(\tau)$ is the trajectory of the detector parametrized by its proper time τ. The Hamiltonian couples the field along the worldline of the detector to an internal degree of freedom of the detector $\hat{\mu}$. In the case that the detector is represented as a two-level system with energy gap Ω, $\hat{\mu}$ is the monopole operator $\hat{\mu}(t) = e^{i\Omega t}\hat{\sigma}^{+} + e^{-i\Omega t}\hat{\sigma}^{-}$ where $\hat{\sigma}^{\pm}$ the operators that map between the ground and excited state. The point-like model can exhibit ultraviolet divergences related to the coincidence limit of the time-ordered n-point functions. One strategy for avoiding the divergences of the point-like model is to introduce a finite extension of the detector-field interaction through a smearing, but as we will see this introduces issues with the covariance and the causality of the model that are due to the extension of the interaction and are absent in the point-like model [92, 11].

The simplest generalization of the point-like interaction Hamiltonian (3.1) involves a linear coupling between the detector observable $\hat{\mu}(t)$ and the scalar field operator $\hat{\phi}(t, \boldsymbol{x})$

$$\hat{H}_{\text{int}} = \lambda \chi(t) \hat{\mu}(t) \otimes \int \mathrm{d}\boldsymbol{x} F(\boldsymbol{x}) \hat{\phi}(t, \boldsymbol{x}) \tag{3.2}$$

where the switching function $\chi(t)$ models the duration of the interaction between field and detector and the smearing function $F(\boldsymbol{x})$ specifies the spatial extension of the interaction (in the proper frame of the detector system) [93, 94]. The support of these functions specifies the spacetime region O over which the detector is coupled to the field, i.e., $O = \text{supp}\chi(t)F(\boldsymbol{x})$. If both the smearing and the switching functions are compactly supported, then the interaction region O is bounded. Note that the interaction region need not coincide with the (initial) localization region of the quantum-mechanical detector system. Commonly both functions (switching and smearing) are introduced as a phenomenological input of the model, especially when the detector system is macroscopic. The switching is modeling the mechanism for switching the interaction on and off, whenever such mechanism is available, [1] and the smearing is modeling the 'size' of the interaction, which in general will not coincide with the apparent size of the detector.

It is perhaps curious that even in the case of an explicitly macroscopic detector system (e.g. in superconducting circuits [95]) the physical intuition that 'the interaction happens where the detector is' is not fulfilled. In [95] the authors investigate the model-dependence of the predictions for different smearing functions and different cut-off functions that determine 'how many' field mode functions are relevant for the detector-field interaction. The result suggests that, in this case, the real shape and size of the macroscopic detector do not affect the prediction as much as the choice of a UV cutoff. This means that one can directly model the feature of finite extension, based on mathematical convenience, without worrying about how the microscopic details of the detector affect the smearing function. In other studies, where the detector is explicitly a microscopic probe (e.g. the electron of an atom coupled to the quantum electromagnetic field), the smearing function has been associated with the microscopic nature of the probe (e.g., the orbitals of a hydrogen atom interacting

[1] For elementary interactions the switching function is harder to motivate, since, for example, the coupling of an electron coupled to the electromagnetic field cannot be 'switched off'.

with the electromagnetic field in a light-matter interaction [70]). It is common to attribute the smearings to the detector (e.g. 'smeared' detector as opposed to pointlike detector) even though smearing the field operator is more accurate mathematically. Conceptually, it is preferable to attribute the smearing (the 'shape' of the interaction) to neither the detector nor the field but to their joint interaction. As they put it in [95], "the shape of the qubit cannot be determined just with an individual description of the qubit itself. Rather, this shape belongs neither to the qubit nor to the line but to both of them in interaction with each other, constituting a property that becomes evident and relevant in and through interactions between the relevant quantum systems".

Overall, the choice of switching and smearing functions is a crucial input of the model that can critically affect its predictions. This choice can be motivated by the underlying (i.e., microscopic) physics, first principles, mathematical convenience, or even aesthetics. On mathematical grounds, the smearing functions were first introduced as a cure to the UV divergences of the point-like model [4] where the detector interacts with the field in a point-like manner. The UV divergences of the point-like model come from the distributional character of the 'field at a point'. Concretely, the response function of a detector at leading order in perturbation theory is a function of the field's Wightman function and can be regulated in different ways through the introduction of suitable switchings and smearings [96, 97]. In this literature, it is typical that the smearing depends on a regulator ϵ (e.g., Gaussian/Lorentzian function) for the purpose of regularizing the response of a point-like detector (e.g., excitation probability) in the limit $\epsilon \to 0$ [96]. Without taking the limit, an infinitely extended smearing function is unphysical since it implies a 'non-local' coupling between the field and the detector in all space.

Finally, let us consider the 'covariant' generalization of the Unruh-DeWitt interaction Hamiltonian [94, 92], where the switching and the smearing come together to form a *spacetime* smearing function $\Lambda(\mathsf{x})$ e.g. $H_{int}(t) = \lambda \int dV \Lambda(\mathsf{x}) \hat{\mu}(\tau(\mathsf{x})) \otimes \hat{\phi}(\mathsf{x})$. In this interaction Hamiltonian both the field and the detector operator are 'smeared' by Λ in the sense that the detector inherits spatial dependence through its proper time $\tau = \tau(\mathsf{x})$ in a general reference frame with coordinates x. For example, if we are considering Lorentz boosts in Minkowski spacetime, $\hat{\mu}(\tau) = \hat{\mu}(\gamma(t - \boldsymbol{v}\boldsymbol{x}))$ in a boosted frame with coordinates $(t, \boldsymbol{x})$. The detector observable $\hat{\mu}$ is only time- (and not space-) dependent in its proper frame, where the spacetime smearing function factorizes like $\Lambda(\mathsf{x}) = \chi(\tau) F(\boldsymbol{x})$ in terms of the switching and the smearing functions. The time duration and the spatial extension of the interaction can only be defined separately in the detector's proper frame (e.g. Fermi normal coordinates in curved spacetime [92]), while they mix in a general reference frame [93, 94, 98]. This 'covariant' form of the interaction Hamiltonian was proposed in [94] for a consistent description of detector physics in curved spacetimes, even though the model fails to be fully covariant due to the non-relativistic nature of the detector [92].

To give a definition of what we mean by a non-relativistic detector model, it will be useful to write the interaction Hamiltonian (density) in the following general form that we introduced in [11] (see also [80])

$$\hat{h}(\mathsf{x}) = \lambda\Lambda(\mathsf{x})\hat{\mathcal{J}}(\mathsf{x}) \otimes \hat{\phi}(\mathsf{x}) \tag{3.3}$$

where $\hat{\mathcal{J}}(\mathsf{x})$ is a current operator that is associated with the detector. This form of the interaction Hamiltonian covers the zoo of detector models that one finds in the contemporary literature (see [80]). In principle, the detector current (through which the particle detector couples to the field) could be derived using an effective field theory approach (e.g.[80, 99, 100]). We say that the detector model is non-relativistic if the detector current is not microcausal over the extension of the interaction region, that is, if

$$\left[\hat{\mathcal{J}}(\mathsf{x}), \hat{\mathcal{J}}(\mathsf{x}')\right] \neq 0 \quad \text{where}\, \mathsf{x}, \mathsf{x}' \in O \,\text{are spacelike separated.} \tag{3.4}$$

This characterization is not about the state of motion of the detector system (how fast it accelerates) or whether it is described by field degrees of freedom. It has to do with how we 'embed' the detector's dynamics in spacetime. For example, a detector system that is described by many discrete degrees of freedom e.g. by the non-relativistic QFT that is used in condensed matter physics will not satisfy (3.4). Or a smeared detector model whose internal dynamics is defined by non-relativistic quantum mechanics will not satisfy (3.4) independently of its state of motion.

In general, the microcausality condition (3.4) will not be satisfied by the detector current operators when one considers spacelike separated points within the extension of the interaction region $O = \text{supp}\Lambda$ due to the non-relativistic dynamics of the detector system [11]. This observation will become important when analyzing the frictions with relativistic causality in the following subsections. Note that the pointlike detector model trivially satisfies (3.4) and is fully causal, since there are no spacelike separated points over the detector's timelike trajectory. Nevertheless, as we mentioned above, the pointlike model suffers from divergences. We see that there is a trade-off between the divergent behavior and the causal behavior of the models: attributing extension to the detector system for avoiding the divergences leads to frictions with relativistic causality. In the next sections we will describe how this can be managed in the context of detector models.

3.2 Signaling between detectors

As we discussed in the previous chapters the relativistic QFT satisfies several notions of relativistic causality at the kinematical and dynamical level. The microcausality is a kinematical condition that roughly states which spacetime regions are in a sense 'independent'. We also discussed that detector models are not necessarily respectful of the notions of relativistic causality of the underlying theory. This raises the question: are the signaling relations between detectors coupled to the field sufficiently constrained by microcausality, or other dynamical assumptions for the QFT? Does microcausality prevent two detectors that are coupled to the field over spacelike separated regions from signaling to each other? We will answer these questions in the general case, including detector-field interactions that are non-compact in space and time (e.g. always 'on') and cases in which more than two detectors are involved.

3.2.1 The role of microcausality

Consider two detectors in spacelike separated regions. For example, consider two two-level systems A,B coupled to the field through the interaction Hamiltonian

$$\hat{H}_{\text{int}} = \sum_{\nu=\text{A,B}} \lambda_\nu \chi_\nu(t) \hat{\mu}_\nu(t) \otimes \int \mathrm{d}\boldsymbol{x} F_\nu(\boldsymbol{x}) \hat{\phi}(\boldsymbol{x}, t) \tag{3.5}$$

Since the two detectors are not directly coupled to each other, the question is: how much signaling can be 'transmitted' through their coupling to the quantum field? Does microcausality prevent them from signaling superluminally?

Previously, it was shown in [37] that after A and B have interacted with the field, and assuming that A interacts with the field before B in some reference frame [2], the state of detector B at leading order in perturbation theory is

$$\hat{\rho}_{\text{B}}^{(2)} = \lambda_{\text{A}} \lambda_{\text{B}} \hat{\rho}_{\text{B,signal}}^{(2)} + \sum_{\nu=\text{A,B}} \lambda_\nu^2 \hat{\rho}_{\nu,\text{noise}}^{(2)} \tag{3.6}$$

where the noise term is local on detector B, and all the influence of the presence of detector A on detector B's density matrix is captured by the 'non-local' term that is proportional to $\lambda_{\text{A}}\lambda_{\text{B}}$. This signaling part of the density matrix can be written as

$$\hat{\rho}_{\text{B,signal}}^{(2)} = 2 \int \mathrm{d}t \mathrm{d}t' \chi_{\text{A}}(t) \chi_{\text{B}}(t') \mathcal{C}(t, t') \hat{d}(t, t') \tag{3.7}$$

[2] As we will see next, this is a non-trivial assumption that does not hold in the general case when the switching functions are not compactly supported over two distinct time intervals.

where

$$\mathcal{C}(t,t') := \int \mathrm{d}\boldsymbol{x}\mathrm{d}\boldsymbol{x}' F_{\mathrm{A}}(\boldsymbol{x}) F_{\mathrm{B}}(\boldsymbol{x}') \langle [\hat{\phi}(t,\boldsymbol{x}), \hat{\phi}(t',\boldsymbol{x}')] \rangle. \tag{3.8}$$

Without going through the details (since we will generalize these expressions below), in (3.7) we denote as $\hat{d}$ an operator on the Hilbert space of B that depends on the states and the internal frequencies of the detectors A,B. The important point is that if both smearings are compactly supported, the integrations in (3.7) and (3.8) are performed over two disjoint and spacelike separated spacetime regions, $\mathrm{supp}\chi_{\mathrm{A}}F_{\mathrm{A}}$ and $\mathrm{supp}\chi_{\mathrm{B}}F_{\mathrm{B}}$. Then, microcausality guarantees that the field commutator vanishes in spacelike separation, and there is no superluminal signaling between the two detectors at second order in perturbation theory [37]. In the case of point-like interactions, this argument can be extended to higher orders in perturbation theory [3] [101].

In [12] we generalized (3.7) and (3.8) for general detector models (possibly in curved spacetimes) using the Hamiltonian density (3.3)

$$\hat{H}_{\mathrm{int}}(\tau) = \sum_{\nu=\mathrm{A,B}} \lambda_\nu \int_{\mathcal{E}(\tau)} \mathrm{d}\mathcal{E}\, \hat{J}_\nu(\mathsf{x}) \otimes \hat{\phi}(\mathsf{x}). \tag{3.9}$$

$\mathcal{E}(\tau)$ is a one-parameter family of spacelike surfaces, where τ is a global function whose level curves represent the planes of simultaneity of the detector's center of mass and (under some assumptions [94]) τ is the detector's proper time. $\mathrm{d}\mathcal{E}$ denotes the family of induced measures on the surfaces $\mathcal{E}(\tau)$. Note that we have assumed that the two detectors share the same proper time. Also, for convenience, we have absorbed the spacetime smearing function in the definition of the detector current operator, i.e., $\hat{J}_\nu(\mathsf{x}) := \Lambda_\nu(\mathsf{x})\hat{\mathcal{J}}_\nu(\mathsf{x})$ (comparing with (3.3)).

If we assume that the state is initially uncorrelated, i.e. $\hat{\rho}_{\mathrm{initial}} = \hat{\rho}_{\mathrm{A}} \otimes \hat{\rho}_{\mathrm{B}} \otimes \hat{\rho}_\phi$, the general expression for signaling is (derived in appendix A.2)

$$\hat{\rho}^{(2)}_{\mathrm{B,sign}} = -\mathrm{i}[\hat{\Sigma}, \hat{\rho}_{\mathrm{B}}] \;\text{ where }\; \hat{\Sigma} = \int\!\!\int \mathrm{d}V \mathrm{d}V' \,\langle \hat{J}_{\mathrm{A}}(\mathsf{x}')\rangle\, G_{\mathrm{R}}(\mathsf{x},\mathsf{x}') \hat{J}_{\mathrm{B}}(\mathsf{x}), \tag{3.10}$$

and where $G_{\mathrm{R}}(\mathsf{x},\mathsf{x}')$ is the retarded Green's function

$$G_{\mathrm{R}}(\mathsf{x},\mathsf{x}') = -\mathrm{i}\theta(\tau(\mathsf{x}) - \tau(\mathsf{x}'))\, \langle [\hat{\phi}(\mathsf{x}), \hat{\phi}(\mathsf{x}')] \rangle\,. \tag{3.11}$$

We see that, for general switching functions (dropping the assumption that the switching functions are compactly supported and non-overlapping), the role of the field commutator in (3.8) is played by the field's retarded Green's function in (3.10).

[3] In [11] we provided a non-perturbative argument for no-signaling based on the causal factorization of compactly supported (spacelike separated) detector-field interactions (see section 3.3).

The operator $\hat{\Sigma}$ can be understood as the current associated with detector B smeared by the propagated expectation value of the current associated with detector A. In the case of the massless Klein-Gordon field in a 3+1 dimensional flat spacetime, for instance, the propagator takes the familiar form of the Lienard-Wiechert potentials

$$G_{\mathrm{R}}[\langle \hat{J}_{\mathrm{A}} \rangle](t, \boldsymbol{x}) = \int \mathrm{d}^3\boldsymbol{x}' \frac{\langle \hat{J}_{\mathrm{A}} \rangle (t_{\mathrm{R}}, \boldsymbol{x}')}{2|\boldsymbol{x} - \boldsymbol{x}'|} \tag{3.12}$$

where $t_{\mathrm{R}} = t - |\boldsymbol{x}'|$ is the retarded time. We see that the operator $\hat{\Sigma}$ carries all the information about the signaling from detector A to B. In the next section we demonstrate that the variance of $\hat{\Sigma}$ bounds the Fisher information of B, i.e., the information that detector B can 'learn' about the coupling of A to the same quantum field.

Notice that if the 'source' $\langle \hat{J}_{\mathrm{A}}(\mathsf{x}') \rangle$ is spacelike separated from the 'receiver' $\hat{J}_{\mathrm{B}}$, $\hat{\Sigma}$ is the zero operator and there is no superluminal signaling. This is because $G_{\mathrm{R}}(\mathsf{x}, \mathsf{x}') \langle \hat{J}_{\mathrm{A}}(\mathsf{x}') \rangle$ is supported in the future lightcone of A's interaction region. Nevertheless, it is quite common in the detector literature to use smearing functions that are not compactly supported. For example, Gaussian smearings are chosen for the sake of computational convenience and analytical results. In their seminal paper on the Unruh effect [65], Unruh and Wald introduce the coupling of the position operator $\hat{\boldsymbol{x}}_t$ (e.g. of an electron) to the field as

$$\hat{H}_{\mathrm{int}} = \lambda \chi(t) \int \mathrm{d}\boldsymbol{x}\, \hat{\phi}(t, \boldsymbol{x}) \otimes \delta(\boldsymbol{x} - \hat{x}_t). \tag{3.13}$$

In this interaction Hamiltonian, the field operator is defined over the spectrum of the position operator of the non-relativistic particle, and the particle's current is $\hat{J}(\mathsf{x}) = \chi(t)\delta(\boldsymbol{x} - \hat{\boldsymbol{x}}_t)$. This type of interaction Hamiltonian can resemble the dipole coupling in the light-matter interaction [70, 64]. To make sense of this (3.13) it is useful to bring it in a more familiar form by representing $\delta(\boldsymbol{x} - \hat{\boldsymbol{x}}_t)$ in the detector's Hilbert space. The Dirac delta distribution $\delta(\boldsymbol{x} - \hat{\boldsymbol{x}}_t)$ is defined over an (interaction picture) orthonormal basis $\{|i_t\rangle\}$ of the Hilbert space of the detector:

$$\begin{aligned} \delta(\boldsymbol{x} - \hat{\boldsymbol{x}}_t) &= \sum_{ij} \langle i_t | \delta(\boldsymbol{x} - \hat{\boldsymbol{x}}_t) | j_t \rangle \, |i_t\rangle\langle j_t| \\ &= \sum_{ij} \int \mathrm{d}^n \boldsymbol{x}' \langle i_t | \delta(\boldsymbol{x} - \hat{\boldsymbol{x}}_t) \, |\boldsymbol{x}'_t\rangle \, \langle \boldsymbol{x}'_t | \, j_t \rangle \, |i_t\rangle\langle j_t| \\ &= \sum_{ij} \psi_i^*(\boldsymbol{x}) \psi_j(\boldsymbol{x}) \, |i_t\rangle\langle j_t| \, . \end{aligned} \tag{3.14}$$

If we define $F_{ij}(\boldsymbol{x}) = \psi_i^*(\boldsymbol{x})\psi_j(\boldsymbol{x})$ the interaction Hamiltonian (3.13) becomes

$$\hat{H}_{\text{UW}} = \lambda\chi(t)\sum_{ij}\int \mathrm{d}^n\boldsymbol{x} F_{ij}(\boldsymbol{x})\hat{\phi}(t,\boldsymbol{x})\otimes |i_t\rangle\langle j_t| \tag{3.15}$$

which consists of multiple terms of smeared field operators coupled to rank-one detector operators that correspond to all possible transitions between the elements of the basis $\{|i_t\rangle\}$, which we can understand as the eigenstates of a chosen observable. Note that the smearing function of the field operator is not introduced by hand in (3.15), in contrast with (3.5). The smearing functions $F_{ij}(\boldsymbol{x})$ are not a freedom of the model, and instead they are determined by the kind of transition we are interested in.

The expectation value of the current in (3.13) is

$$\langle \hat{J}_{\text{A}}\rangle(\mathsf{x}) = \chi_{\text{A}}(t)\langle\delta(\boldsymbol{x}-\hat{\boldsymbol{x}}_t^{\text{A}})\rangle = \chi_{\text{A}}(t)|\psi_{\text{A}}(t,\boldsymbol{x})|^2. \tag{3.16}$$

If we plug this current into (3.10) we see that there is non-zero signaling even if the detectors are ‘centered’ in spacelike separation. Intuitively, the detectors are ‘overlapping’ even when in spacelike separation due to the quantum-mechanical ‘tails’. These ‘tails’ are obscuring relativistic causality when the detectors are put in contact with the underlying relativistic QFT. This contact between the non-relativistic detector system and the relativistic quantum field is a unique feature of non-relativistic detector models.

As we will see next, in [12] we analyzed and quantified the apparent causality violations introduced by two detectors that are *mostly* spacelike separated (when the overlap of their ‘tails’ is ‘small’) from the perspective of quantum metrology. Perhaps counter-intuitively, the causal ‘overlap’ depends not only on how fast the tails decay and on the characteristics of the spacetime, but also on the internal characteristics of the detector systems (e.g., the internal frequencies $\Omega_{\text{A,B}}$). Nevertheless, one can derive frequency-independent bounds to the information that B can gain for detector A’s interaction with the field [12]. Quantifying this cross-talk between distant detector systems is important in the analysis for entanglement harvesting, for which one needs to distinguish between genuine harvesting and the correlations that are established through communication [102, 36].

3.2.2 Signaling estimator

In this section, we generalize the signaling estimator defined in [37] for general detector models, beyond the assumption of compact support for the interaction and for general globally hyperbolic spacetimes. For this purpose, we define a signaling estimator based on an adapted version of the quantum Fisher information to perturbative regimes. In the previous section, we showed that the signaling between two detectors is governed by the operator $\hat{\Sigma}$ in (3.10). This definition, without further assumptions, is merely formal. Indeed, given an arbitrary globally hyperbolic space-time, the retarded Green's function G_{R} is guaranteed to exist as an ordinary distribution acting over *compactly* supported functions [103]. Therefore, it is not a priory guaranteed that the expression (3.10) makes sense if, e.g., the mean value of $\hat{J}_{\text{A}}(\mathsf{x}')$ is not compactly supported. However, it is known that this is not a problem in many of the common cases studied in the literature whenever there are no infrared ambiguities in the theory.

In the spirit of [37] one could be tempted to define a signaling estimator as the norm of the operator $\hat{\Sigma}$ or $\hat{\rho}^{(2)}_{\text{B,signal}}$ in (3.10) [4]. While this would be always well-defined for finite-dimensional particle detectors (e.g. two-level systems), this may not be well-defined for more general models (e.g. harmonic oscillators [104]). In particular, the operator $\hat{\Sigma}$ involves the smearing of the operator $\hat{J}_{\text{B}}(\mathsf{x})$, which does not have well-defined support as an operator in general. To build a meaningful signaling estimator for the general case, we are forced therefore to specify the particular configuration of the state of the detectors.

In what follows, we will define a signaling estimator based on quantum metrology [105], which is precisely concerned with the estimation of a parameter that is (dynamically) encoded in the state of a quantum system. Based on this signaling estimator, we will claim that there is no signaling if B cannot access the parametric information that is encoded in the state of detector A after its interaction with the field. To make it more concrete, we will establish that there will be no signaling if B cannot infer the value of the coupling constant λ_{A} through its local statistics, following the logic of local quantum estimation [105].

A core concept in parameter estimation in quantum metrology is the so-called quantum Fisher information [105, 106]. Given a family of density matrices that are dependent on some parameter, say λ, the quantum Fisher information yields lower bounds on the variance of the distribution of possible values of λ given some certain measurement statistics on the system. When the quantum Fisher information is close to zero, the statistical variance of the optimal parameter estimation grows to infinity, which is a consequence of the Cramer-Rao bound [105].

[4] Note that, $\hat{\rho}^{(2)}_{\text{B,signal}}$ is self-adjoint, but it is not guaranteed to be positive, as it can be seen from (3.10).

In our case, we are going to consider the dependence of the partial state of detector B on λ_{A} so that the quantum Fisher information estimates how much the information about whether A coupled to the field or not (and how strongly) is accessible to B. Note that the whole influence of any parameter of A on B is conditional to the coupling constant λ_{A} being nonzero. We will make direct use of the following result: given a one-parametric family of density matrices $\hat{\rho}(\lambda) = \sum p_m(\lambda) |m(\lambda)\rangle\langle m(\lambda)|$, and a generator $\hat{H}$ such that

$$\mathrm{i}\partial_\lambda \hat{\rho}(\lambda) = [\hat{H}, \hat{\rho}(\lambda)], \tag{3.17}$$

the quantum Fisher information at λ is defined as $\mathcal{F}(\lambda) = \mathrm{tr}(\partial_\lambda \hat{\rho}(\lambda) \hat{L}_\lambda)$ where $\hat{L}_\lambda$ the symmetrized logarithmic derivative of $\hat{\rho}(\lambda)$ [107]. In terms of the eigenstates and eigenvalues of $\hat{\rho}(\lambda)$ the Fisher information can be written as (for detailed derivation, see [106])

$$\mathcal{F}(\lambda) = 2 \sum_{\{p_l+p_m>0\}} \frac{|\langle l|\partial_\lambda \hat{\rho}(\lambda)|m\rangle|^2}{p_l + p_m}, \tag{3.18}$$

where the summation happens over l, m such that $p_l + p_m > 0$ and we have dropped the λ dependence to simplify the notation. Then, the Fisher information is bounded by the variance of the generator $\mathcal{F}(\lambda) \leq 4(\Delta_{\hat{\rho}_\lambda} \hat{H})^2$ [106].

Since in our setup the relevant parameter is the coupling strength of detector A and our signaling estimator will be defined around the regime of weak couplings, we can expand the Fisher information at the lowest orders in the coupling constants and compute the Fisher information around the value $\lambda_{\text{A}} = 0$. To be more explicit, we are interested in the estimation of the parameter λ_{A} (close to zero) from the local statistics of B after the interaction, at leading order in perturbation theory. From equations (3.6), (3.10), we find that

$$\partial_{\lambda_{\text{A}}} \hat{\rho}_{\text{B}}(\lambda_{\text{A}})|_{\lambda_{\text{A}}=0} = -\mathrm{i}\lambda_{\text{B}}[\hat{\Sigma}, \hat{\rho}_{\text{B}}] + \mathcal{O}(\lambda_{\text{B}}^2), \tag{3.19}$$

since λ_{B} is a constant that does not depend on λ_{A}. It is easy to see by direct substitution of (3.19) in (3.18) that the expression for the Fisher information of detector B with respect to the parameter λ_{A} is given by

$$\mathcal{F}_{\text{B}}(\lambda_{\text{A}})|_{\lambda_{\text{A}}=0} = 2\lambda_{\text{B}}^2 \sum_{\{p_l+p_m>0\}} \frac{(p_l - p_m)^2}{p_l + p_m} \left|\langle l|\hat{\Sigma}|m\rangle\right|^2 + \mathcal{O}(\lambda_{\text{B}}^3). \tag{3.20}$$

Note that at leading order, the change on $\hat{\rho}_{\text{B}}(\lambda_{\text{A}})$ with λ_{A} is given by taking the commutator with the self-adjoint operator $\hat{\Sigma}$, which means that at leading order the dependence on λ_{A} is given by the action of a unitary with generator $\hat{\Sigma}$. When the family of density matrices is generated by the action of a unitary group over a pure state, it holds that the quantum Fisher information coincides with four times the variance of the generator of the unitary [106]. This means that, at leading order,

$$\mathcal{F}_{\text{B}}(0) = 4\lambda_{\text{B}}^2 \left(\langle \psi_{\text{B}} | \hat{\Sigma}^2 | \psi_{\text{B}} \rangle - \langle \psi_{\text{B}} | \hat{\Sigma} | \psi_{\text{B}} \rangle^2 \right) + \mathcal{O}(\lambda_{\text{B}}^3). \tag{3.21}$$

When the unitary group acts over general mixed states, the variance gives an upper bound for the quantum Fisher information [106]

$$\mathcal{F}_{\text{B}}(0) \leq 4\lambda_{\text{B}}^2 \left(\langle \hat{\Sigma}^2 \rangle_{\hat{\rho}_{\text{B}}} - \langle \hat{\Sigma} \rangle_{\hat{\rho}_{\text{B}}}^2 \right) + \mathcal{O}(\lambda_{\text{B}}^3). \tag{3.22}$$

Therefore, we can define the following signaling estimator

$$\mathcal{S} = \langle \hat{\Sigma}^2 \rangle_{\hat{\rho}_{\text{B}}} - \langle \hat{\Sigma} \rangle_{\hat{\rho}_{\text{B}}}^2 , \tag{3.23}$$

namely, the variance of $\hat{\Sigma}$, generalizing the estimator defined in [37] to unbounded operators. In our case, the Fisher information estimates signaling by bounding the information that one detector can 'learn' about the coupling of other detectors to the same quantum field. Note that (3.23) is exact (at leading order) for initially pure detector states and provides an upper bound if the initial states are non-pure, as in (3.22).

For the purposes of quantifying whether the setup is devoid of apparent superluminal signaling, having an upper bound to the Fisher information is enough. Concretely, the predictions of a given model are reliable if the upper bound is sufficiently 'small' given the initial states of the detectors and the choice of background spacetime. In this sense, this estimator defines the regime of validity of each model. Nevertheless, this estimator may not be faithfully estimating the amount of signaling that the sender can transmit to the receiver, if simultaneously the variance is large, the detectors' operators are not bounded and the initial states are mixed. In that case, to obtain a faithful measure of signaling one would have to calculate the actual Fisher information (i.e., not only its upper bound) which can be involved depending on the model under consideration.

Substituting Eq. (3.10) into the signaling estimator (3.23), we get

$$\begin{aligned} \mathcal{S} =& \frac{1}{2} \iint \mathrm{d}V \mathrm{d}V' \, G_{\text{R}}[\langle \hat{J}_{\text{A}} \rangle](\mathsf{x}) G_{\text{R}}[\langle \hat{J}_{\text{A}} \rangle](\mathsf{x}') \, \langle \{ \hat{J}_{\text{B}}(\mathsf{x}), \hat{J}_{\text{B}}(\mathsf{x}') \} \rangle \\ & - (G_{\text{R}}[\langle \hat{J}_{\text{B}} \rangle , \langle \hat{J}_{\text{A}} \rangle])^2, \end{aligned} \tag{3.24}$$

where $\{\cdot,\cdot\}$ denotes the anti-commutator and

$$G_{\text{R}}[\langle \hat{J}_{\text{B}}\rangle, \langle \hat{J}_{\text{A}}\rangle] = \int \mathrm{d}V\, G_{\text{R}}[\langle \hat{J}_{\text{A}}\rangle](\mathsf{x})\langle \hat{J}_{\text{B}}(\mathsf{x})\rangle \tag{3.25}$$

is the 'overlap' of the expectation values of the currents, convolved with the retarded Green's function. This estimator captures the main features outlined in [37] to quantify signaling. A first consistency check is that, indeed, if the functions $\langle \hat{J}_{\text{A}}\rangle(\mathsf{x})$, $\langle \hat{J}_{\text{B}}\rangle(\mathsf{x})$, and $\langle\{\hat{J}_{\text{B}}(\mathsf{x}), \hat{J}_{\text{B}}(\mathsf{x}')\}\rangle$ are compactly supported, and if the support of these functions are space-like separated with respect to each other, then the estimator is zero, i.e., there is no signaling between strictly spacelike separated detectors.

The signaling estimator will not be zero if these functions are not compactly supported, but one would expect that detectors that are, in some sense, 'centered' (or 'strongly' supported) around a region cannot significantly influence events outside the future lightcone of this region. Therefore, for any notion of effective localization of a detector, we can define a notion of effective lightcone based on the signaling estimator (3.24). Roughly speaking, two detector interactions 'centered' in spacelike separation can be considered to be effectively spacelike separated if the estimator is negligible. It is useful to consider some examples in which non-compact smearings or switchings arise, to examine which quantities contribute to the signaling estimator between two distant systems. Also, to appreciate the role of the internal dynamics (e.g. the internal frequency of a two-level system, or the spreading of a particle's wavefunction).

Example: two two-level systems interacting with the field

Consider the case of two Unruh-DeWitt detectors that interact with a quantum field in a curved spacetime background. In the case of Unruh-DeWitt detectors, the currents can always be written covariantly as

$$\hat{J}_\nu = \sum_{s=\pm 1} \Lambda_\nu^s \hat{\sigma}_\nu^s \tag{3.26}$$

where the detector index $\nu \in \{\text{A}, \text{B}\}$ and where we have defined

$$\Lambda_\nu^s(\tau_\nu, \boldsymbol{x}_\nu) = \chi_\mu(\tau_\nu) F_\nu(\boldsymbol{x}_\nu) e^{\mathrm{i}s\Omega_\nu \tau_\nu} \tag{3.27}$$

for two different set of coordinates $(\tau_\nu, \boldsymbol{x}_\nu)$ for $\nu \in \{\text{A,B}\}$. The operators $\hat{\sigma}_\nu^{\pm 1} := \hat{\sigma}_\nu^{\pm}$ represent the ladder operators associated with each two-level system.

Given this decomposition, the operator $\hat{\Sigma}$ can be written as

$$\begin{aligned}\hat{\Sigma} &= \sum_{s,s'=\pm1} \hat{\sigma}^s_{\mathrm{B}} \langle \hat{\sigma}^s_{\mathrm{A}} \rangle \, G_{\mathrm{R}}[\Lambda^s_{\mathrm{B}}, \Lambda^{s'}_{\mathrm{A}}] \\ &= \sum_{s,s'=\pm1} \hat{\sigma}^s_{\mathrm{B}} \langle \hat{\sigma}^s_{\mathrm{A}} \rangle \, I(s'\Omega_{\mathrm{A}}, s\Omega_{\mathrm{B}}),\end{aligned} \tag{3.28}$$

where we have defined

$$I(s'\Omega_{\mathrm{A}}, s\Omega_{\mathrm{B}}) := G_{\mathrm{R}}[\Lambda^s_{\mathrm{B}}, \Lambda^{s'}_{\mathrm{A}}]. \tag{3.29}$$

Then, from the definition (3.23), the signaling estimator for this model is given by the variance of the operator $\hat{\Sigma}$ in equation (3.28)

$$\begin{aligned}\mathcal{S} = \sum_{s,s'=\pm1} \langle \hat{\sigma}^s_{\mathrm{A}} \rangle \langle \hat{\sigma}^{s'}_{\mathrm{A}} \rangle \Big[& I(s\Omega_{\mathrm{A}}, \Omega_{\mathrm{B}}) I(s'\Omega_{\mathrm{A}}, -\Omega_{\mathrm{B}}) \\ & - \sum_{u,u'=\pm1} \langle \hat{\sigma}^u_{\mathrm{B}} \rangle \langle \hat{\sigma}^{u'}_{\mathrm{B}} \rangle I(s\Omega_{\mathrm{A}}, u\Omega_{\mathrm{B}}) I(s'\Omega_{\mathrm{A}}, u\Omega_{\mathrm{B}}) \Big].\end{aligned} \tag{3.30}$$

Note that, in the case of two-level systems, one can maximize the signaling between the detectors for all possible states (the optimal value $\mathcal{S}_{max}$ will play a fundamental role in what follows). Indeed, we notice that the second term is negative and vanishes for $\langle \hat{\sigma}^{\pm}_{\mathrm{B}} \rangle = 0$. Then, if we write the expectation value of the ladder operators associated with detector A in polar form, i.e. $\langle \hat{\sigma}^{\pm}_{\mathrm{A}} \rangle = r e^{\pm \mathrm{i}\alpha}$ with $0 \leq r \leq 1$, we realize that the first term can be written as

$$\begin{aligned}\mathcal{S} &= r^2 \left| e^{\mathrm{i}\alpha} I(\Omega_{\mathrm{A}}, -\Omega_{\mathrm{B}}) + e^{-\mathrm{i}\alpha} I(\Omega_{\mathrm{A}}, \Omega_{\mathrm{B}}) \right|^2 \\ &= r^2 \left(|I(\Omega_{\mathrm{A}}, -\Omega_{\mathrm{B}})|^2 + |I(\Omega_{\mathrm{A}}, \Omega_{\mathrm{B}})|^2 \right. \\ &\left. + 2\mathrm{Re}\, e^{\mathrm{i}2\alpha} I(\Omega_{\mathrm{A}}, \Omega_{\mathrm{B}}) I^*(\Omega_{\mathrm{A}}, -\Omega_{\mathrm{B}}) \right).\end{aligned} \tag{3.31}$$

We see that in the signaling estimator there is a contribution coming from an interference term. If we write this interference term in polar form, that is

$$I(\Omega_{\mathrm{A}}, \Omega_{\mathrm{B}}) I^*(\Omega_{\mathrm{A}}, -\Omega_{\mathrm{B}}) = e^{-\mathrm{i}\beta} |I(\Omega_{\mathrm{A}}, \Omega_{\mathrm{B}})| |I(\Omega_{\mathrm{A}}, -\Omega_{\mathrm{B}})|, \tag{3.32}$$

where β is just the principal argument of the complex number (3.32). The signaling estimator can be written then as

$$\begin{aligned}\mathcal{S} = r^2 & \left(|I(\Omega_{\mathrm{A}}, -\Omega_{\mathrm{B}})|^2 + |I(\Omega_{\mathrm{A}}, \Omega_{\mathrm{B}})|^2 \right. \\ & \left. + 2\cos(\alpha - \beta) |I(\Omega_{\mathrm{A}}, \Omega_{\mathrm{B}})| |I(\Omega_{\mathrm{A}}, -\Omega_{\mathrm{B}})| \right),\end{aligned} \tag{3.33}$$

which is maximum when setting $r = 1$ and $\alpha = \beta$. Therefore, the maximum value for the signaling estimator is achieved when the state of the detector B is diagonal in the basis of its free Hamiltonian, and the state of detector A is such that $\langle \hat{\sigma}^{\pm}_{\text{A}} \rangle = e^{\mathrm{i}\beta}$. For these states, the maximum value of the signaling estimator takes the form

$$\mathcal{S}_{\max} = \left(|I(\Omega_{\text{A}}, -\Omega_{\text{B}})| + |I(\Omega_{\text{A}}, \Omega_{\text{B}})| \right)^2 . \tag{3.34}$$

Example: two quantum particles interacting through a quantum field

One can also consider models of the Unruh-Wald type, which describes the interaction between two spinless charged particles through a scalar field and is modeled by Hamiltonians like

$$\hat{H}_{\text{UW}} = \sum_{\nu \in \{A,B\}} \lambda_\nu \chi_\nu(t) \int \mathrm{d}\boldsymbol{x}\, \hat{\phi}(t,\boldsymbol{x}) \otimes \delta(\boldsymbol{x} - \hat{\boldsymbol{x}}_\nu(t)). \tag{3.35}$$

In that case,

$$\hat{J}_\nu(\mathsf{x}) = \chi_\nu(t)\delta(\boldsymbol{x} - \hat{\boldsymbol{x}}_\nu(t)), \tag{3.36}$$

which leads to

$$\begin{aligned} \langle \hat{J}_{\text{A}} \rangle (\mathsf{x}) &= \chi_{\text{A}}(t) \left\langle \delta(\boldsymbol{x} - \hat{\boldsymbol{x}}_{\text{A}}(t)) \right\rangle \\ &= \chi_{\text{A}}(t) |\psi_{\text{A}}(t,\boldsymbol{x})|^2, \end{aligned} \tag{3.37}$$

where $\psi_{\text{A}}(t,\boldsymbol{x})$ is the A's wave function and

$$\begin{aligned} &\langle \hat{J}_{\text{B}}(\mathsf{x}) \hat{J}_{\text{B}}(\mathsf{x}') \rangle \\ &= \chi_{\text{B}}(t)\chi_{\text{B}}(t') \left\langle \delta(\boldsymbol{x} - \hat{\boldsymbol{x}}_{\text{B}}(t))\delta(\boldsymbol{x}' - \hat{\boldsymbol{x}}_{\text{B}}(t')) \right\rangle \\ &= \chi_{\text{B}}(t)\chi_{\text{B}}(t')\psi^*_{\text{B}}(t,\boldsymbol{x})\psi_{\text{B}}(t',\boldsymbol{x}')\mathcal{G}_{\text{B}}(t-t',\boldsymbol{x},\boldsymbol{x}'), \end{aligned} \tag{3.38}$$

where

$$\mathcal{G}_{\text{B}}(t-t',\boldsymbol{x},\boldsymbol{x}') = \langle \boldsymbol{x} | \hat{U}_{\text{B,free}}(t-t') | \boldsymbol{x}' \rangle . \tag{3.39}$$

$\hat{U}_{\text{B,free}}$ is the unitary operator associated with the internal (uncoupled) dynamical evolution of B, and therefore $\mathcal{G}_{\text{B}}$ is the uncoupled propagator of detector B. In contrast to the case that we will review in the next section, unless there are further constraints, there is no optimization with respect to the states that will set the second term of (3.24) to zero, since

$$G_{\mathrm{R}}[\langle \hat{J}_{\mathrm{B}}\rangle\,,\langle \hat{J}_{\mathrm{A}}\rangle] = \int \mathrm{d}t\mathrm{d}\boldsymbol{x}\chi_{\mathrm{B}}(t)|\psi_{\mathrm{B}}(t,\boldsymbol{x})|^2 \times \int \mathrm{d}t'\mathrm{d}\boldsymbol{x}' G_{\mathrm{R}}(t,\boldsymbol{x},t',\boldsymbol{x}')\chi_{\mathrm{A}}(t')|\psi_{\mathrm{A}}(t',\boldsymbol{x}')|^2 \tag{3.40}$$

is non-zero in general for valid normalized states of A, B. We can still derive a bound from the first term of (3.24) we derive the bound

$$\mathcal{S} \leq \int \mathrm{d}t\mathrm{d}\boldsymbol{x} \int \mathrm{d}t'\mathrm{d}\boldsymbol{x}' G_{\mathrm{R}}[\langle \hat{J}_{\mathrm{A}}\rangle](t,\boldsymbol{x}) G_{\mathrm{R}}[\langle \hat{J}_{\mathrm{A}}\rangle](t',\boldsymbol{x}') \times \mathcal{G}_{\mathrm{B}}(t-t',\boldsymbol{x},\boldsymbol{x}')\chi_{\mathrm{B}}(t)\chi_{\mathrm{B}}(t')\psi^*_{\mathrm{B}}(t,\boldsymbol{x})\psi_{\mathrm{B}}(t',\boldsymbol{x}') \tag{3.41}$$

where

$$G_{\mathrm{R}}[\langle \hat{J}_{\mathrm{A}}\rangle](t,\boldsymbol{x}) = \int \mathrm{d}t'\mathrm{d}\boldsymbol{x}' G_{\mathrm{R}}(t,\boldsymbol{x},t',\boldsymbol{x}')\chi_{\mathrm{A}}(t')|\psi_{\mathrm{A}}(t',\boldsymbol{x}')|^2. \tag{3.42}$$

We see that in this case $G_{\mathrm{R}}[\langle \hat{J}_{\mathrm{A}}\rangle]$ is a convolution of the field's retarded propagator, detector A's switching function and the probability density of A. If A's wavefunction at $t = 0$ is compactly supported on a spatial region Δ and detector A interacts with the field only at $t = 0$ with a delta switching function $\chi_{\mathrm{A}}(t) = \delta(t)$ then

$$G_{\mathrm{R}}[\langle \hat{J}_{\mathrm{A}}\rangle](t,\boldsymbol{x}) = \int_{\Delta} \mathrm{d}\boldsymbol{x}' G_{\mathrm{R}}(t,\boldsymbol{x},0,\boldsymbol{x}')|\psi_{\mathrm{A}}(0,\boldsymbol{x}')|^2 \tag{3.43}$$

is compactly supported in the causal future of Δ. On the other hand, if the switching function has a non-zero extension in time e.g. a finite time extension $\epsilon > 0$, $G_{\mathrm{R}}[\langle \hat{J}_{\mathrm{A}}\rangle](t,\boldsymbol{x})$ is supported on the whole $t > 0$ plane due to the instantaneous spreading of A's wavefunction [108], i.e., the fact that $\psi_{\mathrm{A}}(\epsilon,\boldsymbol{x}')$ has support everywhere for all $\epsilon > 0$. The upper bound of (3.41) can be written as

$$\mathcal{S} \leq \int \mathrm{d}t\mathrm{d}\boldsymbol{x} \int \mathrm{d}t'\mathrm{d}\boldsymbol{x}' \psi_{\mathrm{AB}}(t',\boldsymbol{x}')\mathcal{G}_{\mathrm{B}}(t-t',\boldsymbol{x},\boldsymbol{x}')\psi^*_{\mathrm{AB}}(t,\boldsymbol{x}) \tag{3.44}$$

where

$$\psi_{\mathrm{AB}}(t,\boldsymbol{x}) := G_{\mathrm{R}}[\langle \hat{J}_{\mathrm{A}}\rangle](t,\boldsymbol{x})\chi_{\mathrm{B}}(t)\psi_{\mathrm{B}}(t,\boldsymbol{x}) \tag{3.45}$$

This is the 'overlap' of the propagated current of A (3.37) with B's wavefunction and switching function. Due to the non-relativistic internal dynamics of A, B this overlap will generally be non-zero, i.e., there is no valid choice of the states of A, B such that the upper bound vanishes, and in fact, there is always some signaling. This is a major difference with respect to the previous example of the two-level system, where one could choose valid states such that $\mathcal{S} = 0$ (e.g. for $\langle \hat{\mu}_{\text{A}} \rangle = 0$). In this sense, the influence of the tails is more pernicious in the Unruh-Wald model than it is for the usual Unruh-DeWitt detector.

Example: signaling between a spin and a continuous pointer variable

Here we consider a setup similar to the thought experiments in [38] which investigate the interplay between complementarity and relativistic causality. The 'sender' is a spin system locally coupled to the field in a spacetime region, and the 'receiver' is a continuous pointer variable linearly coupled to the field in another spacetime region. Namely,

$$\hat{J}_{\text{A}}(\boldsymbol{x}, t) = \chi_{\text{A}}(t) F_{\text{A}}(\boldsymbol{x}) \hat{\sigma}_{\text{A}}(t) \tag{3.46}$$

$$\hat{J}_{\text{B}}(\boldsymbol{x}, t) = \chi_{\text{B}}(t) F_{\text{B}}(\boldsymbol{x}) \hat{k}_{\text{B}}(t) \tag{3.47}$$

where $\hat{\sigma}_{\text{A}}$ is the internal degree of freedom of the sender that couples to the field and $\hat{k}_{\text{B}}$ is the conjugate to the pointer variable of the receiver $\hat{x}_{\text{B}}$, i.e., $[\hat{x}_{\text{B}}, \hat{k}_{\text{B}}] = \mathrm{i}\hbar\mathbb{I}$. We consider, for simplicity, that the pointer variable does not have internal dynamics,i.e. $\hat{k}_{\text{B}}(t) = \hat{k}_{\text{B}}$, so that it only 'shifts' based on its interaction with the field and the influence of A. Taking into account the definitions (3.23) and (3.25)

$$\mathcal{S} = \text{Var}\, G_{\text{R}}[\hat{J}_{\text{B}}, \langle \hat{J}_{\text{A}} \rangle], \tag{3.48}$$

and that in this case

$$G_{\text{R}}[\hat{J}_{\text{B}}, \langle \hat{J}_{\text{A}} \rangle] = \hat{k}_{\text{B}} G_{\text{R}}[\chi_{\text{B}} F_{\text{B}} \hat{J}_{\text{B}}, \langle \hat{J}_{\text{A}} \rangle], \tag{3.49}$$

we get that

$$\mathcal{S} = \left(\int \mathrm{d}\boldsymbol{x}\mathrm{d}t\; \chi_{\text{B}}(t) F_{\text{B}}(\boldsymbol{x}) G_{\text{R}}[\langle \hat{J}_{\text{A}} \rangle](\boldsymbol{x}, t) \right)^2 (\Delta k_{\text{B}})^2 \tag{3.50}$$

where $(\Delta k_{\text{B}})^2 = \langle \hat{k}_{\text{B}}^2 \rangle - \langle \hat{k}_{\text{B}} \rangle^2$ and assuming for convenience that the conjugate pointer variable is centered around zero ($\langle \hat{k}_{\text{B}} \rangle = 0$). We see that the initial variance of the conjugate pointer variable modulates the amount of signaling from A to B, times the integral prefactor in (3.50) that quantifies the causal overlap of the sender and the receiver. Notice that if the pointer variable conjugate momentum starts in an eigenstate of $\hat{k}_{\mathrm{B}}$, then $\Delta k_{\mathrm{B}} = 0$ and there is no signaling. This is expected since in that case the state of the receiver is an eigenstate of the local interaction Hamiltonian and there is no internal dynamics, so time evolution on the partial system of the receiver is trivial.

Notice that if we consider that the sender is coupled to the field for all times (adiabatic switching) even if the sender and receiver are fixed at two distant spatial locations $\boldsymbol{x}_{\mathrm{A}}, \boldsymbol{x}_{\mathrm{B}}$ they are not spacelike separated if the interaction of A is always 'on'. In this case, the prefactor of (3.50) is given by

$$\int \mathrm{d}t\mathrm{d}t' \chi_{\mathrm{B}}(t) G_{\mathrm{R}}^{\mathrm{AB}}(t,t') \chi_{\mathrm{A}}(t') \langle \hat{\sigma}_{\mathrm{A}}(t') \rangle, \tag{3.51}$$

where $G_{\mathrm{R}}^{\mathrm{AB}}(t,t') = G_{\mathrm{R}}(t, \boldsymbol{x}_{\mathrm{B}}, t', \boldsymbol{x}_{\mathrm{A}})$. In [38] it was shown that the distinguishability of B (i.e. the ability of B to distinguish between the up and down state of the spin A) depends on the overlap $\int \mathrm{d}t\mathrm{d}t' \chi_{\mathrm{B}}(t) G_{\mathrm{R}}^{\mathrm{AB}}(t,t') \chi_{\mathrm{A}}(t')$. Comparing with the prefactor (3.51) that goes into our signaling estimator, we see that this definition does not take into account the effect of the internal dynamics of the spin and the choice of the initial state. This suggests that there might be cases where the ability of A to signal to B is different than e.g. the distinguishability, or some other measure of causality. Thus, it would be interesting to investigate the effect of the internal dynamics and choice of initial state on the causality of the setup.

3.2.3 Signaling estimator for smeared UDW detectors: gap dependence and resonant phenomena

In this section we will analyze of the signaling estimator for the more familiar case of the usual smeared Unruh-DeWitt detector. Namely, the interaction Hamiltonian of two inertial comoving two-level systems interacting with a scalar field is given by

$$\hat{H}_{\mathrm{sm}} = \sum_{\nu=\mathrm{A,B}} \lambda_\nu \chi_\nu(t) \hat{\mu}_\nu(t) \otimes \int \mathrm{d}\boldsymbol{x}\, F_\nu(\boldsymbol{x}) \hat{\phi}(t, \boldsymbol{x}), \tag{3.52}$$

where $\hat{\mu}_\nu(t) = e^{\mathrm{i}\Omega_\nu t} \hat{\sigma}_\nu^+ + e^{-\mathrm{i}\Omega_\nu t} \hat{\sigma}_\nu^-$, where $\sigma_\nu^+, \sigma_\nu^-$ are the ladder operators and Ω_ν is the energy gap between the two levels. We want to ask: which ingredients that define the detector-field interaction are relevant for the signaling estimator? In previous studies the effect of the internal frequencies was neglected, so we wanted to know how the internal frequencies of the two-level systems affect their ability to signal.

To particularize the general expression for the signaling estimator (3.30) to the two-level Unruh-DeWitt detector in flat space-time of $n+1$ dimensions, we consider the current

$$\hat{J}_{\nu}(\mathsf{x}) = \chi_{\nu}(t)F_{\nu}(\boldsymbol{x})\hat{\mu}_{\nu}(t). \tag{3.53}$$

where $\nu = \mathrm{A}, \mathrm{B}$. In flat spacetime, the field propagators are translationally invariant, meaning that they only depend on the difference of spacetime points. This allows us to write the expression for the functions $I(\Omega, \Omega')$ appearing in the expression for the optimal signaling estimator defined in (3.30) as

$$\begin{aligned} I(\Omega, \Omega') = & \int \mathrm{d}t\mathrm{d}\boldsymbol{x}\, G_{\mathrm{R}}(t, \boldsymbol{x}) \int \mathrm{d}\boldsymbol{y} F_{\mathrm{A}}(\boldsymbol{y}) F_{\mathrm{B}}(\boldsymbol{x}+\boldsymbol{y}) \\ & \times \int \mathrm{d}s \chi_{\mathrm{A}}(s) e^{\mathrm{i}\Omega s} \chi_{\mathrm{B}}(t+s) e^{\mathrm{i}\Omega'(t+s)}, \end{aligned} \tag{3.54}$$

which is derived by simply substituting $G_{\mathrm{R}}(\mathsf{x}, \mathsf{x}') = G_{\mathrm{R}}(\mathsf{x} - \mathsf{x}')$ and performing a change of variables.

Further, the propagators in flat spacetime are tempered distributions, which in particular implies that they admit a Fourier transform. In terms of the Fourier transform the integral takes the form

$$I(\Omega, \Omega') = \int \mathrm{d}\boldsymbol{k}\mathrm{d}k^0\, \tilde{G}_{\mathrm{R}}(\boldsymbol{k}, k_0) \tilde{F}^*_{\mathrm{B}}(\boldsymbol{k}) \tilde{F}_{\mathrm{A}}(\boldsymbol{k}) \tilde{\chi}^*_{\mathrm{B}}(k_0 + \Omega') \tilde{\chi}_{\mathrm{A}}(k_0 + \Omega), \tag{3.55}$$

where $\tilde{F}_{\mathrm{A,B}}$ is the n-dimensional Fourier transforms of $F_{\mathrm{A,B}}$ and $\tilde{\chi}_{\mathrm{A,B}}$ is the one-dimensional Fourier transforms of $\chi_{\mathrm{A,B}}$:

$$\tilde{F}(\boldsymbol{k}) = \frac{1}{(2\pi)^{n/2}} \int \mathrm{d}^n\boldsymbol{x} e^{-\mathrm{i}\boldsymbol{k}\boldsymbol{x}} F(\boldsymbol{x}) \tag{3.56}$$

$$\tilde{\chi}(k_0) = \frac{1}{(2\pi)^{1/2}} \int \mathrm{d}\boldsymbol{t} e^{-\mathrm{i}k_0 t} \chi(t). \tag{3.57}$$

For the Klein-Gordon field the Fourier transform of the retarded propagator is given by

$$\tilde{G}_{\mathrm{R}}(\boldsymbol{k}, k^0) = \frac{1}{(2\pi)^{(n+1)/2}} \frac{1}{-(k^0 - \mathrm{i}\epsilon)^2 + \omega_{\boldsymbol{k}}^2} \tag{3.58}$$

where $\omega_{\boldsymbol{k}}^2 = m^2 + \boldsymbol{k}^2$ and ϵ is a regulator that will have to be taken to go to zero after the integrals are performed. Therefore, the integral in (3.55) is given by

$$
\begin{aligned}
I(\Omega,\Omega') = &\frac{1}{(2\pi)^{n+1}} \lim_{\epsilon\to 0^+} \int \mathrm{d}\boldsymbol{k}\, \tilde{F}^*_{\text{B}}(\boldsymbol{k})\tilde{F}_{\text{A}}(\boldsymbol{k}) \\
&\times \int \mathrm{d}k_0 \frac{\tilde{\chi}^*_{\text{B}}(k^0+\Omega')\tilde{\chi}_{\text{A}}(k^0+\Omega)}{-(k^0-\mathrm{i}\epsilon)^2+\omega_{\boldsymbol{k}}^2}.
\end{aligned}
\tag{3.59}
$$

We will analyze the dependence of the estimator on the frequencies $\Omega_{\text{A}}, \Omega_{\text{B}}$. In [12] we also analyzed the behavior of the estimator as the smearing and switching functions of the detectors vary. We will study the dependence of the signaling estimator in the particular case of two UDW detectors that interchange signals through a massless scalar field in a flat spacetime of 3+1 dimensions. In order to simplify the explicit evaluation, we recall that the two detectors are comoving and at rest in some inertial frame, and further assume that the switching and smearing functions are given by Gaussian functions. The centers of the Gaussians that define the detectors' spacetime smearing will be separated by a constant spacetime vector, which in the comoving coordinate frame takes the explicit components $\mathsf{z} = (z_0, \boldsymbol{z})$. We call $z_0 := \Delta$ the time distance and $|\boldsymbol{z}| := L$ the spatial distance. Both smearing functions have spatial width R and temporal width T (see (A.44) and (A.50) in appendix A.3).

Recall that the maximum signaling between the two detectors (equation (3.34)) is given by

$$
\mathcal{S}_{\max} = \left(|I(\Omega_{\text{A}}, -\Omega_{\text{B}})| + |I(\Omega_{\text{A}}, \Omega_{\text{B}})|\right)^2. \tag{3.60}
$$

Through simple analysis (see Appendix A.3), particularizing the Green function in expression (3.54) to the case of a massless scalar field in 3+1 dimensions we get

$$
I(\Omega_{\text{A}}, \Omega_{\text{B}}) = \frac{R}{\sqrt{\pi}L} e^{-\frac{L^2}{4R^2}} \int_0^\infty \mathrm{d}u\, u e^{\frac{-u^2}{4}} (e^{\frac{uL}{2R}} - e^{-\frac{uL}{2R}}) I_p(Ru, \Omega_{\text{A}}, \Omega_{\text{B}}) \tag{3.61}
$$

where

$$
I_p(L, \Omega_{\text{A}}, \Omega_{\text{B}}) = -\frac{T}{8\pi^{3/2}L} e^{-\frac{T^2(\Omega_{\text{B}}-\Omega_{\text{A}})^2}{4}} e^{-\frac{(L+\Delta)^2}{4T^2}} e^{\mathrm{i}(\Omega_{\text{B}}+\Omega_{\text{A}})L} \tag{3.62}
$$

happens to be the value of $I(\Omega_{\text{A}}, \Omega_{\text{B}})$ for two pointlike detectors with identical switching separated by a spatial distance L. In other words, we can express $I(\Omega_{\text{A}}, \Omega_{\text{B}})$ as a spatial integral of the expression for pointlike detectors.

Regarding the behavior of the signaling estimator with respect to the detectors's energy gap, in general, we would expect that signaling decreases in the limit of very large gap of either detector. The fact that the signaling estimator vanishes in the large gap limit is clear from equation (3.59), since in this case the signaling estimator is given by an integral of smooth functions and the dependence on the internal frequencies of the detectors is just a translation in momentum space. For smooth, integrable smearing profiles and switching functions the limit of the integral when Ω_{A} or Ω_{B} goes to infinity can be taken inside the integral, and this limit vanishes. We would also expect the signaling estimator to be the largest in resonance (that is, when $|\Omega_{\text{A}}| = |\Omega_{\text{B}}|$).

It is interesting to consider the pointlike limit. In this case (see Figure 3.1) the signaling does not vanish when $\Omega \to \infty$ while in resonance $|\Omega_{\text{A}}| = |\Omega_{\text{B}}| = \Omega$. This is expected since in the pointlike limit the smearing function becomes a delta which is not a smooth function and the arguments given above do not apply. Furthermore, in the pointlike limit, the signaling estimator reaches an asymptote as the resonant frequency Ω grows (with every other parameter fixed) independently of the initial state of detector A if the state is of the form

$$\hat{\rho}_{\text{A}} = \frac{1}{2}(\mathbb{1} + \cos\alpha\hat{\sigma}_x + \sin\alpha\hat{\sigma}_y). \tag{3.63}$$

The value of the upper bound for the signaling estimator (3.34) behaves, as expected, as an enveloping curve for different values of the initial state of A. Whereas the behavior of the maximum signaling is monotonic, the behavior of the particular signaling estimator for a fixed state of detector A (Eq. (3.30)) becomes oscillating. We relate this phenomenon, already observed in [102], with the fact that for a fixed state and an interaction duration long enough, the detector interacts mostly with some frequencies of the field around its internal frequency Ω_{A}. Therefore, if the detectors are separated by a fixed spacial distance, it is to be expected that signaling will be maximum when they are separated by one of these wavelengths. This can be seen directly from equation (3.62), where we see that the signaling estimator will oscillate in Ω exactly with frequency L^{-1}.

We also observe a resonance phenomenon in Figure 3.2 where both detectors are pointlike separated by spatial distance L and both switchings are Gaussian functions of width T. Detector B (the receiver) is centered around zero. As expected, we see that the signaling is higher when A is centered on the (smeared) past light cone of B, and when the two frequencies match.

Finally, in Figure 3.3 we see the effect of the spatial smearing on signaling when the centers of the two detector-field interactions are in spacelike separation. To

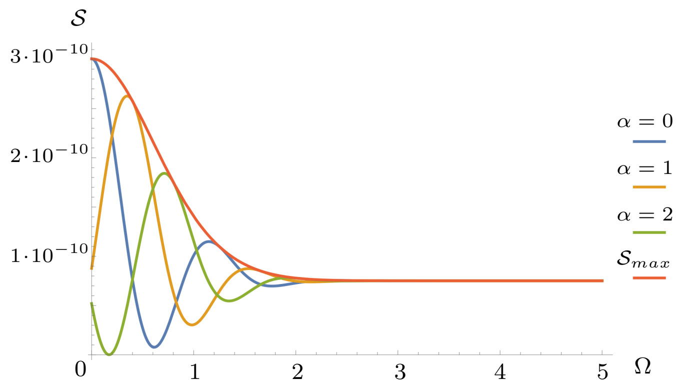

Figure 3.1: The three oscillatory curves (blue, orange, green) give the signaling estimator for different states of detector A as parametrized by a phase α ($\alpha = 0, 1, 2$) as a function of the detectors' gap $\Omega_{\text{A}} = \Omega_{\text{B}} = \Omega$. The red curve that is decreasing monotonically is the $\mathcal{S}_{\text{max}}$. This is for pointlike detectors that are separated by L and both switchings are peaked around zero with temporal width T, and such that $L/T = 5$. Notice that the signaling estimator at this spatial separation is $\mathcal{S} \ll 1$, so the detectors are effectively out of causal contact.

see how much the Gaussian smearing affects signaling and when we stop being in effective spacelike separation as we increase the smearing width R, we study the worst-case scenario: in order to maximize the signaling estimator, we choose the frequencies such that the detectors are in resonance, and we plot the signaling as a function of the spatial width R of the smearing for constant $L/T = 4$. One would expect that the bigger the width the causal connection between the tails of the two smearings grows, which would lead to more signaling. Indeed, this is what happens until some critical value, after which the signaling decreases (see figure 3.3). This might seem counter-intuitive, but since the smearing function is a normalized density, the interaction overall vanishes in the limit of $R/T \to \infty$. In this sense, even though we would expect that the tails are indeed enabling signaling between the detectors, we also observe a 'dilution' effect. Similar behavior was also observed previously in [37].

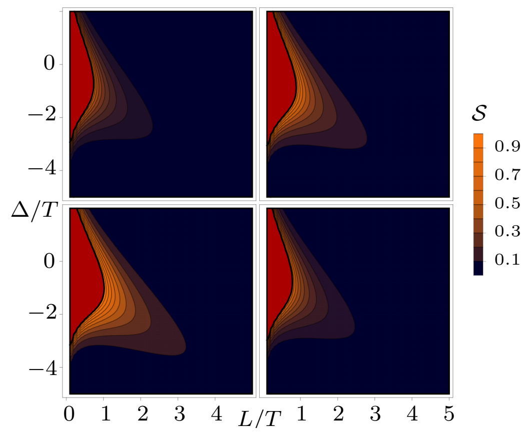

Figure 3.2: Signaling estimator between two pointlike detectors that are separated by a spatial distance L and with a time-lapse Δ. The peaks of the switchings are separated by time Δ with temporal width T. Detector B (the receiver) has internal gap $\Omega_{\text{B}} = 2$ and is centered around zero. The color bar quantifies how much signal it can receive from A depending on where A is in space and time, for different values of $\Omega_{\text{A}}(\Omega_{\text{A}} = 1, 2, 3, 4$ from up left to down right).

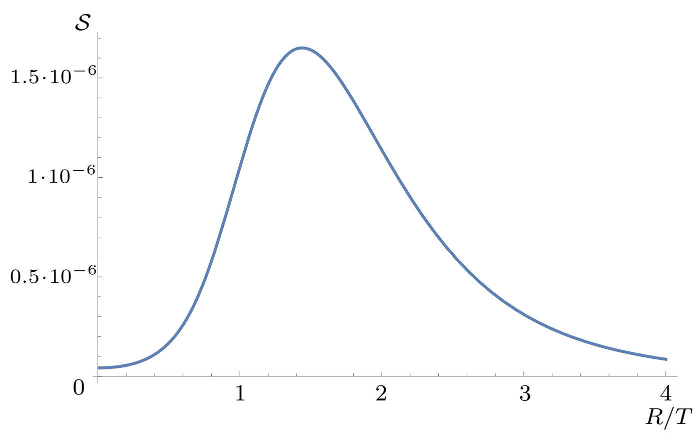

Figure 3.3: Signaling estimator for two smeared detectors with spatial smearings of width R and whose centers are separated by a spatial distance L. The spatial separation L is such that $L/T = 4$ (where T is the duration of the interaction). Both switching functions are peaked around $t = 0$, so the two detectors are effectively spacelike separated ($\mathcal{S} \ll 1$). We have set $\Omega_{\text{A}} = \Omega_{\text{B}} = \Omega$.

3.3 Impossible measurements: a pragmatic approach

To summarize, so far we have analyzed signaling in general bipartite scenarios. At the level of perturbation theory we have seen that compactly supported detector-field interactions behave causally in spacelike separation, and we defined a signaling estimator that captures (at leading order) the 'overlap' between detectors that are 'mostly' spacelike separated. We concluded that at the level of perturbation theory, this 'cross-talk' between detectors can be managed. Yet, we saw that the geometric intuition that the signaling has to be 'small enough' if the centers are 'far enough' (in comparison to the width of e.g. the Gaussian spacetime smearings) only captures the *upper bound* to signaling. If one wants to calculate the actual signaling, one has to take into account the internal dynamics and the internal states of the detector systems in a given configuration.

In other words, how 'far' two distant two-level systems depends non-trivially on the internal frequencies (which perhaps can be explained as a resonance or interference effect) and on how much coherence there is in their internal states (intuition about the state dependence of another (related) signaling estimator can be found in [109]). This can be thought as a difference between the spacetime notion of distance (set by the lightcone structure) and the quantum informational notion of distance that is defined through our signaling estimator. In fact, for a given model, one can draw effective lightcones that capture the signaling possibilities. The discrepancy between the two notions can be argued to be 'small' but further study is required in different spacetimes e.g. in cosmology.

Finally, we observed a 'dilution' effect when one considers bigger widths of the spatial smearing. That is, there is a critical value for the effective size of the interaction beyond which the signaling estimator drops despite the overlap increasing. This might imply that this model is bound to describe microscopic detector systems, or probes, and cannot describe realistically the macroscopic limit of a macroscopic device (this can be evaluated case-by-case by analyzing which scales contribute to the critical size of the interaction). This might not be a surprise, since intuitively it should not be possible to describe a macroscopic device only by 'spreading' out a single degree of freedom, but one would have to consider a bigger number of degrees of freedom distributed in space.

Overall, the infinite 'tails' of the interactions in space and time do not pose a methodological threat at the level of perturbation theory. Nevertheless, the non-perturbative arguments that we gave in [11] only hold for compact detector-field interactions because they are based on the causal factorization property. This property holds for causally orderable compactly supported interactions (see appendix A.1) thanks to microcausality, and it does not hold for non-compact interactions because strictly speaking, they are not causally orderable [5]. This obscures the causality arguments at the non-perturbative level.

In bipartite scenarios, we showed that causal factorization guarantees no superluminal signaling and no retrocausation [11]. Consider two compactly supported and causally orderable detector-field interactions governed by a Hamiltonian $\hat{H}_{int}$ of the general form (3.9). We denote by $\hat{S}_{\text{A+B}}$ the joint scattering map that corresponds to both interactions

$$\hat{S}_{\text{A+B}} = \mathcal{T}e^{i\hat{H}_{int}}. \tag{3.64}$$

In general, it is not clear whether this map factorizes, namely whether it can be written as a product of $\hat{S}_{\text{A}}$ and $\hat{S}_{\text{B}}$ that correspond to each detector-field interaction. We showed that if the two interactions A,B are causally orderable and say A precedes B (see the definitions and the proof in A.1) then

$$\hat{S}_{\text{A+B}} = \hat{S}_{\text{B}}\hat{S}_{\text{A}}, \tag{3.65}$$

which is called the causal factorization property. This implies that

$$\begin{aligned}\hat{\rho}_{\text{A}} &= \text{tr}_{\text{B},\phi}(\hat{S}_{\text{A+B}}\ \hat{\rho}_0\ \hat{S}^\dagger_{\text{A+B}}) = \text{tr}_{\text{B},\phi}(\hat{S}_{\text{B}}\hat{S}_{\text{A}}\ \hat{\rho}_0\ \hat{S}^\dagger_{\text{A}}\hat{S}^\dagger_{\text{B}}) \\ &= \text{tr}_{\text{B},\phi}(\hat{S}_{\text{A}}\ \hat{\rho}_0\ \hat{S}^\dagger_{\text{A}}\hat{S}^\dagger_{\text{B}}\hat{S}_{\text{B}}) = \text{tr}_{\text{B},\phi}(\hat{S}_{\text{A}}\ \hat{\rho}_0\ \hat{S}^\dagger_{\text{A}}).\end{aligned} \tag{3.66}$$

so A cannot receive a signal from B that is not in its causal past (no retrocausation). Similarly, in the special case that the two are spacelike separated, it holds that

$$\hat{S}_{\text{B}}\hat{S}_{\text{A}} = \hat{S}_{\text{A}}\hat{S}_{\text{B}} \tag{3.67}$$

and there is no superluminal signaling (independently of the state $\hat{\rho}_0$). Nevertheless, as we will see in the next subsection, in more complicated scenarios that involve more than two regions, causal factorization does not guarantee no-signaling in spacelike separation. Yet, in the case of weak coupling one can quantify the causality violation by means of the relevant scales of the problem, and see that it is sub-leading order in perturbation theory.

[5] We say that A and B are causally orderable if $\mathcal{J}^-(\mathcal{O}_{\text{A}}) \cap \mathcal{O}_{\text{B}}$ or $\mathcal{J}^-(\mathcal{O}_{\text{B}}) \cap \mathcal{O}_{\text{A}}$ are empty, where by $\mathcal{J}^-$ we denote the causal past.

3.3.1 Impossible dynamics?

Consider the impossible measurement scenario for local regions O_1, O_2, and O_3 depicted in Fig. 3.2. A unitary 'kick' is implemented over region O_1, possibly through the coupling of a detector to the field, which then can be disregarded. In particular, the initial state of the detectors plus field has the form $\hat{\rho}_{\text{initial}} = \hat{U}\hat{\rho}_0\hat{U}^\dagger$ where $\hat{\rho}_0$ is an arbitrary state of the joint system, and $\hat{U} = \mathbb{1}_{\text{A}} \otimes \mathbb{1}_{\text{B}} \otimes \hat{U}_\phi$ is an arbitrary unitary acting on the field's Hilbert space. Two detectors A, B interact with the field over the regions $O_{2,3}$ respectively.

If detector A were not coupled to the field, the expectation values of observables $\hat{D}_{\text{B}}$ of detector B that are evaluated after both interactions are 'off' would not depend on U_ϕ, since B only interacts with the field in the causal complement of O_1. In [11] we derived that in the presence of detector A, the condition that B's observables $\hat{D}_{\text{B}}$ are not sensitive to the local 'kick' $\hat{U}_\phi$ is

$$\hat{V}^\dagger \hat{D}_{\text{B}} \hat{V} = \hat{D}_{\text{B}}, \tag{3.68}$$

where

$$\hat{V} = \hat{S}_{\text{A+B}} \hat{U} \hat{S}^\dagger_{\text{A+B}}. \tag{3.69}$$

Condition (3.68) is equivalent to

$$[\hat{D}_{\text{B}}, \hat{V}] = 0. \tag{3.70}$$

Using the causal factorization condition (since $O_{2,3}$ are causally orderable) condition (3.68) becomes[6]

$$[\hat{S}^\dagger_{\text{B}} \hat{D}_{\text{B}} \hat{S}_{\text{B}}, \hat{S}_{\text{A}} \hat{U} \hat{S}^\dagger_{\text{A}}] = 0. \tag{3.71}$$

To make sense of (3.71) we can think of $\hat{S}^\dagger_{\text{B}} \hat{D}_{\text{B}} \hat{S}_{\text{B}}$ as an induced observable that resides on region O_3 and $\hat{S}_{\text{A}} \hat{U} \hat{S}^\dagger_{\text{A}}$ as the local 'kick' propagated through the coupling to A.

Next, we have to examine the localization of $\hat{S}_{\text{A}} \hat{U} \hat{S}^\dagger_{\text{A}}$. That is, how does the coupling to detector A 'propagate' the local 'kick' over region O_1 to region O_3? Crucially, it turns out that the localization of $\hat{S}_{\text{A}} \hat{U} \hat{S}^\dagger_{\text{A}}$ includes the forward lightcone of region O_2 (see Fig. 3.2) and, as a result, the expectation values for detector B in O_3 will depend on the local 'kick'. By expanding condition (3.71) one finds that this is because $[\hat{J}_{\text{A}}(\mathsf{x}), \hat{J}_{\text{A}}(\mathsf{x}')] \neq 0$ for spacelike separated points within the extension of region O_2 (i.e., $\operatorname{supp}\Lambda$) due to the non-relativistic dynamics. This result

[6] where we have omitted the tensor product with the identities $\mathbb{1}_{\text{A}}$ (in the first input) and $\mathbb{1}_{\text{B}}$ (in the second) to simplify the notation.

links superluminal signaling with superluminal propagation within the device that is implementing the measurement. The physical intuition is that, when a detector is spatially extended, the information propagating inside the detector is not constrained to travel subluminally since the detector is a non-relativistic system. In Sec. 3.1 we argued that $\text{supp}\Lambda$ cannot be straightforwardly interpreted as the region occupied by the detector, but the main point is that if the detector current $\hat{J}_{\text{A}}$ were another relativistic field, and as such obeyed microcausality, then its coupling to the field over region O_2 would not change the localization of the local 'kick' over O_1 and no observable in O_3 would be sensitive to the 'kick'.

This structural issue of using non-relativistic detector models, namely that they are defined using currents that do not obey a microcausality condition, can be tolerated by conducting a rigorous analysis of the regimes of validity of the models. That is, the severity of the causality violations in physically reasonable situations can be quantified. This is not only necessary for justifying the use of the models, but also for making sense of this abstract type of causality violations in concrete scenarios that can represent 'realistic' detection experiments. Since for point-like detectors there is no superluminal propagation, one can disregard this kind of faster-than-light signaling for 'small enough' detectors. Whether a detector is small or not will depend, of course, on the parameters of the problem. One can also argue, in terms of the coupling strength, that in the weak coupling limit the Sorkin-type problem is of at least $\mathcal{O}(\lambda^n)$ when n detectors are involved. As explained above, the signaling between any two detectors A and B is of second order, i.e., of order $\lambda_{\text{A}}\lambda_{\text{B}}$ (see Eq. (3.6)). This is because the λ_{A}^2 and λ_{B}^2 terms are 'local' to each detector and do not allow for the detectors to 'see each other'. Similarly, in the tripartite case of detectors A, B and C in the Sorkin-type configuration, the coupling constants have to be combined for detector C to 'see' A through B, and so the superluminal signaling is of at least third order $\lambda_{\text{A}}\lambda_{\text{B}}\lambda_{\text{C}}$. In fact, in [11] we showed explicitly that, for UDW-type detectors in the tripartite scenario, the superluminal signaling is of *fourth* order in perturbation theory [11] while most relevant calculations are of second order in the coupling constant.

To summarize, the detector models approach addresses Sorkin-type scenarios by adopting a pragmatic attitude towards modeling the measurement of relativistic quantum fields. Because detector models are not fully relativistic, superluminal signaling is in principle possible when tripartite measurement scenarios are modeled using either compactly or non-compactly coupled detectors. However, superluminal signaling in these Sorkin-type scenarios is for all practical purposes (FAPP) ruled out by perturbative calculations that show that the effect can be treated as happening outside the regime of validity of the model.

3.3.2 Impossible measurements in the detector-based measurement theory

In the previous section, we exposed the *dynamical* understanding of how the 'impossible measurements' arise in extended detector-field interactions in concrete models, without making explicit use of any measurement theory. In particular, there was no reference to measurement outcomes (only the evaluation of an expectation value) and state update. In this section, we will demonstrate the consequences of this dynamical argument for the detector-based measurement theory [14], by checking whether the detector-induced state update satisfies the causality condition (2.4) by Borsten et al.

A detector-based measurement theory for QFT that specifies the state update rules for the field that are induced by projective measurements on the detectors has been developed by Polo-Gómez, Garay, and Martín-Martínez in [14]. In terms of the reductio argument in Sec. 2.2.1, the detector models approach rejects assumption P3 that ideal measurement theory is applied directly to the field system. Instead, projective measurements (modeled by rank-1 projection operators) are only performed on detectors, and a generalized Lüders' rule (defined through POVM elements, first introduced in [13] for the Unruh-DeWitt model) is induced as a state update rule for the field. Measurements on the field are carried out by first allowing the detector and field to interact in some region, and then measuring the detector in the causal future of this region when the detector and field are no longer coupled (for all practical purposes).

Assume that the initial state of the detector-field system is a separable state represented by the density operator $\rho = \rho_d \otimes \rho_\phi$. Given the interaction Hamiltonian $\hat{H}_{int}$ between the field and the detector, the evolved state is $\hat{S}_1 \rho \hat{S}_1^\dagger$, where $\hat{S}_1 = \mathcal{T}\exp\left[-i\int_{-\infty}^{t_1} \mathrm{d}t \hat{H}_{int}(t)\right]$ and t_1 is a time after which the detector-field interaction is turned off. At a later time $t_2 \geq t_1$ a projective measurement $\hat{P}(t_2)$ (denoted as $\hat{P}_2$) is applied to the detector system and the total state is updated as follows

$$\rho' = \frac{(\hat{P}_2 \otimes \mathbb{1})\hat{S}_1 \rho \hat{S}_1^\dagger (\hat{P}_2 \otimes \mathbb{1})}{\mathrm{tr}\left((\hat{P}_2 \otimes \mathbb{1})\hat{S}_1 \rho \hat{S}_1^\dagger\right)} \tag{3.72}$$

Note that the unitary scattering operator $\hat{S}_1$ is supported over the interaction region, while the projection operator only depends on time since there is not an explicit spatial dependence of the detector operators. The readout of the detector happens at an arbitrary time point after the detector-field interaction is 'off' and as a result there cannot be any effect on the state of the field. This seems to suggest an epistemic (rather than ontic) interpretation of the state update of the field [7], namely that the state of the field does not 'collapse' in any way but rather we update our information about what the state of the field was all along (given the initial state of the field and the involved detector-field dynamical interactions).

Assuming that the initial state of the detector is $\rho_d = |\psi\rangle\langle\psi|$ and that after the interaction with the field the detector is projected by means of the rank-one projector $|i\rangle\langle i|$ (e.g. onto the $i-$th energy eigenstate of the detector) and by tracing out the detector system, equation (3.72) gives

$$\rho'_\phi = \frac{\hat{M}_{i,\psi}\rho_\phi\hat{M}^\dagger_{i,\psi}}{\mathrm{tr}_\phi\left(\rho_\phi\hat{M}^\dagger_{i,\psi}\hat{M}_{i,\psi}\right)}, \tag{3.73}$$

where

$$\hat{M}_{i,\psi} := \langle i|\hat{S}_1|\psi\rangle \tag{3.74}$$

the POVM elements. In the regime of weak coupling between the field and the detector one can use the Dyson expansion for the scattering operator $\hat{S}_1$, using $\hat{H}_{int}$ from (3.5)

$$\hat{M}_{i,\psi} = \langle i|\psi\rangle\mathbb{1} + \lambda\hat{M}^{(1)}_{i,\psi} + \lambda^2\hat{M}^{(2)}_{i,\psi} + \mathcal{O}(\lambda^3) \tag{3.75}$$

where

$$\hat{M}^{(1)}_{i,\psi} = -i\int \mathrm{d}t\mathrm{d}\boldsymbol{x}\chi(t)F(\boldsymbol{x})\langle i|\hat{\mu}(t)|\psi\rangle\hat{\phi}(t,\boldsymbol{x}) \tag{3.76}$$

$$\hat{M}^{(2)}_{i,\psi} = -\int \mathrm{d}t\mathrm{d}t'\theta(t-t')\chi(t)\chi(t')\int \mathrm{d}\boldsymbol{x}\mathrm{d}\boldsymbol{x}'F(\boldsymbol{x})F(\boldsymbol{x}')\langle i|\hat{\mu}(t)\hat{\mu}(t')|\psi\rangle\hat{\phi}(t,\boldsymbol{x})\hat{\phi}(t',\boldsymbol{x}').$$

In the case of non-selective measurements, Polo-Gómez et al. [14] show that

$$\begin{aligned}\rho^{(ns)}_\phi &= \sum_i \hat{M}_{i,\psi}\rho_\phi\hat{M}^\dagger_{i,\psi}\\ &= \mathrm{tr}_d\left(\hat{S}_1(|\psi\rangle\langle\psi|\otimes\rho_\phi)\hat{S}^\dagger_1\right)\end{aligned} \tag{3.77}$$

This is important for our purposes, since the analysis of the Sorkin-type problem does not rely on selective, but on non-selective (trace-preserving) measurements. We see

[7] In [27] we discuss this point further, as a common feature of candidate state update rules in relativistic spacetime.

that by summing over all possible outcomes i the updated state only depends on the dynamical coupling between the field and the detector. Polo-Gómez et al. explain that "this is because the projective measurement acts only on the detector once the interaction has been switched off, and it does not provide additional information since being non-selective the outcome is not known". This point is important for showing that the expectation values of observables $\hat{A}$ that are defined in spacelike separation from the detector-field interaction region do not change due to the non-selective measurement. That is,

$$\mathrm{tr}_\phi(\rho_\phi \hat{A}) = \mathrm{tr}_\phi(\rho_\phi^{(ns)} \hat{A}) \tag{3.78}$$

since $[\hat{S}_1, \hat{A}] = 0$ thanks to the fields obeying the microcausality condition.

Nevertheless, as we will demonstrate, a non-selective state update of the type (3.77) can enable 'impossible measurements' when more than two regions are involved. Consider the Sorkin-type scenario in Fig. 3.2, with $\hat{A} \in \mathcal{A}(O_3)$ and the non-selective measurement happening over region O_2 (that is, $\hat{M}_{i,\psi} \in \mathcal{A}(O_2)$). Call $\rho_\phi^{(ns)} := \mathcal{E}_2[\rho_\phi]$. Then, we will show that the map $\mathcal{E}_2$ does not satisfy the condition (2.4) by Borsten et al. [55]. Note that, $\mathcal{E}_2$ (as defined in (3.77)) does not correspond to an idealized projective measurement (a non-selective state update rule that is defined by means of projectors, see (2.1)). The criterion (2.4) by Borsten et al still applies, since as they mention in [44] the criterion holds for any 'valid' state update map $\mathcal{E}_2$ (that leads to well-defined expectation values $\langle \hat{A} \rangle$ over region O_3). This includes state update maps that follow from particular probe prescriptions, as in the detector-based measurement theory that we consider here, or the FV framework [29]. In our case, since the effect of the non-selective measurement only depends on the unitary scattering map ($\hat{S}_1$ in (3.77), $\hat{S}_{\text{A}}$ in what follows) the dynamical analysis of the previous section applies almost straightforwardly.

Using the notation of the previous section, first we rewrite condition (3.71) as:

$$[\hat{S}_{\text{A}}^\dagger \hat{S}_{\text{B}}^\dagger \hat{D}_{\text{B}} \hat{S}_{\text{B}} \hat{S}_{\text{A}}, \hat{U}] = 0, \tag{3.79}$$

where $\hat{U}$ represents the unitary 'kick' in O_1. Taking the trace of the action of this operator on any state $\rho_{\text{A}} \rho_{\text{B}} \rho_\phi$ of the total system yields

$$\mathrm{tr}_{\text{A,B},\phi} \left([\hat{S}_{\text{A}}^\dagger \hat{S}_{\text{B}}^\dagger \hat{D}_{\text{B}} \hat{S}_{\text{B}} \hat{S}_{\text{A}}, \hat{U}] \, \rho_{\text{A}} \rho_{\text{B}} \rho_\phi \right) = 0 \tag{3.80}$$

$$\mathrm{tr}_{\text{A},\phi} \left([\hat{S}_{\text{A}}^\dagger (\mathrm{tr}_{\text{B}} \hat{S}_{\text{B}}^\dagger \hat{D}_{\text{B}} \hat{S}_{\text{B}} \rho_{\text{B}}) \hat{S}_{\text{A}}, \hat{U}] \, \rho_{\text{A}} \rho_\phi \right) = 0. \tag{3.81}$$

Defining

$$\hat{\Phi}_{\text{B}} := \text{tr}_{\text{B}} \left(\hat{S}_{\text{B}}^{\dagger} \hat{D}_{\text{B}} \hat{S}_{\text{B}} \rho_{\text{B}} \right), \tag{3.82}$$

the induced field observable that corresponds to the measurement of the expectation value of the detector observable $\hat{D}_{\text{B}}$, we have that

$$\text{tr}_{\text{A},\phi} \left([\hat{S}_{\text{A}}^{\dagger} \hat{\Phi}_{\text{B}} \hat{S}_{\text{A}}, \hat{U}]\, \rho_{\text{A}} \rho_{\phi} \right) = 0. \tag{3.83}$$

Performing the trace over detector A, this equation can be written as

$$\text{tr}_{\phi} \left([\mathcal{E}_2^d(\hat{\Phi}_{\text{B}}), \hat{U}]\, \rho_{\phi} \right) = 0 \tag{3.84}$$

where

$$\mathcal{E}_2^d(\hat{\Phi}_{\text{B}}) := \text{tr}_{\text{A}} \left(\hat{S}_{\text{A}}^{\dagger} \hat{\Phi}_{\text{B}} \hat{S}_{\text{A}} \rho_{\text{A}} \right) \tag{3.85}$$

is the dual non-selective map. If we demand that Eq. (3.84) holds for all states of the field ρ_{ϕ} we get the condition

$$[\mathcal{E}_2^d(\hat{\Phi}_{\text{B}}), \hat{U}] = 0, \tag{3.86}$$

which is precisely in the form of the criterion (2.4) for the induced maps and observables (3.85) and (3.82) that we defined above. What we showed is that in this case, condition (3.86) is equivalent to the dynamical condition (3.79) that we derived in the previous section, and fails to be satisfied for the reasons that we exposed there. Written in this form, the violation of (3.86) shows that, in general, the (dual) state update map $\mathcal{E}_2^d$ does not define an observable in the causal complement of O_1, where the unitary 'kick' $\hat{U}$ is supported (see also analysis in [59]).

Chapter 4

What do detectors detect?

In this chapter, we will first examine in detail how particle detector models can be used to retrieve particle-like phenomenology in the weak coupling regime (and long-time limit). In particular, we will examine the phenomenon of resonance and time-of-arrival. Roughly, these two notions correspond to energy versus time measurements, and we will argue that these two measurements are complementary. Then we will examine the strong coupling regime using a continuous pointer variable that is locally coupled to the quantum field, and we will show explicitly that it gets correlated with field averages. We will also consider the solvable QBM model to see how it reproduces particle/field phenomenology in the weak/strong coupling regimes, respectively. Solvable models can be useful for characterizing the detector's response also in intermediate regimes, where one expects that neither the field nor the particle concept can be used to fully characterize the detector's response. Intermediate regimes are experimentally accessible in implementations with tunable couplings, for example, in circuit quantum electrodynamics [110], cavity optomechanics [111], Rydberg atoms [112] or in analogue gravity experiments with Bose-Einstein condensates [113].

In the previous chapters, we discussed that there is a variety of models that are defined through specifying what system is coupled and how it is coupled to the field, as well as how the dynamics is solved. Perturbation theory is typically used in the usual Unruh-DeWitt-type models. One can also achieve non-perturbative results for instantaneous interactions (with δ switching, see e.g. [33, 114, 115]). In this chapter, we provide a non-perturbative pointer variable analysis for an arbitrary switching function which, as we will show, 'weights' the field time-average that the pointer variable gets correlated with. Then, we compare the results of this analysis with the predictions of the solvable model based on QBM.

Each model has advantages and disadvantages, and while the results are in principle comparable, we will see that not all methods are applicable in exactly the same set-ups. For example, we will see that a quantum harmonic oscillator coupled to the field can be assigned an arbitrary smearing and switching function in the pointer variable analysis, but only in the limit that the oscillator is gapless (trivial internal dynamics). In the case of a finite gap, we can solve the full dynamics of a quantum harmonic oscillator coupled to the field using the QBM model, but this is solvable only for constant switching functions and pointlike smearings. Then, for weak couplings, one can try to compare the QBM model with the pointlike UDW in the long-time limit (and two-level approximation) and as we will see there are some essential differences e.g. in the study of resonance. As we will demonstrate, in contrast to standard perturbation techniques, the improved model based on QBM recovers the relativistic Breit-Wigner resonant behavior in the weak coupling regime.

4.1 Strong coupling: pointer variables correlated with field averages

von Neumann introduced a simple measurement model in which the pointer variable $\hat{X}$ of some apparatus gets correlated with an observable $\hat{A}$ of the measured system through the dynamical interaction between system and apparatus [46], but the analysis is easily extended to continuous pointer variables (see e.g. [116]). Assume that the interaction Hamiltonian is $\hat{H}_{int} = \delta(t)\hat{A} \otimes \hat{P}$ where $\hat{P}$ is the conjugate pointer variable, i.e., $[\hat{X}, \hat{P}] = i$. For an ideal apparatus, it is assumed that the pointer variable only changes during the interaction with the microscopic system. Once this interaction is over, the pointer variable $\hat{X}$ is frozen, i.e., $[\hat{H}_A, \hat{X}] = 0$. It is typical to assume the stronger condition that the self-dynamics of the apparatus is negligible, i.e., $\hat{H}_A = 0$. Also, we assume that $\hat{X}$ is a continuous pointer variable, so that it gets correlated with a continuous system observable $\hat{A}$ through the time-evolution operator

$$\hat{U}(t) = e^{-i\hat{H}_s t} \int \mathrm{d}a |a\rangle\langle a| \otimes e^{-ia\hat{P}} \tag{4.1}$$

where we see that each possible value of $\hat{A}$ is correlated with the displacement operator $\exp[-ia\hat{P}]$ that displaces the pointer variable by the value a. One can derive that the probability distribution of the pointer variable after the interaction with the system is [116]

$$P(x) = \int \mathrm{d}a w(x-a) |\langle a|\Psi_0\rangle|^2 \tag{4.2}$$

where $|\Psi_0\rangle$ the state of the system before the (instantaneous) interaction and

$$w(x-a) = \langle\psi_0|\hat{E}_{x-a}|\psi_0\rangle \tag{4.3}$$

where $|\psi_0\rangle$ the initial state of the apparatus and $\hat{E}_x$ the eigenprojections of the pointer variable. We see that the initial probability distribution $|\langle a|\Psi_0\rangle|^2$ over the outcomes α is modified through the integral kernel $w(x-a)$ that encodes the characteristics of the pointer variable and the interaction.

Here, we generalize the analysis above to the case of scalar field measurements. The idea is that for strong couplings, a continuous pointer variable is correlated with field averages. Using the pointer variable method for suitable detector-field coupling, we can calculate the detector's probability distribution non-perturbatively beyond instantaneous interactions (δ switchings).

Consider a one-dimensional pointer variable $\hat{P}$ coupled to a smeared field operator $\hat{\Phi}[F] := \int \mathrm{d}\boldsymbol{x} F(\boldsymbol{x})\hat{\Phi}(\boldsymbol{x})$ for a time interval that is dictated by a switching function $\chi(t)$ as described by the interaction Hamiltonian

$$\hat{H}_{int} = \chi(t)\hat{\Phi}[F] \otimes \hat{P}. \tag{4.4}$$

The full Hamiltonian in the Schrödinger picture is

$$\hat{H}(t) = \hat{H}_0 + \chi(t)\hat{\Phi}[F] \otimes \hat{P} \tag{4.5}$$

where $\hat{H}_0 = \hat{H}_{\text{F}} \otimes \mathbb{1} + \mathbb{1} \otimes \hat{H}_{\text{D}}$. The time evolution operator is

$$\hat{U}(t) = \mathcal{T} e^{-i\int^t \mathrm{d}s \hat{H}(s)} \tag{4.6}$$

which we can decompose as (see appendix B.1)

$$\hat{U}(t) = e^{-i\hat{H}_0 t}\hat{S}(t) \tag{4.7}$$

where

$$\hat{S}(t) = \mathcal{T} e^{-i\int^t \mathrm{d}s \chi(s)\hat{\Phi}_s[F]\otimes\hat{P}(s)}. \tag{4.8}$$

The main 'trick' of the pointer variable analysis is to make use of the simple form that the operator $\hat{S}(t)$ takes when it acts on eigenstates $|p\rangle$ of the conjugate pointer variable $\hat{P}$, namely that

$$\begin{aligned}\hat{S}(t)\left(\mathbb{1}_f \otimes |p\rangle\langle p|\right) &= \mathcal{T} e^{-i\int^t \mathrm{d}s \chi(s)\hat{\Phi}_s[F]\otimes\hat{P}(s)}\left(\mathbb{1}_F \otimes |p\rangle\langle p|\right) \\ &= \mathcal{T} e^{-ip\int^t \mathrm{d}s \chi(s)\hat{\Phi}_s[F]} \otimes |p\rangle\langle p| \end{aligned} \tag{4.9}$$

if it holds that

$$[\hat{H}_{\mathrm{D}}, \hat{P}] = 0, \tag{4.10}$$

which is easy to see using the Dyson expansion for the time-ordered exponential. We see that the self-dynamics of the detector does not have to be zero (or negligible with respect to the interaction with the field) as long as it commutes with the conjugate pointer variable. That is, the detector can be a free particle [1] with $\hat{H}_{\mathrm{D}} = \hat{P}^2/2M$.

We want to know the probability distribution of the possible values x of the pointer variable $\hat{X}$ that is conjugate to the variable $\hat{P}$, i.e., $[\hat{X}, \hat{P}] = i$. Using (4.9) we find that (see appendix B.1 for derivation)

$$p(t,x) = \frac{1}{2\pi}\int \mathrm{d}p\mathrm{d}p' e^{i(p-p')x}\psi(p,t)\psi^*(p',t)\mathrm{tr}_f\left(\hat{U}_\chi(t,p)\rho_0\hat{U}^\dagger_\chi(t,p')\right), \tag{4.11}$$

where we have defined

$$\hat{U}_\chi(t,p) := \mathcal{T}e^{-ip\int^t \mathrm{d}s\chi(s)\hat{\Phi}_s[F]}. \tag{4.12}$$

We have denoted as $\hat{\rho}_0$ the initial state of the field and

$$\psi(p,t) = \langle p|e^{-it\hat{H}_{\mathrm{D}}}|\psi_0\rangle \tag{4.13}$$

where $|\psi_0\rangle$ the detector's initial state. Notice that if we switch off the interaction, Hamiltonian (4.11) simply gives $p(t,x) = |\psi(t,x)|^2$. The effect of the interaction on the statistics of the pointer variable is encoded in the integral kernel $\mathrm{tr}_f(...)$ in (4.11).

Now the question is: Which field observable is the pointer variable correlated with? We will show that the pointer variable is correlated with time-averaged field amplitudes. We can rewrite (4.11) as

$$p(t,x) = \mathrm{tr}_f(\hat{\Pi}_x(t)\hat{\rho}_0\hat{\Pi}^\dagger_x(t)), \tag{4.14}$$

where

$$\hat{\Pi}_x(t) := \frac{1}{\sqrt{2\pi}}\int \mathrm{d}p e^{ipx}\psi(p)\hat{U}_\chi(t,p). \tag{4.15}$$

This defines a set of POVMs that corresponds to the possible outcomes of the pointer variable (see (B.28) in appendix B.1). Using the Magnus expansion (B.23) we can rewrite (4.15) as

$$\hat{\Pi}_x(t) = \frac{1}{\sqrt{2\pi}}\int \mathrm{d}p\,\psi(p)\exp\left(-\frac{i}{2}p^2c_t + ip(x-\hat{\Phi}_{\mathrm{sm}}(t))\right) \tag{4.16}$$

[1] which will allow us to compare with the case of a harmonic oscillator in the limit $\Omega \to 0$ (section 4.3.2).

where

$$c_t = -i\int^t \mathrm{d}s \int^s \mathrm{d}s'\chi(s)\chi(s')[\hat{\Phi}_s[F], \hat{\Phi}_{s'}[F]] \tag{4.17}$$

and

$$\hat{\Phi}_{\mathrm{sm}}(t) := \int^t \mathrm{d}s\chi(s)\hat{\Phi}_s[F]. \tag{4.18}$$

The expression (4.16) seems to suggest that the pointer variable is correlated with (a function of) the time-averaged smeared field operator $\hat{\Phi}_{\mathrm{sm}}(t)$. For example, when the field is in the vacuum state (4.14) is

$$p_0(t,x) = \int \mathrm{d}p\mathrm{d}p' e^{ix(p-p')}\psi(p)\psi^*(p')e^{-c_t(p^2+p'^2)}\langle 0|e^{-i(p-p')\hat{\Phi}_{\mathrm{sm}}(t)}|0\rangle. \tag{4.19}$$

The case of Gaussian measurements

For example, if we assume a Gaussian profile for the wavefunction of the pointer variable

$$\psi(p) = \frac{1}{(2\pi\sigma_p^2)^{1/4}}\exp\left[-\frac{p^2}{4\sigma_p^2}\right] \tag{4.20}$$

we get (see appendix B.1)

$$\hat{\Pi}_x^\dagger(t)\hat{\Pi}_x(t) = \frac{1}{\sqrt{2\pi}\Sigma_t}\exp\left[-\frac{\left(x-\hat{\Phi}_{\mathrm{sm}}(t)\right)^2}{2\Sigma_t^2}\right] \tag{4.21}$$

where

$$\Sigma_t = \sqrt{\frac{1}{4\sigma_p^2} + c_t^2\sigma_p^2}. \tag{4.22}$$

So we see that the 'resolution' of the average-field measurements depends on the initial spread of the conjugate pointer variable and the smeared field Green's function. Then

$$\langle\hat{x}(t)\rangle = \frac{1}{\sqrt{2\pi}\Sigma_t}\langle\int \mathrm{d}x\, x\exp\left[-\frac{\left(x-\hat{\Phi}_{\mathrm{sm}}(t)\right)^2}{2\Sigma_t^2}\right]\rangle \tag{4.23}$$

and we get that

$$\langle\hat{x}(t)\rangle = \langle\hat{\Phi}_{\mathrm{sm}}(t)\rangle = \langle\int^t \mathrm{d}s\chi(s)\hat{\Phi}_s[F]\rangle. \tag{4.24}$$

Indeed the average pointer variable at time t is equal to the (up to t) time-average of the smeared field operator. For example, if the field is initially in the vacuum state, or any fixed-particle state, it holds that $\langle\hat{\Phi}\rangle = 0$ and so the average pointer variable is not shifted. Then, one has to look at higher moments of the pointer variable

$$\begin{aligned} \langle \hat{x}^n(t)\rangle &= \int \mathrm{d}x\, x^n p(x,t) \\ &= \int \mathrm{d}x\, x^n \,\mathrm{tr}_f\left(\hat{\Pi}_x^\dagger(t)\hat{\Pi}_x(t)\hat{\rho}_0\right). \end{aligned} \tag{4.25}$$

In this case, based on (4.21) it is easy to check that

$$\langle \hat{x}^2(t)\rangle = \langle \hat{\Phi}^2_{\mathrm{sm}}(t)\rangle + \Sigma_t^2 \tag{4.26}$$

which implies that the variance of the pointer variable is 'tracking' the variance of the field average (in the sense of [117])

$$(\Delta\hat{x}(t))^2 = (\Delta\hat{\Phi}_{sm}(t))^2 + \Sigma_t^2 \tag{4.27}$$

where

$$\Sigma_t^2 = \frac{1}{4\sigma_p^2} + c_t^2\sigma_p^2 \tag{4.28}$$

It is easy to see that the uncertainty relation for the pointer variable leads to a lower bound for the noise Σ_t,

$$\Sigma_t \geq \sqrt{|c_t|}. \tag{4.29}$$

The lower bound to the noise does not depend on the initial state of the pointer variable, but only on the localization area of the field, and the strength of the coupling as encoded in c_t. In appendix B.2 we calculate how the noise scales in the case of masless field in 3+1 dimensions. Assuming spacetime smearings of widths σ_t, σ_x in space and time accordingly, then

$$\Sigma_t \geq \frac{\lambda}{\sqrt{2\pi}}\sqrt{\frac{\sigma_t}{\sigma_x(\sigma_t^2+\sigma_x^2)}}. \tag{4.30}$$

In this example, this bound expresses the limitation to the field measurability in terms of the size of the spacetime region that the pointer variable is addressing and the coupling strength.

Connection to histories approach: time-extended propositions

In the usual quantization procedure, one quantizes single-time observables of the classical configuration space of a system. A classical observable $A(t)$ is mapped to a quantum observable $\hat{A}(t)$ that is a self-adjoint operator with spectral projections that correspond to all possible values. It is not obvious in this standard account how one can measure time-averages of observables, or what is the quantum counterpart of a time-averaged classical observable (see e.g. [118]). It is worth pointing out that the proposition that corresponds to $\int^t \mathrm{d}s\chi(s)\langle\hat{\Phi}_s[F]\rangle = a$ is represented by the class operator [79]

$$\begin{aligned} C(a) &= \int \frac{\mathrm{d}k}{2\pi} e^{-ika} e^{it\hat{H}} \mathfrak{T} e^{ik\int^t \mathrm{d}s\chi(s)\hat{\Phi}_s[F]} \\ &= \int \frac{\mathrm{d}k}{2\pi} e^{-ika} U_\chi(t,k) \end{aligned} \tag{4.31}$$

which went into the POVM that we defined above (see (4.12)). To this extent, we can convince ourselves that this POVM corresponds to a smeared field time average. We see that time-extended propositions for a given system are not associated with projectors, but with class operators (products of projector operators in the Heisenberg picture) which are *not* projectors (since a given projector does not commute with itself at different times). This implies that when one is interested in formulating time-extended propositions (or measurements) histories-based formalisms arise naturally because there are no 'yes-no' type questions for time-extended observables.

4.1.1 Example: detecting a coherent state

In this section we consider the response of the continuous pointer variable to a coherent state

$$|f\rangle = e^{i\hat{\Phi}_0[f]}|0\rangle \tag{4.32}$$

where

$$\hat{\Phi}_0[f] = \int \mathrm{d}\boldsymbol{x} f(\boldsymbol{x})\hat{\Phi}(0,\boldsymbol{x}). \tag{4.33}$$

The expectation value of the pointer variable is

$$\begin{aligned} \langle\hat{x}(t)\rangle &= \langle f|\hat{\Phi}_{\mathrm{sm}}(t)|f\rangle \\ &= \langle f|\int^t \mathrm{d}s\chi(s)\hat{\Phi}_s[F]|f\rangle. \end{aligned} \tag{4.34}$$

Using the BCH formula

$$\langle f|\hat{\Phi}_{\rm sm}(t)|f\rangle = -i\int^{t} {\rm d}s\chi(s)[\hat{\Phi}_0[f],\hat{\Phi}_s[F]] \tag{4.35}$$

For Gaussian functions

$$f(\boldsymbol{x}) = \frac{1}{(\sqrt{2\pi}\sigma)^3}e^{-\frac{\boldsymbol{x}^2}{2\sigma^2}} \tag{4.36}$$

$$F(\boldsymbol{x}) = \frac{1}{(\sqrt{2\pi}\sigma)^3}e^{-\frac{(\boldsymbol{x}-\boldsymbol{L})^2}{2\sigma^2}}. \tag{4.37}$$

In appendix B.3, for a massless field in 3+1 dimension we derive that

$$\langle f|\hat{\Phi}_{\rm sm}(t)|f\rangle = \int^{t} {\rm d}s\chi(s)C(s) \tag{4.38}$$

where

$$C(s) = \frac{\pi^2\sqrt{2\pi}\sigma}{L}\left(\exp\left[-\frac{(L+s)^2}{4\sigma^2}\right]-\exp\left[-\frac{(L-s)^2}{4\sigma^2}\right]\right), \tag{4.39}$$

and $\hat{\Phi}_{\rm sm}(t)$ is defined in (4.34). If we assume a switching function that is compactly supported only in the time interval $[t_-,t_+]$ where $t_\pm = T\pm\Delta$, we get that

$$\langle f|\hat{\Phi}_{\rm sm}|f\rangle = \frac{\sqrt{2}\pi^3\sigma^2}{L}\left(\text{erf}\left[\frac{t_++L}{2\sigma}\right]-\text{erf}\left[\frac{t_-+L}{2\sigma}\right]+\text{erf}\left[\frac{t_+-L}{2\sigma}\right]-\text{erf}\left[\frac{t_--L}{2\sigma}\right]\right). \tag{4.40}$$

This is equal to the expectation value of the pointer variable. We would expect that the signal peaks at $T=L$, roughly, when the coherent state arrives at the location where the pointer variable is coupled to the field. Indeed, the first two and the last two terms can be thought of as smoothened box functions around $T\pm L$. This means that the detector has to be placed on the future or past lightcone of the localized coherent state (centered at zero).

More generally, instead of (4.32) we can also define a coherent state in terms of a field operator $\hat{\Phi}$ that is also smeared in time [2]. Let us smear the field operator with the spacetime smearing function $f(\mathsf{x})$

$$i\hat{\Phi}[f] = i\int {\rm d}\mathsf{x}f(\mathsf{x})\int {\rm d}\boldsymbol{k}\left(u_{\boldsymbol{k}}(\mathsf{x})\hat{a}_{\boldsymbol{k}}+u^*_{\boldsymbol{k}}(\mathsf{x})\hat{a}^\dagger_{\boldsymbol{k}}\right) = \hat{a}^\dagger[\lambda]-\hat{a}[\lambda] \tag{4.41}$$

[2]Inspired by the writing $\hat{\Phi}(f) = \hat{a}(KEf)+\hat{a}^*(KEf)$ where f smooth compactly supported in Minkowski spacetime, Ef a solution to the equation of motion and $K:\text{sol}\to\mathcal{H}$ the one particle Hilbert space [66].

for

$$\lambda(\boldsymbol{k}) = i\int \mathrm{d}\mathsf{x} f(\mathsf{x}) u^*_{\boldsymbol{k}}(\mathsf{x}) \tag{4.42}$$

and so we can bring the coherent state in the usual form [3]

$$e^{i\hat{\Phi}[f]}\,|0\rangle = e^{\hat{a}^\dagger[\lambda]-\hat{a}[\lambda]}\,|0\rangle\,. \tag{4.43}$$

Then, we consider the covariant commutation relations [60]

$$[\hat{\Phi}[f], \hat{\Phi}[f']] = iE(f, f') \tag{4.44}$$

and (abusing notation) we denote $\hat{\Phi}_{\mathrm{sm}}(t)$ in (4.34) as $\hat{\Phi}[\chi F]$. Then it follows from the BCH formula that

$$\langle f|\hat{\Phi}[\chi F]|f\rangle = E(f, \chi F) \tag{4.45}$$

and

$$\langle f|\hat{\Phi}^2[\chi F]|f\rangle = \langle 0|\hat{\Phi}^2[\chi F]|0\rangle + E^2(f, \chi F). \tag{4.46}$$

Then subtracting the square of (4.45) from the square of (4.46) we calculate the variance

$$(\Delta\hat{\Phi}_{\mathrm{sm}}(t))_f = (\Delta\hat{\Phi}_{\mathrm{sm}}(t))_0 \tag{4.47}$$

The variance is the same as in the vacuum state (since the coherent state is basically like a displaced vacuum state, the mean value is different than in the vacuum but the variance is not).

Finally, we can consider the full probability distribution of the pointer variable when the field is in a coherent state. Using the Weyl relations

$$e^{i\hat{\Phi}[f]}e^{i\hat{\Phi}[f']} = e^{-\frac{i}{2}E(f,f')}e^{i\hat{\Phi}[f+f']} \tag{4.48}$$

Given a coherent state $|f\rangle$ of the field, the probability distribution for the pointer variable is

$$p_f(x,t) = \int \mathrm{d}p\mathrm{d}p' e^{ix(p-p')}\psi(p)\psi^*(p')e^{-c_t(p^2+p'^2)}e^{i(p'-p)E(f,\chi F)}\langle 0|e^{i(k'-k)\hat{\Phi}[\chi F]}|0\rangle. \tag{4.49}$$

Looking at the full probability, the question is: Can this detector distinguish $|f\rangle$ from the vacuum, and how well? The intuition from the analysis and the examples above is

[3] Note that this state is different than the time-ordered version $\mathcal{T}e^{i\hat{\Phi}[f]}\,|0\rangle$ that is created by a source term in the Hamiltonian.

that this continuous pointer variable should be able to distinguish the coherent state from the vacuum since it measures field amplitudes, and the pointer variable behaves as an 'antenna'. Yet, we notice something counter-intuitive: comparing (4.49) with the vacuum probability (4.19) we see that the f-dependence appears only as a phase that depends on the overlap $E(f, \chi F)$. That is, when the spacetime support of the pointer variable coincides with the support of the coherent state the phase vanishes (since $E(f, f) = 0$) and the probability becomes equal to the vacuum probability. In other words, it seems like when we couple the pointer variable 'on top' of the coherent state, the detector becomes transparent. We were able to explain this by taking into account that when f is time extended, it entails not only $\hat{\Phi}$ elongation but also $\dot{\hat{\Phi}} = \hat{\Pi}$ elongation. At the same time, since the coupling of the pointer variable to the field is time-extended by means of the switching, the pointer variable is sensitive to both quadratures. One can analyze the response of the pointer variable by means of how 'aligned' the relative angles of the coherent state and the coupling. The full analysis will appear elsewhere [119].

4.2 Weak coupling: detecting a wavepacket

The particle-like interpretation of the detector's response mostly relies on the phenomenon of resonance, that is, the detector 'clicking' (only) when the detector's energy gap 'matches' the energy of an (almost) monochromatic wavepacket. It is curious that this simple phenomenon is not so well understood in the literature of detector models in QFT. Here, we partially review previous analysis of wavepacket detection in the Unruh-DeWitt model [120, 120] and then we present our analysis in the next subsection. Then, we consider wavepacket detection in the solvable model based on QBM, and we argue that the perturbative techniques cannot capture the resonant behavior in full.

Consider the interaction Hamiltonian

$$\hat{H}_{int} = \lambda \chi(t) \hat{\mu}(t) \hat{\Phi}_t[F] \tag{4.50}$$

where

$$\hat{\mu}(t) = e^{i\omega t}\sigma^+ + e^{-i\omega t}\sigma^- \tag{4.51}$$

the monopole operator of a two-level system. The excitation probability is

$$P^+(\infty) = \lambda^2 \int_{-\infty}^{\infty} \mathrm{d}t\mathrm{d}t' \chi(t)\chi(t') e^{-i\omega(t-t')} \mathrm{tr}(\hat{\Phi}_t[F]\hat{\Phi}_{t'}[F]\hat{\rho}_0). \tag{4.52}$$

We can decompose the smeared field two-point function as

$$\begin{aligned}\operatorname{tr}(\hat{\Phi}_t[F]\hat{\Phi}_{t'}[F]\hat{\rho}_0)& \\ = \int \mathrm{d}\boldsymbol{x}\mathrm{d}\boldsymbol{x}' F(\boldsymbol{x})F(\boldsymbol{x}')&\left(W(\boldsymbol{x},t;\boldsymbol{x}',t') + W(\boldsymbol{x}',t';\boldsymbol{x},t) + W_0(\boldsymbol{x},t;\boldsymbol{x}',t')\right)\end{aligned} \tag{4.53}$$

where

$$W(\boldsymbol{x},t;\boldsymbol{x}',t') := \int \mathrm{d}\boldsymbol{k}\mathrm{d}\boldsymbol{k}' u^*_{\boldsymbol{k}}(\boldsymbol{x},t)u_{\boldsymbol{k}'}(\boldsymbol{x}',t')\rho_{\boldsymbol{k},\boldsymbol{k}'} \tag{4.54}$$

$$W_0(\boldsymbol{x},t;\boldsymbol{x}',t') := \int \mathrm{d}\boldsymbol{k} u_{\boldsymbol{k}}(\boldsymbol{x},t)u^*_{\boldsymbol{k}}(\boldsymbol{x}',t') \tag{4.55}$$

and

$$\rho_{\boldsymbol{k},\boldsymbol{k}'} = \operatorname{tr}(\hat{a}_{\boldsymbol{k}'}\hat{\rho}_0\hat{a}^\dagger_{\boldsymbol{k}}) \tag{4.56}$$

the one-particle reduced density matrix. This demonstrates that all the information of the response for a single detection (leading order to perturbation theory) is encoded in (4.56). So the detector response is the same for multi-particle states that give the same one-particle reduced density matrix. For a one-particle wavepacket state

$$|\psi\rangle = \int \mathrm{d}\boldsymbol{k}\tilde{\psi}(\boldsymbol{k})\hat{a}^\dagger_{\boldsymbol{k}}|0\rangle \tag{4.57}$$

(4.56) is simply $\rho_{\boldsymbol{k},\boldsymbol{k}'} = \tilde{\psi}(\boldsymbol{k})\tilde{\psi}^*(\boldsymbol{k}')$. If we define

$$\psi(\boldsymbol{x},t) := \int \mathrm{d}\boldsymbol{k} u_{\boldsymbol{k}}(\boldsymbol{x},t)\tilde{\psi}(\boldsymbol{k}), \tag{4.58}$$

which happens to be the Newton-Wigner wavefunction of the wavepacket [121], the excitation probability factorizes as follows

$$P^+(\infty) = P_0 + P_\psi \tag{4.59}$$

where P_0 the vacuum excitation probability (see equation 4.64 below) and

$$P_\psi = \left|\int \mathrm{d}t e^{i\omega t}\chi(t)\int \mathrm{d}\boldsymbol{x} F(\boldsymbol{x})\psi(\boldsymbol{x},t)\right|^2 + \left|\int \mathrm{d}t e^{i\omega t}\chi(t)\int \mathrm{d}\boldsymbol{x} F(\boldsymbol{x})\psi^*(\boldsymbol{x},t)\right|^2. \tag{4.60}$$

Writing explicitly the Newton-Wigner wavefunction as

$$\psi(\boldsymbol{x},t) = \frac{1}{\sqrt{2\pi}^3}\int \frac{\mathrm{d}\boldsymbol{k}}{\sqrt{2\omega_{\boldsymbol{k}}}}\tilde{\psi}(\boldsymbol{k})e^{i\boldsymbol{k}\boldsymbol{x}-i\omega_{\boldsymbol{k}}t} \tag{4.61}$$

If we write (4.60) in momentum space we can recognize the usual 'counter-rotating' and 'co-rotating' terms

$$P_\psi = \left| \int \frac{\mathrm{d}\boldsymbol{k}}{4\pi\sqrt{2\omega_{\boldsymbol{k}}}} \tilde{F}(\boldsymbol{k}) \tilde{\chi}(\omega + \omega_{\boldsymbol{k}}) \tilde{\psi}(\boldsymbol{k}) \right|^2 + \left| \int \frac{\mathrm{d}\boldsymbol{k}}{4\pi\sqrt{2\omega_{\boldsymbol{k}}}} \tilde{F}(\boldsymbol{k}) \tilde{\chi}(\omega - \omega_{\boldsymbol{k}}) \tilde{\psi}(\boldsymbol{k}) \right|^2, \tag{4.62}$$

where $\tilde{\chi}, \tilde{F}$ the usual Fourier transforms of the switching and smearing functions. We want to analyze whether there is a resonance effect when the wavepacket is almost monochromatic at the wavelength of the detector energy gap. We also want to analyze the role that the spatial smearing plays in the wavepacket detection, as we have not assumed that the detector is pointlike.

For the study of resonance, it is typical to assume that the interaction is always on, namely $\chi(t) = 1$. In this case, the first 'counter-rotating' term vanishes (because $\tilde{\chi}(\omega + \omega_{\boldsymbol{k}}) = \delta(\omega + \omega_{\boldsymbol{k}})$) and we have that

$$P_\psi = \left| \int \frac{\mathrm{d}\boldsymbol{k}}{4\pi\sqrt{2\omega_{\boldsymbol{k}}}} \tilde{F}(\boldsymbol{k}) \tilde{\psi}(\boldsymbol{k}) \delta(\omega - \omega_{\boldsymbol{k}}) \right|^2. \tag{4.63}$$

Also, notice that in the long-time limit the vacuum contribution to the excitation probability vanishes for similar reasons. Indeed, from (4.52),(4.53),(4.55) and (4.59) we get that

$$\begin{aligned} P_0 &= \lambda^2 \int \mathrm{d}t \mathrm{d}t' \chi(t) \chi(t') e^{-i\omega(t-t')} \int \mathrm{d}\boldsymbol{x} \mathrm{d}\boldsymbol{x}' F(\boldsymbol{x}) F(\boldsymbol{x}') \int \mathrm{d}\boldsymbol{k} u_{\boldsymbol{k}}(\boldsymbol{x}, t) u^*_{\boldsymbol{k}}(\boldsymbol{x}', t') \\ &= \frac{\lambda^2}{2(2\pi)^3} \int \frac{\mathrm{d}\boldsymbol{k}}{|\boldsymbol{k}|} |\tilde{F}(\boldsymbol{k})|^2 |\tilde{\chi}(\omega + \omega_{\boldsymbol{k}})|^2. \end{aligned} \tag{4.64}$$

This means that the vacuum contribution is also 'counter-rotating' and will vanish in the long-time limit, or more precisely in the adiabatic limit (where one assumes a family of switching functions of finite width T and takes the limit $T \to \infty$ [122, 123]).

The spatial smearing effectively selects the frequencies of the field that the detector is sensitive to. In [64] it was observed that if the support of the Fourier transform of the smearing $\tilde{F}(k)$ does not include the detector gap ω, or if $\tilde{F}(\pm\omega)$ is negligible, the excitation probability is also negligible, that is, the detector becomes transparent to the resonant frequency. This is easy to see, especially in the long-time limit, through (4.63). For example, consider a massless scalar field in $1+1$ dimensions, a Gaussian smearing function

$$F(x) = \frac{1}{\sqrt{2\pi}\sigma_x} e^{-\frac{x^2}{2\sigma_x^2}} \tag{4.65}$$

and a Gaussian wavepacket

$$\tilde{\psi}(\boldsymbol{k}) = \frac{1}{\sqrt{2\pi}\sigma_{\boldsymbol{k}}} e^{-\frac{(\boldsymbol{k}-\boldsymbol{k}_0)^2}{2\sigma_{\boldsymbol{k}}^2}} . \tag{4.66}$$

Through equation (4.63) we see that the excitation probability is given by the overlap of $\tilde{F}(\boldsymbol{k})$ with $\tilde{\psi}(\boldsymbol{k})$ evaluated at the frequency of the detector:

$$P_\psi = \frac{\lambda^2 e^{-\omega^2\sigma_x^2}}{4\pi^4\sigma_{\boldsymbol{k}}^2\omega} e^{-\left(\frac{\omega-\omega_0}{\sigma_{\boldsymbol{k}}}\right)^2} , \tag{4.67}$$

where $\omega_0 = \omega_{\boldsymbol{k}_0}$. The exponential factor $e^{-\omega^2\sigma_x^2}$ captures the effect of the detector's size on the excitation probability. We see that for the probability to not be significantly smaller than the one of the pointlike model, it has to hold that $1/\sigma_x \gg \omega$. That is, the width of the Fourier of the smearing function $\tilde{F}(\boldsymbol{k})$ should be much larger than the detector energy gap. We also see that the excitation probability peaks at $\omega = \omega_0$. Nevertheless, in arbitrary spacetime dimension, it is not clear whether the excitation probability increases in amplitude the more monochromatic the wavepacket, i.e., in the limit $\sigma_{\boldsymbol{k}} \to 0$, and counter-intuitive behavior of the monochromatic limit was pointed out in [124]. We will elaborate on this issue in the next subsection.

4.2.1 Resonance and time-of-arrival in the smeared UDW model

There are two related but distinct questions: Does the excitation probability 'peak' when the wavepacket is (almost) monochromatic at the frequency of the detector gap? and, does the excitation probability 'peak' at the expected 'time-of-arrival' of the wavepacket at the detector location? As we will see, in some sense these two questions are complementary. The intuition is that 'the more monochromatic the wavepacket the better the measurement of its energy (due to resonance) and the worse the time-of-arrival measurement (since an almost monochromatic wavepacket is very delocalized). In what follows, we will refine this intuition by analyzing what it is that the detector's response is sensitive to.

We will assume that the detector is centered around $\boldsymbol{L}$, and the wavepacket is centered at zero with momentum $\boldsymbol{k}_0$. This will help us understand resonance and time-of-arrival in terms of the configuration of the detector and the wavepacket in space and time. We define the overlap of the spatial smearing with the Newton-Wigner wavefunction of the wavepacket

$$\psi_F(t) := \int \mathrm{d}\boldsymbol{x} F(\boldsymbol{x})\psi(\boldsymbol{x},t) \tag{4.68}$$

and so the expression (4.60) becomes

$$P_\psi = \lambda^2 \left| \int \mathrm{d}t e^{i\omega t}\chi(t)\psi_F(t)\right|^2 + \lambda^2 \left| \int \mathrm{d}t e^{i\omega t}\chi(t)\psi_F^*(t)\right|^2 . \tag{4.69}$$

As we argued above, we will assume that the width of the Fourier $\tilde{F}(\boldsymbol{k})$ is much larger than the momentum of the wavepacket $1/\sigma_x \gg |\boldsymbol{k}_0|$ so that $\tilde{F}(\boldsymbol{k}) \simeq \tilde{F}(\boldsymbol{k}_0)$ since we will assume an almost monochromatic wavepacket

$$\begin{aligned}\psi_F(t) &= \int \frac{\mathrm{d}\boldsymbol{k}}{\sqrt{2\omega_{\boldsymbol{k}}}} \tilde{F}(\boldsymbol{k}) e^{i\boldsymbol{k}\boldsymbol{L} - i\omega_{\boldsymbol{k}} t}\tilde{\psi}(\boldsymbol{k}) \\ &\simeq \tilde{F}(\boldsymbol{k}_0)\psi(\boldsymbol{L},t). \end{aligned} \tag{4.70}$$

Since the almost monochromatic wavepacket is around $\boldsymbol{k}_0$ we can change variables $\boldsymbol{k} = \boldsymbol{k}_0 + \boldsymbol{q}$ with $|\boldsymbol{q}| \ll 1$ to estimate that

$$\psi(\boldsymbol{L},t) \simeq u_{\boldsymbol{k}_0}(\boldsymbol{L},t)\tilde{\psi}(\boldsymbol{L} - \boldsymbol{v}_0 t) \tag{4.71}$$

where $\boldsymbol{v}_0 = \boldsymbol{k}_0/\omega_{\boldsymbol{k}_0}$ the 'relativistic velocity' (see appendix C). The function $\tilde{\psi}$ is peaked at $\boldsymbol{q} = 0$ so the maximum value is obtained for some time t when $\boldsymbol{L}$ and $\boldsymbol{v}_0$ are aligned (that is, when we 'shoot' the wavepacket towards the detector's center) and the problem essentially reduces to a one-dimensional problem. This is compatible with our observation in [15] that the detector is detecting spherical waves (and not plane waves) and might explain some of the counter-intuitive behavior that was observed in [124] that in particular spacetime dimensions that excitation probability *decreases* in the monochromatic limit.

For the rest, we denote $L := |\boldsymbol{L}|$ and $v_0 := |\boldsymbol{v}_0|$. If we choose $\chi, \tilde{\psi}$ to be Gaussian functions with width σ_t, σ_k we get

$$P_\psi = \frac{\lambda^2 |F(k_0)|^2}{2\omega_0} \frac{\sigma_{k,t}^2}{2\sigma_k^2\sigma_t^2} e^{-\frac{(T - L/v_0)^2}{(\sigma_t^2 + \tilde{\sigma}_k^2)}} \left(e^{-2\sigma_{k,t}^2(\omega-\omega_0)^2} + e^{-2\sigma_{k,t}^2(\omega+\omega_0)^2}\right) \tag{4.72}$$

where

$$\mu_{k,t} = \frac{T\tilde{\sigma}_k^2 + \frac{L}{v_0}\sigma_t^2}{\sigma_t^2 + \tilde{\sigma}_k^2} \tag{4.73}$$

and

$$\sigma_{k,t} = \frac{\sigma_t \tilde{\sigma}_k}{\sqrt{\sigma_t^2 + \tilde{\sigma}_k^2}}, \tag{4.74}$$

where $\tilde{\sigma}_k = \sigma_k / v_0$. We see that indeed the probability peaks at $\omega = \omega_0$ and that there is an overall factor that peaks at the expected 'time-of-arrival' time $T = L/v_0$.

Time-of-arrival probability of a relativistic particle

Note that the probability (4.72) that we derived in the example above is not a genuine time-of-arrival probability, it is the *total* probability at $t \to \infty$. Temporal probabilities are not easy to define since there is no time operator. For example, in non-relativistic quantum mechanics $|\psi(t, \boldsymbol{x})|^2$ is a probability density over $\boldsymbol{x}$ labeled by t (but not over t) which means that we cannot integrate it over a time interval to decide how likely it is that the particle will be found at these times. A method for defining temporal probabilities in the context of detector models in QFT can be found in [80, 125]. The QTP program is based on the formalism of the decoherent histories approach, and it constructs temporal probabilities that are associated with propositions about the first-crossing time of the particle, given that suitable conditions are satisfied by the switching function (interpreted as a time sampling at leading order). It leads to the same expressions as we derived above at leading order in perturbation theory, but it provides the tools for assigning the temporal coarse-graining that is needed for constructing well-defined temporal probabilities.

Assuming a switching function that is peaked around T we have that

$$P_T = \lambda^2 \int \mathrm{d}s\mathrm{d}s' \chi(s-T)\chi(s'-T) e^{-i\omega(s-s')} |\tilde{F}(\boldsymbol{k}_0)|^2 \psi(\boldsymbol{L}, s)\psi^*(\boldsymbol{L}, s'). \tag{4.75}$$

We perform the following change of variables: $S = (s+s')/2$ and $\xi = s - s'$. We get

$$P_T = \int \mathrm{d}S \chi(s-T) P(s) \tag{4.76}$$

where

$$P(s) = \lambda^2 |\tilde{F}(\boldsymbol{k}_0)|^2 \int \mathrm{d}\xi e^{i\omega\xi} \chi(\xi) \psi^*(L, s + \frac{\xi}{2}) \psi(L, s - \frac{\xi}{2}). \tag{4.77}$$

The probability (4.76) is a well-defined probability with respect to the time T, namely it satisfies Kolmogorov additivity when integrated over time-intervals ΔT if

$\Delta T \gg \sigma_t$ the width of the Gaussian sampling function χ. This means that time-of-arrival propositions are additive only if they are very coarse-grained. If we increase the resolution of the time-measurements the consistency condition is not satisfied.

The time-of-arrival-like factor (see (4.72)) does not arise in the standard analysis of resonance because it is typical to assume the long-time limit i.e., $\chi(t) = 1$ (for killing the counter-rotating term and the vacuum noise term) to obtain a sharp peak at the resonant frequency. From this perspective, it is problematic that we assumed a Gaussian switching function of finite width in the example above. By doing so, we realize the two-fold role that this timescale plays if we view it as a time resolution: the more we want to resolve the time-of arrival, we introduce noise and in that sense we lose the resonant behavior. At the same time, for small time-resolution, the expression is not even a well-defined temporal probability (since the additivity condition cannot be satisfied).

4.3 The QBM model

In the previous sections, we saw that a field-particle interaction with an UDW coupling corresponds (i) to a measurement of field properties if the apparatus Hamiltonian is negligible compared to the interaction term, and (ii) to a measurement of particle properties if the interaction is weak compared to the apparatus Hamiltonian. In this section, we present a model that incorporates both cases for different parameter regimes. Furthermore, this model, being solvable, can be used to characterize intermediate regimes. This analysis is base on QBM models, which admit exact solutions for their time evolution [126, 127, 128, 129, 130].

4.3.1 Solving the Heisenberg equations of motion

For this purpose, we consider the analogy between the detector interacting with the quantum field and the quantum Brownian motion (QBM) model in which a quantum harmonic oscillator of frequency Ω_0 is interacting with a bath of harmonic oscillators indexed by i, which is described by the Hamiltonian

$$\hat{H} = \frac{\hat{P}^2}{2M} + \frac{1}{2}M\omega_0^2\hat{X}^2 + \sum_i \left(\frac{\hat{p}_i^2}{2m_i} + \frac{1}{2}m_i\omega_i^2\hat{q}_i^2\right) + \hat{X} \otimes \sum_i c_i\hat{q}_i \tag{4.78}$$

The Unruh-DeWitt type Hamiltonian that couples linearly the quadrature of the quantum harmonic oscillator to the quantum field with interaction Hamiltonian

$$\hat{H}_{\text{int}} = \lambda \hat{X} \otimes \hat{\Phi}(\boldsymbol{x}_0) \tag{4.79}$$

is of the QBM form (4.78) if we decompose the field as a bath of quantum harmonic oscillators indexed by $i = \boldsymbol{k}$, defined in appendix D. Choosing $\boldsymbol{x}_0 = 0$ the coupling constants are

$$c_{\boldsymbol{k}} = \frac{\lambda}{\sqrt{\omega_{\boldsymbol{k}}}}. \tag{4.80}$$

The form of the coupling constants specifies the spectral density of the environment. In this case (see appendix D.1) the spectral density is

$$I(\omega) = \pi\lambda^2\omega. \tag{4.81}$$

The spectral density being proportional to ω is the definition of an Ohmic environment. That is, through the QBM model we see that a quantum harmonic oscillator linearly coupled to a scalar quantum field in a pointlike manner 'perceives' the field as an Ohmic environment. This is not the case e.g. for a smeared coupling between the field and the oscillator[4]. The QBM model can be solved for a certain class of environments, and the Ohmic case is the simplest. The model is easily solvable because the Heisenberg equations of motion are linear

$$\dot{\hat{X}}(t) = \hat{P}(t)/M \tag{4.82}$$

$$\dot{\hat{P}}(t) = -M\omega_0^2\hat{X}(t) - \lambda\hat{\Phi}(t,0) \tag{4.83}$$

$$\left(\Box - m^2\right)\hat{\Phi}(t,\boldsymbol{x}) = -\lambda\delta(\boldsymbol{x})\hat{X}(t). \tag{4.84}$$

The solution to (4.84) is

$$\hat{\Phi}(t,\boldsymbol{x}) = \hat{\Phi}_0(t,\boldsymbol{x}) - \lambda\int_0^t \mathrm{d}t' G_{\text{R}}(t,\boldsymbol{x};t',0)\hat{X}(t') \tag{4.85}$$

where $\hat{\Phi}_0(t,\boldsymbol{x})$ is the free solution and G_{R} is the retarded propagator of the Klein-Gordon equation. Combining this with the equations of motion above we get

$$\ddot{\hat{X}}(t) + \omega_0^2\hat{X}(t) - \frac{\lambda^2}{M}\int_0^t \mathrm{d}t' G_{\text{R}}(t',\boldsymbol{x};t,0)\hat{X}(t) = -\lambda\hat{\Phi}(t,0) \tag{4.86}$$

[4] When more than one oscillator is involved, one can argue that extended interactions are effectively pointlike if they are small with respect to the relative distance of the oscillators [131].

which is a linear integro-differential equation with an operator-valued source.

The solutions to the Heisenberg equations of motion can be found through the Laplace transform (see e.g. [116]) and are given by

$$\hat{X}(t) = \dot{u}(t)\hat{X}(0) + \frac{1}{M}u(t)\hat{P}(0) - \frac{\lambda}{M}\int_0^t \mathrm{d}t' u(t-t')\hat{\Phi}_0(t') \tag{4.87}$$

where $\hat{\Phi}_0(t)$ is the solution to the free Klein Gordon equation and $u(t)$ is the homogeneous solution which satisfies

$$\ddot{u}(t) + \omega_0^2 u(t) - \frac{\lambda^2}{M}\int_0^t \mathrm{d}t' G_{\mathrm{R}}(t, x_0; t', x_0)u(t') = 0. \tag{4.88}$$

By integration by parts, we write this as

$$\ddot{u}(t) + \bar{\omega}^2 u(t) + \frac{\lambda^2}{M}\int_0^t \mathrm{d}t' \gamma(t-t')\dot{u}(t') = 0 \tag{4.89}$$

where γ is the so-called dissipation kernel, and $\bar{\omega}^2 = \omega_0^2 - \frac{2\lambda^2}{M}\gamma(0)$ is the renormalized frequency of the detector. The field's Green function in (4.88) is the derivative of the so-called dissipation kernel (see e.g. [132]) which is related to the spectral density as follows

$$\gamma(t) = \int_0^\infty \frac{\mathrm{d}\omega}{2\pi}\frac{I(\omega)}{\omega}\cos(\omega t) \tag{4.90}$$

For a massless field and a pointlike coupling

$$\gamma(t) = \frac{\lambda^2}{4\pi}\delta(t), \tag{4.91}$$

and equation (4.89) becomes

$$\ddot{u} + 2\Gamma\dot{u} + \bar{\omega}^2 u = 0, \tag{4.92}$$

where $\Gamma = \frac{\lambda^2}{8\pi M}$. The solutions are

$$u(t) = \begin{cases} \frac{\sin(\omega t)}{\omega}e^{-\Gamma t}, & \omega = \sqrt{\bar{\omega}^2 - \Gamma^2}, \quad \Gamma < \bar{\omega} \\ \frac{\sinh(\Gamma' t)}{\Gamma'}e^{-\Gamma t}, & \Gamma' = \sqrt{\Gamma^2 - \bar{\omega}^2}, \quad \Gamma > \bar{\omega} \end{cases} \tag{4.93}$$

This means that one can analyze the behavior of this model in the weak/strong coupling regimes in terms of the underdamped/overdamped solutions of a classical damped harmonic oscillator (see e.g.[126]).

From (4.87) we see that

$$\hat{X}(t) = \hat{X}_0(t) - \frac{\lambda}{M}\hat{\Phi}_u(t) \tag{4.94}$$

where

$$\hat{X}_0(t) := \dot{u}(t)\hat{X}(0) + \frac{1}{M}u(t)\hat{P}(0) \tag{4.95}$$

and

$$\hat{\Phi}_u(t) := \int_0^t \mathrm{d}t' u(t-t')\hat{\Phi}_0(t'). \tag{4.96}$$

So we see that the position operator of the harmonic oscillator is basically tracking the field operator smeared/convolved in time with the homogeneous solution. The more the dissipation (stronger coupling $\Gamma > \bar{\omega}$) the more local in time this 'tracking' becomes. For the variances, it holds that

$$(\Delta\hat{X}(t))^2 = (\Delta\hat{X}_0(t))^2 + \frac{\lambda^2}{M^2}(\Delta\hat{\Phi}_u(t))^2. \tag{4.97}$$

4.3.2 The strong coupling regime: field measurements

We want to see to what extent we can reproduce the result of this section of this chapter, where we established that the pointer variable is correlated with field averages. First, we will have to solve the QBM model with a coupling to the momentum of the harmonic oscillator and then consider the case in which the bare frequency of the harmonic oscillator is zero (gapless pointer variable). In the previous section we saw that the quadratures of the harmonic oscillator are 'tracking' the field operator convoluted with $u, \dot{u}$ i.e. the homogeneous solution and its derivative. This is a convolution that is non-local in time, depending on how strong the dissipation is. Of course, the dissipation did not appear in the pointer variable analysis, and we should be able to recover that the pointer variable is correlated with the time average of $\hat{\Phi}(t, \boldsymbol{x}_0)$ weighted by $\chi(t) = 1$.

In appendix D.2 we solve the equations of motion for the QBM model with interaction Hamiltonian

$$\hat{H}_{int} = \lambda\hat{\Phi}(t, \boldsymbol{x}_0) \otimes \hat{P}(t) \tag{4.98}$$

We find (see (D.34)) that $u(t)$ satisfies

$$\ddot{u}(t) + \omega_0^2(1 - 2M\gamma(0))u(t) + 2\pi\lambda^2 M\omega_0^2\dot{u}(t) = 0 \tag{4.99}$$

which for $\omega_0 = 0$ simply reduces to $\ddot{u}(t) = 0$, and as a result we have that $\dot{u}(t) = 1$ (no dissipation). Then, the solution to the equation of motion for the conjugate pointer variable $\hat{X}(t)$ is reduced to

$$\hat{X}(t) = \hat{X}(0) - \int_0^t \mathrm{d}t'\hat{\Phi}_0(t'). \tag{4.100}$$

As we argued in the first section, this corresponds to strong coupling in the sense that the self-dynamics of the oscillator is negligible. We see that without internal dynamics there cannot be dissipation, even for strong couplings.

4.3.3 The weak coupling regime: resonance

The homogeneous solution for $\Gamma < \bar{\Omega}$ is

$$u(t) = \frac{\sin(\omega t)}{\omega} e^{-\Gamma t} \tag{4.101}$$

where

$$\omega = \sqrt{\bar{\omega}^2 - \Gamma^2}. \tag{4.102}$$

We will assume that the state of the field is a wavepacket, and we will calculate the average energy of the harmonic oscillator (in the long-time limit) which is

$$\langle \hat{H} \rangle = \frac{(\Delta \hat{P})^2}{2M} + \frac{M\bar{\omega}^2(\Delta \hat{X})^2}{2}. \tag{4.103}$$

This is because $\langle \hat{X} \rangle = \langle \hat{P} \rangle = 0$ for Fock states of the field, and as a result $(\Delta \hat{X}) = \langle \hat{X}^2 \rangle$ and $(\Delta \hat{P}) = \langle \hat{P}^2 \rangle$. Note that we have used the renormalized and not the bare frequency of the harmonic oscillator in (4.103). The variance of $\hat{X}$ is given by (4.97) where $\Delta \hat{X}_0(t) = 0$ for $t \gg 1/\Gamma$ (see appendix D.3). Assuming that the environment is in a one-particle state

$$|\psi\rangle = \int \mathrm{d}\boldsymbol{p}\psi(\boldsymbol{p} - \boldsymbol{p}_0)\hat{a}_{\boldsymbol{p}}^\dagger |0\rangle \tag{4.104}$$

Then,

$$(\Delta\hat{\Phi}_u(t))^2 = (\Delta\hat{\Phi}_0(t))^2 + (\Delta\hat{\Phi}_{u,\psi}(t))^2 \tag{4.105}$$

where $(\Delta\hat{\Phi}_{u,\psi}(t))$ the state-dependent part, which is

$$(\Delta\hat{\Phi}_{u,\psi}(t))^2 = \left|\int \mathrm{d}\boldsymbol{p}\psi^*(\boldsymbol{p}-\boldsymbol{p}_0)v_{u,\boldsymbol{p}}(t)\right|^2 \tag{4.106}$$

where

$$v_{u,\boldsymbol{p}}(t) := \int_0^t \mathrm{d}t' u_{\boldsymbol{p}}(t',\mathbf{0})u(t-t') \tag{4.107}$$

and $u_{\boldsymbol{p}}$ comes from the usual mode decomposition of the field. Further, we assume that the wavepacket is centered around $\boldsymbol{L}$, namely $\psi(\boldsymbol{p}-\boldsymbol{p}_0) = \chi(\boldsymbol{p}-\boldsymbol{p}_0)e^{i\boldsymbol{p}\boldsymbol{L}}$, where $\psi(\boldsymbol{p}-\boldsymbol{p}_0)$ is peaked around $\boldsymbol{p}_0$ (almost monochromatic) and that $\Gamma|\boldsymbol{L}| \gg 1$. We get we get

$$(\Delta\hat{\Phi}_{u,f}(t))^2 = \frac{|\psi(t,\boldsymbol{L})|^2}{8\bar{\Omega}^2|\boldsymbol{p}_0|}\left(\frac{1}{\Gamma - i(|\boldsymbol{p}_0|+\bar{\Omega})} - \frac{1}{\Gamma - i(|\boldsymbol{p}_0|-\bar{\Omega})}\right)^2. \tag{4.108}$$

where

$$\psi(t,\boldsymbol{L}) = \int \mathrm{d}\boldsymbol{p}\chi(\boldsymbol{p}-\boldsymbol{p}_0)e^{-i|\boldsymbol{p}|t}e^{-i\boldsymbol{p}\boldsymbol{L}}. \tag{4.109}$$

Performing a similar calculation for $\Delta\hat{P}(t)$, which is related to $\Delta\hat{\Phi}_{\dot{u}}$, (see appendix D.3) we get

$$\langle\hat{H}(t)\rangle = \frac{|\Psi(t,\boldsymbol{L})|^2}{2M|\boldsymbol{p}_0|}\frac{|\boldsymbol{p}_0|^2+\bar{\omega}^2}{|(\bar{\omega}^2-|\boldsymbol{p}_0|^2)-2i\Gamma|\boldsymbol{p}_0||^2} + \mathcal{N} \tag{4.110}$$

where the vacuum noise is given by

$$\mathcal{N} = \frac{\lambda^2}{2M}(\Delta\hat{\Phi}_{\dot{u},0}(t))^2 + \frac{\lambda^2\bar{\omega}^2}{2M}(\Delta\hat{\Phi}_{u,0}(t))^2. \tag{4.111}$$

If we perform the calculation with a smeared coupling between the harmonic oscillator and the field (as in [133]) we see that the noise term diverges logarithmically as the size of the smearing vanishes. It can be shown that this noise term represents the leading order behavior of the noise in a power expansion involving a UV cutoff. What we see in (4.110) is that when the dissipation Γ is small (weak coupling) the energy of the oscillator sharply peaks when the wavepacket's energy coincides with the renormalized frequency of the oscillator. The form of the resonance is of the (expected) Breit-Wigner form [134]. Indeed, resonances of the Breit-Wigner type require absorption effects that can only be captured non-perturbatively, or after a partial resummation of the perturbative series [116].

4.4 Detecting a massive particle

In most of the calculations above, we assumed a massless scalar field (even though most expressions are general) for the purpose of modeling the interaction of a detector with the electromagnetic field. The formalism is quite general, so in principle it can be applied to the case of a massive scalar field to model the detection of a massive field excitation, or a massive particle (at least for small masses, as in the case of neutrinos). The analysis of the localized detection of a massive particle in quantum field theory yields some interesting qualitative results that are absent in the quantum mechanical description.

To motivate what follows, consider a non-relativistic quantum particle of non-zero mass m. Its state admits a spatial representation by means of spatial wavefunction $\psi(\boldsymbol{x}, t)$. Then the probability of detecting the quantum-mechanical particle at location $\boldsymbol{x}$ and at time t is given by the square amplitude of its wavefunction $|\psi(\boldsymbol{x}, t)|^2$. In comparison, when a relativistic quantum particle is described as an excitation of a massive scalar field, the $|\psi|^2$ description is not available for two distinct but related reasons: the absence of a spatial wavefunction, and the non-availability of idealized measurements. Instead, one has to model dynamically the detection process, by choosing a coupling between the detector and the massive scalar field. The observation is that the localized detection of a relativistic quantum particle, e.g. in which locations it is most likely to be detected, can be highly dependent on the physical interaction that we choose for probing it. This seems to suggest that localization of a quantum-field-theoretical state is a relational property that can only be defined in relation to another physical system that is interacting with the quantum field in one way or another [135, 8].

4.4.1 Where something is (found) depends on how you look for it

Consider an initial state, in which the field is in a wavepacket state and the detector initially in the ground state

$$|\Psi_0\rangle = |g\rangle \otimes \int \mathrm{d}\boldsymbol{y}\psi(\boldsymbol{y})\hat{a}^\dagger_{\boldsymbol{y}} |0\rangle \tag{4.112}$$

where

$$\hat{a}_{\boldsymbol{y}} := \int \mathrm{d}\boldsymbol{k} e^{-i\boldsymbol{k}\boldsymbol{y}} \hat{a}_{\boldsymbol{k}} \tag{4.113}$$

the non-relativistic localization scheme [5]. We assume an interaction Hamiltonian of the form

$$\hat{H}_{int} = \lambda\chi(t)\hat{\mu}(t) \otimes \int \mathrm{d}\boldsymbol{x} F(\boldsymbol{x})\hat{\mathcal{O}}(\boldsymbol{x}, t) \tag{4.114}$$

where $\hat{\mathcal{O}}(\boldsymbol{x}, t)$ is a (composite) operator of a massive scalar Klein Gordon field. We will compare the probability of the wavepacket detection (which is associated with the detector's excitation probability) for two different choices of the operator $\hat{\mathcal{O}}$: 1. $\hat{\mathcal{O}}(\boldsymbol{x}, t) = \hat{\phi}(\boldsymbol{x}, t)$ and 2. $\hat{\mathcal{O}}(\boldsymbol{x}, t) = \partial_{\boldsymbol{x}}\hat{\phi}(\boldsymbol{x}, t)$.

To demonstrate the point in the simplest way possible, we will look at the probability amplitude that the detector goes to the excited state and the field to the ground state[6] (the calculation of the full excitation probability can also be found in appendix C.2). Using the Dyson expansion of the interaction Hamiltonian, the probability amplitude of this transition is

$$\Pi(\tau) = \lambda \int^{\tau} \mathrm{d}t \chi(t)\langle e|\hat{\mu}(t)|g\rangle \int \mathrm{d}\boldsymbol{x} \int \mathrm{d}\boldsymbol{y} F(\boldsymbol{x})\psi(\boldsymbol{y})\langle 0|\hat{\mathcal{O}}(t, \boldsymbol{x})\hat{a}^{\dagger}_{\boldsymbol{y}}|0\rangle + \mathcal{O}(\lambda^2). \tag{4.115}$$

To evaluate the integral kernel $\langle 0|\hat{\mathcal{O}}(t, \boldsymbol{x})\hat{a}^{\dagger}_{\boldsymbol{y}}|0\rangle$ it is useful to express the field operator in terms of the non-relativistic localization scheme

$$\hat{\phi}(t, \boldsymbol{x}) = \int \mathrm{d}\boldsymbol{z} \left(K(t, \boldsymbol{x} - \boldsymbol{z})\hat{a}_{\boldsymbol{z}} + K^*(t, \boldsymbol{x} - \boldsymbol{z})\hat{a}^{\dagger}_{\boldsymbol{z}}\right) \tag{4.116}$$

where the integral kernels are given by the Fourier transform of the field's mode functions

$$K(t, \boldsymbol{x} - \boldsymbol{z}) = \int \mathrm{d}\boldsymbol{k} v_{\boldsymbol{k}}(t, \boldsymbol{x}) e^{-i\boldsymbol{k}\boldsymbol{z}}. \tag{4.117}$$

The following function arises in the excitation probability amplitude

$$\tilde{\psi}(x, t) := \int \mathrm{d}\boldsymbol{y} K(t, \boldsymbol{x} - \boldsymbol{y})\psi(\boldsymbol{y}). \tag{4.118}$$

It is easy to check (see appendix C.2) that $\tilde{\psi}$ is the Newton-Wigner wavefunction of the relativistic particle. For the case of coupling to the amplitude $\hat{\mathcal{O}}(\boldsymbol{x}, t) = \hat{\phi}(\boldsymbol{x}, t)$ the probability amplitude is

$$\Pi_{\mathrm{ampl}}(\tau) = -\lambda \int^{\tau} \mathrm{d}t \chi(t)\langle e|\hat{\mu}(t)|g\rangle \int \mathrm{d}\boldsymbol{x} F(\boldsymbol{x})\tilde{\psi}(t, \boldsymbol{x}) + \mathcal{O}(\lambda^2). \tag{4.119}$$

[5] note that $\psi(\boldsymbol{y})$ does not admit the wavefunction interpretation in general, for details see [45].

[6] Note that first order in the coupling constant λ for the probability amplitude corresponds to second order for the probability (which is typically the leading order).

We see that the detection probability amplitude Π_{ampl} depends on the overlap of the Newton-Wigner wavefunction of the particle with the smearing function (which roughly represents where the detector is). For the case of the derivative coupling $\hat{\mathcal{O}}(\boldsymbol{x},t) = \partial_{\boldsymbol{x}}\hat{\phi}(\boldsymbol{x},t)$, using that

$$\partial_{\boldsymbol{x}}\hat{\phi}(t,\boldsymbol{x}) = \int \mathrm{d}\boldsymbol{z}\left(\partial_{\boldsymbol{x}}K(t,\boldsymbol{x}-\boldsymbol{z})\hat{a}_{\boldsymbol{z}} + \partial_{\boldsymbol{x}}K^*(t,\boldsymbol{x}-\boldsymbol{z})\hat{a}^\dagger_{\boldsymbol{z}}\right) \tag{4.120}$$

we get that

$$\Pi_{\text{der}}(\tau) = -\lambda\int^{\tau}\mathrm{d}t\chi(t)\langle e|\hat{\mu}(t)|g\rangle\int\mathrm{d}\boldsymbol{x}F(\boldsymbol{x})\partial_{\boldsymbol{x}}\tilde{\psi}(t,\boldsymbol{x}) + \mathcal{O}(\lambda^2). \tag{4.121}$$

Comparing with (4.119) we see that detection probability amplitude Π_{der} is sensitive to the *spatial derivative* of the Newton-Wigner wavefunction, rather than the amplitude. This is counter-intuitive from a quantum-mechanical perspective, since the spatial derivative of a wavefunction is irrelevant for the $|\psi|^2$ prescription of the detection probability.

The observation that the localized detection of a relativistic quantum particle in which locations it is most likely to be detected, can be highly dependent on the physical interaction that we choose for probing it, can shed light on the localization problem in QFT. The localization problem stems from the fact that it is difficult to define a good notion of particle localization in relativity, without running into problems with relativistic causality. This is supported by no-go results, like the one by Malament (that we reviewed in the introduction) and others [5, 6, 108]. Attempts of relativistic generalizations of the wavefunction (or the position operator), like the Newton Wigner wavefunction (by means of which we have written the detector's response to a wavepacket) are not fully relativistic (for example, the corresponding Newton Wigner operator defined in [39] does not obey microcausality). The observation above suggests that the non-existence of a relativistic wavefunction, or a relativistic position operator, is a feature that one can incorporate by explicitly modeling the detection process, since a relativistic position operator would have to be interaction-dependent. In this sense, detector models are useful in analyzing the localization problem. It will be interesting to further investigate the above observation for realistic interactions (electromagnetic, weak, gravitational) for charged and/or massive particles or neutrinos, to see how far one could 'stretch' the discrepancy between the different wavefunctions (local probability distributions) that one can attach to a single particle.

Chapter 5

Summary and conclusions

Sorkin's impossible measurements problem highlights that the straightforward extension of standard measurement theory to QFT leads to friction with relativistic causality, for reasons that are independent of other foundational issues in relativistic quantum theory (covariance, entanglement, Reeh-Schlieder property etc). We presented the 'impossible measurement' problem as a reductio argument, that is, we identified the premises that go into the examples that were previously presented in the literature, and lead to the unacceptable conclusion (superluminal signaling). The logical structure of a reductio argument is useful for disentangling this problem from other foundational issues, and for classifying the various responses. We also provided a brief historical introduction to the topic of local measurements in QFT, by revisiting the initial historical episodes in the 1930's.

The overall diagnosis is that 'impossible measurements' arise from a 'mismatch' between the causal structure of spacetime and the operations that we assume 'on top'. The causal structure of spacetime is encoded in a partial order, which is straightforwardly defined for pointlike events, but not so straightforwardly for events that reside over extended spacetime regions (e.g. measurements of field averages). One moral is that microcausality is not sufficient for blocking the 'impossible measurements' scenarios when more than two regions are involved. One can impose ad hoc conditions on the allowed operations, or the allowed configuration of regions, but these conditions are hard to motivate physically. Detector models are useful for making sense of these abstract ad hoc conditions. We quantified the problem by means of the relevant scales in a given detection set-up, and we provide a case-by-case analysis for arguing that the problem can be pushed outside the regime of validity of each model.

With this analysis, one can appreciate the role that the dynamics plays in defining, or restricting, the possible operations. This blurs the strict dichotomy between dynamics and measurements or operations, which is particularly relevant for the formulation of the measurement problem. We showed that, pragmatically, 'how much' causality violation one gets depends on 'how local' the dynamics is. In scattering-like treatments, like the detector models approach and the FV-framework in algebraic QFT, the problem/resolution boils down to the non-locality/locality of the scattering map respectively: in the detector models approach one can give pragmatic arguments for excluding 'impossible measurements' FAPP, while in the FV framework 'impossible measurements' are fully eliminated in principle thanks to the nice locality property of the dynamics. Finally, the 'impossible measurements' problem is still an open problem beyond (generalized) scattering theory, in histories-based formalisms.

In chapter 3 we provided an exhaustive analysis of signaling for the variety of detector models in QFT. The main question was the following: Is the quantum informational notion of causality, that follows from the signaling relations between detectors, compatible with the spacetime notions of causality? In bipartite scenarios and for causally orderable interactions, we proved that superluminal signaling and retrocausation are blocked thanks to the causal factorization of the detector-field interactions, which is a non-perturbative result. In the perturbative regime, we applied tools from quantum metrology to define a new signaling estimator (leading order in perturbation theory) that can be used to analyze causality for noncompact detecor-field interactions that are not causally orderable. Based on this estimator, we showed that the signaling possibilities are restricted by the field's retarded Green's function and as a result, for compactly supported interactions, the Fisher information is non-zero only in the forward lightcone of the coupling region. For noncompact interactions, we showed that the causal overlap non-trivially depends on the internal states (coherence in the energy basis) and on the internal frequencies. Nevertheless, one can derive a maximum bound to the signaling estimator and draw effective lightcones, that reflect the signaling possibilities, on a case-by-case basis. Quantifying this 'cross-talk' between detectors that are 'mostly' spacelike separated in a given spacetime, is important for causality considerations, as well as for entanglement harvesting.

In chapter 4 we addressed the following questions: If QFT does not admit a straightforward particle (or field) ontology, how can we interpret the detector's response using the notion of field or particle? What do particle detectors detect, and in what regimes? The Unruh-DeWitt particle detector is usually thought of as a detector that 'clicks' (gets excited) depending on its state of motion and the state of the field. For fixed-particle states of the field, e.g. momentum-space wavepackets, we demonstrated when, where and why the detector 'clicks' in terms of resonance and 'time-of-arrival'. We showed that in the monochromatic limit, the detector 'clicks' at a time-of-arrival given by the relativistic velocity. Studying resonance in the smeared Unruh-DeWitt model, we found that response depends on the overlap of the smearing function with the wavepacket's Newton Wigner wavefunction. We showed that a clean particle-like phenomenology can only be achieved in the long-time limit (constant switching), which kills the noise and the counter-rotating terms (that is, the detector 'clicks' only at the 'right' frequency and only due to the wavepacket). Trying to view the long-time limit as a limiting case of the adiabatic switching (e.g. Gaussian switching function) we found the following trade-off: the more we resolve the 'time-of-arrival' the worst the description of resonance, because the noise is zero only in the long-time limit. In this sense, time and energy measurements are complementary in this set-up.

Regarding field measurements, we considered detectors that do not 'click', that is, a gapless continuous pointer variable that 'shifts' according to the smeared field time-average and is insensitive to the particle content of the field. This pointer variable analysis works non-perturbatively, in the limit that the detector-field interaction is much stronger than the detector's self-dynamics. Then, we considered a quantum harmonic oscillator detector, that can both 'click' and 'shift' because it is characterized by discrete energy levels but also continuous variables. For pointlike couplings, the dynamics is solvable (in the long-time limit) using the formalism of quantum Brownian motion. This improved detector model based on QBM, being solvable, can be used to characterize the weak, strong and intermediate regime. A new feature that comes into play (due to the non-negligible self-dynamics of the detector), is the *dissipation* induced by the field environment. The qualitative behavior of this model depends on the relation between two parameters: the dissipation and the renormalized frequency of the detector (given a cut-off scale). For strong dissipation, one gets that the continuous pointer variable of the detector is 'tracking' the field averages. For weak dissipation, we get resonance in the Breit-Wigner form. This is an improved description of resonance with respect to the perturbative treatment, because it takes into account at least a partial ressumation. It is also an improved description of field measurements with respect to the pointer variable analysis, where we had to assume a gapless detector. The gap causes the dissipation with the environment, and the dissipation determines how close the continuous variable is 'tracking' the field. This analysis sheds new light on the field/particle duality in QFT measurements induced by detector models.

Finally, we considered the localized detection of a massive wavepacket. In previous work, we showed explicitly that a massive scalar field can be described as a continuously infinite tensor product of local harmonic oscillators $\hat{a}_x, \hat{a}_x^\dagger$ with frequency given by the mass of the field. The fact that there is no position operator for the relativistic particle implies that there is no $|\psi|^2$ prescription for its localized detection. In fact, the probability of detection in a particular location highly depends on the physical interaction that we are using to probe the local excitation of the field. We showed, for example, for derivative coupling of the detector to $\partial_x \hat{\Phi}$ the detection probability depends on the spatial derivative (rather than the amplitude) of the wavepacket's Newton-Wigner wavefunction. This is an example of a localization feature that is absent in the quantum mechanical description, that is worth investigating further. It should be possible to define 'induced' interaction-dependent position operators for relativistic particles in QFT.

Epilogue: Localization, signaling and the QFT measurement problem

No-go results by Sorkin and Malament imply that there can be no localized *ideal* measurements in relativistic spacetime due to friction with relativistic causality. For local QFT measurements, these results necessitate the introduction of probe systems that are dynamically coupled to the field. The fact that localization in QFT can only be defined through a particular dynamical interaction with a probe, can lead to a whole new intuition about the microscopic world and how we interact with it. Further, several results suggest that relativistic QFT does not admit a particle ontology. Considerations of QFT in curved spacetime support the argument that 'QFT is a theory about fields'(e.g. [66]), but the field ontology is also not easy to establish, especially beyond the scalar field theory. Perhaps this suggests that the field/particle concepts, that are describing very well the ontology of our classical world, are not quite applicable in the quantum regime. These concepts are reflected in the quantum theories through the process of quantization, but they cannot be used to establish an ontology for the quantum theory, and they only arise phenomenologically.

Classical theories come with 'obvious' ontological commitments, and the entities predicted by the theory are the 'carriers' of causal relations. This intuitive physical picture breaks down in QFT, since it is not clear what the theory is 'about' and measurement theory/frameworks/models are necessary for extracting its empirical content. The details of how the empirical content of QFT is extracted is important for considerations of causality, as well as for the very formulation of the measurement problem (see discussion in [73]). In this thesis, we emphasized the role that the dynamics plays both for the signaling relations that are compatible with QFT and for retrieving field/particle phenomenology. Dynamics is ‘physical’ (intrinsic to the theory) and it can bring back some of the intuitive physical picture of the world, its causal structure and the entities that reside in it. An emerging theme in the Philosophy of QFT literature is that, indeed, dynamics seems to be playing an important role for the formulation of the measurement problem in QFT (see discussion in [136, 137, 138]).

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APPENDICES

Appendix A

Signaling between detectors

A.1 Causal factorization of detector-field interactions

In this appendix we provide a proof of the causal factorization of the scattering operator for compactly supported detectors. The proof relies on elementary properties of unitary propagators. Consider a general time-dependent interaction Hamiltonian of the form,

$$\hat{H}(t) = \hat{H}_{\text{A}}(t) + \hat{H}_{\text{B}}(t), \tag{A.1}$$

and its associated Schrodinger equation

$$\partial_t |\psi(t)\rangle_{\text{A+B}} = -\mathrm{i}(\hat{H}_{\text{A}}(t) + \hat{H}_{\text{B}}(t))\,|\psi(t)\rangle\,, \tag{A.2}$$

or, more conveniently, in its integral form

$$\begin{aligned} |\psi(t)\rangle_{\text{A+B}} &= |\psi(t')\rangle_{\text{A+B}} \\ &\quad - \mathrm{i}\int_{t'}^{t} \mathrm{d}t (\hat{H}_{\text{A}}(t'') + \hat{H}_{\text{B}}(t''))\,|\psi(t'')\rangle_{\text{A+B}}\,. \end{aligned} \tag{A.3}$$

By recursively applying this integral equation, disregarding domain issues, we can formally write the evolution of the state as the action of a two-parametric group of unitary operators, also known as the unitary propagator, $\hat{U}_{\text{A+B}}(t,t')$:

$$\begin{aligned} |\psi(t)\rangle = \hat{U}_{\text{A+B}}(t,t')\,|\psi(t')\rangle = \sum_n \frac{(-\mathrm{i})^n}{n!}\int_{t'}^{t}\mathrm{d}t_1\cdots\int_{t'}^{t}\mathrm{d}t_n \\ \times\, \mathcal{T}(\hat{H}_{\text{A}}(t_1) + \hat{H}_{\text{B}}(t_1))\dots(\hat{H}_{\text{A}}(t_n) + \hat{H}_{\text{B}}(t_n))\,|\psi(t')\rangle\,, \end{aligned} \tag{A.4}$$

where the second line is the Dyson expansion of the operator $\hat{U}_{\text{A+B}}(t,t')$. The time ordering of two time-dependent operators $\hat{A}(t)$ and $\hat{B}(t)$ as

$$\mathcal{T}\hat{A}(t)\hat{B}(t') := \theta(t-t')\hat{A}(t)\hat{B}(t') + \theta(t'-t)\hat{B}(t')\hat{A}(t), \tag{A.5}$$

where the definition is similar for higher orders. It is useful to define unitary propagators that are associated to local evolution for each detector, that is

$$\hat{U}_\nu(t,t')\,|\psi(t')\rangle = \sum_n \frac{(-\mathrm{i})^n}{n!}\int_{t'}^{t}\mathrm{d}t_1\cdots\int_{t'}^{t}\mathrm{d}t_n \mathcal{T}\hat{H}_\nu(t_1)\ldots\hat{H}_\nu(t_n)\,|\psi(t')\rangle\,, \tag{A.6}$$

where $\nu \in \{\text{A},\text{B}\}$. In order to describe the dynamics of the detection process, we are particularly interested in the scattering operator, that is, the limit

$$\hat{S}_{\text{A+B}} = \lim_{t'\to-\infty}\lim_{t\to\infty}\hat{U}_{\text{A+B}}(t,t') \tag{A.7}$$

when the Hamiltonians are given by the expressions

$$\hat{H}_{\text{A,B}}(t) = \int_{\mathcal{E}(t)}\mathrm{d}\mathcal{E}\;\hat{h}_{\text{A,B}}(\mathsf{x}). \tag{A.8}$$

Consider that the supports Λ_{A} and Λ_{B} of the two detector-field interactions are causally orderable with A preceding B with respect to some foliation. Then we want to show that the scattering map factorizes, i.e.

$$\hat{S}_{\text{A+B}} = \hat{S}_{\text{B}}\hat{S}_{\text{A}}. \tag{A.9}$$

To show this we find the Schrodinger equation of the factorized dynamics and prove that it coincides with the full dynamics. Then we will use a uniqueness argument to prove that therefore the dynamics coincide. Consider the family of states

$$|\psi(t)\rangle_{\text{AB}} = \hat{U}_{\text{B}}(t,-\infty)\hat{U}_{\text{A}}(t,-\infty)|\psi\rangle_0 \tag{A.10}$$

where $|\psi\rangle_0$ is a fixed vector. It holds that

$$\begin{aligned}&\partial_t|\psi(t)\rangle_{\text{AB}}\\ &= -\mathrm{i}\left(\hat{H}_{\text{B}}(t) + \hat{U}_{\text{B}}(t,-\infty)\hat{H}_{\text{A}}(t)\hat{U}^\dagger_{\text{B}}(t,-\infty)\right)|\psi(t)\rangle_{\text{AB}}\,.\end{aligned} \tag{A.11}$$

Let us first distinguish two trivial cases. First, consider that A precedes B, with respect to the foliation $\mathscr{T}(\mathsf{x})$. Then, there exists a number t_c such that

$$\hat{H}_{\text{B}}(t_c) = \hat{H}_{\text{A}}(t_c) = 0 \tag{A.12}$$

and

$$\hat{H}_{\mathrm{B}}(t) = 0 \qquad t < t_c \tag{A.13}$$
$$\hat{H}_{\mathrm{A}}(t) = 0 \qquad t > t_c. \tag{A.14}$$

This implies that

$$\hat{U}_{\mathrm{B}}(t,-\infty)\hat{H}_{\mathrm{A}}(t)\hat{U}^{\dagger}_{\mathrm{B}}(t,-\infty) = \hat{H}_{\mathrm{A}}(t) \tag{A.15}$$

for all t, since $\hat{U}_{\mathrm{A}}(t,-\infty)$ is only different from the identity operator, $\hat{\mathbb{I}}$, when $\hat{H}_{\mathrm{B}}(t) = 0$. Second, if the supports are spacelike separated, then

$$[\hat{H}_{\mathrm{B}}(t), \hat{H}_{\mathrm{A}}(t')] = 0 \tag{A.16}$$

for all $t, t' \in \mathbb{R}$, and therefore

$$\begin{aligned}\hat{U}_{\mathrm{B}}(t,-\infty)\hat{H}_{\mathrm{A}}(t)\hat{U}^{\dagger}_{\mathrm{B}}(t,-\infty) = &\sum_n \frac{(-\mathrm{i})^n}{n!}\int_{-\infty}^{t}\mathrm{d}t_1 \cdots \int_{-\infty}^{t}\mathrm{d}t_n \\ &\times \mathcal{T}\Big[\ldots[\hat{H}_{\mathrm{A}}(t), \hat{H}_{\mathrm{B}}(t_1)]\ldots, \times \hat{H}_{\mathrm{B}}(t_n)\Big] = \hat{H}_{\mathrm{A}}(t).\end{aligned} \tag{A.17}$$

More generally if the detectors are causally orderable, essentially (assuming, without proof, that the adjoint action of the unitary evolution can be carried inside the integral (A.18)) then

$$\hat{U}_{\mathrm{B}}(t,-\infty)\hat{H}_{\mathrm{A}}(t)\hat{U}^{\dagger}_{\mathrm{B}}(t,-\infty) = \int_{\mathcal{E}(t)}\mathrm{d}\mathcal{E}\; \hat{U}_{\mathrm{B}}(t,-\infty)\hat{h}_{\mathrm{A}}(\mathsf{x})\hat{U}^{\dagger}_{\mathrm{B}}(t,-\infty). \tag{A.18}$$

For each $\mathsf{x} \in \operatorname{supp}(\Lambda_{\mathrm{A}})$, $\hat{h}_{\mathrm{A}}(\mathsf{x})$ is either causally connected or spacelike separated to the support of B, so we can choose the corresponding proof from the two ones given above to show that it remains unchanged under the adjoint action of $\hat{U}_{\mathrm{B}}(t,-\infty)$. Therefore

$$\hat{U}_{\mathrm{B}}(t,-\infty)\hat{H}_{\mathrm{A}}(t)\hat{U}^{\dagger}_{\mathrm{B}}(t,-\infty) = \int_{\mathcal{E}(t)}\mathrm{d}\mathcal{E}\; \hat{h}_{\mathrm{A}}(\mathsf{x}) = \hat{H}_{\mathrm{A}}(t).$$

Altogether, the conclusion is that if A precedes B for some observer then $|\psi(t)\rangle_{\mathrm{AB}}$ fulfills (A.2), and since (A.2) is a linear differential equation, the vector

$$|\varphi(t)\rangle = |\psi(t)\rangle_{\mathrm{AB}} - |\psi(t)\rangle_{\mathrm{A+B}} \tag{A.19}$$

also fulfils (A.2). Now, setting $|\varphi(-\infty)\rangle = 0$ implies $|\varphi(t)\rangle = 0$ for all t, since the solution is unique and $|\varphi(t)\rangle = 0$ is a solution with initial condition $|\varphi(-\infty)\rangle = 0$. Therefore, we have shown that

$$|\psi(t)\rangle_{\text{A+B}} = |\psi(t)\rangle_{\text{AB}} = \hat{U}_{\text{A}}(t,-\infty)\hat{U}_{\text{B}}(t,-\infty)|\psi\rangle_0, \tag{A.20}$$

for all t, and more concretely,

$$\hat{S}_{\text{A+B}}|\psi\rangle = |\psi(\infty)\rangle_{\text{A+B}}\Big|_{|\psi(-\infty)\rangle_{\text{A+B}}=|\psi\rangle} = \hat{S}_{\text{A}}\hat{S}_{\text{B}}|\psi\rangle, \tag{A.21}$$

for all states in the Hilbert space.

A.2 Derivation of operator $\hat{\Sigma}$

Consider the case of two general detectors, A and B, which interact with the field according to the interaction Hamiltonian

$$\sum_{\nu=\text{A,B}} \hat{H}_\nu(\tau) = \sum_{\nu=\text{A,B}} \int_{\mathcal{E}(\tau)} \mathrm{d}\mathcal{E}\ \hat{h}_\nu(\mathsf{x}). \tag{A.22}$$

where in this case the corresponding Hamiltonian densities will be given by

$$\hat{h}_{\text{A}}(\mathsf{x}) = \lambda_{\text{A}} \hat{J}_{\text{A}}(\mathsf{x}) \otimes \mathbb{1}_{\text{B}} \otimes \hat{\phi}(\mathsf{x}) \tag{A.23}$$

and

$$\hat{h}_{\text{B}}(\mathsf{x}) = \lambda_{\text{B}} \mathbb{1}_{\text{A}} m \otimes \hat{J}_{\text{B}}(\mathsf{x}) \otimes \hat{\phi}(\mathsf{x}). \tag{A.24}$$

The joint evolution in the interaction picture of the detectors and the field can be described as a unitary operator acting over the joint initial state of the field-detectors system $\hat{\rho}_{\text{initial}}$. Then the state in the asymptotic future will be given by the transformation

$$\hat{\rho}_{\text{final}} = \hat{U}_{\text{A+B}}\ \hat{\rho}_{\text{initial}}\ \hat{U}^\dagger_{\text{A+B}}. \tag{A.25}$$

The unitary implementing the time evolution can be formally written in terms of the Dyson series

$$\hat{U}_{\text{A+B}} = \sum_n \frac{(-\mathrm{i})^n}{n!} \int_{-\infty}^{\infty} \mathrm{d}\tau_1 \cdots \int_{-\infty}^{\infty} \mathrm{d}\tau_n \mathcal{T}\left(\hat{H}_{\text{A}}(\tau_1) + \hat{H}_{\text{B}}(\tau_1)\right) \ldots \left(\hat{H}_{\text{A}}(\tau_n) + \hat{H}_{\text{B}}(\tau_n)\right). \tag{A.26}$$

This means that we can rewrite (A.25) as

$$\hat{U}_{\text{A+B}}\,\hat{\rho}_{\text{initial}}\,\hat{U}^{\dagger}_{\text{A+B}} = \sum_n \frac{(-\mathrm{i})^n}{n!}\int \mathrm{d}\tau_n \mathcal{T}\left[\hat{H}(\tau_n), \dots [\hat{H}(\tau_1), \hat{\rho}_{\text{initial}}]\dots\right]. \tag{A.27}$$

If the couplings are weak, we can truncate the series at next to leading order

$$\begin{aligned}\hat{U}_{\text{A+B}}\,\hat{\rho}_{\text{initial}}\,\hat{U}^{\dagger}_{\text{A+B}} = \hat{\rho}_{\text{initial}} &- \mathrm{i}\int_{-\infty}^{\infty}\mathrm{d}\tau[\hat{H}_{\text{A}}(\tau)+\hat{H}_{\text{B}}(\tau), \hat{\rho}_{\text{initial}}] \\ &- \frac{1}{2}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\mathrm{d}\tau\mathrm{d}\tau'\mathcal{T}\left[\hat{H}_{\text{A}}(\tau)+\hat{H}_{\text{B}}(\tau),\; [\hat{H}_{\text{A}}(\tau')+\hat{H}_{\text{B}}(\tau'), \hat{\rho}_{\text{initial}}]\right] \\ &+ \mathcal{O}(\lambda^3).\end{aligned} \tag{A.28}$$

where the time-ordering operator is defined as follows

$$\mathcal{T}\hat{A}(t)\hat{B}(t') := \theta(t-t')\hat{A}(t)\hat{B}(t') + \theta(t'-t)\hat{B}(t')\hat{A}(t) \tag{A.29}$$

for two time-dependent operators $\hat{A}(t)$ and $\hat{B}(t)$. The local statistics of the detector B will be given by the partial trace

$$\hat{\rho}_{\text{B}} = \mathrm{tr}_{\text{A},\phi}(\hat{U}_{\text{A+B}}\,\hat{\rho}_{\text{initial}}\,\hat{U}^{\dagger}_{\text{A+B}}). \tag{A.30}$$

and the signaling term can be defined as

$$\hat{\rho}^{(2)}_{\text{B,signal}} = \frac{\partial^2}{\partial\lambda_{\text{A}}\partial\lambda_{\text{B}}}\mathrm{tr}_{\text{A},\phi}(\hat{U}_{\text{A+B}}\,\hat{\rho}_{\text{initial}}\,\hat{U}^{\dagger}_{\text{A+B}})|_{\lambda_{\text{A}}=\lambda_{\text{B}}=0}. \tag{A.31}$$

Note that, given any operator $\hat{O}$, it follows that

$$\mathrm{tr}_{\text{A},\phi}([\hat{H}_{\text{A}}(\tau), \hat{O}]) = 0 \tag{A.32}$$

since $\hat{H}_{\text{A}}(\tau)$ only depends on operators of detector A and the field, and thereby can be permuted within the partial trace. This allows us to disregard multiple terms in (A.32), thereby leading to

$$\begin{aligned}\hat{\rho}_{\text{B}} = \mathrm{tr}_{\text{A},\phi}(\hat{\rho}_{\text{initial}}) &- \mathrm{i}\int_{-\infty}^{\infty}\mathrm{d}\tau\mathrm{tr}_{\text{A},\phi}[\hat{H}_{\text{B}}(\tau), \hat{\rho}_{\text{initial}}] \\ &- \frac{1}{2}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\mathrm{d}\tau\mathrm{d}\tau'\mathrm{tr}_{\text{A},\phi}\mathcal{T}\left[\hat{H}_{\text{B}}(\tau), [\hat{H}_{\text{B}}(\tau'), \hat{\rho}_{\text{initial}}]\right] \\ &- \frac{1}{2}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\mathrm{d}\tau\mathrm{d}\tau'\mathrm{tr}_{\text{A},\phi}\mathcal{T}\left[\hat{H}_{\text{B}}(\tau), [\hat{H}_{\text{A}}(\tau'), \hat{\rho}_{\text{initial}}]\right] \\ &+ \mathcal{O}(\lambda^3),\end{aligned} \tag{A.33}$$

or, more conveniently, we can use the Jacobi identity in the last commutator and again the cyclic property of the partial trace acting over $\hat{H}_{\text{A}}(\tau)$:

$$\begin{aligned}
&\hat{\rho}_{\text{B}} = \text{tr}_{\text{A},\phi}\hat{\rho}_{\text{initial}} - \text{i}\int_{-\infty}^{\infty} \text{d}\tau \text{tr}_{\text{A},\phi}[\hat{H}_{\text{B}}(\tau), \hat{\rho}_{\text{initial}}] \\
&- \frac{1}{2}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} \text{d}\tau\text{d}\tau' \text{tr}_{\text{A},\phi}\mathcal{T}\left[\hat{H}_{\text{B}}(\tau), [\hat{H}_{\text{B}}(\tau'), \hat{\rho}_{\text{initial}}]\right] \\
&- \frac{1}{2}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} \text{d}\tau\text{d}\tau' \text{tr}_{\text{A},\phi}\left[\mathcal{T}[\hat{H}_{\text{B}}(\tau), \hat{H}_{\text{A}}(\tau')], \hat{\rho}_{\text{initial}}\right] \\
&+ \mathcal{O}(\lambda^3).
\end{aligned} \tag{A.34}$$

Note that the first three terms do not contribute to the signaling term (A.31), since they do not depend on λ_{A}. Therefore the signaling term will be given by

$$\hat{\rho}^{(2)}_{\text{B,signal}} = -\frac{1}{2}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} \text{d}\tau\text{d}\tau' \int_{\mathcal{E}(\tau)} \text{d}\mathcal{E} \int_{\mathcal{E}(\tau')} \text{d}\mathcal{E}' \text{tr}_{\text{A},\phi}\left[\hat{J}_{\text{A}}(\mathsf{x}') \otimes \hat{J}_{\text{B}}(\mathsf{x}) \otimes \mathcal{T}[\hat{\phi}(\mathsf{x}), \hat{\phi}(\mathsf{x}')], \hat{\rho}_{\text{initial}}\right]. \tag{A.35}$$

Now, consider that the state is initially uncorrelated, i.e. $\hat{\rho}_{\text{initial}} = \hat{\rho}_{\text{A}} \otimes \hat{\rho}_{\text{B}} \otimes \hat{\rho}_{\phi}$. This gives the following compact expression

$$\hat{\rho}^{(2)}_{\text{B,signal}} = -\text{i}[\hat{\Sigma}, \hat{\rho}_{\text{B}}] \tag{A.36}$$

where we have defined the operator

$$\begin{aligned}
\hat{\Sigma} &= \int\int \text{d}V\text{d}V' \langle \hat{J}_{\text{A}}(\mathsf{x}')\rangle\, G_{\text{R}}(\mathsf{x}, \mathsf{x}')\hat{J}_{\text{B}}(\mathsf{x}) \\
&= \int \text{d}V G_{\text{R}}[\langle \hat{J}_{\text{A}}\rangle](\mathsf{x})\hat{J}_{\text{B}}(\mathsf{x}).
\end{aligned} \tag{A.37}$$

Here $G_{\text{R}}(\mathsf{x}, \mathsf{x}')$ is the retarded Green function

$$G_{\text{R}}(\mathsf{x}, \mathsf{x}') = -\text{i}\theta(\tau(\mathsf{x}) - \tau(\mathsf{x}'))\, \langle [\hat{\phi}(\mathsf{x}), \hat{\phi}(\mathsf{x}')]\rangle\,, \tag{A.38}$$

whereas $\text{d}V$ denotes the element of volume with respect to the background metric

$$\text{d}V = \text{d}\mathsf{x}^{n+1}\sqrt{|g|}, \tag{A.39}$$

and where $\sqrt{|g|}$ is the determinant of the metric. Note that we have used the fact that

$$\begin{aligned}
&\int \text{d}\tau \int \text{d}\mathcal{E}(\tau) = \int \text{d}\mathsf{x}^{n+1}\sqrt{|g|}\int \text{d}\tau\delta(\tau(\mathsf{x}) - \tau) \\
&= \int \text{d}\mathsf{x}^{n+1}\sqrt{|g|} = \int \text{d}V.
\end{aligned} \tag{A.40}$$

A.3 Signaling estimator for Gaussian smearings and switchings in 3+1 dimensional Minkowski spacetime

In $3+1$ dimensions the Green's function is

$$G_{\mathrm{R}}(t-t',\boldsymbol{x}-\boldsymbol{x}') = -\frac{1}{4\pi}\frac{\delta(t-t'-|\boldsymbol{x}-\boldsymbol{x}'|)}{|\boldsymbol{x}-\boldsymbol{x}'|} \tag{A.41}$$

We will consider Gaussian smearing and switching functions to evaluate the expression

$$I(\Omega_{\mathrm{A}},\Omega_{\mathrm{B}}) = \int \mathrm{d}t\mathrm{d}t' \chi_{\mathrm{B}}(t)e^{\mathrm{i}\Omega_{\mathrm{B}}t}\chi_{\mathrm{A}}(t')e^{-\mathrm{i}\Omega_{\mathrm{A}}t'}\mathcal{C}(t-t') \tag{A.42}$$

where

$$\mathcal{C}(t-t') = -\frac{1}{4\pi}\int \mathrm{d}\boldsymbol{x}\mathrm{d}\boldsymbol{x}' F_{\mathrm{B}}(\boldsymbol{x})F_{\mathrm{A}}(\boldsymbol{x}')\frac{\delta(t-t'-|\boldsymbol{x}-\boldsymbol{x}'|)}{|\boldsymbol{x}-\boldsymbol{x}'|} \tag{A.43}$$

We consider that B is centered around zero, i.e.,

$$F_{\mathrm{B}}(\boldsymbol{x}) = \frac{1}{(\sqrt{2\pi}R)^3}e^{-\frac{\boldsymbol{x}^2}{2R^2}} \tag{A.44}$$

and A is centered around $\boldsymbol{z}$ with the same width R. Let's introduce the following non-orthogonal change of variables: we keep the variable $\boldsymbol{x}$ and we define a new integration variable $\boldsymbol{y} = \boldsymbol{x}-\boldsymbol{x}'$ (the Jacobian is 1). We then first perform the integral with respect to $\boldsymbol{x}$ in (A.43) which is a convolution of the two Gaussian smearings

$$\int \mathrm{d}\boldsymbol{x} F_{\mathrm{B}}(\boldsymbol{x})F_{\mathrm{A}}(\boldsymbol{x}-\boldsymbol{y}) = \frac{1}{(\sqrt{\pi}2R)^3}e^{-\frac{(\boldsymbol{y}+\boldsymbol{z})^2}{4R^2}} \tag{A.45}$$

Then

$$\mathcal{C}(t-t') = -\frac{1}{4\pi}\int \mathrm{d}\boldsymbol{y}e^{-\frac{(\boldsymbol{y}+\boldsymbol{z})^2}{4R^2}}\frac{\delta(t-t'-|\boldsymbol{y}|)}{|\boldsymbol{y}|} \tag{A.46}$$

Calling $\tau = t-t'$ and $|\boldsymbol{z}| := L$

$$\mathcal{C}(\tau) = -\frac{e^{-\frac{L^2}{4R^2}}}{2}\int_0^\infty |\boldsymbol{y}|\mathrm{d}|\boldsymbol{y}|e^{-\frac{|\boldsymbol{y}|^2}{4R^2}}\frac{2R^2}{L|\boldsymbol{y}|}\left(e^{\frac{L|\boldsymbol{y}|}{2R^2}} - e^{-\frac{L|\boldsymbol{y}|}{2R^2}}\right)\delta(\tau-|\boldsymbol{y}|) \tag{A.47}$$

Altogether,

$$I(\Omega_{\text{A}},\Omega_{\text{B}}) = \frac{4\pi e^{-\frac{L^2}{4R^2}}}{(2\sqrt{\pi})^3 LR}\int |\boldsymbol{y}|\mathrm{d}|\boldsymbol{y}|e^{-\frac{|\boldsymbol{y}|^2}{4R^2}}\left(e^{\frac{L|\boldsymbol{y}|}{2R^2}} - e^{-\frac{L|\boldsymbol{y}|}{2R^2}}\right) I_p(|\boldsymbol{y}|,\Omega_{\text{A}},\Omega_{\text{B}}) \tag{A.48}$$

where I_p is the expression of I for two pointlike detectors with identical switchings separated by a distance $|\boldsymbol{y}|$,

$$\begin{aligned} I_p(&|\boldsymbol{y}|,\Omega_{\text{A}},\Omega_{\text{B}}) \\ &= -\frac{1}{4\pi}\int \mathrm{d}t\mathrm{d}t'\chi_{\text{B}}(t)e^{\mathrm{i}\Omega_{\text{B}}t}\chi_{\text{A}}(t')e^{-\mathrm{i}\Omega_{\text{A}}t'}\frac{\delta(t-t'-|\boldsymbol{y}|)}{|\boldsymbol{y}|} \\ &= -\frac{1}{4\pi}\int \mathrm{d}t\mathrm{d}\tau\chi_{\text{B}}(t)e^{\mathrm{i}(\Omega_{\text{B}}-\Omega_{\text{A}})t}\chi_{\text{A}}(t-\tau)e^{\mathrm{i}\Omega_{\text{A}}\tau}\frac{\delta(\tau-|\boldsymbol{y}|)}{|\boldsymbol{y}|} \\ &= -\frac{e^{\mathrm{i}\Omega_{\text{A}}|\boldsymbol{y}|}}{4\pi|\boldsymbol{y}|}\int \mathrm{d}t\chi_{\text{B}}(t)\chi_{\text{A}}(t-|\boldsymbol{y}|)e^{\mathrm{i}(\Omega_{\text{B}}-\Omega_{\text{A}})t} \end{aligned} \tag{A.49}$$

Now we evaluate this for Gaussian switching functions

$$\chi_{\text{B}}(t) = \frac{1}{\sqrt{2\pi}}e^{-\frac{t^2}{2T^2}} \tag{A.50}$$

and A displaced by Δ we have

$$I_p(|\boldsymbol{y}|,\Omega_{\text{A}},\Omega_{\text{B}}) = -\frac{e^{\mathrm{i}\Omega_{\text{A}}|\boldsymbol{y}|}}{8\pi^2|\boldsymbol{y}|}\int \mathrm{d}te^{\mathrm{i}(\Omega_{\text{B}}-\Omega_{\text{A}})t}e^{-\frac{t^2}{2T^2}}e^{-\frac{(t-(|\boldsymbol{y}|+\Delta))^2}{2T^2}} \tag{A.51}$$

which is a Gaussian integral that gives

$$I_p(|\boldsymbol{y}|,\Omega_{\text{A}},\Omega_{\text{B}}) = \frac{T\sqrt{\pi}}{8\pi^2|\boldsymbol{y}|}e^{\frac{\mathrm{i}(\Omega_{\text{A}}+\Omega_{\text{B}})|\boldsymbol{y}|}{2}}e^{\frac{i(\Omega_{\text{B}}-\Omega_{\text{A}})\Delta}{2}}e^{-\frac{(\Omega_{\text{B}}-\Omega_{\text{A}})^2T^2}{4}}e^{-\frac{(|\boldsymbol{y}|+\Delta)^2}{4T^2}} \tag{A.52}$$

Finally, changing variables $u := |\boldsymbol{y}|/R$ we get

$$I(\Omega_{\text{A}},\Omega_{\text{B}}) = \frac{R}{\sqrt{\pi}L}e^{-\frac{L^2}{4R^2}}\int_0^\infty \mathrm{d}u\; ue^{\frac{-u^2}{4}}(e^{\frac{uL}{2R}} - e^{-\frac{uL}{2R}})I_p(Ru,\Omega_{\text{A}},\Omega_{\text{B}}) \tag{A.53}$$

where

$$I_p(L,\Omega_{\text{A}},\Omega_{\text{B}}) = -\frac{T}{8\pi^{3/2}L}e^{-\frac{T^2(\Omega_{\text{B}}-\Omega_{\text{A}})^2}{4}}e^{-\frac{(L+\Delta)^2}{4T^2}}e^{\mathrm{i}(\Omega_{\text{B}}+\Omega_{\text{A}})L}. \tag{A.54}$$

Appendix B

Field measurements

B.1 Pointer variable analysis

We consider a one-dimensional pointer variable $\hat{P}$ coupled to a smeared field operator $\hat{\Phi}[F] := \int \mathrm{d}\boldsymbol{x} F(\boldsymbol{x})\hat{\Phi}(\boldsymbol{x})$ for a time interval that is dictated by a switching function $\chi(t)$ as described by the interaction Hamiltonian

$$\hat{H}_{int} = \chi(t)\hat{\Phi}[F] \otimes \hat{P}. \tag{B.1}$$

The full Hamiltonian in the Schroedinger picture is

$$\hat{H}(t) = \hat{H}_0 + \chi(t)\hat{\Phi}[F] \otimes \hat{P} \tag{B.2}$$

where $\hat{H}_0 = \hat{H}_{\text{F}} \otimes \mathbb{I} + \hat{H}_{\text{D}}$. The time evolution operator is

$$\hat{U}(t) = \mathcal{T}e^{-i\int^t \mathrm{d}s\hat{H}(s)} \tag{B.3}$$

where the explicit time dependence of the Hamiltonian is introduced by the switching function. Since generally $[\hat{H}_F, \hat{H}_{int}] \neq 0$ and $[\hat{H}_D, \hat{H}_{int}] \neq 0$ the time evolution operator does not factorize. For this, we define

$$\hat{S}(t) := e^{i\hat{H}_0 t}\hat{U}(t). \tag{B.4}$$

The time derivative of $\hat{S}(t)$ is

$$\begin{aligned}\partial_t\hat{S}(t) &= i\hat{H}_0 e^{i\hat{H}_0 t}\hat{U}(t) + e^{i\hat{H}_0 t}\partial_t\hat{U}(t) \\ &= i\hat{H}_0 e^{i\hat{H}_0 t}\hat{U}(t) + e^{i\hat{H}_0 t}\left(-i\hat{H}_0 - i\hat{H}_{int}\right)\hat{U}(t) \\ &= -ie^{i\hat{H}_0 t}\hat{H}_{int}\hat{U}(t).\end{aligned} \tag{B.5}$$

To get $\hat{S}(t)$ on the right-hand-side we introduce the free evolution operator and its inverse and by using (B.4) we get

$$\begin{aligned}\partial_t \hat{S}(t) &= -i\chi(t) e^{i\hat{H}_0 t}\left(\hat{\Phi}[F]\otimes\hat{P}\right)e^{-i\hat{H}_0 t}\hat{S}(t)\\ &= -i\chi(t)\left(\hat{\Phi}_t[F]\otimes\hat{P}(t)\right)\hat{S}(t)\end{aligned}\tag{B.6}$$

where

$$\hat{\Phi}_t[F] := \int \mathrm{d}\boldsymbol{x} F(\boldsymbol{x})\hat{\Phi}(\boldsymbol{x},t)\tag{B.7}$$

$$\hat{P}(t) = e^{i\hat{H}_D t}\hat{P}e^{-i\hat{H}_D t}\tag{B.8}$$

are the Heisenberg (or interaction) picture operators. The equation of motion

$$\partial_t \hat{S}(t) = -i\chi(t)\left(\hat{\Phi}_t[F]\otimes\hat{P}(t)\right)\hat{S}(t)\tag{B.9}$$

is solved by

$$\hat{S}(t) = \mathcal{T}e^{-i\int^t \mathrm{d}s\chi(s)\hat{\Phi}_s[F]\otimes\hat{P}(s)}\tag{B.10}$$

which is the so-called scattering operator, but for finite time t.

Going back to the definition (B.4) we get that

$$\hat{U}(t) = e^{-i\hat{H}_0 t}\hat{S}(t)\tag{B.11}$$

where $\hat{S}(t)$ is given by (B.10). Namely, we have shown that

$$\mathcal{T}e^{-i\int_0^t \mathrm{d}s\left(\hat{H}_0+\chi(s)\hat{\Phi}[F]\otimes\hat{P}\right)} = e^{-i\hat{H}_0 t}\mathcal{T}e^{-i\int_0^t \mathrm{d}s\chi(s)\hat{\Phi}_s[F]\otimes\hat{P}(s)}.\tag{B.12}$$

If $\hat{\rho}_0$ the initial state of the field and $\hat{\rho}_{\text{D}} = |\psi_0\rangle\langle\psi_0|$ the initial state of the detector, the reduced density matrix of the detector due to the interaction is

$$\begin{aligned}\hat{\rho}_{\text{D}}(t) &= \mathrm{tr}_f\left(\hat{U}(t)(\hat{\rho}_0\otimes|\psi_0\rangle\langle\psi_0|)\hat{U}^\dagger(t)\right)\\ &= \int \mathrm{d}p\mathrm{d}p'\mathrm{tr}_f\left(e^{-i\hat{H}_0 t}\hat{S}(t)|p\rangle\langle p|(\hat{\rho}_0\otimes|\psi_0\rangle\langle\psi_0|)|p'\rangle\langle p'|\hat{S}^\dagger(t)e^{i\hat{H}_0 t}\right)\end{aligned}\tag{B.13}$$

where we have used the definition (B.4) and we have inserted two resolutions of identity in the basis of the conjugate pointer variable.

Here comes the main 'trick' of the pointer variable analysis: by using the Dyson expansion of the time-ordered exponential one can see that

$$
\begin{aligned}
\hat{S}(t)\left(\mathbb{1}_f \otimes |p\rangle\langle p|\right) &= \mathcal{T}e^{-i\int^t \mathrm{d}s\chi(s)\hat{\Phi}_s[F]\otimes\hat{P}(s)}\left(\mathbb{1}_F \otimes |p\rangle\langle p|\right)\\
&= \mathcal{T}e^{-ip\int^t \mathrm{d}s\chi(s)\hat{\Phi}_s[F]} \otimes |p\rangle\langle p|\,,
\end{aligned}
\tag{B.14}
$$

if it holds that

$$
[\hat{H}_{\mathrm{D}}, \hat{P}] = 0. \tag{B.15}
$$

We see that the self-dynamics of the detector does not have to be zero (or negligible with respect to the interaction with the field) as long as it commutes with the conjugate pointer variable. That is, the detector can be a free particle with

$$
\hat{H}_{\mathrm{D}} = \frac{\hat{P}^2}{2M} \tag{B.16}
$$

which will allow us to compare with the case of a harmonic oscillator in the limit $\Omega \to 0$. Using (B.14)

$$
\begin{aligned}
&\rho_{\mathrm{D}}(t)\\
&= \int \mathrm{d}p\mathrm{d}p'\,\mathrm{tr}_f\left(e^{-i\hat{H}_0 t}\mathcal{T}e^{-ip\int^t \mathrm{d}s\chi(s)\hat{\Phi}_s[F]}\,|p\rangle\langle p|\left(\hat{\rho}_0\,|\psi_0\rangle\langle\psi_0|\right)|p'\rangle\langle p'|\,\mathcal{T}e^{ip'\int^t \mathrm{d}s\chi(s)\hat{\Phi}_s[F]}e^{i\hat{H}_0 t}\right)\\
&= \int \mathrm{d}p\mathrm{d}p'\,\mathrm{tr}_f\left(e^{-i\hat{H}_{\mathrm{F}} t}\mathcal{T}e^{-ip\int^t \mathrm{d}s\chi(s)\hat{\Phi}_s[F]}\hat{\rho}_0\,|p\rangle\langle p|\psi(t)\rangle\langle\psi(t)|p'\rangle\langle p'|\,\mathcal{T}e^{ip'\int^t \mathrm{d}s\chi(s)\hat{\Phi}_s[F]}e^{i\hat{H}_{\mathrm{F}} t}\right)\\
&= \int \mathrm{d}p\mathrm{d}p'\,\mathrm{tr}_f\left(\hat{U}_\chi(t,p)\hat{\rho}_0\hat{U}_\chi^\dagger(t,p')\right)\psi(p,t)\psi^*(p',t)\,|p\rangle\langle p'|
\end{aligned}
\tag{B.17}
$$

where we have defined

$$
\hat{U}_\chi(t,p) := \mathcal{T}e^{-ip\int^t \mathrm{d}s\chi(s)\hat{\Phi}_s[F]}. \tag{B.18}
$$

Since the Hamiltonian of the system is simply the free Hamiltonian (B.16) we can substitute $\psi(p,t) = \psi(p)$. So both the free dynamics of the detector and the field (due to the cyclic property of the trace) have dropped out.

Now we want to know the probability distribution of the possible values x of the pointer variable $\hat{X}$ that is conjugate to the variable $\hat{P}$, namely

$$
[\hat{X}, \hat{P}] = i. \tag{B.19}
$$

Using the Fourier transform we get that

$$
p(t,x) = \frac{1}{2\pi}\int \mathrm{d}p\mathrm{d}p' e^{i(p-p')x}\psi(p)\psi^*(p')\mathrm{tr}_f\left(\hat{U}_\chi(t,p)\rho_0\hat{U}_\chi^\dagger(t,p')\right). \tag{B.20}
$$

We see that the factor $\mathrm{tr}_f\left(\hat{U}_\chi(t,p)\rho_0\hat{U}_\chi^\dagger(t,p')\right)$ plays the role of an integral kernel that depends on the interaction Hamiltonian and the state of the field and affects the statistics of the pointer variable. If we define

$$\hat{\Pi}_x(t) := \frac{1}{\sqrt{2\pi}}\int \mathrm{d}k e^{ikx}\psi(k)\hat{U}_\chi(t,k) \tag{B.21}$$

then,

$$p(t,x) = \mathrm{tr}_f(\hat{\Pi}_x(t)\hat{\rho}_0\hat{\Pi}_x^\dagger(t)). \tag{B.22}$$

We use the Magnus expansion to decompose the time-ordered exponential (B.18) as

$$\mathcal{T}e^{-ip\int^t \mathrm{d}s\chi(s)\hat{\Phi}_s[F]} = e^{-ip\int^t \mathrm{d}s\chi(s)\hat{\Phi}_s[F]}e^{-\frac{1}{2}\mathcal{C}_t p^2} \tag{B.23}$$

where

$$\mathcal{C}_t = \int^t \mathrm{d}s\int^s \mathrm{d}s'\chi(s)\chi(s')[\hat{\Phi}_s[F],\hat{\Phi}_{s'}[F]] \tag{B.24}$$

the time-ordered commutator of the smeared field operators. We see that $\mathcal{C}_t$ is imaginary ($\mathcal{C}_t^* = -\mathcal{C}_t$) so we define c_t s.t.

$$\mathcal{C}_t = ic_t. \tag{B.25}$$

We will also call

$$\hat{\Phi}_{\mathrm{sm}}(t) := \int^t \mathrm{d}s\chi(s)\hat{\Phi}_s[F]. \tag{B.26}$$

Then (B.21) becomes

$$\hat{\Pi}_x(t) = \frac{1}{\sqrt{2\pi}}\int \mathrm{d}p\,\psi(p)\exp\left(-\frac{i}{2}p^2c_t + ip(x-\hat{\Phi}_{\mathrm{sm}}(t))\right) \tag{B.27}$$

From (B.21) we can see that this defines a set of POVMs for the measurement of the pointer variable. Indeed

$$\begin{aligned}\int \mathrm{d}x\,\hat{\Pi}_x^\dagger(t)\hat{\Pi}_x(t) &= \frac{1}{2\pi}\int \mathrm{d}x\int \mathrm{d}p\mathrm{d}p' e^{i(p-p')x}\psi(p)\psi^*(p')\hat{U}_\chi(t,p)\hat{U}_\chi^\dagger(t,p')\\ &= \frac{1}{2\pi}\int \mathrm{d}x\int \mathrm{d}p\mathrm{d}p'\psi(p)e^{-\frac{i}{2}p^2c_t}\psi^*(p')e^{-\frac{i}{2}p'^2c_t}e^{i(p-p')(x-\hat{\Phi}_{\mathrm{sm}}(t))}\\ &= \int \mathrm{d}p\mathrm{d}p'\delta(p-p')\psi(p)e^{-\frac{i}{2}p^2c_t}\psi^*(p')e^{-\frac{i}{2}p'^2c_t}e^{i(p-p')\hat{\Phi}_{\mathrm{sm}}(t)}\\ &= \int \mathrm{d}p|\psi(p)|^2\mathbb{1}_f = \mathbb{1}_f\end{aligned} \tag{B.28}$$

where we have used that the initial state of the pointer variable is square normalized and that $\exp[\lambda\hat{\Phi}] = \mathbb{1}_f$ for $\lambda = 0$.

For example, if we assume a Gaussian wavefunction of width σ_p for the pointer variable

$$\psi(p) = \frac{1}{(2\pi\sigma_p^2)^{1/4}} \exp\left[-\frac{p^2}{4\sigma_p^2}\right] \tag{B.29}$$

then

$$\hat{\Pi}_x(t) = \frac{1}{(2\pi\sigma_p^2)^{1/4}} \frac{1}{\sqrt{2\pi}} \int \mathrm{d}p \exp\left[-\frac{\alpha_t p^2}{2} + ip\left(x - \hat{\Phi}_{\mathrm{sm}}(t)\right)\right] \tag{B.30}$$

where

$$\alpha_t := \frac{1}{2\sigma_p^2} + ic_t. \tag{B.31}$$

This is the Fourier transform of a Gaussian with a complex coefficient, and it converges since $\mathrm{Re}(\alpha_t) > 0$. So we have

$$\hat{\Pi}_x(t) = \frac{1}{(2\pi\sigma_p^2)^{1/4}} \frac{1}{\sqrt{\alpha_t}} \exp\left[-\frac{\left(x - \hat{\Phi}_{\mathrm{sm}}(t)\right)^2}{2\alpha_t}\right] \tag{B.32}$$

and

$$\hat{\Pi}_x^\dagger(t)\hat{\Pi}_x(t) = \frac{1}{\sqrt{2\pi}\sigma_p} \left|\frac{1}{\sqrt{\alpha_t}}\right|^2 \exp\left[-\mathrm{Re}(1/\alpha_t)\left(x - \hat{\Phi}_{\mathrm{sm}}(t)\right)^2\right] \tag{B.33}$$

which becomes

$$\hat{\Pi}_x^\dagger(t)\hat{\Pi}_x(t) = \frac{1}{\sqrt{2\pi}\Sigma_t} \exp\left[-\frac{\left(x - \hat{\Phi}_{\mathrm{sm}}(t)\right)^2}{2\Sigma_t^2}\right] \tag{B.34}$$

where

$$\Sigma_t = \sqrt{\frac{1}{4\sigma_p^2} + c_t^2\sigma_p^2}. \tag{B.35}$$

B.2 Noise calculation: massless 3+1 case

We want to evaluate

$$c = \int \mathrm{d}s\mathrm{d}s' \int \mathrm{d}\boldsymbol{x}\mathrm{d}\boldsymbol{x}' F(\boldsymbol{x})F(\boldsymbol{x}')\chi(s)\chi(s')G_{\mathrm{R}}(s - s', \boldsymbol{x} - \boldsymbol{x}') \tag{B.36}$$

for Gaussian switching

$$\chi(t) = \frac{\lambda}{\sqrt{2\pi}\tau} e^{-\frac{t^2}{2\tau^2}} \tag{B.37}$$

and Gaussian smearing

$$F(\boldsymbol{x}) = \frac{1}{\sqrt{2\pi}^3 l^3} e^{-\frac{\boldsymbol{x}^2}{2l^2}} \tag{B.38}$$

and for the massless scalar field in 3+1 dimensions where

$$G_{\mathrm{R}}(s - s', \boldsymbol{x} - \boldsymbol{x}') = -\frac{1}{4\pi} \frac{\delta(s - s' - |\boldsymbol{x} - \boldsymbol{x}'|)}{|\boldsymbol{x} - \boldsymbol{x}'|} \tag{B.39}$$

We write (B.36) as

$$c = \int \mathrm{d}s\mathrm{d}s' \chi(s)\chi(s') \int \mathrm{d}\boldsymbol{x}\mathrm{d}\boldsymbol{x}' F(\boldsymbol{x})F(\boldsymbol{x}')\mathcal{C}(s - s') \tag{B.40}$$

where

$$\mathcal{C}(s - s') = \int \mathrm{d}\boldsymbol{x}\mathrm{d}\boldsymbol{y} F(\boldsymbol{x})F(\boldsymbol{x} - \boldsymbol{y}) \frac{\delta(s - s' - |\boldsymbol{y}|)}{|\boldsymbol{y}|} \tag{B.41}$$

and $\boldsymbol{y} = \boldsymbol{x} - \boldsymbol{x}'$. Doing the convolution of the Gaussians we get

$$\mathcal{C}(s - s') = -\frac{1}{4\pi(\sqrt{\pi}2l)^3} \int \mathrm{d}\boldsymbol{y} \exp\left(-\frac{\boldsymbol{y}^2}{4l^2}\right) \frac{\delta(s - s' - |\boldsymbol{y}|)}{|\boldsymbol{y}|}. \tag{B.42}$$

Calling $r = |\boldsymbol{y}|$ and $t = s - s'$

$$\mathcal{C}(t) = -\frac{1}{(\sqrt{\pi}2l)^3} \int \mathrm{d}r r \exp\left(-\frac{r^2}{4l^2}\right) \delta(t - r) \tag{B.43}$$

and

$$\begin{aligned} c &= -\frac{1}{(\sqrt{\pi}2l)^3} \int \mathrm{d}s\mathrm{d}t \chi(s)\chi(s - t) \int \mathrm{d}r r \exp\left(-\frac{r^2}{4l^2}\right) \delta(t - r) \\ &= -\frac{1}{(\sqrt{\pi}2l)^3} \int \mathrm{d}r r \exp\left(-\frac{r^2}{4l^2}\right) \int \mathrm{d}s \chi(s)\chi(s - r) \\ &= -\frac{\lambda^2}{(\sqrt{\pi}2l)^3 \sqrt{\pi}2\tau} \int \mathrm{d}r r \exp\left(-\frac{r^2}{4l^2} - \frac{r^2}{4\tau^2}\right) \\ &= -\frac{\lambda^2 \tau}{8\pi^2 l(\tau^2 + l^2)}. \end{aligned} \tag{B.44}$$

B.3 Response of pointer variable to coherent state

We want to evaluate

$$\langle f|\hat{\Phi}_{\mathrm{sm}}(t)|f\rangle = -i\int^{t} \mathrm{d}s\chi(s)[\hat{\Phi}_0[f], \hat{\Phi}_s[F]] \tag{B.45}$$

where f, F Gaussian smearing functions of width σ that are centered around $0, \boldsymbol{L}$ respectively, so that $L := |\boldsymbol{L}|$ is the spatial distance between the centers of the initial coherent state and the interaction region of the pointer variable. For a massless field in $3+1$ dimensions the commutator is

$$[\hat{\Phi}(t,\boldsymbol{x}), \hat{\Phi}(t',\boldsymbol{x}')] = \frac{i}{4\pi}\frac{\delta(t-t'+|\boldsymbol{x}-\boldsymbol{x}'|) - \delta(t-t'-|\boldsymbol{x}-\boldsymbol{x}'|)}{|\boldsymbol{x}-\boldsymbol{x}'|}. \tag{B.46}$$

Following [37] we define

$$\begin{aligned} C(t-t') &= i[\hat{\Phi}_t[f], \hat{\Phi}_{t'}[F]] \\ &= i\int \mathrm{d}\boldsymbol{x}\mathrm{d}\boldsymbol{x}' f(\boldsymbol{x})F(\boldsymbol{x}')[\hat{\Phi}(t,\boldsymbol{x}), \hat{\Phi}(t',\boldsymbol{x}')] \end{aligned} \tag{B.47}$$

for the Gaussian functions

$$f(\boldsymbol{x}) = \frac{1}{(\sqrt{2\pi}\sigma)^3} e^{-\frac{\boldsymbol{x}^2}{2\sigma^2}} \tag{B.48}$$

$$F(\boldsymbol{x}) = \frac{1}{(\sqrt{2\pi}\sigma)^3} e^{-\frac{(\boldsymbol{x}-\boldsymbol{L})^2}{2\sigma^2}}. \tag{B.49}$$

All together,

$$C(t-t') = -\frac{1}{32\pi^4\sigma^6}\sum_{j=1,-1} j\int \mathrm{d}\boldsymbol{x}\mathrm{d}\boldsymbol{x}' e^{-\frac{\boldsymbol{x}^2}{2\sigma^2}} e^{-\frac{(\boldsymbol{x}'-\boldsymbol{L})^2}{2\sigma^2}} \frac{\delta(t-t'+j|\boldsymbol{x}-\boldsymbol{x}'|)}{|\boldsymbol{x}-\boldsymbol{x}'|} \tag{B.50}$$

to account for the two terms in the commutator (B.47). The two terms correspond to the advanced and the retarded Green's function, and will both contribute for general switching function. Changing variables to $\boldsymbol{u} = \boldsymbol{x} + \boldsymbol{x}'$ and $\boldsymbol{v} = \boldsymbol{x} - \boldsymbol{x}'$ we get

$$C(t-t') = -\frac{1}{64\pi^4\sigma^6}\sum_{j=1,-1} j\int \mathrm{d}\boldsymbol{u}\mathrm{d}\boldsymbol{v} e^{-\frac{(\boldsymbol{u}+\boldsymbol{v})^2}{8\sigma^2}} e^{-\frac{(\boldsymbol{u}-\boldsymbol{v}-2\boldsymbol{L})^2}{8\sigma^2}} \frac{\delta(t-t'+j|\boldsymbol{v}|)}{|\boldsymbol{v}|} \tag{B.51}$$

which can be further simplified as

$$C(t-t') = -\frac{e^{-\frac{L^2}{2\sigma^2}}}{64\pi^4\sigma^6}\sum_{j=1,-1} j\int \mathrm{d}\boldsymbol{u}\mathrm{d}\boldsymbol{v} e^{-\frac{|\boldsymbol{u}|^2+2\boldsymbol{u}\boldsymbol{L}}{4\sigma^2}} e^{-\frac{|\boldsymbol{v}|^2-2\boldsymbol{v}\boldsymbol{L}}{4\sigma^2}} \frac{\delta(t-t'+j|\boldsymbol{v}|)}{|\boldsymbol{v}|} \tag{B.52}$$

First, we solve the integral over d$\boldsymbol{u}$

$$2\pi\int_0^\infty \mathrm{d}|\boldsymbol{u}||\boldsymbol{u}|^2 e^{-\frac{|\boldsymbol{u}|^2}{4\sigma^2}}\int_{-1}^1 \mathrm{d}(\cos\theta) e^{\frac{|\boldsymbol{u}|L\cos\theta}{2\sigma^2}} = \frac{8\pi\sigma^2}{L}\int_0^\infty \mathrm{d}|\boldsymbol{u}||\boldsymbol{u}|e^{-\frac{|\boldsymbol{u}|^2}{4\sigma^2}}\sinh\left(\frac{L|\boldsymbol{u}|}{2\sigma^2}\right) \tag{B.53}$$

where

$$\int_0^\infty \mathrm{d}|\boldsymbol{u}||\boldsymbol{u}|e^{-\frac{|\boldsymbol{u}|^2}{4\sigma^2}}\sinh\left(\frac{L|\boldsymbol{u}|}{2\sigma^2}\right) = 4\sigma^3\sqrt{2\pi^3}e^{\frac{L^2}{4\sigma^2}}. \tag{B.54}$$

We put this back into (B.52) and we perform the integration over d$\boldsymbol{v}$ for each term

$$\begin{aligned} &-j\frac{\pi\sqrt{2\pi^3}e^{-\frac{L^2}{4\sigma^2}}}{2\sigma L}\int_0^\infty \mathrm{d}|\boldsymbol{v}||\boldsymbol{v}|e^{-\frac{|\boldsymbol{v}|^2}{4\sigma^2}}\int_{-1}^1 \mathrm{d}(\cos\theta)e^{-\frac{|\boldsymbol{v}|L\cos\theta}{2\sigma^2}}\delta(t-t'+j|\boldsymbol{v}|) \\ &= j\frac{\pi\sigma\sqrt{2\pi^3}e^{-\frac{L^2}{4\sigma^2}}}{L}\int_0^\infty \mathrm{d}|\boldsymbol{v}|e^{-\frac{|\boldsymbol{v}|^2}{4\sigma^2}}\sinh\left(\frac{|\boldsymbol{v}|L}{2\sigma^2}\right)\delta(t-t'+j|\boldsymbol{v}|) \\ &= j\frac{\pi\sigma\sqrt{2\pi^3}}{L}e^{-\frac{L^2}{4\sigma^2}}e^{-\frac{(t-t')^2}{4\sigma^2}}\sinh\left(\frac{jL(t-t')}{2\sigma^2}\right). \end{aligned} \tag{B.55}$$

All together we have that

$$C(t-t') = \frac{\pi\sqrt{2\pi^3}\sigma}{L}e^{-\frac{L^2+(t-t')^2}{4\sigma^2}}\left(\sinh\left(\frac{L(t-t')}{2\sigma^2}\right) - \sinh\left(\frac{jL(t'-t)}{2\sigma^2}\right)\right) \tag{B.56}$$

which takes the form

$$C(t-t') = \frac{\pi\sqrt{2\pi^3}\sigma}{L}\left(\exp\left[\frac{[L-(t'-t)]^2}{4\sigma^2}\right] - \exp\left[\frac{[L+(t'-t)]^2}{4\sigma^2}\right]\right) \tag{B.57}$$

where we see an explicit pick on the ligntcone. Now, from (B.45) we see that

$$\langle f|\hat{\Phi}_{\mathrm{sm}}(t)|f\rangle = \int^t \mathrm{d}s\chi(s)C(s). \tag{B.58}$$

Appendix C

Wavepacket detection

C.1 Resonance and time-of-arrival

Here we will make the almost-monochromatic approximation for the NW wavefunction of the wavepacket. We define $\boldsymbol{q} = \boldsymbol{k} - \boldsymbol{k}_0$, and we will keep terms that are $\mathcal{O}(|\boldsymbol{q}|)$. We Taylor expand the field frequencies around $\boldsymbol{k}_0$ as follows

$$\omega_{\boldsymbol{k}} = \sqrt{(\boldsymbol{k}_0 + \boldsymbol{q})^2 + m^2} \simeq \sqrt{\omega_{\boldsymbol{k}_0}^2 + 2\boldsymbol{k}_0\boldsymbol{q}} \simeq \omega_{\boldsymbol{k}_0}\left(1 + \frac{\boldsymbol{k}_0\boldsymbol{q}}{\omega_{\boldsymbol{k}_0}^2} + \mathcal{O}(|\boldsymbol{q}|^2)\right). \tag{C.1}$$

So at leading order we get that

$$\omega_{\boldsymbol{k}} = \omega_{\boldsymbol{k}_0} + \boldsymbol{v}_0\boldsymbol{q} \tag{C.2}$$

where

$$\boldsymbol{v}_0 = \boldsymbol{k}_0/\omega_{\boldsymbol{k}_0} \tag{C.3}$$

the relativistic velocity, which is basically the wavepacket's group velocity. Then

$$\begin{aligned}\psi(\boldsymbol{L}, t) &= \int \frac{\mathrm{d}\boldsymbol{k}}{\sqrt{2\omega_{\boldsymbol{k}}}} e^{i(\boldsymbol{k}\boldsymbol{L} - \omega_{\boldsymbol{k}} t)}\tilde{\psi}(\boldsymbol{k} - \boldsymbol{k}_0) \\ &\simeq \frac{1}{\sqrt{2\omega_{\boldsymbol{k}_0}}}\int \mathrm{d}\boldsymbol{q} e^{i(\boldsymbol{k}_0 + \boldsymbol{q})\boldsymbol{L} - i\omega_{\boldsymbol{k}_0} t - i\boldsymbol{v}_0\boldsymbol{q}t}\tilde{\psi}(\boldsymbol{q}) \\ &= e^{i\boldsymbol{k}_0\boldsymbol{L} - it\omega_{\boldsymbol{k}_0}}\tilde{\psi}(\boldsymbol{L} - \boldsymbol{v}_0 t)\end{aligned} \tag{C.4}$$

So overall we get that

$$P_\psi = \tag{C.5}$$
$$\lambda^2 \frac{|F(\boldsymbol{k}_0)|^2}{2\omega_0} \left(\left| \int \mathrm{d}t e^{i(\omega-\omega_0)t} \chi(t-T)\tilde{\psi}(\boldsymbol{L}-\boldsymbol{v}_0 t) \right|^2 + \left| \int \mathrm{d}t e^{i(\omega+\omega_0)t} \chi(t-T)\tilde{\psi}(\boldsymbol{L}-\boldsymbol{v}_0 t) \right|^2 \right)$$

where ω the energy gap of the detector and $\omega_0 = \omega_{\boldsymbol{k}_0}$ the dominant frequency of the wavepacket. The function $\tilde{\psi}$ is picked at $\boldsymbol{q} = 0$ so the maximum value is obtained for some time t when $\boldsymbol{L}$ and $\boldsymbol{v}_0$ are aligned (that is, when we 'shoot' the wavepacket towards the detector's center) and the problem essentially reduces to a one-dimensional problem. For the rest we denote $L := |\boldsymbol{L}|$ and $v_0 := |\boldsymbol{v}_0|$. If we choose $\chi, \tilde{\psi}$ to be Gaussian functions with width σ_t, σ_k we get

$$\chi(t-T)\tilde{\psi}(L-v_0 t) = \frac{1}{2\pi v_0 \tilde{\sigma}_k \sigma_t} e^{-\frac{(t-T)^2}{2\sigma_t^2}} e^{-\frac{(t-L/v_0)^2}{2\tilde{\sigma}_k^2}} \tag{C.6}$$

where $\tilde{\sigma}_k = \sigma_k / v_0$. The two Gaussians can be combined

$$\chi(t-T)\tilde{\psi}(L-v_0 t) = \frac{1}{2\pi v_0 \tilde{\sigma}_k \sigma_t} e^{-\frac{(T-L/v_0)^2}{2(\sigma_t^2+\tilde{\sigma}_k^2)}} e^{-\frac{(t-\mu_{k,t})^2}{2\sigma_{k,t}^2}} \tag{C.7}$$

where

$$\mu_{k,t} = \frac{T\tilde{\sigma}_k^2 + \frac{L}{v_0}\sigma_t^2}{\sigma_t^2 + \tilde{\sigma}_k^2} \tag{C.8}$$

and

$$\sigma_{k,t} = \frac{\sigma_t \tilde{\sigma}_k}{\sqrt{\sigma_t^2 + \tilde{\sigma}_k^2}}. \tag{C.9}$$

After the Fourier transform we get

$$P_\psi = \frac{\lambda^2 |F(k_0)|^2}{2\omega_0} \frac{\sigma_{k,t}^2}{2\sigma_k^2 \sigma_t^2} e^{-\frac{(T-L/v_0)^2}{(\sigma_t^2+\tilde{\sigma}_k^2)}} \left(e^{-2\sigma_{k,t}^2(\omega-\omega_0)^2} + e^{-2\sigma_{k,t}^2(\omega+\omega_0)^2} \right). \tag{C.10}$$

C.2 Detecting massive wavepackets

Let's consider an initial state

$$|\Psi_0\rangle = |g\rangle \otimes \int \mathrm{d}\boldsymbol{y} \psi(\boldsymbol{y}) \hat{a}_{\boldsymbol{y}}^\dagger |0\rangle \tag{C.11}$$

where

$$\hat{a}_{\boldsymbol{y}} := \int \mathrm{d}\boldsymbol{k} e^{-i\boldsymbol{k}\boldsymbol{y}} \hat{a}_{\boldsymbol{k}} \tag{C.12}$$

the non-relativistic localization scheme. The interaction Hamiltonian is of the form

$$\hat{H}_{int} = \lambda\chi(t)\hat{\mu}(t) \otimes \int \mathrm{d}\boldsymbol{x} F(\boldsymbol{x})\hat{\mathcal{O}}(\boldsymbol{x}, t) \tag{C.13}$$

where $\hat{\mathcal{O}}(\boldsymbol{x}, t)$ is a composite operator of a massive scalar Klein Gordon field. Using the Dyson expansion of the time evolution operator we get

$$|\Psi(\tau)\rangle = \left(\mathbb{1} + \int^{\tau} \mathrm{d}t \hat{H}_{int}(t) + \int^{\tau} \mathrm{d}t_1 \int^{t_1} \mathrm{d}t_2 \hat{H}_{int}(t_1)\hat{H}_{int}(t_2) + ...\right)|\Psi_0\rangle \tag{C.14}$$

First order in λ we get

$$\begin{aligned} |\Psi(\tau)\rangle &= |g\rangle \otimes \int dy \psi(y) \hat{a}^{\dagger}_{\boldsymbol{y}} |0\rangle \\ &+ \lambda \int^{\tau} \mathrm{d}t \chi(t) \int \mathrm{d}\boldsymbol{x} F(\boldsymbol{x})\hat{\mathcal{O}}(\boldsymbol{x}, t) \int \mathrm{d}\boldsymbol{y} \psi(\boldsymbol{y}) \hat{a}^{\dagger}_{y} |0\rangle \otimes \hat{\mu}(t) |0\rangle + ... \end{aligned} \tag{C.15}$$

For example, if we are interested in the probability amplitude Π that the detector gets excited and the field gets to its ground state (first order in λ, second order for the probability)

$$\Pi(\tau) = \lambda \int^{\tau} \mathrm{d}t \chi(t) \langle e|\hat{\mu}(t)|g\rangle \int \mathrm{d}\boldsymbol{x} \int \mathrm{d}\boldsymbol{y} F(\boldsymbol{x})\psi(\boldsymbol{y})\langle 0|\hat{\mathcal{O}}(t, \boldsymbol{x})\hat{a}^{\dagger}_{\boldsymbol{y}}|0\rangle + \mathcal{O}(\lambda^2) \tag{C.16}$$

If we express the field operator in terms of the non-relativistic localization scheme we take the non-local relation

$$\hat{\Phi}(t, \boldsymbol{x}) = \int \mathrm{d}\boldsymbol{z} \left(K(t, \boldsymbol{x} - \boldsymbol{z})\hat{a}_{\boldsymbol{z}} + K^*(t, \boldsymbol{x} - \boldsymbol{z})\hat{a}^{\dagger}_{\boldsymbol{z}}\right) \tag{C.17}$$

where

$$K(t, \boldsymbol{x} - \boldsymbol{z}) = \int \mathrm{d}\boldsymbol{k} v_{\boldsymbol{k}}(t, \boldsymbol{x}) e^{-i\boldsymbol{k}\boldsymbol{z}} \tag{C.18}$$

and $v_{\boldsymbol{k}}(t, \boldsymbol{x})$ the modes over which we expand the field operator. In the case of derivative coupling

$$\hat{\mathcal{O}}(t, \boldsymbol{x}) = \partial_{\boldsymbol{x}} \hat{\Phi}(t, \boldsymbol{x}), \tag{C.19}$$

the field integral Kernel is

$$\int \mathrm{d}\boldsymbol{x} \int \mathrm{d}\boldsymbol{y} F(\boldsymbol{x})\psi(\boldsymbol{y})\langle 0|\partial_{\boldsymbol{x}}\hat{\Phi}(t,\boldsymbol{x})a^{\dagger}_{\boldsymbol{y}}|0\rangle. \tag{C.20}$$

The 'trick' is that by integrating by parts we get that

$$\begin{aligned}\Pi(\tau) &= \lambda \int^{\tau} \mathrm{d}t\chi(t)\langle e|\hat{\mu}(t)|g\rangle \int \mathrm{d}\boldsymbol{y}\psi(\boldsymbol{y}) \int \mathrm{d}\boldsymbol{x}\partial_{\boldsymbol{x}} F(\boldsymbol{x})K(t,\boldsymbol{x}-\boldsymbol{y}) \\ &= -\lambda \int^{\tau} \mathrm{d}t\chi(t)\langle e|\hat{\mu}(t)|g\rangle \int \mathrm{d}\boldsymbol{x} F(\boldsymbol{x})\partial_{\boldsymbol{x}}\tilde{\psi}(t,\boldsymbol{x}),\end{aligned} \tag{C.21}$$

where

$$\tilde{\psi}(x,t) := \int \mathrm{d}\boldsymbol{y} K(t,\boldsymbol{x}-\boldsymbol{y})\psi(\boldsymbol{y}). \tag{C.22}$$

We realize that this is the wavepacket convoluted with the field's Wightman function, or equivalently the Newton-Wigner wavefunction

$$\tilde{\psi}(t,\boldsymbol{x}) = \int \mathrm{d}\boldsymbol{y} \left(\int \mathrm{d}\boldsymbol{k} \frac{e^{-i\omega_{\boldsymbol{k}}t}e^{-i\boldsymbol{k}(\boldsymbol{y}-\boldsymbol{x})}}{\sqrt{2\omega_{\boldsymbol{k}}}} \right) \psi(\boldsymbol{y}) = \int \frac{\mathrm{d}\boldsymbol{k}}{\sqrt{2\omega_{\boldsymbol{k}}}} e^{i\boldsymbol{k}\boldsymbol{x}-i\omega_{\boldsymbol{k}}t}\psi(\boldsymbol{k}). \tag{C.23}$$

For example, if we consider ultra non-relativistic particle excitations e.g. the Fourier transform of $\psi(\boldsymbol{x})$ is compactly supported in $[-\Lambda,\Lambda]$ where $\Lambda << m$, then

$$\tilde{\psi}(\boldsymbol{x},t) := \frac{e^{-imt}}{\sqrt{2m}}\psi(\boldsymbol{x}) \tag{C.24}$$

Then the probability amplitude becomes

$$\Pi_{\text{der}} = \frac{\lambda}{\sqrt{2m}} \int \mathrm{d}t e^{-imt}\chi(t)\langle e|\hat{\mu}(t)|g\rangle \int \mathrm{d}\boldsymbol{x} F(\boldsymbol{x})\partial_{\boldsymbol{x}}\psi(\boldsymbol{x}) \tag{C.25}$$

Under the same approximation, for the interaction Hamiltonian that involves $\hat{\mathcal{O}}(t,\boldsymbol{x}) = \hat{\Phi}(t,\boldsymbol{x})$ we get

$$\Pi_{\text{ampl}} = \frac{\lambda}{\sqrt{2m}} \int \mathrm{d}t e^{-imt}\chi(t)\langle e|\hat{\mu}(t)|g\rangle \int \mathrm{d}\boldsymbol{x} F(\boldsymbol{x})\psi(\boldsymbol{x}) \tag{C.26}$$

Note that this is a feature of the detection process in QFT that is relevant even in the ultra-non-relativistic limit. In that limit, the non-locality between the field operator and the non-relativistic localization scheme in (4.116) is 'switched-off'. If the Fourier transform of $\psi(\boldsymbol{y})$ in (4.112) is compactly supported in $[-\Lambda, \Lambda]$ where $\Lambda << m$, then

$$\tilde{\psi}(\boldsymbol{x}, t) := \frac{e^{-imt}}{\sqrt{2m}}\psi(\boldsymbol{x}), \tag{C.27}$$

and the probability amplitude becomes

$$\Pi_{\text{der}} = \frac{\lambda}{\sqrt{2m}} \int \mathrm{d}t e^{-imt}\chi(t)\langle e|\hat{\mu}(t)|g\rangle \int \mathrm{d}\boldsymbol{x} F(\boldsymbol{x})\partial_{\boldsymbol{x}}\psi(\boldsymbol{x}) + \mathcal{O}(\lambda^2). \tag{C.28}$$

Under the same (crude) approximation, for the spatial derivative coupling we get

$$\Pi_{\text{ampl}} = \frac{\lambda}{\sqrt{2m}} \int dt e^{-imt}\chi(t)\langle e|\hat{\mu}(t)|g\rangle \int \mathrm{d}\boldsymbol{x} F(\boldsymbol{x})\psi(\boldsymbol{x}) + \mathcal{O}(\lambda^2). \tag{C.29}$$

Also, if we look at any possible transition for the field, the excitation probability of the detector is

$$\begin{aligned} P_{g\to e}(\tau) &= \lambda^2 \int^{\tau} \mathrm{d}t\mathrm{d}t'\chi(t)\chi(t')e^{i\Omega(t-t')} \int \mathrm{d}\boldsymbol{x}\mathrm{d}\boldsymbol{x}' F^*(\boldsymbol{x})F(\boldsymbol{x}')\langle\psi|\hat{\Phi}(t,\boldsymbol{x})\hat{\Phi}(t',\boldsymbol{x}')|\psi\rangle \\ &= 2\lambda^2 \left| \int^{\tau} \mathrm{d}t e^{-i\Omega t}\chi(t) \int \mathrm{d}\boldsymbol{x} F(\boldsymbol{x})\tilde{\psi}^*(\boldsymbol{x},t) \right|^2 \\ &+ \lambda^2 \int^{\tau} \mathrm{d}t\mathrm{d}t'\chi(t)\chi(t')e^{i\Omega(t-t')} \int \mathrm{d}\boldsymbol{x}\mathrm{d}\boldsymbol{x}' F(\boldsymbol{x})K(\boldsymbol{x}-\boldsymbol{x}', t-t')F^*(\boldsymbol{x}') \end{aligned} \tag{C.30}$$

where we have used that ψ is square integrable and that $\int dz K(x-z,t)K(z-x',t') = K(x-x', t-t')$ for the evaluation of the second term. In the case of derivative coupling we get

$$\begin{aligned} P_{g\to e}(\tau) = 2\lambda^2 &\left| \int^{\tau} \mathrm{d}t e^{-i\Omega t}\chi(t) \int \mathrm{d}\boldsymbol{x} F(\boldsymbol{x})\partial_x\tilde{\psi}^*(t,\boldsymbol{x}) \right|^2 \\ &+ \lambda^2 \int^{\tau} \mathrm{d}t\mathrm{d}t'\chi(t)\chi(t')e^{i\Omega(t-t')} \int \mathrm{d}\boldsymbol{x}\mathrm{d}\boldsymbol{x}' F(\boldsymbol{x})F^*(\boldsymbol{x}') \\ &\times \int \mathrm{d}\boldsymbol{z}\, \partial_{\boldsymbol{x}}K(\boldsymbol{x}-\boldsymbol{z},t)\partial_{\boldsymbol{x}'}K(\boldsymbol{x}'-\boldsymbol{z},t'). \end{aligned} \tag{C.31}$$

Appendix D

The QBM model

D.1 The coupling of a quantum harmonic oscillator to a quantum field

We will consider one quantum harmonic oscillator interacting with a field environment

$$\hat{H}_{env} = \int \mathrm{d}\boldsymbol{k}\, \omega_{\boldsymbol{k}} \hat{a}^{\dagger}_{\boldsymbol{k}} \hat{a}_{\boldsymbol{k}} \tag{D.1}$$

where

$$\hat{a}_{\boldsymbol{k}} = \sqrt{\frac{\omega_{\boldsymbol{k}}}{2}} \hat{q}_{\boldsymbol{k}} + i\sqrt{\frac{1}{2\omega_{\boldsymbol{k}}}} \hat{p}_{\boldsymbol{k}} \tag{D.2}$$

such that $[\hat{q}_{\boldsymbol{k}}, \hat{p}_{\boldsymbol{k}'}] = \delta_{\boldsymbol{k},\boldsymbol{k}'}$.

We can perform the canonical transformation $\hat{q}'_{\boldsymbol{k}} = \sqrt{\omega_{\boldsymbol{k}}}\hat{q}_{\boldsymbol{k}}$ and $\hat{p}'_{\boldsymbol{k}} = \hat{p}_{\boldsymbol{k}}/\sqrt{\omega_{\boldsymbol{k}}}$ in terms of which

$$\hat{a}_{\boldsymbol{k}} = \frac{1}{\sqrt{2}}(\hat{q}'_{\boldsymbol{k}} + i\hat{p}'_{\boldsymbol{k}}). \tag{D.3}$$

Then

$$\hat{H}_{env} = \int \mathrm{d}\boldsymbol{k}\, \frac{\omega_{\boldsymbol{k}}}{2} \left(\hat{q}'^2_{\boldsymbol{k}} + \hat{p}'^2_{\boldsymbol{k}}\right) \tag{D.4}$$

which can be brought in the QBM form with $m_{\boldsymbol{k}} = 1/\omega_{\boldsymbol{k}}$ and $\Omega_{\boldsymbol{k}} = \omega_{\boldsymbol{k}}$. The interaction Hamiltonian is

$$\begin{aligned}\hat{H}_{int} &= \lambda \hat{\Phi}(\boldsymbol{x}_0) \otimes \hat{X} \\ &= \lambda \int \mathrm{d}\boldsymbol{k} e^{i\boldsymbol{k}\boldsymbol{x}_0} \hat{\Phi}_k \otimes \hat{X} \\ &= \lambda \int \frac{\mathrm{d}\boldsymbol{k}}{\sqrt{\omega_{\boldsymbol{k}}}} \left(\cos(\boldsymbol{k}\boldsymbol{x}_0) \hat{q}'_{\boldsymbol{k}} + i \sin(\boldsymbol{k}\boldsymbol{x}_0) \hat{p}'_{\boldsymbol{k}} \right) \otimes \hat{X} \end{aligned} \tag{D.5}$$

If we set $\boldsymbol{x}_0 = 0$

$$\hat{H}_{int} = \lambda \int \frac{\mathrm{d}\boldsymbol{k}}{\sqrt{\omega_{\boldsymbol{k}}}} \hat{q}'_{\boldsymbol{k}} \otimes \hat{X}. \tag{D.6}$$

Note that the form that the coupling takes a different form depending on the point in which we couple the harmonic oscillator (in a different location the harmonic oscillator is not only coupled to the $\hat{q}'$ s but also to the $\hat{p}'$ s of the environment).Compared with the QBM interaction Hamiltonian (4.78) the coupling constants are

$$c_{\boldsymbol{k}} = \frac{\lambda}{\sqrt{\omega_{\boldsymbol{k}}}}. \tag{D.7}$$

The spectral density that is associated with the bath of quantum harmonic oscillators in the environment is given by

$$I(\omega) = \pi \int \mathrm{d}\boldsymbol{k} \frac{c_{\boldsymbol{k}}^2}{2 m_{\boldsymbol{k}} \omega_{\boldsymbol{k}}} \delta(\omega - \omega_{\boldsymbol{k}}) \tag{D.8}$$

which gives

$$I(\omega) = \pi^2 \lambda^2 \omega. \tag{D.9}$$

The fact that the spectral density of the scalar field environment is proportional to ω (Ohmic environment) simplifies significantly the calculations. Crucially, this holds only in the pointlike interaction. In the case of smeared couplings the quantum field environment is effectively Ohmic only in some regimes.

If we consider the interaction Hamiltonian

$$H_{int} = \lambda \hat{\Phi}[F] \otimes \hat{X} \tag{D.10}$$

Using the decomposition above

$$H_{int} = \lambda \int \frac{\mathrm{d}\boldsymbol{k}}{\sqrt{\omega_{\boldsymbol{k}}}} \left(\beta(\boldsymbol{k}) q'_{\boldsymbol{k}} + s(\boldsymbol{k}) p'_{\boldsymbol{k}} \right) \otimes \hat{X} \tag{D.11}$$

where

$$c(\boldsymbol{k}) = \int \mathrm{d}\boldsymbol{x} F(\boldsymbol{x}) \cos(\boldsymbol{k}\boldsymbol{x}) \tag{D.12}$$

$$s(\boldsymbol{k}) = \int \mathrm{d}\boldsymbol{x} F(\boldsymbol{x}) \sin(\boldsymbol{k}\boldsymbol{x}) \tag{D.13}$$

If we choose $F(\boldsymbol{x})$ centered around zero then $s(\boldsymbol{k}) = 0$ and we get that

$$c_{\boldsymbol{k}} = \frac{\lambda c(\boldsymbol{k})}{\sqrt{\omega_{\boldsymbol{k}}}} \tag{D.14}$$

It is easy to check that in the case of Gaussian smearing,

$$F(\boldsymbol{x}) = \frac{1}{l^3 \sqrt{2\pi}^3} e^{-\frac{|\boldsymbol{x}|^2}{2l^2}}, \tag{D.15}$$

the spectral density is

$$I(\omega) \sim \frac{\lambda}{l^2} \omega e^{-\frac{\omega^2}{l^2}} \tag{D.16}$$

so the environment is not Ohmic.

D.2 QBM coupled to momentum

Consider the Hamiltonian

$$\hat{H} = \hat{H}_{env} + \frac{\hat{P}^2}{2M} + \frac{M\omega_0^2}{2} \hat{X}^2 + \lambda \hat{P}(t) \otimes \hat{\Phi}(t, \boldsymbol{x}_0) \tag{D.17}$$

The equations of motion for the harmonic oscillator are

$$\frac{\mathrm{d}\hat{X}(t)}{\mathrm{d}t} = \frac{\hat{P}(t)}{M} - \lambda \hat{\Phi}(t, \boldsymbol{x}_0) \tag{D.18}$$

$$\frac{\mathrm{d}\hat{P}(t)}{\mathrm{d}t} = M\omega_0^2 \hat{X}(t) \tag{D.19}$$

and the combined equations of motion for the field give

$$\Box \hat{\Phi}(t) = \lambda \hat{P}(t) \delta(\boldsymbol{x} - \boldsymbol{x}_0) \tag{D.20}$$

we will call $\hat{\Phi}_0(t,\boldsymbol{x})$ the solution to the homogeneous equation $\Box\hat{\Phi}_0(t,\boldsymbol{x}) = 0$. The full solution is

$$\hat{\Phi}(t,\boldsymbol{x}) = \hat{\Phi}_0(t,\boldsymbol{x}_0) + \lambda \int^t G_{\rm R}(\boldsymbol{x},t;\boldsymbol{x}_0,s)\hat{P}(s)\mathrm{d}s \tag{D.21}$$

Combining the equations we get that

$$\frac{\mathrm{d}^2\hat{X}(t)}{\mathrm{d}^2 t} - M\omega_0^2\hat{X}(t) = \lambda\frac{\mathrm{d}\hat{\Phi}_0(x_0,t)}{\mathrm{d}t} + \lambda^2\frac{\mathrm{d}}{\mathrm{d}t}\int^t G_{\rm R}(x_0,t;x_0,s)\hat{P}(s)\mathrm{d}s \tag{D.22}$$

In general, equation (D.22) does not take the usual integrodifferential form. It is better to use the canonical transformation

$$\hat{X}' = \hat{P} \tag{D.23}$$

$$\hat{P}' = -\hat{X} \tag{D.24}$$

The interaction Hamiltonian (D.17) becomes

$$\hat{H} = \hat{H}_{env} + \frac{\hat{P'}^2}{2M'} + \frac{M'\omega_0'^2}{2}\hat{X}'^2 + \lambda\hat{X}'\otimes\hat{\Phi}(t,\boldsymbol{x}_0) \tag{D.25}$$

where

$$M' = \frac{1}{M\omega_0^2} \tag{D.26}$$

$$\omega_0' = \omega_0. \tag{D.27}$$

We will set $\boldsymbol{x}_0 = 0$ and we will denote $\hat{\Phi}_0(t) := \hat{\Phi}_0(t,\mathbf{0})$. Then the solutions are

$$\hat{X}'(t) = \dot{u}(t)\hat{X}'(0) + \frac{u(t)}{M'}\hat{P}'(0) - \frac{\lambda}{M'}\int_0^t \mathrm{d}t' u(t-t')\hat{\Phi}_0(t') \tag{D.28}$$

and

$$\hat{P}'(t) = M'\ddot{u}\hat{X}'(0) + \dot{u}(t)\hat{P}'(0) - \lambda\int_0^t \mathrm{d}t'\dot{u}(t-t')\hat{\Phi}_0(t') \tag{D.29}$$

where $u(t)$ such that

$$\ddot{u}(t) + \bar{\Omega}'^2 u(t) + \frac{2}{M'}\int_0^t \gamma(t-t')\dot{u}(t') = 0 \tag{D.30}$$

where

$$\dot{\gamma}(t-t') = \eta(t-t') \tag{D.31}$$

and

$$\eta(t-t') := \lambda^2 G_{\mathrm{R}}(x_0, t; x_0, t'). \tag{D.32}$$

The frequency $\bar{\omega}'$ is

$$\bar{\omega}'^2 = \omega'^2 - \frac{2\gamma(0)}{M'} \tag{D.33}$$

Going back to the original variables we get that = where u satisfies

$$\ddot{u}(t) + \bar{\omega}'^2 u(t) + \frac{2}{M'}\int_0^t \gamma(t-t')\dot{u}(t')\mathrm{d}t' = 0. \tag{D.34}$$

From the pointlike spectral density we get

$$\gamma(t-t') = \pi\lambda^2\delta(t-t') \tag{D.35}$$

so we have

$$\ddot{u}(t) + \omega_0^2(1 - 2M\gamma(0))u(t) + 2\pi\lambda^2 M\omega_0^2\dot{u}(t) = 0 \tag{D.36}$$

or

$$\ddot{u}(t) + \bar{\Omega}'^2 u(t) + \Gamma\dot{u}(t) = 0 \tag{D.37}$$

which we solve demanding the initial conditions $\dot{u}(0) = 1$ and $u(0) = 0$. The solution is

$$u(t) = \frac{\sin(\omega t)}{\omega} e^{-\frac{\Gamma t}{2}} \tag{D.38}$$

where

$$\omega := \sqrt{\frac{\Gamma^2}{4} - \bar{\omega}'^2} \tag{D.39}$$

Again we see that for $\omega_0 = 0$ we get that $\dot{u} = 1$ (no dissipation).

D.3 Resonance in the QBM

The homogeneous solution for $\Gamma < \bar{\omega}$ is

$$u(t) = \frac{\sin(\omega t)}{\omega} e^{-\Gamma t} \tag{D.40}$$

where

$$\omega = \sqrt{\bar{\omega}^2 - \Gamma^2}. \tag{D.41}$$

The average energy of the harmonic oscillator is

$$\langle \hat{H} \rangle = \frac{(\Delta \hat{P})^2}{2M} + \frac{M\bar{\omega}^2(\Delta \hat{X})^2}{2}. \tag{D.42}$$

This is because $\langle \hat{X} \rangle = \langle \hat{P} \rangle = 0$ for Fock states of the field, and as a result $(\Delta \hat{X}) = \langle \hat{X}^2 \rangle$ and $(\Delta \hat{P}) = \langle \hat{P}^2 \rangle$. Note that we have used the renormalized and not the bare frequency of the harmonic oscillator in (4.103).

The variance $\Delta \hat{X}$ is given by (4.97) where

$$\begin{aligned}(\Delta \hat{X}_0(t))^2 &= \ddot{u}^2(t)\langle \hat{X}^2(0)\rangle + \dot{u}^2(t)\langle \hat{P}^2(0)\rangle \\ &\quad + \dot{u}(t)\ddot{u}(t)\left(\langle \hat{X}(0)\hat{P}(0)\rangle + \langle \hat{P}(0)\hat{X}(0)\rangle\right)\end{aligned} \tag{D.43}$$

which vanishes for $t \gg 1/\Gamma$ because there is an overall prefactor $e^{-\Gamma t}$. Then,

$$(\Delta \hat{\Phi}_u(t))^2 = (\Delta \hat{\Phi}_0(t))^2 + (\Delta \hat{\Phi}_{u,\psi}(t))^2 \tag{D.44}$$

where

$$\begin{aligned}(\Delta \hat{\Phi}_{u,0}(t))^2 &= \int \mathrm{d}\boldsymbol{p}\, |\psi(\boldsymbol{p}-\boldsymbol{p}_0)|^2 \int \mathrm{d}\boldsymbol{k}\, |v_{u,\boldsymbol{k}}(t)|^2 \\ &= \int \mathrm{d}\boldsymbol{k}\, |v_{u,\boldsymbol{k}}(t)|^2\end{aligned} \tag{D.45}$$

is independent of the state of the wavepacket (since ψ is normalized) and also time-independent. On the other hand

$$(\Delta \hat{\Phi}_{u,\psi}(t))^2 = \left| \int \mathrm{d}\boldsymbol{p} \psi(\boldsymbol{p}-\boldsymbol{p}_0) v_{u,\boldsymbol{p}}(t) \right|^2 \tag{D.46}$$

is the state-dependent part of the 'signal' where

$$v_{u,\boldsymbol{p}}(t) := \int_0^t \mathrm{d}t' u_{\boldsymbol{p}}(t', \boldsymbol{0}) u(t-t'), \tag{D.47}$$

the convolution of the field's modes with the homogeneous solution. Plugging in (D.40):

$$v_{u,\boldsymbol{p}}(t) = -\frac{ie^{-\Gamma t}}{2\omega\sqrt{2\omega_{\boldsymbol{p}}}} \left(e^{i\omega t} \int_0^t \mathrm{d}t' e^{-i(\omega_{\boldsymbol{p}}+\omega)t'+\Gamma t'} - e^{-i\omega t} \int_0^t \mathrm{d}t' e^{-i(\omega_{\boldsymbol{p}}-\omega)t'+\Gamma t'} \right). \tag{D.48}$$

So overall,

$$\Delta\hat{\Phi}^2_{u,\psi}(t) = \left| \frac{1}{2i\omega} \int \mathrm{d}\boldsymbol{p} \frac{\psi(\boldsymbol{p}-\boldsymbol{p}_0)}{\sqrt{2\omega_{\boldsymbol{p}}}} \left(\frac{e^{-i\omega_{\boldsymbol{p}}t} - e^{i\bar{\omega}t-\Gamma t}}{\Gamma - i(\omega_{\boldsymbol{p}}+\omega)} - \frac{e^{-i\omega_{\boldsymbol{p}}t} - e^{-i\omega t-\Gamma t}}{\Gamma - i(\omega_{\boldsymbol{p}}-\omega)} \right) \right|^2, \quad \text{(D.49)}$$

If we assume $\psi(\boldsymbol{p}-\boldsymbol{p}_0) = \chi(\boldsymbol{p}-\boldsymbol{p}_0)e^{i\boldsymbol{p}\boldsymbol{L}}$, where χ is picked around $\boldsymbol{p} = \boldsymbol{0}$ and $\Gamma|\boldsymbol{L}| \gg 1$ (and for $t >> 1/\Gamma$)

$$\begin{aligned} \Delta\hat{\Phi}^2_{u,\psi}(t) = &\left| \frac{1}{2i\omega\sqrt{2|\boldsymbol{p}_0|}} \left(\frac{1}{\Gamma - i(|\boldsymbol{p}_0|+\omega)} - \frac{1}{\Gamma - i(|\boldsymbol{p}_0|-\omega)} \right) \right|^2 \\ &\times \left| \int \mathrm{d}\boldsymbol{p}\chi(\boldsymbol{p}-\boldsymbol{p}_0)e^{i\boldsymbol{p}\boldsymbol{L}}e^{-i|\boldsymbol{p}|t} \right|^2 \end{aligned} \quad \text{(D.50)}$$

Defining

$$\Psi(t,\boldsymbol{L}) = \int \mathrm{d}\boldsymbol{p}\chi(\boldsymbol{p}-\boldsymbol{p}_0)e^{-i|\boldsymbol{p}|t}e^{-i\boldsymbol{p}\boldsymbol{L}} \quad \text{(D.51)}$$

we get

$$(\Delta\hat{\Phi}_{u,\psi}(t))^2 = \frac{|\Psi(t,\boldsymbol{L})|^2}{8\omega^2|\boldsymbol{p}_0|} \left| \frac{1}{\Gamma - i(|\boldsymbol{p}_0|+\omega)} - \frac{1}{\Gamma - i(|\boldsymbol{p}_0|-\omega)} \right|^2, \quad \text{(D.52)}$$

or, simplifying

$$(\Delta\hat{\Phi}_{u,\psi}(t))^2 = \frac{|\Psi(t,\boldsymbol{L})|^2}{2|\boldsymbol{p}_0|} \frac{1}{|(\bar{\omega}^2 - |\boldsymbol{p}_0|^2) - 2i\Gamma|\boldsymbol{p}_0||^2}, \quad \text{(D.53)}$$

Similarly,

$$(\Delta\hat{P}(t))^2 = (\Delta\hat{P}_0(t))^2 - \lambda^2(\Delta\hat{\Phi}_{\dot{u}}(t))^2 \quad \text{(D.54)}$$

where

$$\begin{aligned} (\Delta\hat{P}_0(t))^2 = &M^2\ddot{u}^2(t)\Delta(\hat{X}(0))^2 + \dot{u}^2(t)(\Delta\hat{P}(0))^2 \\ &+ M\ddot{u}(t)\dot{u}(t)\left(\langle\hat{X}(0)\hat{P}(0)\rangle + \langle\hat{P}(0)\hat{X}(0)\rangle\right) \end{aligned} \quad \text{(D.55)}$$

which should also vanish for $t \gg 1/\Gamma$. Then

$$(\Delta\hat{\Phi}_{\dot{u}}(t))^2 = (\Delta\hat{\Phi}_{\dot{u},0}(t))^2 + (\Delta\hat{\Phi}_{\dot{u},\psi}(t))^2 \quad \text{(D.56)}$$

where

$$(\Delta\hat{\Phi}_{\dot{u},0}(t))^2 = \int \mathrm{d}\boldsymbol{k}|v_{\dot{u},\boldsymbol{k}}(t)|^2 \quad \text{(D.57)}$$

and

$$(\Delta\hat{\Phi}_{\dot{u},f}(t))^2 = \left|\int \mathrm{d}\boldsymbol{p}\psi(\boldsymbol{p}-\boldsymbol{p}_0)v_{\dot{u},\boldsymbol{p}}(t)\right|^2 \tag{D.58}$$

is the state-dependent part, where

$$v_{u,\boldsymbol{p}}(t) := \int_0^t \mathrm{d}t' u_{\boldsymbol{p}}(t',\mathbf{0})\dot{u}(t-t'). \tag{D.59}$$

The derivative of the homogeneous solution can be written as

$$\dot{u}(t) = \frac{\bar{\omega}}{\omega}e^{-\Gamma t}\cos(\omega t+\phi) \tag{D.60}$$

where ϕ is such that

$$\tan(\phi) = \frac{\Gamma}{\omega}. \tag{D.61}$$

Then

$$v_{\dot{u},\boldsymbol{p}}(t) = \frac{\bar{\omega}e^{-\Gamma t}}{2\omega\sqrt{2\omega_{\boldsymbol{p}}}}\left(e^{i(\omega t+\phi)}\int_0^t \mathrm{d}t' e^{-i(\omega_{\boldsymbol{p}}+\omega)t'+\Gamma t'} + e^{-i(\omega t+\phi)}\int_0^t \mathrm{d}t' e^{-i(\omega_{\boldsymbol{p}}-\omega)t'+\Gamma t'}\right). \tag{D.62}$$

Following the same steps as above, we get

$$(\Delta\hat{\Phi}_{u,\psi}(t))^2 = \frac{|\Psi(t,\boldsymbol{L})|^2}{2}\frac{|\boldsymbol{p}_0|}{|(\bar{\omega}^2-|\boldsymbol{p}_0|^2)-2i\Gamma|\boldsymbol{p}_0||^2}, \tag{D.63}$$

Overall, we have that

$$\langle\hat{H}(t)\rangle = \frac{|\Psi(t,\boldsymbol{L})|^2}{2M|\boldsymbol{p}_0|}\frac{|\boldsymbol{p}_0|^2+\bar{\omega}^2}{|(\bar{\omega}^2-|\boldsymbol{p}_0|^2)-2i\Gamma|\boldsymbol{p}_0||^2} + \mathcal{N} \tag{D.64}$$

where the vacuum noise is given by

$$\mathcal{N} = \frac{\lambda^2}{2M}(\Delta\hat{\Phi}_{\dot{u},0}(t))^2 + \frac{\lambda^2\bar{\omega}^2}{2M}(\Delta\hat{\Phi}_{u,0}(t))^2. \tag{D.65}$$