Quantifying the Impact of Transit Reliability on Users Cost - A Simulation Based Approach

by

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I hereby declare that I am the sole author of this thesis. This is a true copy of the thesis, including any required final revisions, as accepted by my examiners.

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Abstract

The role of public transportation increases as travel demand increases due to the growth in population and economics. The importance of providing a balanced public transportation has increased. In Ontario, Canada, the provincial government investing more than $17B in transit projects by the year of 2020 [28]. Consequently, planners and engineers motivated to pay more attention to mode split (mode choice) models used to estimate transit ridership. In most existing mode choice models, the likelihood of a trip maker using a transit mode (e.g. transit) is based on the generalized cost (GC) of using transit mode relative to the generalized cost of all other available modes.

In conventional generalized cost formulations, transit costs are considered deterministic. It is quite evident, however, that great variability exists in the reliability of transit service and, as a result, the actual costs experienced by users. Efforts are ongoing to incorporate the costs of reliability in mode choice models by extending formulations to include penalties for arriving prior to or later than a desired arrival time.

Transit operators strive to provide reliable service to retain and attract more users. Unreliable service can adversely affect the user by arriving late or early at their destination, waiting longer at their boarding station, and spending more time than expected in the transit vehicle. Unreliable service will also increase the user's anxiety associated with the uncertainty and discomfort. All these factors should be considered explicitly within the generalized cost (GC) function in order to accurately capture the GC of transit service relative to other modes and to ensure that these factors are not incorporated within the mode specific constant.

In this study, a GC model is developed that explicitly represents service reliability. Service reliability is represented in the model as penalties associated with passengers' late arrival, early arrival, departure time shifting, waiting time, and anxiety. Furthermore, a methodology of utilizing field data to capture service reliability is defined. A Monte-Carlo simulation framework has been developed using the proposed GC function to quantify the impact of transit reliability on transit user cost.

The proposed framework was applied on the iXpress service in the Regional of Waterloo in Ontario, Canada, utilizing Automated Vehicle Location (AVL) system data from the Regional Municipality of Waterloo to estimate service reliability. All the coefficients included in the proposed GC are assumed based on the relative importance of each penalty to scheduled in vehicle time by considering different passenger classes. In this research, the transit passengers are assumed to belong to one of three passenger classes based on their risk tolerance. From the results, it was found that increasing reliability of arrivals at a station can decrease transit users generalized costs significantly. We further posit that including uncertainty in the calculation of generalized costs may provide better estimates for mode split in travel forecasting models.

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Dedication

This is dedicated to the the light of my life my lovely wife Reham and my angels Omar, Ahmad, and the new expected baby.

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Glossary

AAT	The actual time that a transit unit arrives at a station
AD	Arrival delay: the difference between the passenger necessary arrival time at destination station and the transit unit actual arrival time at the station.
ADT	The actual time that a transit unit departs from a station
AVL	Automatic vehicle location: it is a system used to track vehicle movements and stops and store the spatial and temporal data
CV	Coefficient of variation: a statistical measure of the data dispersion (standard deviation/mean)
GC	Generalized cost is the total cost associated with a trip including monetary costs (service fare) and nonmonetary costs (such as waiting time)
h	Transit service headway: the time interval between two subsequent vehicles on a route in the same direction
IVT	In vehicle time: the travel time taken by a TU to travel from origin station to destination station
NAT	The time that a passengers prefers to arrive at his/her destination at.
PsAT	The time that a passenger arrives at his/her boarding station at
RP	Revealed preference survey
SAT	The scheduled time that a transit unit has to arrive at a station at
SD	Schedule delay: the difference between the TU actual arrival time and schedule arrival time or the difference between the TU actual departure time and schedule departure time
SDT	The scheduled time that a transit unit has to depart at from a station
SIVT	Scheduled in-vehicle time: the scheduled travel time between
SP	Stated preference survey
TTV	Travel time variability:
TU	Transit Unit: a set of n vehicles traveling physically as a one unit [55]
WT	Waiting time: the difference between passenger arrival time at origin station and the transit unit departure time from the station

Chapter 1

Introduction

1.1 Introduction

Travel demand is increasing with the continued growth in population and the economy. This leads to additional vehicles, trips, and traffic congestion on the road network. Therefore, the importance of encouraging travelers to shift to other mobility modes such as public transit is growing. Public transit is an essential component of a transportation system that can alleviate congestion. It offers mobility and accessibility for people to perform their daily tasks: work, school and recreational activities etc. Hence, demand for more efficient and reliable public transit services is rising. Service reliability has been recognized as one of the most important service attributes by transit users and providers. From a service provider perspective, service reliability is one of the important and determining indications of service performance. User perceive service reliability as the uncertainty associated with waiting time, travel time, and arrival time for a given trip.

While improving service reliability attracts more customers, unreliable service can lead to a reduction in revenue due to passengers who had unpleasant service experiences shifting to other modes. Generally, transit users are restricted by timing constraints at their destination (e.g. work, school, or medical appointments.) Therefore, they can be affected by the consequences associated with arriving late at their destination (Bates et al. 2001). The trip maker's decision in transport mode choice and departing time is influenced by the degree of service reliability and the trip maker's willingness to experience delays.

The choice of transport mode prediction (mode choice) is considered as one of the most important models in transport planning process [35]. Mode choice models used to estimate the portion of trips over each of the available transport modes between different origin-destination (OD) pairs and can be applied at two different levels: zonal (aggregate) and household (disaggregate) levels. Although Several models have been developed, the most widely used is the multinomial logit model [27](Equation 1.1.

$$P_{ti} = \frac{e^{U_i}}{\sum_{i=1}^{n} e^{U_i}} \tag{1.1}$$

where

P_{ti}	Probability of trip maker t choosing mode i
U_i	The utility function associated with using the transportation mode i
n	The available transport modes

From the previous equation, it can be observed that each mode (i) has a specific cost and benefits that represented by the utility function of this mode and can be evaluated by itself. However, the choice of a each mode is based on its utility or cost relative to the utility or cost of all available modes. As the number of transport modes increases, the probability of choosing each mode decreases with no changing on its utility or cost.

The utility function of each transport mode is impacted by several factors such as trip maker socio-economic factors (income, cars ownership, age, etc.) and mode attributes factors (in-vehicle time, access time, out of pocket cost, etc.). Generally, the utility function has a linear form combining all variables considered as mode attributes and individual characteristics [35](see Equation 1.2).

$$U_i = \beta_0 + \beta_1 X_1 + \beta_2 X_2 + ... + \beta_a X_a + \alpha_0 + \alpha_1 Y_1 + \alpha_2 Y_2 + ... + \alpha_b Y_b \tag{1.2}$$

where

U_i	The utility function associated with using the transportation mode i
X_1, X_2, X_a	Mode attributes
Y_1, Y_2, Y_b	Individual's characteristics
$\beta_1, \beta_2, \beta_b$	Parameters for mode attributes
$\alpha_1, \alpha_2, \alpha_a$	Parameters for individual's characteristics

Variables included in the utility function have different units. Therefore in order to estimate the relative importance of each variable included in the utility function to others, all the variables are converted to a common unit. This conversion process results in a linear function of the summation of all converted variables called generalized cost (GC) function [23]. This generalized cost function replaces the utility function in Equation 1.2.

As a consequent, it can be stated that the likelihood of a trip maker using a particular transport mode (e.g. transit) is based on the generalized cost (GC) of using that mode relative to the generalized cost of all other available modes. Typically, GC functions include in-vehicle time, out of vehicle time, out of pocket cost, transfer penalties, and a mode-specific constant (bias). The mode-specific constant is supposed to represent the factors that are difficult to quantify such as comfort of ride, reliability, etc. Figure 1.1 demonstrates the out of vehicle time and in vehicle time through a conceptual deterministic trip by a passenger from an origin to a destination.

Although some average measures of reliability may be captured within the mode specific constant, reliability is not explicitly represented in most mode choice models. Therefore, considering reliability within GC functions will help to accurately predict transit ridership and to quantify the impact of transit service reliability on both passengers' GC and mode choice forecasting.

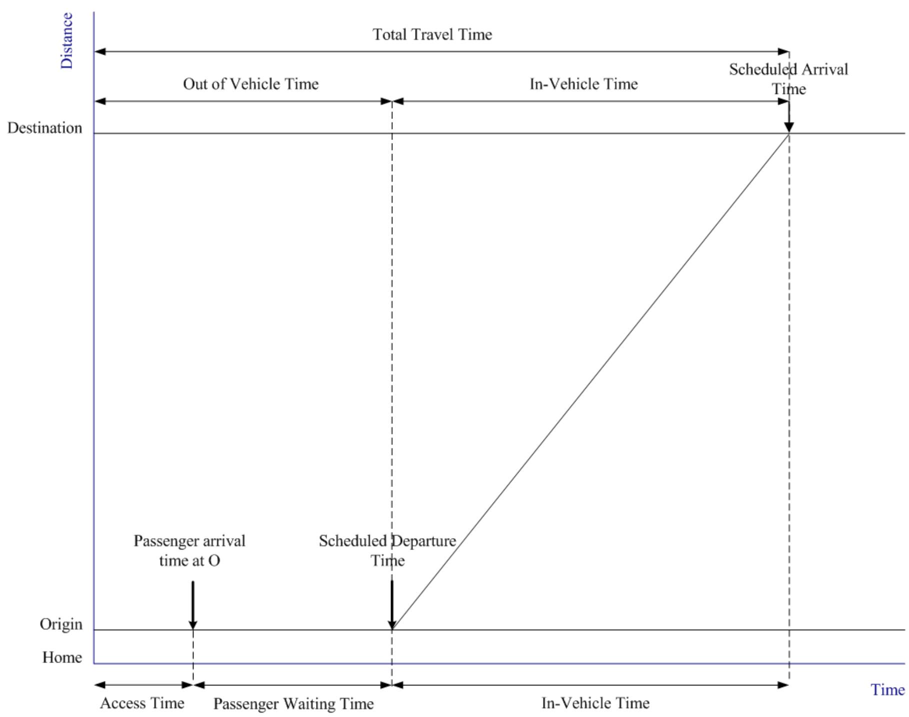


Figure 1.1: A Space - Time diagram for a Conceptual Trip

1.2 Motivation

Transit operators strive to provide a reliable service to retain and attract more users. Unreliable service can adversely affect users by arriving late or early at their destination, waiting longer at their boarding station, and spending more time in the transit vehicle than expected. Unreliable service will also increase users' anxiety associated with the uncertainty and discomfort (Bates et al. 2001). All these factors should be considered explicitly within the GC function in order to accurately capture the GC of transit service relative to other modes.

While consideration of service reliability in GC calculation is important, the challenge is how to quantify reliability. Reliability is commonly represented by schedule adherence, variation in travel time, and variation in headway. All this information can be obtained through archived automatic vehicle location system (AVLS) data. AVLS has been used as one of the intelligent transportation systems to monitor and improve the service reliability of many transit operators. AVLS can provide a large amount of data that can be effectively analyzed to improve reliability.

Recently, studying the effects of travel time reliability (TTR) on travelers' behavior has been given considerable attention. Though some studies consider reliability in a mode choice model, most concentrate on its impacts on route choice and departure time choice. In addition, most of the results are based on estimations using either stated preference (SP) or revealed preference (RP) data analysis (Hollander 2005).

In this study, a GC model is developed that explicitly represents service reliability. Service reliability is manifest in the model as penalties associated with passengers' late arrival, early arrival, departure time shifting, and waiting time. Furthermore, a methodology of utilizing AVLS data to capture service reliability is defined. By considering different passenger behavior, the impact of service reliability on passenger GC is quantified using a Monte Carlo simulation model that considers passenger arrival time, bus departure time from origin, passenger preferred arrival time, and actual arrival time at destination.

1.3 Goals and objectives

The main goal of this study is to develop a methodology that explicitly incorporates transit service reliability within the calculation of GC and quantifies the impact that service reliability has on user GC.

This thesis has the following objectives:

1. Use AVLS data to quantify the degree of transit service reliability in terms of travel time and schedule adherence;
2. Develop a comprehensive GC function that explicitly considers service reliability;
3. Assess the impact that an unreliable system has on the GC of transit users' who have different risk tolerance levels;
4. Assess the effects of an unreliable system on passengers' behavior in choosing their departure time and how that choice impacts their GC.

All of these objectives should help service providers and planners quantify the impact of reliability on passengers' behavior. Subsequently, they can enhance the service and predict more accurately passengers' GC and mode split.

1.4 Thesis Organization

Chapter 2 reviews previous research related to reliability within the generalized cost and utility functions. Chapter 3 presents the proposed GC formulation; discusses the method used to evaluate the proposed formulation and the analysis needed to calibrate model inputs to archived AVLS data. Chapter 4 describes the study area and case study. Chapter 5 demonstrates the results obtained from the simulation model. Chapter 6 summarizes the conclusions of this study, discusses the study limitations, and identifies potential future extensions to the work.

Chapter 2

Literature Review

The previous studies on incorporating service reliability effects on passengers' cost are reviewed in this chapter. The chapter, consisting of three main sections, is organized as follows. First, section 1 introduces the concept of service reliability and its effects on passengers' behavior. Section 2 reviews the work that has been done to incorporate reliability into the calculation of the GC model. Finally, section 3 summarizes the limitations found in these models and identifies the need for further research.

2.1 Transit Service Reliability Concept

Transit service reliability is an essential service attribute and is a concern for both transit agencies and users. In a transit service context, reliability has been defined differently by different researchers. Polus [40] and Abkowitz [1] defined service reliability as the consistent and invariable performance of the transportation system over a period of time. Turnquist and Blume [53] defined reliability as the ability of the transportation system to adhere to schedule, maintain a regular headway and provide a consistent travel time. Kimpel [20] agreed with Turnquist and Blume when he defined reliability as the departure delay, running time variation, and headway delay regarding to schedule. While these definitions commonly describe service reliability, they indicate different perceptions of reliability by transit providers and passengers.

Transit passengers consider service reliability as one of the important factors for service quality [37], [49], [50]. From the passenger's perspective, reliability is commonly measured by the delay within the service schedule [1]. Lack of reliability adversely affects transit passengers because of the uncertainty associated with 1) transit unit departure delay, which results in additional waiting time, and 2) in-vehicle time, which results in arrival delay ([4], [2]).

As a consequence of service unreliability, the passenger's anxiety and discomfort will be aggravated [2]. Therefore, the attractiveness of transit relative to other modes will decline that in turn will lead to a decrease in transit ridership. A major weakness of existing GC formulations is the lack of explicit consideration of service reliability.

2.1.1 Service Reliability and Passenger's Waiting Time

Waiting time is defined as the difference between the passenger arrival time and the transit unit departure time at a station. The expected waiting time depends on the distribution of both the user arrival time and transit unit departure time. It was found that transit users value the waiting time twice as much as the in-vehicle time [54]. Hence, waiting time is an important service attribute that can reduce the attractiveness of transit service.

For frequent service (i.e. short service headway), the expected waiting time is commonly used in planning application as shown in Equation 2.1.

$$E[WT] = 0.5 * h \tag{2.1}$$

where

$E[WT]$	Average waiting time
h	Scheduled headway

In contrast, for infrequent service, the average waiting time for users is less than half of the headway. However, for an unreliable service with long headway, the average waiting time is greater than half of the headway. This occurs because:

1. Passengers tend to arrive earlier to account for the possibility that the TU will depart earlier than scheduled departure time; and

2. When the TU's departure is delayed and the headway is large, the waiting time increases.

[1]On-time performance: how closely the actual performance is to the schedule

Osuna and Newell [36] derived the the expected waiting time under the assumption of random arrival time for passengers as shown in Equation 2.2

$$E[WT] = 0.5E[h](1 + CV_h^2) \tag{2.2}$$

where

$E[WT]$	Average waiting time
$E[h]$	Average service headway
CV_h	Coefficient of variation in headway (standard deviation/mean)

Although Equation 2.2 relates service reliability to users' average waiting time, the average waiting time is not the best indicator of a passenger waiting cost. Additionally, the passengers may not arrive randomly as was assumed.

Many researchers have assumed that passengers are arriving randomly and independently of the service schedule at at the station. However, it has been suggested by some researchers [53], and we concur, that users (under some conditions) tend to minimize their waiting time by consulting schedules and arriving at a station at a selected time prior to the TU scheduled departure time.

This relationship between waiting time and unreliability was modeled by Bowman and Turnquist [4]. They focused on the effect of schedule deviation in bus arrival time at a particular station on passengers' utility function which acts as passenger arrival probability at the same station. Bowman and Turnquist suggested that with a highly reliable station, the peak of passenger arrival rate is obviously observed before the transit unit scheduled departure time by a short period (with a 20-min headway, the peak was at 2.4 min) prior to scheduled departure time. As the reliability declines, the probability of missing the bus increases; subsequently, the peak arrival rate shifts earlier from the scheduled departure time and spreads over the headway.

Figure 2.1[2] illustrates the calibrated model by Bowman and Trunquist and the observed passenger arrival time for a 20-min headway service. They suggested that with the random arrival model, the average waiting time is overestimated. On the other hand, their developed model (shown in Equation 2.3) represents the observed average waiting time much better.

$$E[WT(t)] = [1 - P(t)] * WT(t) + P(t) * WT'(t) \tag{2.3}$$

[2]The source of this figure is reference [53]

where

$E[WT(t)]$	Expected waiting time for an arrival at t
$P(t)$	Probability of TU arrives prior to time t
$WT(t)$	Expected waiting time given the TU arrives after time t
$WT'(t)$	Expected waiting time given the TU arrives before time t

However, the authors assume that all users have the same risk tolerance on perceiving the waiting time. In addition, they have considered that all passengers are aware of the service schedule.

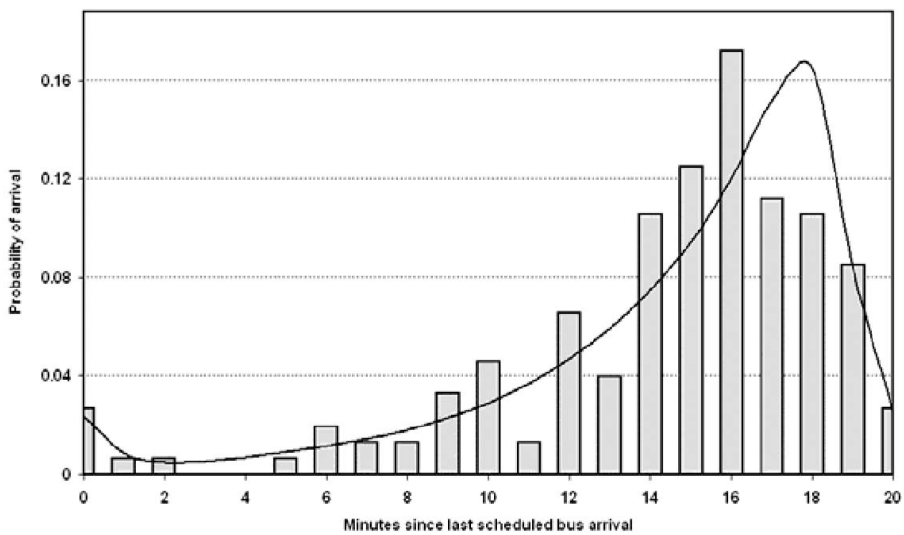


Figure 2.1: Passenger Arrival Distribution for a 20 min Headway Service

2.1.2 Service Reliability and Travel Time

In-vehicle time is defined as the time needed for a transit unit to travel from a user's boarding station to the alighting station. For transit operations in shared rights-of-way, in-vehicle times are subject to variability caused by many factors: traffic, route, passengers, and transit operational characteristics [25]. Those factors also include day-to-day travel demand variation, traffic congestion variation, capacity of the road, signalized intersections, passenger volume at stop, etc.

Increased total travel time variability (TTV), including both wait time and in-vehicle time, diminishes the users' ability to predict accurately their arrival time. For some passengers, an arrival time either earlier or later than their desired time is undesirable consequences. Thus, studies have shown that passengers place a higher value on TTV than on the in-vehicle time ([30], [18], [2], [39], [41]).

Overall, travel time variability represents the key to service reliability. Variations in departure time and in arrival time impact passengers' waiting time and in-vehicle time, respectively.

2.2 Service Reliability in Discrete Choice Model

Generally, the uncertainty associated with travel time is considered as an extra cost for the user. However, this cost is not explicitly captured in most of the mode choice models. This may lead to the overestimation or underestimation of the model-predicted number of transit passengers.

For several years, many researchers have been studying the effect of service reliability on passengers' behavior. Gaver [16] was the first to include the service reliability concept within the GC function. He suggested that passengers will minimize the probability of arriving late at their destination by changing their departure time. The work of Gaver and others can be classified into two different approaches namely: The Mean-Variance Approach and The Scheduling Approach.

2.2.1 Mean-Variance Approach

The mean variance approach posits that the travel time variability has a direct effect on the passenger as a source of inconvenience. In this approach, the travel time variability and mean travel time are explicitly included within the GC function. It assumes that the passenger has prior experience with the service for a specific trip (between a specific pair of stations). Thus, passengers estimate the mean and the variance (or standard deviation) of the travel time for their trip.

In the field of transportation, the first model using this approach was specified by Jackson and Jucker [19] where the user explicitly considers both the travel time variability and mean travel time. In their model, the user's objective is to maximize the following utility function shown in Equation 2.4:

$$U = \alpha E[tt] + \tau V(tt) + \delta C \tag{2.4}$$

where

$E[tt]$	Expected travel time between specific pair stations
$V(tt)$	Variance of travel time
C	The cost associated with using the transportation mode
α, τ, δ	The parameters that measures the influence of each variable

In a stated preferences survey conducted by Jackson and Jucker, transit passengers were asked to choose between two alternatives they had been given as paired comparison questions. The parameter τ was used as a surrogate for the passengers'level of risk aversion.

The mean variance approach has been effectively used in problems of portfolio selection in financial markets. In the transportation field, it has been widely implemented to evaluate the travel time variability on the mode choice model [18]. However, in the presence of scheduling constraints, this approach cannot capture the behavior of departure time choice due to the lack of arrival delay consideration within the model.

2.2.2 Scheduling Approach

The scheduling approach is based on capturing the cost associated with arriving early or late at a destination relative to the passenger's necessary arrival time and allowing passengers to optimize their departure time to minimize their own cost [18]. That represents the concept of the scheduling approach.

Gaver [16] first introduced the idea of the TTV effect on the departure time choice behavior. Gaver [16], Knight[21], and Pells [38] claim that the travel time variability effect can be captured through the safety margin (slack time) added by users as a reaction to travel time variability associated with the trip. They assume that the trip maker selects a safety margin seeking an optimal trade-off between early and late arrival penalties that are included in the GC function.

Pells defined the safety margin as the difference between the mean arrival time and the necessary arrival time at destination such as the work start time [38]. He calibrated two different choice models by conducting a stated preference survey. The first model was used to value the slack time and the second was used to value late arrival time. He found that for trips to work, the cost of TTV is correlated to the slack time added to the trip as a safety margin. In addition, he observed that, as the TTV decreases, the slack time allowed by the trip maker also decreased.

Unlike Pells, Polak [39] defined the safety margin as the difference between the scheduled travel time and the expected travel time estimated by the trip maker. He assumed that the passenger has a historical knowledge of expected travel time. If the expected travel time is equal to the scheduled travel time, the safety margin will be zero.

Cosslett and McFadden [27] empirically studied the tradeoff between mean travel time, arrival delay, and the probability of arriving late for the automobile users going to work. A sample of data from the Urban Travel Demand Forecasting Project(UTDFP)[3] was used. The probability of being late was treated explicitly within the following utility function:

$$U(T) = -\alpha IVT(T) - \beta ADE(T) - \gamma P_L(T) \tag{2.5}$$

where

$U(T)$	The utility associated with a passenger's arrival time T
$IVT(T)$	In-vehicle time in minutes corresponding to arrival time T
$ADE(T)$	Arrival delay early time at work in minutes when $T <$ the official work time
$P_L(T)$	The probability of arriving late if the planned arrival time is T
α	The cost per minute of travel time
β	The cost per minute of early arrival
γ	The cost of probability of late arrival

The passenger's objective was to maximize the utility function. The probability of arriving late was estimated by a normal random variable with mean $IVT(T)$ and standard deviation $\sigma = a[IVT(T) - IVT(0)]$, where $IVT(0)$ is the off-peak travel time and a is a constant.

This study was one of the first studies to model passenger behavior as a function of TTV. The main contribution of this study is to model the behavior of the auto drivers for journey to work trips by expressing their trade-off between arriving earlier than their necessary time, the additional travel time, and the probability of being late.

Cosselett and McFadden concentrated on the difference of the trip maker sensitivity to the penalty of arriving early and arriving late. However, the method of

[3] This is a sub-sample of the data compiled by the Urban Travel Demand forecasting Project conducted by the Institute of Transportation Studies, University of California-Berkeley under the supervision of McFadden

estimating the variable represents the lateness penalty, which associated with the TTV, is inconsistent with the method of estimating the earliness penalty in terms. In addition, through the developed model, it was implied that the departure time decision is fully represented within the model by not including a constant within the model [1].

Based on earlier work by Gaver [16] and Cosselett [27], Small [44] developed a discrete choice model based on the premise that the passenger will experience some disutility due to arrival delay either with early or late arrival. He defined the arrival delay (AD) as the deviation from the passenger necessary arrival time (NAT) in minutes rounded to the nearest five minutes. Small's estimated GC function is

$$GC(th) = \alpha IVT + \beta ADE + \gamma ADL + \theta D_L \tag{2.6}$$

where

$GC(th)$	The generalized cost associated with a passenger's departure time from home th
th	Passenger's departure time from home
IVT	In-vehicle time in minutes
ADE	Arrival delay early time in minutes defined as MAX{-AD,0}
ADL	Arrival delay late time in minutes defined as MAX{AD,0}
D_L	Dummy variable equal to 1 if $ADL \neq 0$ and equal to 0 otherwise
θ	The additional discrete penalty due to late arrival
α	The cost per minute of travel time
β	The cost per minute of early arrival
γ	The cost per minute of late arrival

Small claims that the time before the passenger's necessary arrival time (NAT) is considered less onerous compared to the time after NAT, which means people prefer to arrive early rather than arrive late. Thus, in addition to the discrete lateness penalty, the linear function associated with the penalty of arriving late is steeper than the one associated with early arrival. Moreover, people prefer grater travel time rather than arriving late. Consequently, Small measured the coefficients based on $\beta > \alpha > \gamma$. Figure 2.2 illustrates the arrival delay disutility functions provided by Small.

As observed, unlike Cosslett, Small's utility function has consistent definitions for both early and late arrival penalties. However, the TTV effect is not captured

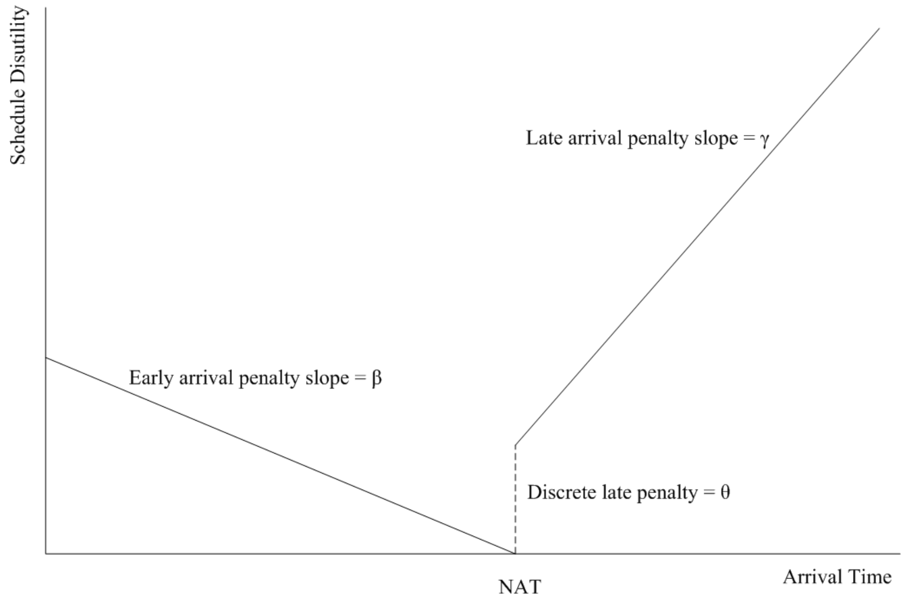


Figure 2.2: Small's Formulation of Arrival Delay Disutility

within the developed model. In addition, it was assumed that the passenger can be certain to arrive at destination at NAT.

Based on the work of Gaver [16] and Small [44], Noland and Small [32]extended the model of scheduling choice. They explicitly considered TTV within the GC function model. Their model is as follow:

$$E[GC] = \alpha E(IVT) + \beta E(ADE) + \gamma E(ADL) + \theta P_L \tag{2.7}$$

where

$E[GC]$	Expected generalized cost
$E[IVT]$	Expected In-vehicle time in minutes
$E[ADE]$	Expected arrival delay early time in minutes defined as MAX{-AD,0}
$E[ADL]$	Expected arrival delay late time in minutes defined as MAX{AD,0}
P_L	The probability of being late
θ	The additional discrete penalty due to late arrival
α	The cost per minute of travel time
β	The cost per minute of early arrival
γ	The cost per minute of late arrival

All the terms included in the model are based on the distribution of the travel time. They introduced IVT as the summation of three components:

1. Free Flow travel time (T_f) between home and work and
2. Extra travel time due to recurrent congestion ($T_x(t_h)$) that is a function of departure time from home.
3. Extra travel time due to non-recurrent congestion ($T_r(t_h)$) that is a function of departure time at home.

They assume that T_x can be modeled as the safety margin the passengers allow to avoid arriving late at their destination and T_r is modeled as a random variable that is independent of recurrent congestion and departure time. They have evaluated the model using two different distributions for T_r: an exponential and a uniform distribution. They have derived the optimal expected costs by replacing the term T_r by the mean of T_r to indicate the cost associated with travel time variability (TTV) within the model as shown in Equation 2.8.

$$E[GC]^* = \alpha(T_f + T_x + b) + \theta P_L^* + b\left\{\beta \ln\left[\frac{\theta + b(\beta+\gamma)}{b(\beta - \alpha\Delta)}\right] - \frac{\theta(\beta - \alpha\Delta)}{\theta + b(\beta+\gamma)} - \alpha\Delta\right\} \tag{2.8}$$

In this case, the probability of being late, P_L^*, is estimated by

$$P_L^* = \frac{b(\beta - \alpha\Delta)}{\theta + b(\beta+\gamma)} \tag{2.9}$$

where

$E[GC]^*$	Expected generalized cost
b	Mean of T_r
Δ	The change in the recurrent congestion profile corresponding to the departure time (increases as the departure time delays)
P_L^*	The optimal probability of being late

Noland and Small compared the result of $E[GC]^*$ generated by the two assumed T_r distributions. It was found that the results are not significantly different; the largest difference was \$0.73 when AD=30. In addition, the cost associated with schedule delay, when a uniform distribution was assumed, had no significant effect on the $E[GC]^*$. However, the proportion when exponential distribution was assumed, consistently represented almost a half of $E[GC]^*$, 46-48%. In both distributions, as the standard deviation increased, the proportion of cost of the probability of being late,P_L^*, within the expected cost function decreased due to the greater safety margin that was allowed by the trip maker to avoid late arrival.

Noland et al [33] assumed that the TTV itself can be a source of inconvenience due to the inability of the trip maker to plan accurately. Therefore, based on the model developed by Noland and Small shown in Equation 2.7, Noland et al [33] extended the scheduling model by adding a new term called Planning Cost, C_P. Therefore, the total expected cost is given as Equation 2.10, the sum of expected cost due to arrival delay, $E[GC]_{AD}$ expressed in Equation 2.7 as $E[GC]$ and C_P.

$$E[GC] = E[GC]_{AD} + C_P \tag{2.10}$$

They assume that C_P is a function of the standard deviation of the extra travel time due to non-recurrent congestion, $\sigma(T_r)$ with δ as a coefficient. Hence, the model is given as follows:

$$E[GC] = \alpha E(IVT) + \beta E(ADE) + \gamma E(ADL) + \theta P_L + \delta f(\sigma) \tag{2.11}$$

The model in Equation 2.12 implies the consideration of both approaches. Regarding C_P, Noland et al evaluated the model by measuring TTV in two different ways. First, they included the travel time standard deviation within the model and found δ was statistically significant, but with an illogical sign (positive.) Second, they included the travel time coefficient of variation (standard deviation/mean) instead of the standard deviation and found that although the sign was logical, the value of δ was not statistically significant. In addition, the lateness probability term, PL, was consistently significant over all the evaluated models. Therefore, when the model is considering the arrival delay penalties, there is no need to consider a separate measure of TTV such as travel time standard deviation. The source of inconvenience and stress due to TTV is effectively represented by the late arrival and adherence to strict schedules of the trip maker.

Through several hypothetical simulations, Noland et al [33] analyzed the effect of TTV on trip maker cost and behavior. They found that as the probability of non-recurrent congestion decreases, the cost associated with early arrival increases, β. Moreover, as the TTV increases, the cost associated with lateness probability, θ, increases as well as the cost of expected travel time, α. In contrast, the planning cost, C_P, has a small impact on the total cost by the negligible variation over different nonrecurrent congestion probabilities. These findings suggest the importance of scheduling delay measures in the utility function [31].

Based on the model developed by Noland et al in Equation 2.12, Small et al have modified the model in two ways, found to be essential distinctions to the model [45]. The first is suggesting that the GC does not vary linearly with the arrival delay early (ADE) term. They suggested a quadratic relationship to be included within the model; thus, the term $E(ADE)^2$ was added as a new variable. The second is to add another term which represents the probability of extra late arrival,P_{XL}, under the condition of reported flexible arrival time at work. This term is referring to the probability of exceeding a time point set by the trip maker as the latest possible arrival time at work before facing serious consequences of being late. The model was given as follows:

$$E[GC] = \alpha E(IVT) + \beta E(ADE) + \gamma E(ADL) + \theta_1 P_L + \delta f(\sigma) + \beta_2 E(ADE)^2 + \theta_2 P_{XL} \tag{2.12}$$

The coefficients of schedule delay in the new model followed the expected effect on trip maker, $\beta > \alpha > \gamma$ with positive impact on the total expected generalized cost ($E[GC]$). With regards to the new variables, Small et al found both of them are statistically significant. The coefficient of the expected arrival delay early ($E(ADE)$) has a small negative magnitude and as well as the coefficient $E(ADE)^2$, but with a positive sign. The model implies that the passengers will be pleased by arriving a few minutes early (i.e. 3minutes earlier than theirNAT). However, as long as the arrival time increasingly deviates from their NAT,the GC increases. Like Noland et al [33], Small et al found that the variable measuring the standard deviation of travel time has a negligible contribution on total expected generalized cost ($E[GC]$). In addition, it is not statistically significant and has an illogical positive sign.

Bates et al [2] and Noland and Polak [31] show that the mean-variance and scheduling approach are equivalent under some certain assumptions:

1. Travel time variability has an exponential distribution with parameter b,
2. There is no disutility (cost) associated with arriving late (i.e.$\theta = 0$), and
3. The additional in-vehicle time (IVT) resulting from recurrent congestion travel time (T_r) is the same for all departure times.

When these three assumptions are made, the GC is a linear function of travel time standard deviation (b) as shown in Equation 2.13 which is similar to the mean-variance approach.

$$E[GC]^* = \alpha\overline{IVT} + b\,\beta \ln\left(1 + \frac{\gamma}{\beta}\right) \tag{2.13}$$

where

$\overline{IVT}$ Average in-vehicle travel time

Under these assumptions, the GC no longer depends on the distribution of T_r. If all the model coefficients, α, β, and γ, are positive, EC^* is a increasing linear function of mean and standard deviation of the total travel time. The three mentioned simplification assumptions are not likely to occur in reality; however, a part of this assumption might occur by some modes [31]. For instance, when the service is running in the dedicated bus lane i.e public transportation, it will not be affected by non-recurring traffic. Thus, $\delta = 0$ which simplifies the model; however, the model is not linear in travel time standard deviation [31].

All the previous reviewed scheduling models were proposed for automobile users and assume continuous departure times. The majority of the studies demonstrate that the scheduling approach is better than the mean-variance approach in reflecting the effect of TTV on passengers' behavior. However, mean-variance models have been used more widely in practice due to their straight foreword implementation.

2.3 Reliability in Public Transportation

In contrast to the auto-based models discussed in the previous section, departure times choices for public transportation passengers are discrete (scheduled departure times). Due to the discreteness of the choice, the optimal departure time is not necessarily equal to the optimal departure time in the continuous case [13].

Among the previous research findings, the majority concur that the travel time variability (TTV) effect can be captured only by the scheduling delay measures within the utility function. However, Bates et al. [2] disagreed with this argument when considering public transportation system passengers. They suggest that passengers may dislike the delay incorporated in the service schedule, to accommodate this, and therefore they considered two new variables in the $E[GC]$ function namely: schedule delay early (SDE) and schedule Delay late (SDL).

$$SD = AAT - SAT \tag{2.14}$$

$$SDE = \begin{cases} SD & If\ SD < 0 \\ 0 & Otherwise \end{cases} \tag{2.15}$$

and

$$SDL = \begin{cases} SD & If\ SD > 0 \\ 0 & Otherwise \end{cases} \tag{2.16}$$

where

SD	Schedule adherence delay
SAT	Schedule arrival time
AAT	Actual arrival time

Therefore, the expected utility function is given as

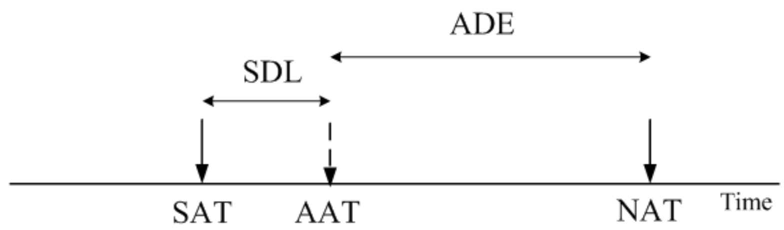
$$E[GC] = \alpha E(IVT) + \beta E(ADE) + \gamma E(ADL) + \theta P_L + \beta_2 E(SDE) + \gamma_2 E(SDL) \tag{2.17}$$

Bates et al [2] have conducted a stated preference survey asking respondents to choose between two services with different characteristics. The result obtained from the model revealed some differences when compared to models used for car users. They found that with the consideration of the mean or the standard deviation of schedule delay the coefficients were significant and have logical signs.

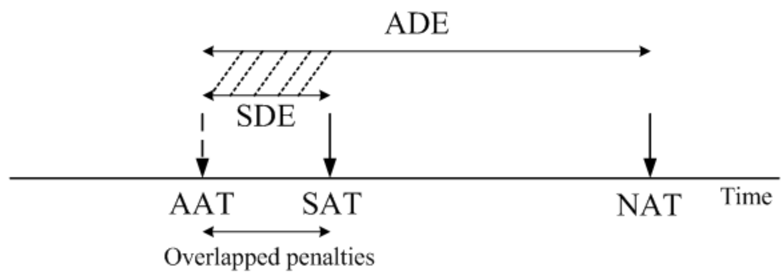
Bates observed that, for rail services, the distribution of travel time was not adequately represented by standard mathematical distribution which led to incorrect interpretation [2]. In addition, they found that all coefficients are statistically significant and have the right sign, which suggests the importance of the modification in the model.

In the case of where actual arrival time (AAT) is later than scheduled arrival time SAT but earlier than a passenger's necessary arrival time (NAT), the expected arrival delay early penalty (ADE) decreases as the AAT moves towards the NAT. On the other hand, the disutility associated with schedule adherence delay due to late arrival (SDL) in regards to scheduled arrival time (SAT) increases regardless of NAT [2] i.e. people dislike being later than expected, even if they are not later than needed. Figure 2.3(a) illustrates the importance of adding the adherence schedule delay term in the model with such a case.

The model developed by Bates et al [2] based on the earlier work of Noland and Small [32] shown in Equation 2.17 is the most adequate and comprehensive model discovered in the review of the literature that can be used to incorporate service



(a) Case1: When SAT < AAT < NAT



(b) Case2: When AAT < SAT < NAT

Figure 2.3: Adherence Schedule Delay Estimation

reliability within the GC function for public transportation. However, there remain several weaknesses associated with this model:

1. In some cases, the arrival delay (AD) and schedule delay (SD) will overlap; in contrast to the previous example, when actual arrival time (AAT) is earlier than the scheduled (SAT) and NAT, the difference between AAT and SAT will be double counted within the arrival delay and schedule delay penalties as shown in Figure 2.3(b),

2. The transit passenger's anxiety associated with deviations from the schedule of inter station travel times while enroute to the destination station is not considered within the model, and

3. The waiting time penalty is not incorporated within the model.

2.4 Summary

This chapter has focused on the work that has been directed at understanding the impact of service reliability on trip makers' behavior and cost. First, it presented the different definitions of service reliability provided by researchers which simply can be defined in the public transit context as transit units (TUs) departing on time, having a reasonable travel time, arriving at scheduled arrival time. It has also addressed the objective and the importance of the explicit consideration of service reliability within the calculation of GC and the utility functions.

Two different approaches have been pursued by the existing work, namely mean-variance approach and scheduling approach. Although the mean variance approach is widely used due to its ease of implementation, most of the authors agree that the scheduling approach is more representative of travel time variability (TTV). A review of existing scheduling models was conducted and discussed. Limitations associated with the reviewed models were identified. Although the significance of all the variables included within the different discussed models, the impact of TTV has not been properly considered through them.

The research proposed herein is aimed to extend the previous generalized cost model of Bates et al. This research strives to develop a methodology that can quantify the impact of service reliability on the individual trip maker. The next chapter outlines the proposed simulation framework using the new extended scheduling model.

In the next chapter, the proposed GC model that incorporates service reliability is introduced in addition to the explanation of research methodology.

Chapter 3

Research Methodology

Passengers' behavior is influenced by transit service travel time variability (TTV), as discussed in Chapter 2. In the public transportation field, trip maker's perception of TTV depends on previous experience with the transit service. TTV is perceived as a disutility of a journey by the trip maker in addition to the disutility associated with travel time itself.

As discussed in the previous chapters, transit service reliability is important for both transit users and service providers. In general, transit service reliability affects the users' behavior and mode choice which consequently has a direct effect on the service provider.

In this chapter, a simulation framework utilizing AVLS data is illustrated and explained in two sections. In first section, a new generalized cost model for public transit users is developed that recognizes a number of the limitations associated with the existing scheduling models discussed in the previous chapter. Section 2 outlines the simulation model used to quantify the impact of transit reliability on transit user cost.

3.1 Proposed Generalized Cost Model

As discussed in Chapter 2, the existing scheduling models reviewed have not properly captured the TTV impact within the generalized cost and utility functions. In addition, most of these models were developed to be used for car users with only a few that considered the specific nature of the public transport. In this section, based on the seminal work of Small [44], Noland and Small [32], and Bates [2], the GC model is extended to consider the three limitations discussed in Chapter two:

1. Incorporating the waiting time penalty within the model;
2. Incorporating the schedule delay penalty in an alternative form;
3. Incorporating the transit passenger's anxiety associated with deviations from the schedule of inter station travel times while enroute.

The proposed generalized cost function considering reliability (GC_R) of transit service is as follows

$$GC_R = \alpha_1 WT + \alpha_2 SIVT + \alpha_3 IVTD + \alpha_4 ADE + \alpha_5 ADL + \theta\delta + \alpha_6 ANX + \alpha_7 SD \tag{3.1}$$

where

GC_R	The generalized cost considering reliability (minutes)
WT	Passenger waiting time at boarding station (minutes)
$SIVT$	The scheduled in-vehicle time between O-D stations (minutes)
$IVTD$	The in-vehicle time delay (minutes)
ADE	Arrival delay early time (minutes)
ADL	Arrival delay late time (minutes)
θ	The additional discrete penalty due to late arrival
δ	Late dummy variable equals 1 when $ADL > 0$ and 0 otherwise
ANX	Anxiety penalty of a transit passenger during the trip
$SD_{0.9}$	Schedule adherence delay penalty (minutes)
α_i	The relative importance of each variable

In subsequent sections, each of the variables associated with service reliability included in the proposed GC_R model is discussed in detail.

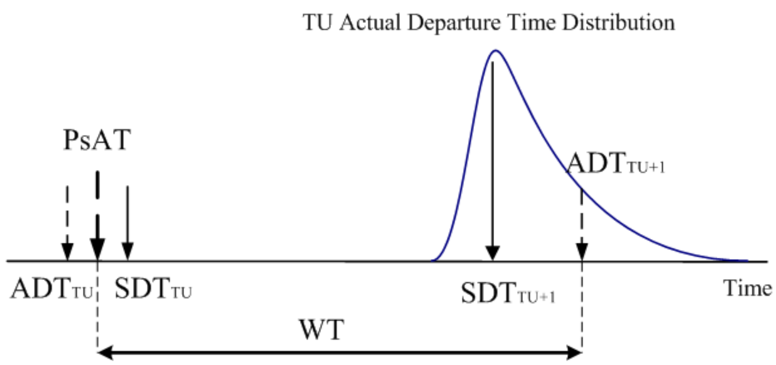
3.1.1 Waiting Time Penalty

The scheduling model developed by Bates et al [2] considered the impact of service reliability on the arrival time by assuming that the service is reliable at boarding stations (meaning, the TU always departs on time). In reality, when transit headway is long and service is reliable, most transit passengers arrive at their origin stations shortly prior to the scheduled departure time (SDT) to minimize their waiting time (WT) [53]. Naturally, this assumes that passengers are aware of the service reliability.

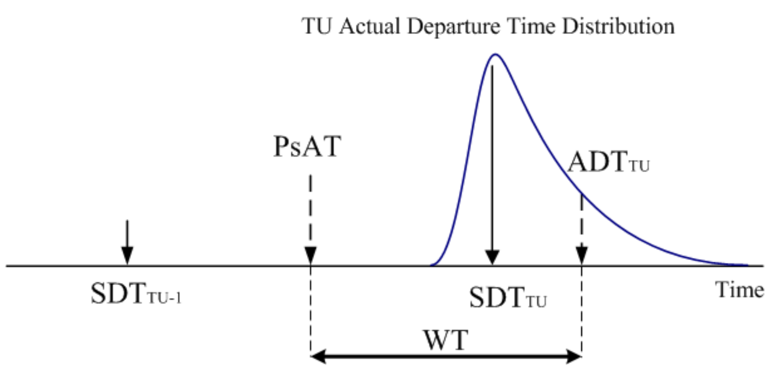
In the light of unreliability at boarding stations, individual waiting time is explicitly treated as an additional term within the GC_R model. At the individual transit passenger level, WT is calculated as the difference between TU actual departure time, ADT, and passenger arrival time, PsAT, at a particular station as expressed in Equation 3.2.

$$WT = ADT - PsAT \tag{3.2}$$

In an unreliable transit system, schedule adherence is typically poor and TUs depart later (or less commonly earlier) than scheduled departure time (SDT). When the TU departs earlier than SDT, the passenger waiting time (WT) is reduced unless the TU departs before the passenger arrival time ($PsAT$), in this case dramatically increased (Figure 3.1(a)). When the TU departs later than SDT, WT also increased (Figure 3.1(b)).



(a) Case1: PsAT > ADT



(b) Case2: PsAT < ADT

Figure 3.1: Passenger Waiting Time at Boarding Station

Table 3.1: The three cases that WT Penalty Varies According to them

ADT	PsAT	WT Penalty
$< SDT$	$< ADT$	Low
$> SDT$	$< ADT$	High
$< SDT$	$> ADT$	Very High

3.1.2 In-vehicle Time

The in-vehicle time IVT term in the proposed model was disaggregated into two components: the scheduled in-vehicle travel time ($SIVT$) and the in-vehicle travel time delay ($IVTD$). The $SIVT$ is the service expected travel time between O-D stations pair. The $IVTD$ term may take on either positive or negative values depending on the variation in the travel time relative to the the scheduled travel time (STT).

$$IVTD = ATT - STT \tag{3.3}$$

where

$IVTD$	In-vehicle travel time delay (minutes)
ATT	Actual travel time for the trip (minutes)
STT	Scheduled travel time for the trip (minutes)

3.1.3 Arrival Delay Penalty and Discrete Lateness Penalty

Transit users face the discreteness of the service departure time. That means passengers have to comply with the service schedule by adjusting their departure time in order to arrive at their destination prior to their necessary arrival time (NAT). It is assumed that NAT is the latest time travelers can arrive at their destination without being late. Based on the approach introduced by Small [44], passengers will incur extra cost associated by arriving at their destination earlier or later than their NAT. For early arrival, the cost decreases as actual arrival time (AAT) approaches passenger's necessary arrival time (NAT). On the other hand, if actual arrival time (AAT) is after passenger's necessary arrival time (NAT), the cost increases as a function of arrival delay (AD) in addition to a discrete cost of not arriving on time as shown in Figure 3.2. In general, early arrival and late arrival are valued by users depending on the purpose of their trip. Regarding a passenger's necessary arrival time (NAT), arrival delay (AD) is expressed as follow:

$$AD = AAT - NAT \tag{3.4}$$

$$ADE = \begin{cases} -AD & If\ AD < 0 \\ 0 & Otherwise \end{cases} \tag{3.5}$$

and

$$ADL = \begin{cases} AD & If\ AD > 0 \\ 0 & Otherwise \end{cases} \tag{3.6}$$

where

AD	Arrival delay in minutes
ADE	Arrival delay early in minutes
ADL	Arrival arrival delay late in minutes

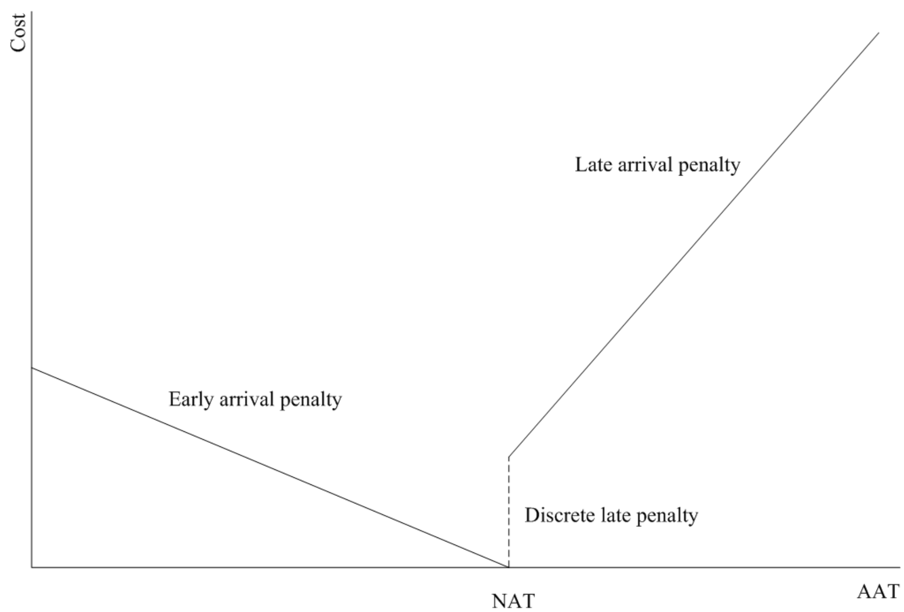


Figure 3.2: Small's Formulation of Early and Late Arrival Penalties

In the case of arriving later than passenger's NAT ($ADL > 0$), the late dummy variable will be equal to zero and therefore an additional discrete penalty (θ) is added to the late arrival penalty.

3.1.4 Anxiety Penalty

After transit passengers board the TU, they may experience some anxiety associated with the possibility of arriving late at their destination. The level of anxiety experienced by passengers during the trip varies along the route according to on time performance of the TU until they arrive at their destination station. Passengers evaluate the service performance at boarding stations and at each intermediate station on the route by estimating the probability of arriving late at their destination (i.e. estimate the probability of arriving later than their NAT). Mathematically, in this research, it is proposed that anxiety can be expressed as the probability of $ADT_O + ATT_{OD} > NAT$ multiplied by the time that this probability of being late is experienced, which is ATT between O-D pair or ATT between the stop stations in route during the trip, as shown in Equation 3.7.

$$ANX_{(i,i+1)} = P_{Late_{(i)}} \times ATT_{(i,i+1)} \tag{3.7}$$

$$P_{Late_{(i)}} = \int_{NAT-SDT_i}^{\infty} f(ATT_{i-i_D})\, dT. \tag{3.8}$$

where

i	Station number
i_D	Destination station
$ANX_{(i,i+1)}$	Passenger's Anxiety between station i and and the following station, $i+1$
$P_{Late_{(i)}}$	Passenger's estimation at station i of probability of arriving late at D
$ATT_{(i,i+1)}$	Actual travel time between station i and the following station, $i+1$
$f(ATT_{i-i_D})$	Actual travel time probability distribution function between station i and destination station, iD

Graphically, the probability of being late is shown in Figure 3.3 as the hatched area under the ATT_{OD} distribution curve.

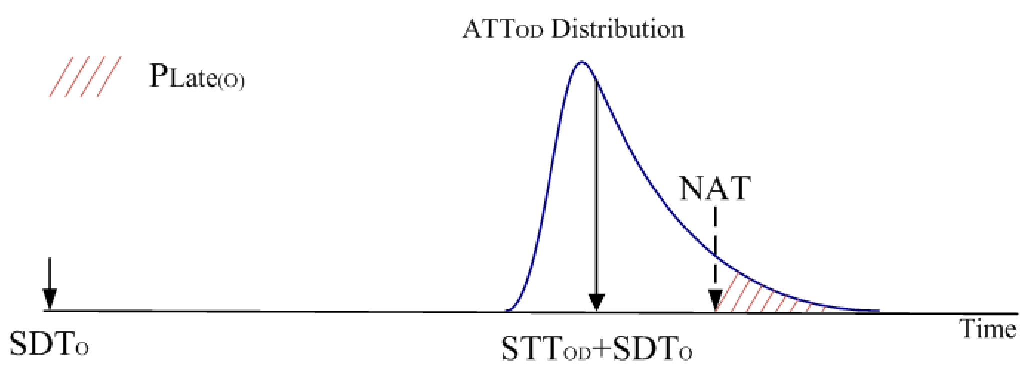


Figure 3.3: The Probability of Being Late depends On TT Distribution

It can be observed that the passengers' anxiety is significantly affected by waiting time and the distribution of travel times. When the TU departs the boarding station late, the probability of arriving late at the destination station increases and passenger anxiety also increases. Anxiety is also impacted by the enroute intermediate stations travel time experienced by TU as this influencing the probability of arriving late at the destination station.

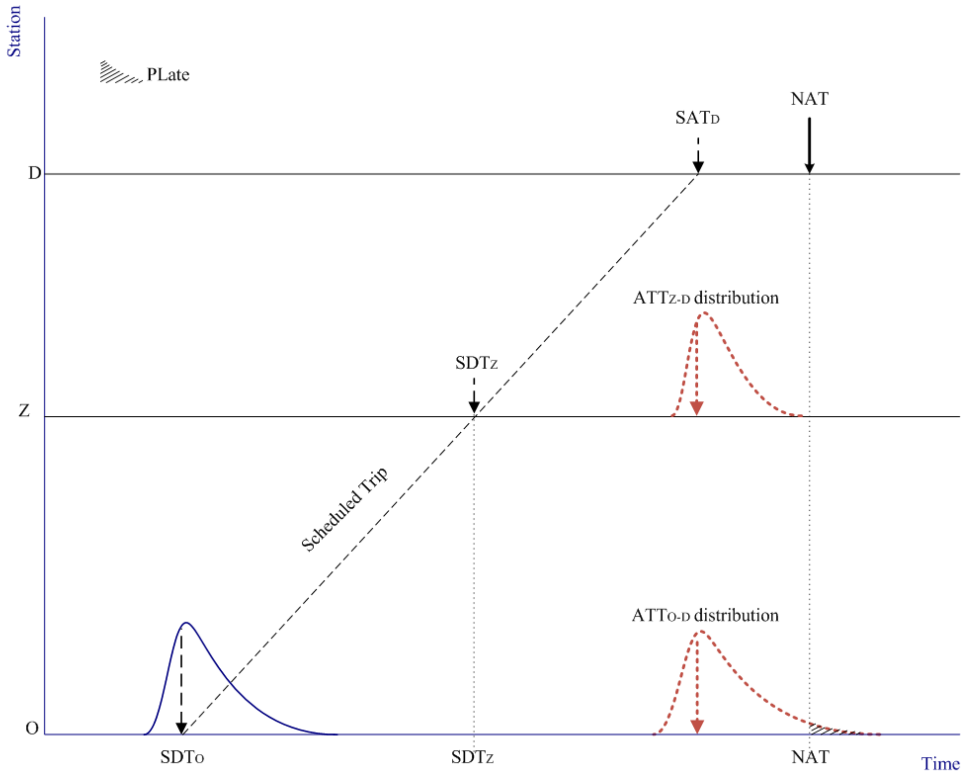
To illustrate, consider a passenger travel from station O to station D on a route that passes through station Z (Figure 3.4(a). The TU is scheduled to depart from station O at SDT_O and arrive at station D at SAT_D. Using the historical distribution of transit travel time from station O to station D, the probability of the passenger arriving at the destination station D after NAT is illustrated in Figure 3.4(a). However, the TU is delayed and departs at time ADT_O; consequently, the distribution of the arrival time of TU at station D is given by the travel time distribution from station O to station D but shifted by the amount of time of $ADT_O - SDT_O$.

Accordingly, the probability of being late increases as shown in figure 3.4(b). At intermediate station Z, the passenger re-evaluates the performance of the service and finds that the travel time from station O to station Z has been longer than scheduled. Using the distribution of travel time from station Z to station D, the distribution of arrival time at station D can be updated. The anxiety penalty this passenger experiences on this trip is shown in Equation 3.9 (see Figure 3.5).

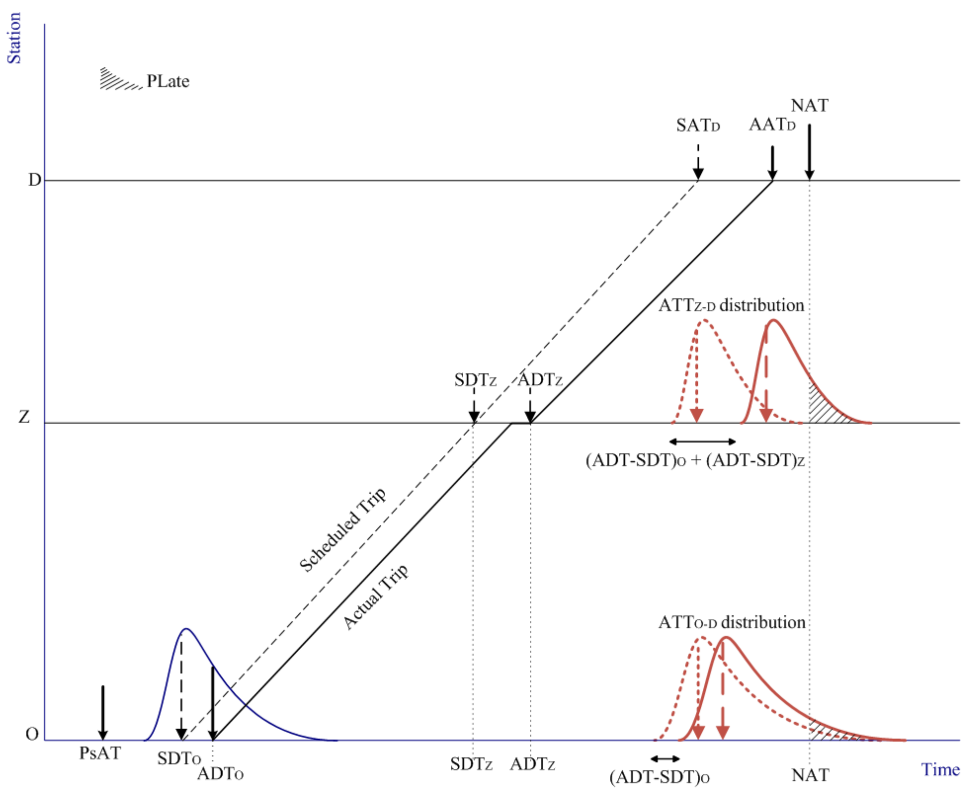
$$ANX = P_{Late_{(O)}} \times ATT_{O-Z} + P_{Late_{(Z)}} \times ATT_{Z-D} \tag{3.9}$$

where

ANX_{O-D}	Passenger's Anxiety between O and the following station
$P_{Late_{(O)}}$	Passenger's estimation at O of probability of arriving late at D
$ATT_{(O-Z)}$	Actual travel time between station O and the intermediate station Z
$P_{Late_{(Z)}}$	Passenger's estimation at Z of probability of arriving late at D
$ATT_{(Z-D)}$	Actual travel time between station Z and the destination station Z



(a) Case1: Scheduled Trip



(b) Case2: Actual Trip

Figure 3.4: Anxiety Penalty Estimation as a Function of TTV and ADT

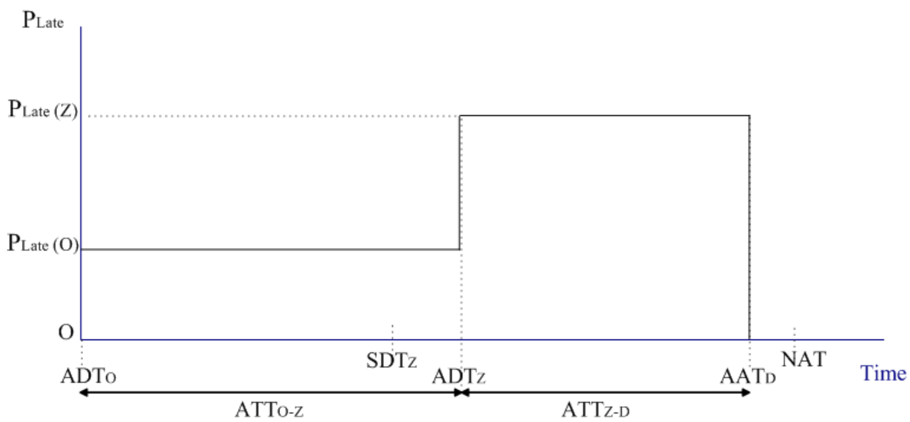


Figure 3.5: The probability of Late at Origin Station and Station Z

3.1.5 Schedule Adherence Penalty

There is some evidence [2] suggesting that transit passengers experince a cost as a result of unreliable service even if their probability of arriving later than their NAT is zero (i.e. ANX=0). In the scheduling model developed by Bates et al [2], this penalty is represented by the term SD (schedule adherence delay). However, as discussed in Chapter 2, this formulation is not suitable for some situations.

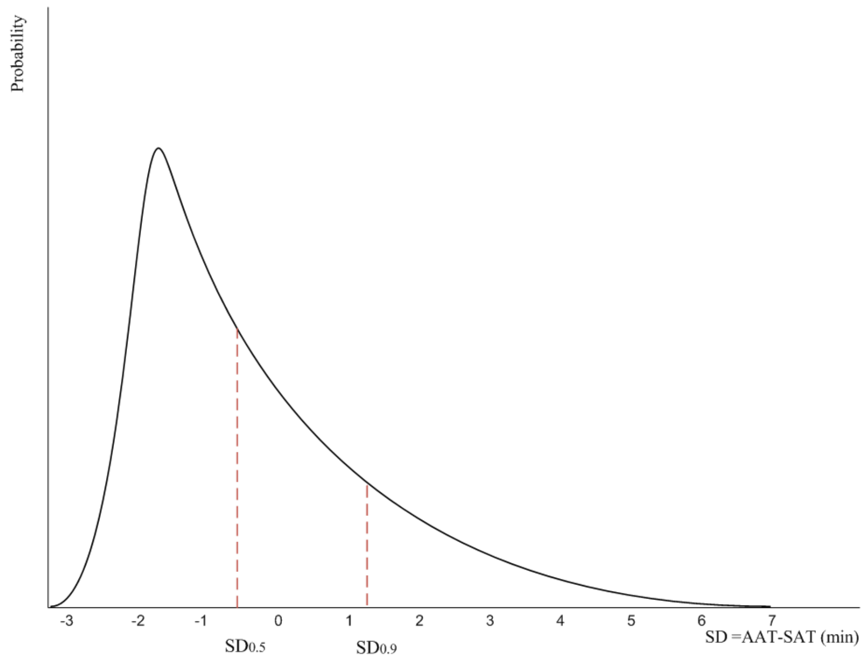
The standard deviation is commonly used to measure the dispersion of data and has been used to represent the level of scheduled adherence. However, arrival time distributions are typically non symmetrical and skewed to the right. For non symmetrical distributions, the standard deviation is less meaningful than a stated percentile.

Thus, the difference between the 90^{th} percentile and 50^{th} is added as a measure of the service variability regarding schedule adherence at destination as expressed in equation 3.10. This percentiles are subjectively chosen on the basis of engineering judgement. Figure 3.6(a) and 3.6(b) illustrates PDF and CDF distribution curves, respectively, for a particular station from which the schedule delay 90^{th} and 50^{th} percentiles can be determined.

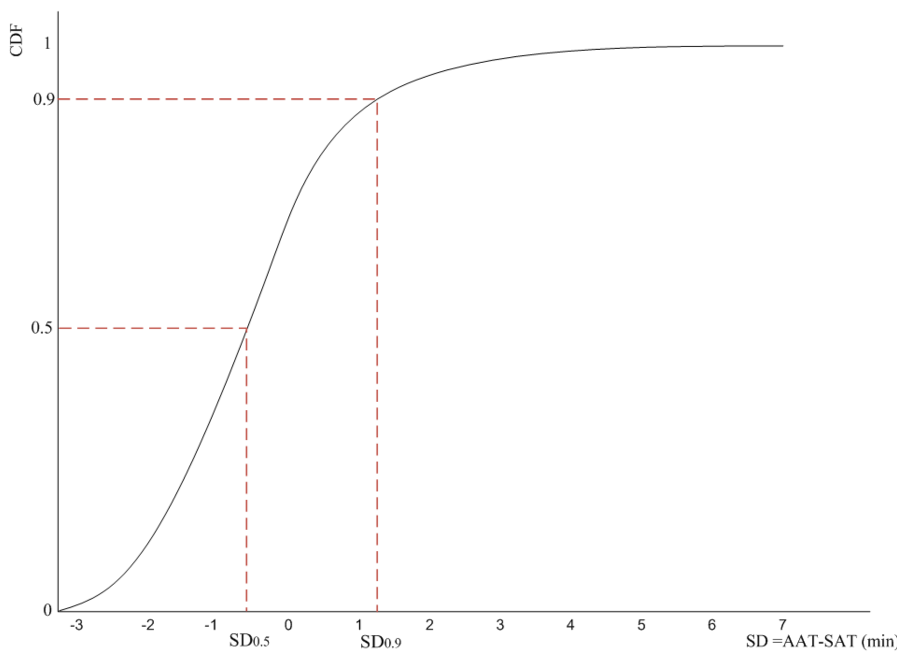
$$SD = SD_{0.9} - SD_{0.5} \tag{3.10}$$

where

$SD_{0.9}$ Schedule delay 90^{th} percentile at destination station
$SD_{0.5}$ Schedule delay 50^{th} percentile at destination station



(a) Case1: PDF



(b) Case2: CDF

Figure 3.6: Distribution Showing the 90^{th} and 50^{th} Percentile for a Data Set

3.2 Simulation Model

By their nature, reliability models are based on probabilistic frameworks. The approach to applying these models is a Monte-Carlo simulation. A Monte-Carlo Simulation approach is categorized as a sampling method that generates two or more random variables as inputs for the model. Those randomly generated variables are based on probability distributions to simulate a sample from the population.

A Monte-Carlo simulation approach has been used in this research to quantify the impact of service reliability on user cost. The model is developed at a passenger level: the cost associated with a single trip for a single passenger is estimated using the proposed GC_R model (equation 3.1). These estimated costs are compared to costs estimated from conventional GC model formulation which is not considering service reliability. The Monte-Carlo simulation is also used in the decision process used by individual transit passenger to select their trip departure time to minimize their expected GC.

3.2.1 Origin and Destination Stations

The Monte-Carlo simulation assumes that the transit network consists of a single transit line with n stations. Each station has an equal probability of being chosen either as an origin or destination station by a passenger.

Thus, the first random variables generated by the simulation are origin and destination station numbers for the individual trip. Those random variables, $RV1$ and $RV2$, are integer numbers and uniformly distributed on the interval (1,n) and they are not equal (i.e. $Origin \neq Destination$). The discrete probability density function is

$$f(i) = \begin{cases} \frac{1}{n-1} & for\, 1 \leqslant i \leqslant n \\ 0 & Otherwise \end{cases}$$

Having generated the random variables, the traveler's trip is identified by origin station, RV1, and destination station, RV2.

3.2.2 Trip Maker's Necessary Arrival Time

Each traveler has a necessary arrival time (NAT), reflecting the latest arrival time at destination station that satisfies passenger's trip objectives. Possible NATs are assumed to be uniformly distributed in the range of the time between two consecutive TU scheduled arrival times.

Assuming that the $SAT_{TU} = 0$ and the next TU scheduled arrival time is $SAT_{TU+1} = h$, where h is the service headway, a passenger's NAT is assumed to be uniformly distributed in the range (0,h). Figure 3.7 illustrates the random variable distribution that refers to the passenger's NAT.

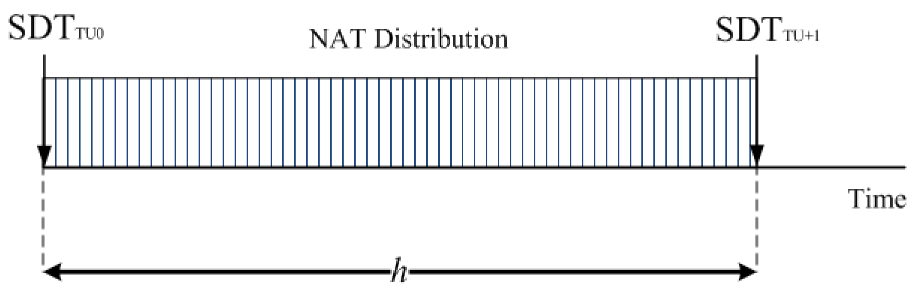


Figure 3.7: NAT Uniform Distribution at Destination between Two Consecutive TUs Arrival Time

The impact of service reliability on traveler cost is essentially based on the passenger's NAT. Most of the reliability influenced penalties in the GC_R function are expressed in relation to the passenger's NAT. Given an NAT, the passenger selects a departure time to minimize his/her expected GC as described in the next section.

3.2.3 Departure Time Choice

Having an assigned NAT, the passenger starts to assess trip departure time alternatives t_D. By evaluating the expected cost of the trip due to the discrete nature of transit service, the passenger assesses the expected cost associated with the transit unit (TU) departure time t_D that arrives at SDT before and nearest to the NAT, the previous transit unit TU_{-1}, and the following transit unit TU_{+1}. Then, the passenger selects the one that provides the lower expected cost (i.e. minimum $E(GC)$).

It is assumed that the traveler has experience with the transit service and therefore has a knowledge of the departure time and travel time distribution. Within the

Monte-Carlo simulation, the distributions are obtained from collected field data for transit units arrival, departure travel times.

The model used to choose departure time alternative ,t_D, in this research is derived from the proposed schedule model(GC_R) expressed in Equation 3.1. For all three t_D alternatives associated with a specific trip, it is assumed that the expected waiting time ($E(WT)$), expected in-vehicle time ($E(IVT)$), and schedule adherence delay penalty(SD) are equal and therefore can be eliminated from the model (this implies that headways are constant and the distribution of both departure time and travel time do not change with the three t_D alternatives). Thus, the model includes only the penalties impacted by the NAT and t_D by Equation 3.12 and 3.13.

$$E(GC) = \alpha_1 E(ADE) + \alpha_2 E(ADL) + \alpha_3 ANX \tag{3.11}$$

where

$E(GC)$	The expected generalized cost
$E(ADE)$	Expected early arrival delay time in minutes
$E(ADL)$	Expected late arrival arrival delay time in minutes
ANX	Anxiety penalty of a transit passenger during the trip
α_i	The relative importance of each variable

Accordingly, for each TU alternative, the expected arrival delay is estimated as the sum of each of the possible values of AD at a specific schedule delay (SD) value multiplied by the probability of obtaining that value. Mathematically, for each t_D, this is given as follows

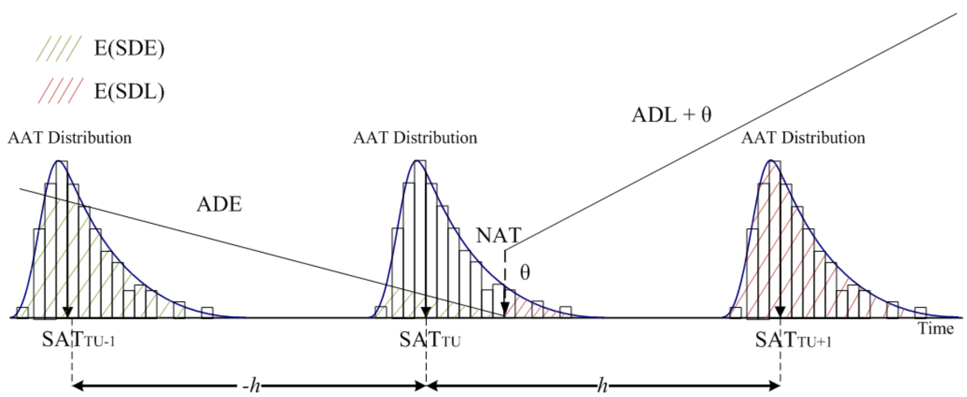
$$E(ADE) = \begin{cases} \sum_{i=1}^{n} (AD_{(AAT)_i} P(AAT)_i) & for\ 1 \leqslant i \leqslant i_{(NAT)} \\ 0 & Otherwise \end{cases} \tag{3.12}$$

$$E(ADL) = \begin{cases} \sum_{i=1}^{n} (AD_{(AAT)_i} + \theta) P(AAT)_i & for\ i_{(NAT)} < i \leqslant n \\ 0 & Otherwise \end{cases} \tag{3.13}$$

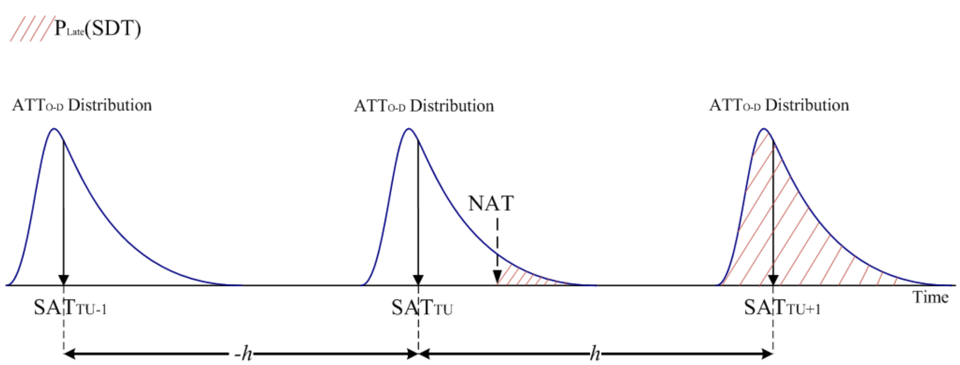
where

n	AAT distribution class intervals number
i	The number of the class interval, generated from the field data
$AD_{(AAT)_i}$	Arrival delay penalty corresponding to interval i
AAT_i	The AAT midpoint of interval i
$P(AAT)_i$	Probability of arriving at any time time within the interval i
$i_{(NAT)}$	The number of class interval includes passenger's NAT
θ	Late arrival discrete penalty

Graphically, the expected schedule penalties are presented in Figure 3.8(a) for three alternatives t_D. In Figure 3.8(b), the anxiety penalty is illustrated for the same three alternatives. The anxiety associated with the previous TU, TU_{-1} will be zero because there is no probability of being late at the destination (i.e. $P_{Late} = 0$). In contrast for TU_{+1}, the probability of being late is equal to one, which means that there is certainty that the passenger will be late, and therefore the anxiety term will have a large value.



(a) Case1: Expected Arrival Delay



(b) Case2: P_{Late} at Destination

Figure 3.8: Expected Schedule Delay Penalties and P_L estimation for O-D Three Departure Time Alternatives

The $E(GC)$ is calculated for each t_D using Equation 3.11 and the t_D provides the minimum cost is chosen by the passenger.

3.2.4 Passenger's Arrival Time

The next parameter generated in the simulation is passenger arrival time at the origin station. It was found from the literature review that for transit service with short headway the passengers arrive randomly throughout the headway. However, with longer headway service, travelers try to minimize their waiting cost by adjusting their arrival time with respect to the schedule departure time (SDT) [4]. It is expected that the amount of time in advance of the schedule departure time (SDT) that passengers arrive is impacted by the service reliability. The number of passengers arriving at origin stations is increasing gradually in the range $(SDT_{TU-1}, (SDT)_{TU})$ until a peak point and then it is decreasing. This can be explained by passengers trying to optimize their arrival time before SDT by a certain period; however, at a point of time, the probability of missing the TU because it departs the station prior to the SDT increases the cost associated with the trip.

Consequently, it is assumed that the distribution of passenger arrival time follows a triangular distribution. Assuming $SDT_{TU} = 0$ and the previous TU departure time $SDT_{TU-1} = -h$, the passenger arrival time ($PsAT$) distribution is shown in Figure 3.9. The probability of a passenger arriving at time t ($PsAT$) is be estimated as

$$f(PsAT|a,\, b,\, c) = \begin{cases} \frac{2(t-a)}{(b-a)(b-c)} & for\, a \leqslant t \leqslant c \\ \\ \frac{2(t-a)}{(b-a)(b-c)} & for\, c \leqslant t \leqslant b \\ \\ 0 & Otherwise \end{cases} \tag{3.14}$$

where

$f(t\|a,\, b,\, c)$	The probability of a passenger arriving at time t
$a,\, b,\, c$	Triangle distribution parameters

For the simulation model, an individual passenger arrival time ($PsAT$) is selected randomly from the triangular distribution. Thus, the inverse of the triangular distribution is used to transform a continuous uniform random variable in the

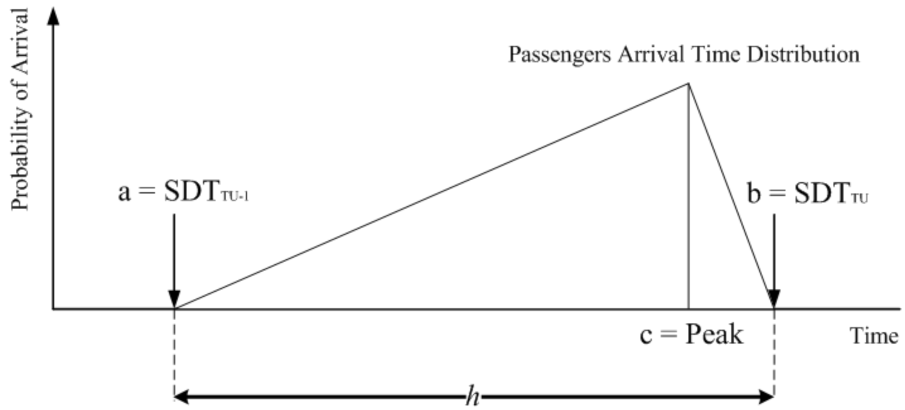


Figure 3.9: Assumed Distribution of Passengers Arrival Time

range (0,1) to a variable that follows the triangular distribution in Figure 3.9. The transforming process is expressed as

$$PsAT = \begin{cases} a + \sqrt{u * (b-a) * (c-a)} & for \sqrt{u * (b-a) * (c-a)} + a < c \\ \\ b - \sqrt{(1-u) * (b-a) * (b-c)} & for \sqrt{u * (b-a) * (c-a)} + a \geqslant c \end{cases} \tag{3.15}$$

where

u	Uniformly distributed random number in the range of (0,1)
$PsAT$	Passenger arrival time
a, b, c	Triangle distribution parameters

3.2.5 Calculation of GC

Returning to the proposed model GC_R in equation 3.1, the GC for a specific trip can be estimated. First, the waiting time penalty is calculated as explained previously as the difference between the passenger arrival time ($PsAT$) and a transit unit actual departure time (ADT). Second, in-vehicle time (IVT) is randomly selected from the actual travel time (ATT) distribution for each O-D station within the journey. For instance, if the passenger is boarding at station O going to station D through station Z as illustrated in Figure 3.10, two ATT random variables are generated. One for the travel time from station O to station Z and the other from station Z to station D. Third, the arrival delay early (ADE) and late (ADL) are computed using Equations 3.4, 3.5, and 3.6. Fourth, the anxiety penalty associated with the trip is estimated using Equations 3.7 and 3.8. Fifth, the schedule delay

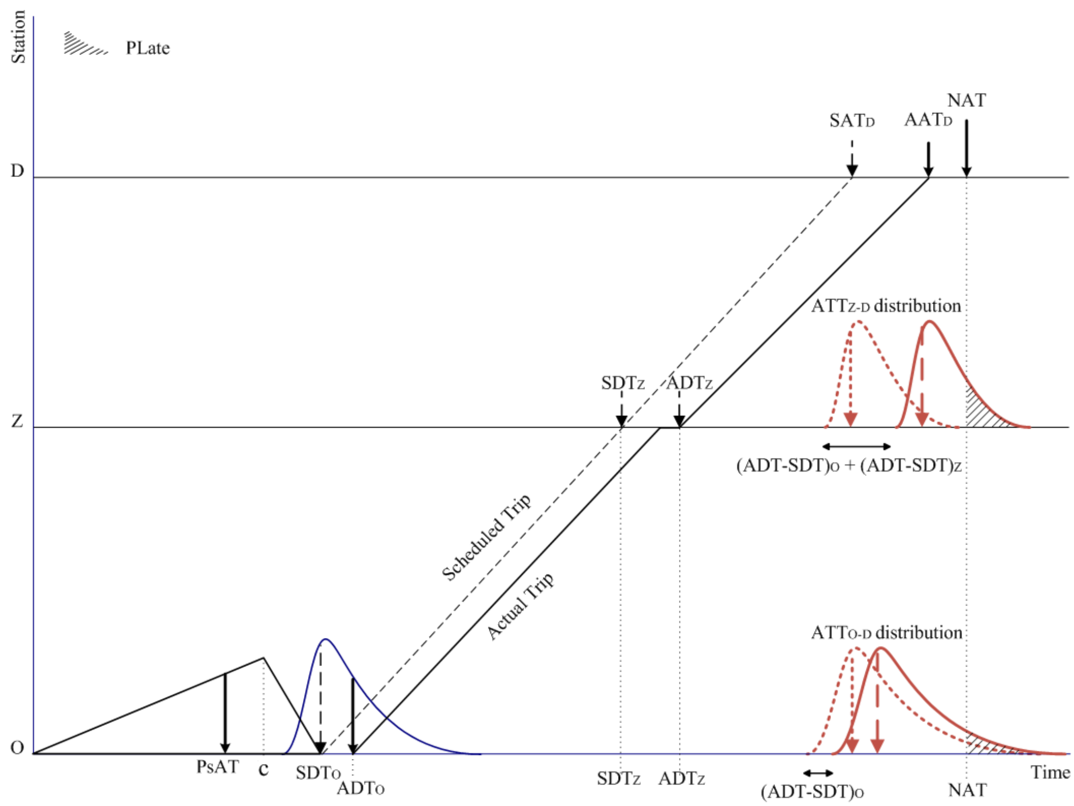


Figure 3.10: Graphical Example for an Individual Traveler Trip

penalty (SD) is calculated based on the actual arrival time (AAT) distribution that has been calibrated from field data. The 90^{th} and 50^{th} percentiles of schedule delay are computed for all station pairs.

Having estimated generalized cost using the proposed model (GC_R), the cost using the conventional generalized cost model (GC_C) is also calculated using equation 3.16.

$$GC_C = (\alpha_1 AT + \alpha_2 WT + \alpha_3 IVT)VOT + f \tag{3.16}$$

where

GC_C	Generalized cost ($\$/trip$)
AT	The access time to the line (minutes)
WT	Waiting time (assumed to be equal to half of the headway) (minutes)
IVT	In-vehicle time (minutes)
VOT	The value of time ($\$/min$)
f	The service fare (\$)
α_i	The relative importance of the term

As a result, the GC for an individual passenger for a single trip for a specific O-

D stations using both GC models (GC_R and GC_C) are computed. By running the simulation for a large number of passengers for each pair of stations, the average GC_R can be estimated. By comparing the average GC_R to the result obtained from GC_C function for the same O-D pair, the impact of service reliability can be quantified.

3.2.6 Numerical Example

A hypothetical numerical example for the simulation model is provided to illustrate the calculation of the GC using the GC_R formulation. Assume that the simulation model generated a passenger traveling from station O to station D through station Z. The passenger's generated necessary arrival time (NAT) is 7:44:00.

The first step is to choose the departure TU that minimizes the $E(GC)$ as defined in Equation 3.11. The schedule for the three alternatives departure TUs is shown in Table 3.2. From the table, the scheduled travel time between station O and station Z is six minutes (i.e. $SIVT_{O-Z}$=6 min) and the scheduled travel time from station Z to station D is three minutes ($SIVT_{Z-D}$=3 min). Therefore, the total scheduled in-vehicle time for the generated trip is nine minutes ($SIVT_{O-D}$=9 min).

Table 3.2: The Transit Service Schedule Departure Time for the Three Departure Time Alternatives

	Schedule Dep/Arr Time		
Station	**TU-1**	**TU**	**TU+1**
O	7:15:00	7:30:00	7:45:00
Z	7:21:00	7:36:00	7:51:00
D	7:24:00	7:39:00	7:54:00

Using Equation 3.11, the $E[GC]$ results are calculated and shown in Table 3.3. Based on the results, the transit unit TU is chosen by the passenger.

Table 3.3: $E[GC]$ for Each Departure Time Alternative

	TU-1	**TU**	**TU+1**
$E[GC]$	20.50	15.30	35.25

The passenger arrival time at boarding station is generated based on the triangle distribution assumed to be four minutes prior to the SDT_{TU} (i.e. $PsAT$ = 7:26:00).

In this case, the parameter C in the triangle distribution is assumed to occurs at time $SDT_{TU} - 3$ minutes.

The distribution of TU actual departure times is known and a hypothetical AAT can be generated. It is assumed to be one minute before the schedule departure time (SDT) (i.e. ADT=7:29:00). Similarly, the actual travel time for each segment in the route is generated using the field data; actual travel time between station O and Z is eight minutes and the actual travel time between Z and D is three minutes (total in-vehicle time IVT=11 min). In this case, the actual arrival time at destination (station D) is calculated (AAT_D=7:40:00) which is earlier than the passenger's NAT. Therefore, there is no late arrival delay penalty (ADL=0) and the early arrival delay calculated as the difference between AAT and the passenger's NAT (i.e. ADE=4 min). The waiting time is estimated as the difference between the actual departure time and the passenger arrival time (WT=3 min). The probability of being late is calculated at station O and station Z ($P_{Late_O} = 0.001$ and $P_{Late_Z} = 0$) and therefore the anxiety is calculated as follows

$$ANX = P_{Late_{(O)}} * IVT_{O-Z} + P_{Late_{(Z)}} * IVT_{Z-D} = 0.008 \qquad (3.17)$$

Finally, the GC using GC_R formulation is calculated based on the weighting or the value of time for each of the variables included within the model.

The next chapter presents an application of the simulation framework using AVLS data from The Regional Municipality of Waterloo to quantify the impact of service reliability on passenger cost.

Chapter 4

Case Study

4.1 The Region of Waterloo

The Regional Municipality of Waterloo, which is located in Southern Ontario, is one of the fastest growing communities in Canada. It consists of the cities of Kitchener, Waterloo, Cambridge and four rural townships. Currently, the population of Waterloo Region is approximately 500,000 but this number is expected to reach 730,000 in the next 20 years [34].

In 2001, the Region of Waterloo developed a Regional Growth Management Strategy (RGMS) to provide direction for long-term growth management within the region. RGMS was built through extensive consultation with the citizens of the community by identifying their goals and outlining a plan to achieve them. One of the basic goals of RGMS is the provision of a suitable transportation system that accommodates the future travel demands and has a positive influence on land use [8]. Grand River Transit[1](GRT) is the public transport operator in the region created by the Region of Waterloo in 2000. GRT replaced city-based service and provided a synchronized service between the cities of Kitchener, Waterloo and Cambridge. Since that time, the ridership has improved dramatically. Recently, in response to community needs, GRT has upgraded their fleet to include buses with low-floors, wheelchair access, bicycle racks on the front etc. Operationally, one of the essential enhancements in the region's transportation system toward providing transportation alternatives is iXpress, the express bus service.

[1]A member of the Canadian Urban Transit Association

4.2 GRT iXpress Service

In 2005, GRT launched a limited stop express bus service running through the three main cities in the region: Waterloo, Kitchener, and Cambridge. The service, branded iXpress, has a length of approximately 33 km with 13 bus stations along the route(Figure 4.1).

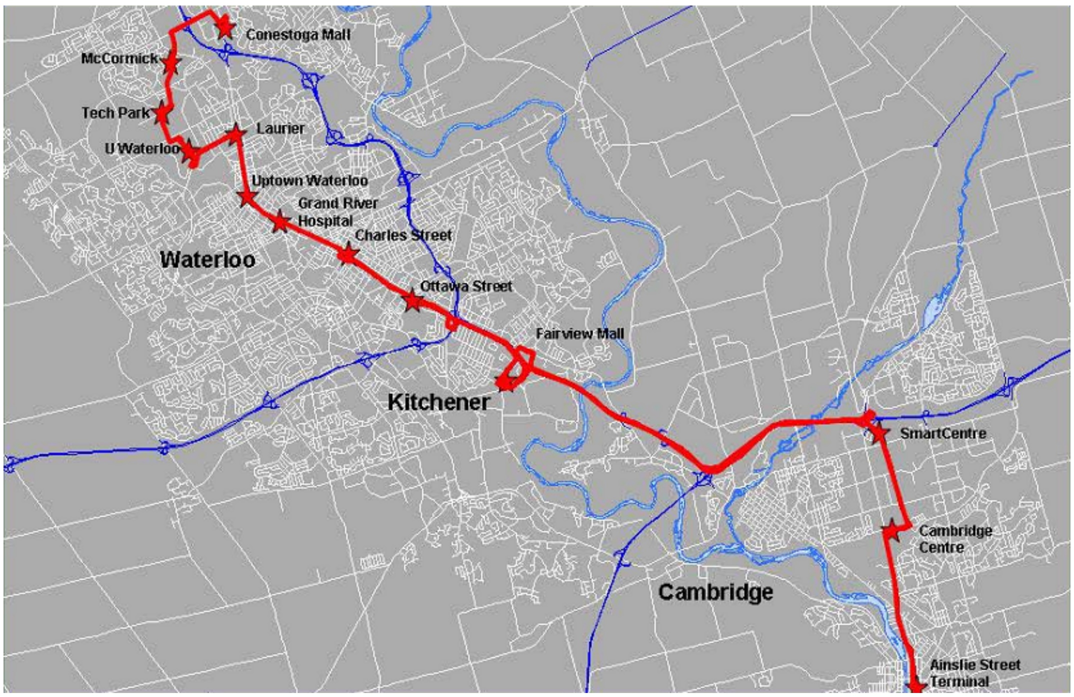


Figure 4.1: Grand River Transit iXpress Service Route and Stations

iXpress route stations are located at the major activity centers within the region serving four downtowns, two universities, five major shopping centers, office complexes, major hospitals, and two transit terminals (Table 4.1). At the commencement of service, the service operated only from Monday through Friday, from 06:00 to 19:00 with two different headways: fifteen-minute headway service during morning and afternoon peak periods and thirty-minute headway during the midday. In the last quarter of 2007, the weekday service hours were extended from 05:40 to 23:00; additionally, the service was expanded to operate on the weekends, Saturdays from 07:30 to 19:30 and Sundays from 10:00 to 18:00.

As part of the service, GRT implemented advanced transit technologies on both the iXpress fleet and route. Transit signal priority (TSP) has been deployed at seventeen intersections along the iXpress route to improve reliability of the service.

Table 4.1: Grand River Transit iXpress Stations and Centers Served

Station #	Station Name	Description
1	Conestoga Mall	A major shopping center in Waterloo
2	McCromic	A community center and public facilities
3	R&T Park	UW Research and Technology Park
4	U Waterloo	University of Waterloo at Davis Center
5	Laurier	Wilfrid Laurier University
6	Uptown Waterloo	King street beside Waterloo Town Square
7	Grand River Hospital	A major hospital in the region
8	Charles Terminal	Downtown kitchener bus terminal
9	Ottawa	Ottawa St and Charles St intersection
10	Fairview	A major shopping mall in Kitchener
11	Smart Centers	A major shopping center in Cambridge
12	Cambridge Center	A major shopping mall in Cambridge
13	Ainslie Terminal	The main terminal in Cambridge

In addition, iXpress stations utilize variable message signs that display real-time information about the arrival time of the next bus. Most importantly for the purpose of this research, each iXpress bus is equipped by a Computer-Aided Dispatch System (CAD) and Automated Vehicle Location System (AVLS). The CAD and AVLS provide and archive real-time temporal and spatial data related to the performance of the iXpress service.

4.2.1 iXpress Archived AVLS Data Analysis

In the past, transit agencies' cost were much greater for data collection to monitor, evaluate and enhance the efficiency of the system. With the introduction of AVLS, compared to the manual data collection approach which needs a vast human resource, transit agencies are able to gather high quality spatial and temporal transit operation data. These data can be transferred directly to the operation control center for real-time applications or be archived for off-line analysis as shown in Figure 4.2 [14]. Spatial data are used for the where query and temporal data are used for the when query about any TU within a transit fleet in a transit service network. For a specific transit service route, TUs travel through multiple stations following a preset plan and schedule. The data captured by AVLS involves a record that includes the time arriving at or departing from each station along the transit route. These data are one of the basic components of the proposed framework in this research.

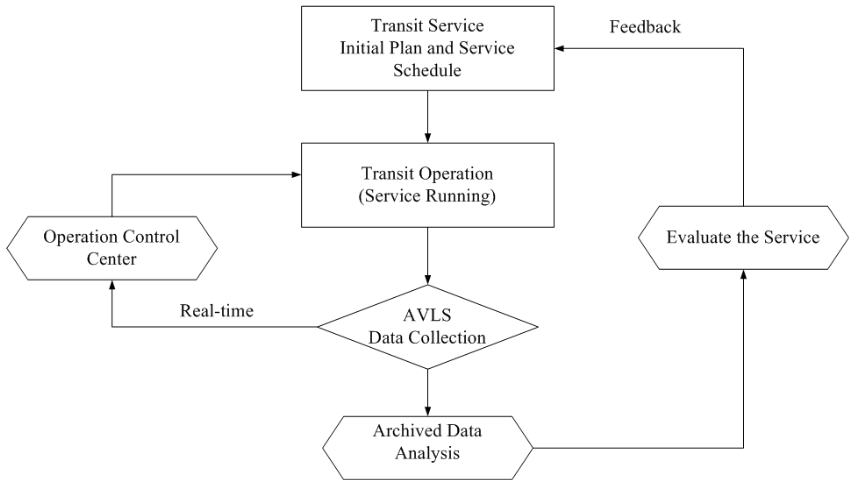


Figure 4.2: AVLS Data Collection, Analysis, and Service Evaluation Process

AVLS data collected from iXpress is imported into databases for off-line analysis to evaluate the performance of the operations. The system is capable of providing standard daily reports and query request reports from the archived AVLS data. In addition, the archived data can be exported to a ".csv" (comma separated values) file format to be used for further analysis by researchers.

For the purpose of this research, the RMOW provided two months of archived iXpress AVLS data: Mar 24^{th} to May 24^{th}, 2008 in 6659 data base format (dbf) files. Each file has archived AVLS data for a particular bus on a particular day. According to the type of analysis required in this research, all the provided dbf files were combined into a single database. Approximately 163,500 data records existed in the database including both Northbound and Southbound directions.

Not all the fields contained in the GRT database were required for the research analysis. Consequently, the fields that are not required were eliminated. Table 4.2 lists the fields kept in the database and used in this research.

The AVLS data were collected and stored automatically in the database. However, the data may contain errors. Inaccuracies may occur either as a result of incorrect location measurement provided by the GPS or an error may occur in the matching of the appropriate scheduled data. Consequently, preliminary tests for data quality assurance were conducted before the data were used for analysis. Each record in the database was checked to determine if the recorded values exceed a predefined threshold. If it does, then this record was eliminated from the database.

The remaining data were stratified by the a.m. and p.m. peak hours (Table 4.3)

Table 4.2: Database's Fields Used for Data Analysis

Field Name (Field Format)	**Description**
Date (Date)	The date of the trip
Bus ID(Number)	The ID of the bus made the trip
Event (Number)	ID given for each activity made by the bus on the route
Trip Start (Number)	ID given for all activities in one complete direction trip [a]
Stop (Text)	Giving the stop name
Actual Arrival (Time)	The bus actual arrival time at the stop
Actual Departure (Time)	The bus actual departure time from the stop
Scheduled Arrival (Time)	The bus scheduled arrival time at the stop
Scheduled Departure (Time)	The bus scheduled departure time from the stop

[a]One direction trip: A trip made by a particular bus starts from the first station moving in the southbound direction to the last station via versa

and were used in this research. Therefore, two Structured Query Language (SQL[2]) statements were coded and executed in MS Access to retrieve the data associated with the peak periods from the original database. The database derived from these two queries formed the database used in the analysis in this research. The data were analyzed at two different levels: Station Level and Segment Level.

Table 4.3: A.M. and P.M. Peak Hours

Peak Period	**Start**	**End**
A.M. Peak	07:00	10:00
P.M. Peak	15:00	19:00

4.2.1.1 Station Level Data Analysis

At the station level, schedule adherence delay (SD) distributions were estimated from the AVLS data by computing the difference in arrival time, DAT, between the actual and scheduled arrival time at a particular station and the difference in departure time, DDT, between the actual and scheduled departure time from a particular station.

$$DAT = AAT - SAT \tag{4.1}$$

[2]SQL is a query language that helps to manage, query, retrieve, and modify databases

$$DDT = ADT - SDT \quad (4.2)$$

where

DAT Difference in arrival time (minutes)

DDT Difference in departure time (minutes)

As observed, each station is servicing two directions, NB and SB. Hence, the distribution of SD is estimated for each station and direction. Queries written in SQL were developed to retrieve the data under specific criteria. For instance, if the distribution of the of the differences of arrival time was required for the Uptown Waterloo station and the bus is traveling in the SB direction (i.e. the previous station was Laurier) then the SQL statements are as follow

1. The field $TripStart_{Uptown} = TripStart_{Laurier}$, which reflects that the bus stoped at the two stations in the same one direction trip
2. The field $Event_{Uptown} > Event_{Laurier}$, which identifies the trip direction

A sample from the previous SQL statements result is given in table 4.4. Similarly,

Table 4.4: A Sample form the Data Retrieved for DAT

Date	**Bus Num**	**Event**	**TripStart**	**Stop**	**DAT**
4/1/2008	2406	1668427	1668403	Laurier	-0:02:42
4/1/2008	2406	1668429	1668403	Uptown Waterloo	-0:02:20
4/1/2008	2408	1668592	1668568	Laurier	0:00:41
4/1/2008	2408	1668595	1668568	Uptown Waterloo	0:01:13
4/1/2008	2409	1668789	1668768	Laurier	0:00:54
4/1/2008	2409	1668791	1668768	Uptown Waterloo	0:01:26
4/1/2008	2409	1668881	1668856	Laurier	-0:01:07
4/1/2008	2409	1668883	1668856	Uptown Waterloo	0:00:15
4/1/2008	2411	1669740	1669718	Laurier	-0:00:20
4/1/2008	2411	1669743	1669718	Uptown Waterloo	-0:00:28
4/1/2008	2411	1669926	1669902	Laurier	0:00:26
4/1/2008	2411	1669928	1669902	Uptown Waterloo	0:01:50

DDT is estimated by creating SQL statements to retrieve data that meet the criteria. Using the same example, DDT data for Laurier station in SB direction is needed. Thus, the criteria is as follow

1. The field $TripStart_{Uptown} = TripStart_{Laurier}$, which reflects that the bus stops at the two stations in the same one direction trip

2. The field $Event_{Laurier} > Event_{Uptown}$, which identifies the trip direction

Having done the previous process, a csv format file is exported for every station in each direction. Hence, schedule adherence delay (SD) distribution at any particular station can easily be found for either departure time or arrival time from data files as shown in Figure 4.3. In total, there are 96 files for departure and arrival SD data that are used later in the simulation.

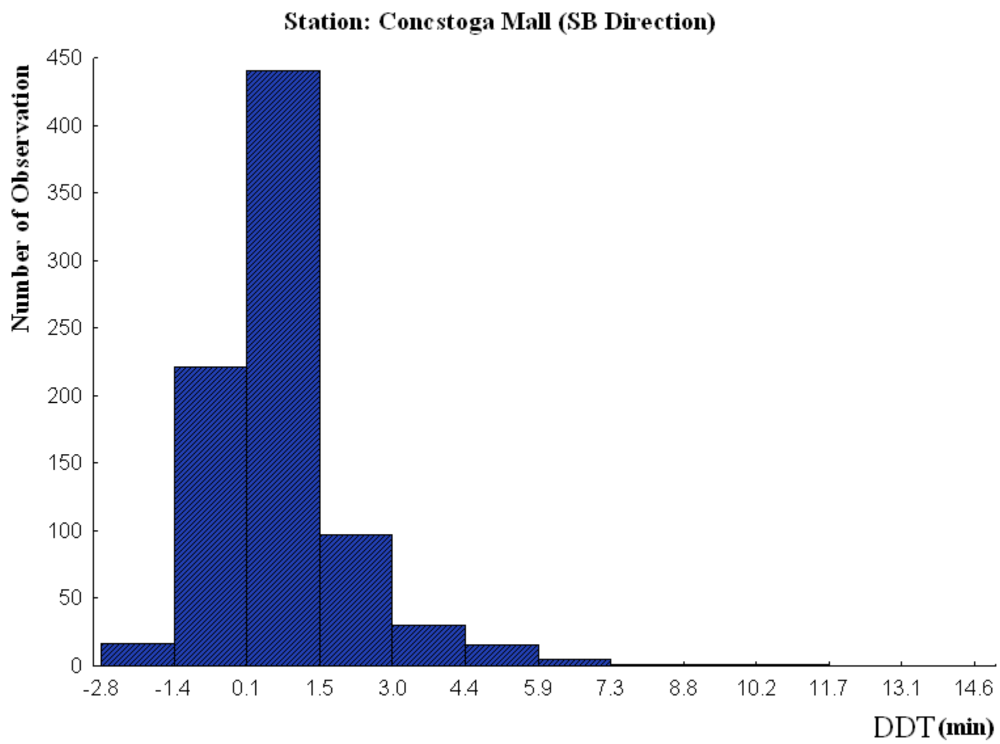


Figure 4.3: Distribution of the Difference Between Actual and Scheduled Departure Time

4.2.1.2 Segment Level Analysis

At the segment level, the distribution of the difference between the scheduled and actual travel times is calculated for each pair of stations. The actual travel time is calculated as the difference between the actual arrival time at a destination station and the actual departure time from the origin station (Equation 4.3). The scheduled travel time is computed the same way using the scheduled time. This calculation needs to be done under certain conditions as follow:

1. The field $TripStart_D = TripStart_O$, which reflects that the trip is made between the O-D stations in the same one direction trip

2. The field $Event_D > Event_O$, which identifies the trip direction

$$ATT_{O-D} = AAT_D - ADT_O \tag{4.3}$$

where

ATT_{O-D}	Actual arrival time between station O and D
AAT_D	Actual arrival time at station D
ADT_O	Actual departure time from station O

Having done the previous calculation, the difference between the actual travel time (ATT) and scheduled travel time (STT) is calculated for each pair of stations and exported to csv file format. From these data, the distribution of the difference in travel time (DTT) can be developed for use in the simulation. In total, the number of possible pair of stations is 156 pairs (13×12) generated in separate files including DTT.

4.3 Passengers Travel Time and Reliability Perceptions

It was discussed previously that each passenger has a cost function depending on different variables such as travel time, necessary arrival time, arrival delay etc. Each passenger chooses the departure time of the bus that minimizes his/her GC function associated with the trip. It is reasonable to expect that the consequences of being late or early at the destination vary according to the purpose of the trip and passenger characteristics.

Consequently, in this research, travelers are classified into three classes according to the cost they incur as a result of arriving at their destination later or earlier than their necessary arrival time (NAT):

1. Risk Averse Passengers (RAP). These are passengers who incur a high cost when late and therefore usually are willing to accept only a small likelihood of being late;

2. Risk Moderate Passengers(RMP). Passengers who incur moderate cost when late and therefore are willing to accept a greater likelihood of being late;

3. Risk Neutral Passengers (RNP). Passengers who incur low cost when late and therefore are wiling to accept the greatest likelihood of being late.

The impact of poor service reliability on each passenger class varies according to the different relative weighting of each penalty associated with the passengers' trips. The impact of poor service reliability on RAP class is grater than RPP class which in turn grater than RNP class. These different weightings are reflected through the use of class specific coefficients in the GC_R model.

A value of one is assigned to scheduled in-vehicle travel time ($SIVT$) coefficient. The importance of each of the other variables in the GC_R function is set relative to $SIVT$ (Equation 4.4).

$$GC_R = \alpha_1 WT + SIVT + \alpha_2 IVTD + \alpha_3 ADE + \alpha_4 ADL + \theta\delta + \alpha_5 ANX + \alpha_6 SD_{0.9} \tag{4.4}$$

It was recommended by many studies that waiting time is weighted two to three times the in-vehicle time [26] [54]. However, there are no studies in the literature that distinguish the waiting time coefficient by passenger type. Consequently, in this study it is assumed that risk-averse passengers assign a higher weight to waiting time than risk-moderate passengers who assign a higher weight than risk-neutral passengers (Figure 4.4). Table 4.6 lists the waiting time coefficient assumed for each passenger class.

The in-vehicle travel time delay ($IVTD$) is assumed to be perceived differently in two ways according to actual arrival time (AAT) by different passenger classes. If AAT is later than passenger's necessary arrival time (NAT), the travel time will be considered as a cause of being late; thus, it will be weighted more than the case when AAT is prior to NAT, which results in early arrival, as shown in Table 4.5.

Table 4.5: The Weight of Late Penalty, α_1, for Different Passengers Classes

	α_2 IVTD Coefficient	
Passenger Class	**AAT<NAT**	**AAT>NAT**
RAP	1.25	2.5
RPP	1.25	2
RMP	1.25	1.5

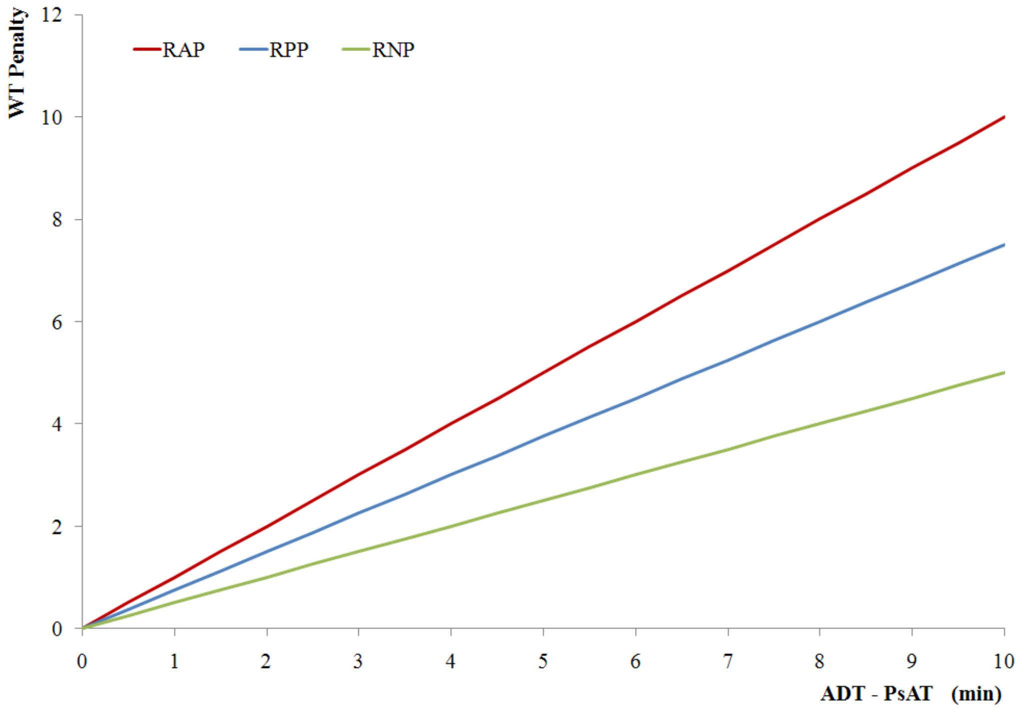


Figure 4.4: Graphical Representation of Waiting Time Penalty

Regarding the arrival delay (AD) either early or late, the penalties for each of the three classes of passenger types were derived from Small's arrival delay formulation [44]. For risk-averse passengers, it is assumed that the cost of being early is lower than the cost of being late; that the cost of arriving early is the lowest of the three passenger classes; and that the cost of arriving late is the highest of the three passenger classes. The discrete lateness penalty (θ) varies as a function of the service headway (h) according to the passenger class. The AD penalties for the three classes of passengers is illustrated in Figure 4.5. Table 4.6 provides the assumed values for AD coefficients and the discrete lateness penalties for the different passengers classes.

Studies in the literature suggest that the waiting time due to the uncertainty of bus arrival time increases the customer's anxiety [29] and therefore the perception of waiting time was found to be 1.15 times the actual waiting time. Since the waiting time is weighted as 2 to 3 times the in-vehicle time, it is assumed that the anxiety coefficient should be weighted 2.3 to 3.45 times the in-vehicle time ($SIVT$). It is assumed that the schedule adherence delay penalty (SD) has the same weight as the anxiety penalty. Similar to other coefficients, the risk-averse passenger class has a higher weighting on these penalties than the other two passenger classes. Table

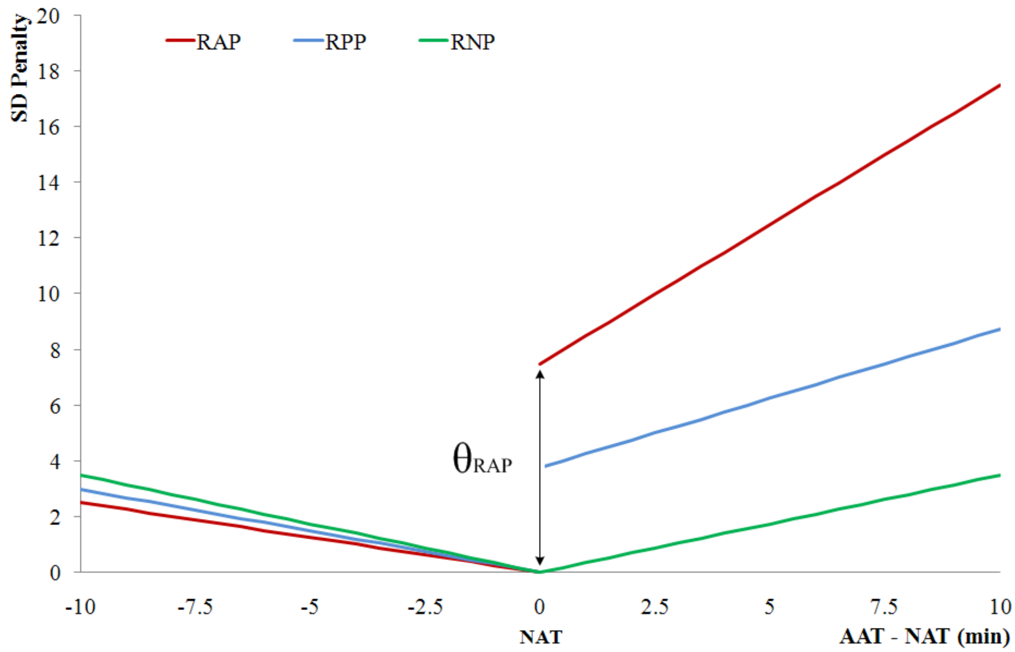


Figure 4.5: Graphical Representation of Schedule Delay Penalties for Passengers Classes

4.6 lists the assumed coefficient values for all variables and all passenger classes.

Table 4.6: GC_R Model Coefficients for Different Passenger Classes

	GC_R **model coefficients**					
Passenger Class	α_1	α_3	α_4	α_5	α_6	θ
RAP	3.5	0.25	1	2	2	0.5 h
RPP	2.85	0.3	0.5	1.5	1.5	0.25 h
RNP	2	0.35	0.35	1	1	0

All the coefficients in the GC_R model have been subjectively chosen within reasonable ranges. Calibrating values for these coefficients using empirical data needs to be done; however, this is considered outside of the scope of this thesis. Most studies that calibrate coefficients of the GC model use either revealed preference (RP) or stated preference (SP) data [18].

Revealed preference data is collected by observing the choices that people actually make in real-world conditions. Stated preference data is collected through experimental or survey situations based on hypothetical situations. The respondents are provided with a set of hypothetical choice options and they are asked to indicate what choice option they would select. The evaluation of passengers' behavior can be extracted through the choices they made based on the trade-off between the variables within the GC function.

There are some drawbacks and limitations associated with each technique. First, RP mirrors the reality of people's behavior. In contrast, SP data lack reality, which means that people's choices are not necessarily what they actually do. However, SP survey is beneficial when introducing new policies or situations that do not yet exist and therefore cannot be obtained via an RP survey. In this research context, the most challenging aspect of designing a SP survey is how to illustrate the reliability terms in a clear manner that allows respondents to intuitively understand them.

4.4 Monte-Carlo Simulation

As discussed in Chapter 3, a Monte-Carlo simulation approach has been adapted for the purpose of this research. The simulation model has been developed in Matlab 2009 software to quantify the impact of service reliability on the passengers' generalized cost. The simulation generates 10,000 passenger trips for each possible O-D pair stations in the iXpress network.

4.4.1 Inputs into Simulation Model

Passengers are equally likely to belong to each of the three passengers classes. The passenger arrival time distribution is different for each passenger class. For risk-averse passengers, it is assumed that the passengers are likely to arrive earlier than the other passenger classes as they wish to reduce the likelihood of missing the bus and incurring a very high penalty for being late (Figure 4.6.) As observed in the figure, there is a possibility that a passenger arrives before the departure time of the bus (TU-1) prior to the target bus(TU). In this case, the passenger will select to ride the previous bus (TU-1). The peak passengers arrival time for different passengers classes have been subjectively specified as shown in Table 4.7.

Furthermore, it is assumed that not all the passengers are aware of the advertised scheduled departure time (SDT). Thus, a portion of each passenger class will arrive randomly at boarding station. It is assumed that the portion of passengers who are not aware of SDT is smaller for the risk-averse passengers than the other two passenger classes (Table 4.7).

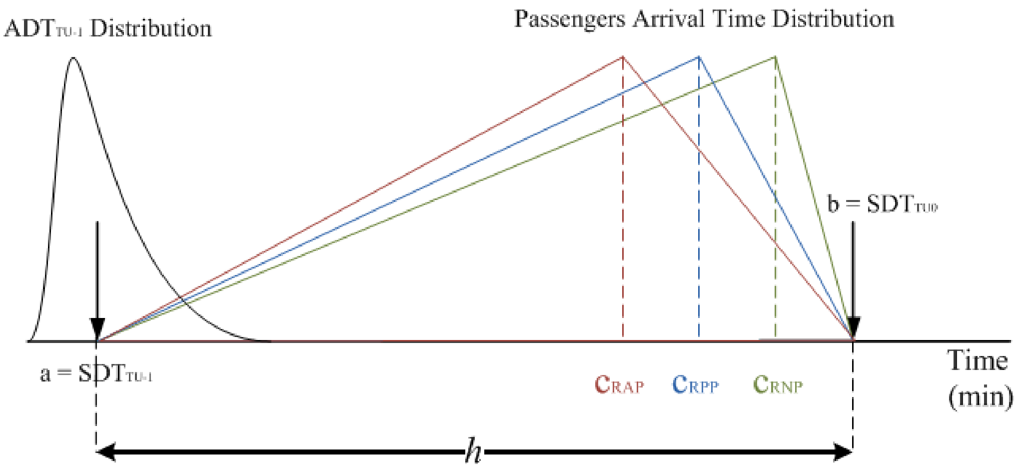


Figure 4.6: Peak Passengers Arrival Time for Different Passengers Classes

Table 4.7: Inputs into Simulation Model Related to Passengers Characteristics

Passenger Class	**Passenger aware of SDT %**	**Peak PsAT (C) (min)**
RAP	98	3.5
RPP	95	2
RNP	93	1

4.4.2 Output of Simulation Model

The simulation models each individual passenger trip. The model consists of four main steps:

1. Passenger generation;
2. Departure time alternatives evaluation;
3. Penalties estimation;
4. Calculation of GC.

The last step in the simulation model is estimating both GC functions: the GC_R and GC_C. At a passenger's trip level, the GC_C function for each O-D pair stations is deterministic and equal for all passengers. However, most of the variables within the GC_R model are stochastic and therefore the GC of a trip between a given pair of stations varies from one passenger to another even when the passengers have the same class. Thus, the average of the GC_R model for each passengers class is estimated and compared to the GC_C. Table 4.8 summarizes the different variables considered by GC_C and GC_R.

Table 4.8: Comparison of the Variables Included in GC_C and GC_R

Variables	GC_C	GC_R
WT	0.5 h	$f(ADT - PsAT)$
IVT	SIVT	$f(SIVT, IVTD)$
ADE	Not considered	$f(AAT - NAT)$
ADL	Not considered	$f(NAT + AAT) + \theta$
Anxiety	Not considered	$f(P_{Late}, DTT)$
SD	Not considered	$f(90^{th} - 50^{th})$

In the next chapter, results from the simulation are presented, discussed, and interpreted.

Chapter 5

Results and Discussion

In this chapter, results from the Monte-Carlo simulation are presented and discussed. The first section provides general statistical results from the simulation model. In Section 2, the conventional GC (GC_C) and the GC considering service reliability (GC_R) estimations are presented. Section 3 discusses the reliability measure used in this research. Finally, the relationship between the estimated GC_R with the service reliability measure is demonstrated.

5.1 Results

A total number of 156,000 passenger trips (10,000 between each O-D pair) were generated by the simulation over the entire iXpress network. One third of the passengers traveling between each O-D pair were of each of the three types namely risk-averse passengers (RAP), risk-moderate passengers (RMP), and risk-neutral passengers (RNP). All these passengers were modeled to select a particular bus for their trip that minimized their expected generalized cost.

Regarding the departure time evaluation process in the model, the results indicate that 68.64% of passengers selected the target bus (i.e. TU); 30.67% selected the previous bus (i.e. TU-1); and 0.69% selected the bus following the target bus (TU+1). Figure 5.1 demonstrates the portion of each passenger class that selected the different departure time alternatives.

As shown in Figure 5.2, almost all passengers (99.27%) that select the previous bus (TU-1) arrive prior to their necessary arrival time (NAT).On the other hand, between 3.7 and 7.2% of passengers who choose the target bus (TU) arrive late.

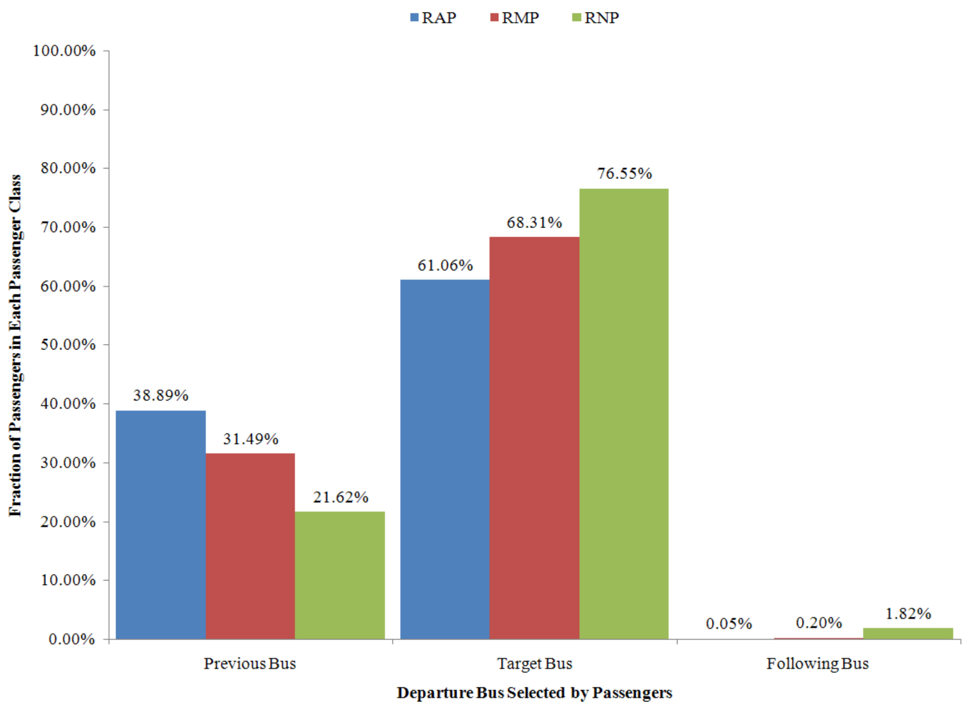


Figure 5.1: Selection of Departure Time Alternatives

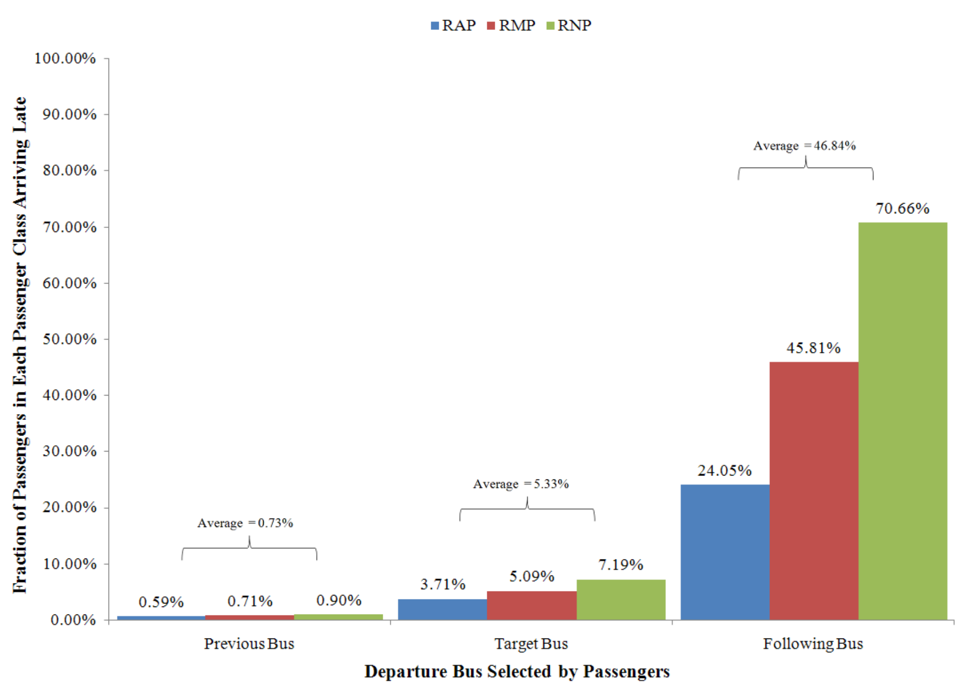


Figure 5.2: Fraction of Passengers Arriving Late at Destination

Between 24 and 70% of passengers who select the following bus (TU+1) arrive after their NAT. As expected, risk-neutral passengers are more likely to arrive late than other passenger classes because they incur the lowest penalty for doing so.

5.2 Generalized Cost Calculation Results

5.2.1 Conventional Generalized Cost(GC_C)

The conventional generalized cost formulation does not consider variability (reliability) of service. Consequently, it is useful to compare values obtained from the conventional generalized cost formulation (GC_C) with these obtained from the proposed reliability generalized cost formulation (GC_R). Therefore, the GC_C has been estimated for each stations pair. In the GC_C, it is assumed that for a given O-D pair, all passengers have the same average waiting time and experience the exact scheduled travel time (STT). Consequently, all passengers making a trip between the given O-D pair stations have the same generalized cost regardless of their actual waiting time, travel time, and risk tolerance characteristics. This is not a realistic assumption as is illustrated later.

In Figure 5.3, a random sample of 30 simulated passengers having the same risk aversion characteristics traveling from Conestoga Mall station to Uptown Waterloo station have been chosen as an example. This trip has a scheduled travel time (STT) of 20 minutes and a service headway (h) of 15 minutes. Hence, GC_C can be estimated as follows

$GC_C = 2.5WT + SIVT$

$WT = 0.5(h)$

$\therefore GC_C = 2.5(7.5) + 20 = 38.75\, minutes$

This cost value, as mentioned before, is assumed to be experienced by all passengers traveling over this section. However, as illustrated by the sample data shown in figure 5.3, there is a large variation in actual in-vehicle travel time and passenger waiting time compared to the values used with the GC_C calculation. Those variations are not captured by GC_C. Consequently, GC_C have been estimated for all possible O-D stations along the iXpress as a first step to be compared with GC_R to quantify the impact of travel time variability on users' generalized cost.

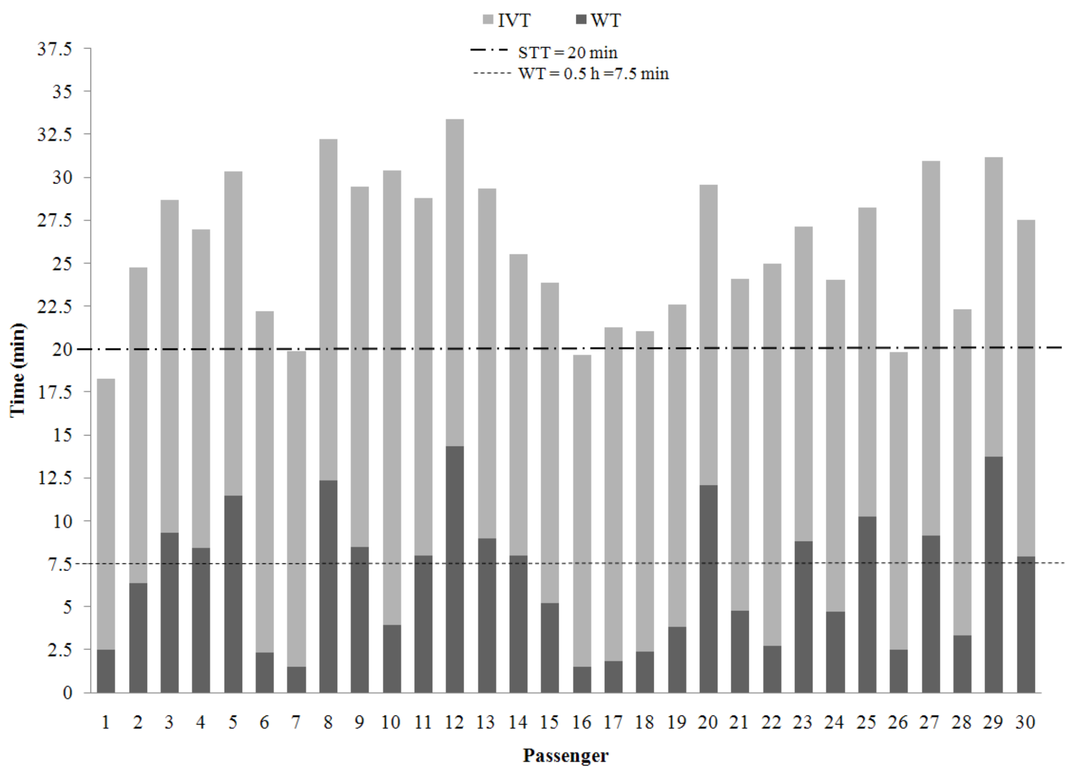


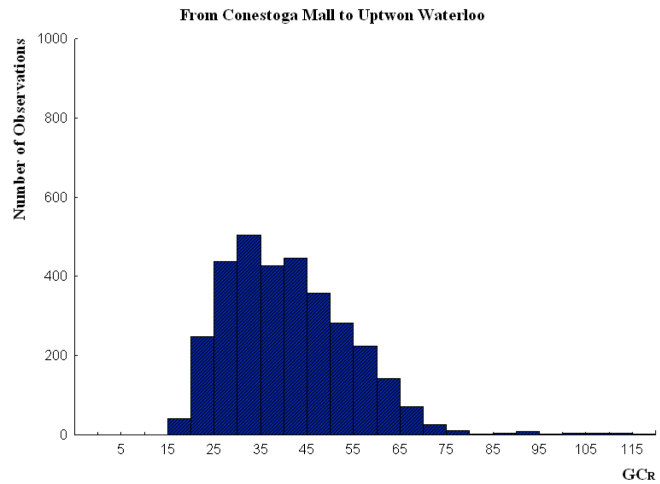
Figure 5.3: Passenger's Actual Waiting Time and In-vehicle Time compared to the Expected Values Considered in the GC_C

5.2.2 Generalized Cost Considering Service Reliability (GC_R)

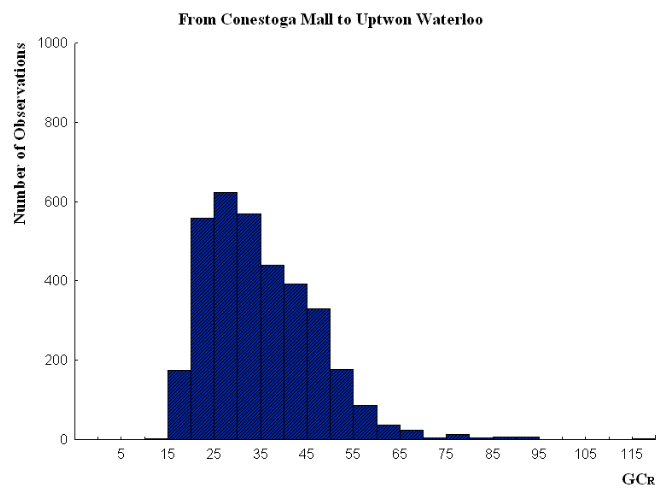
When the travel time variability is considered in the GC_R calculation, it is observed that the GC_R varies from one passenger's trip to another. The variations associated with GC_R result from the variability of passenger arrival time at boarding station, service reliability, passenger's necessary arrival time (NAT), and passenger aversion characteristic.

Considering the same trip as in the previous section (i.e. Conestoga Mall to Uptown Waterloo), the distribution of GC_R for each passenger class was calculated. Figure 5.4(a) illustrates the the distribution of GC_R for risk-averse passengers; Figure 5.4(b) illustrates the distribution for risk-moderate passengers; and Figure 5.4(c) illustrates the distribution for risk-neutral passengers.

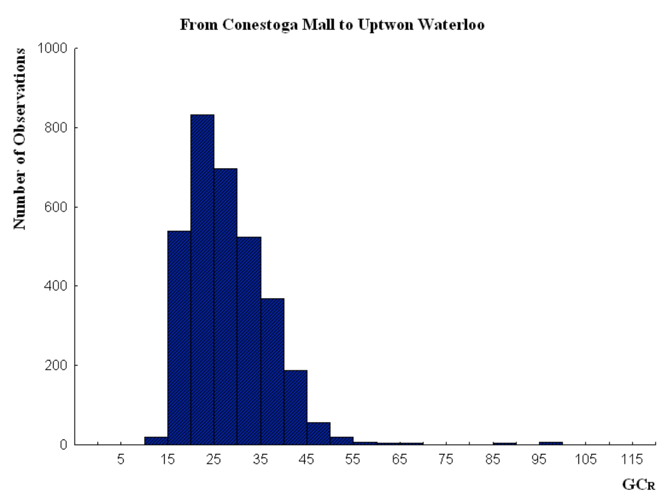
The results show that the distribution of GC_R is not symmetric and therefore standard measures of dispersion such as the standard deviation and coefficient of variation are not adequate descriptors. Consequently, the variation in the generalized cost is quantified by computing the difference between the 90^{th} percentile



(a) Case 1: Risk-Averse Passengers



(b) Case 2: Risk-Moderate Passengers



(c) Case 3: Risk-Neutral Passengers

Figure 5.4: Distribution of GC_R for Each Passenger class (Trip from Conestoga Mall to Uptown Waterloo)

and 50^{th} percentile (Table 5.1). The results show that risk-averse passengers are the most sensitive to service reliability as they experience both the highest average cost and the largest variation. In contrast, risk-neutral passengers are least sensitive to variations service reliability.

Table 5.1: Passenger Class GC Sensitivity to Variability in Transit Service (Conestoga Mall to Uptown Waterloo)

		GC_R	
Passenger Class	90^th %	50^th %	$90^th - 50^th$
Risk-averse (RAP)	59.11	39.69	19.42
Risk-moderate (RMP)	50.73	33.16	17.57
Risk-neutral (RNP)	39.44	26.58	12.86

5.3 Measure of Reliability

In Chapter 2, we noted that the literature has defined service reliability in several different ways. The most important reliability measures are those that reflect the passengers' perspective. Accordingly, a reliable system can be measured as the ability of service to adhere to scheduled departure time, scheduled travel time, and scheduled arrival time. All three of these measurements are influenced by travel time variability (TTV). Also, most of the variables included in the GC_R model such as arrival delay (AD), in-vehicle travel time delay ($IVTD$), and Anxiety (ANX) penalties are impacted by travel time variability. Consequently, we choose to classify transit service reliability in terms of the variability in travel time.

Statistically, TTV can be quantified in several ways: variance, standard deviation, coefficient of variance, etc. Alternatively, for non-symmetric distributions, the difference between two percentiles can give an indication of the spread of the data in addition to a good description of the distribution shape [10]. In the context of this research, three principle approaches have been used as reliability measures [17][12]:

1. Standard deviation or variance of travel time distribution;
2. The difference between the 90^{th} percentile and 50^{th} percentile of travel time distribution;

3. Scheduling delay: arriving earlier or later than NAT.

The relationship between GC_R and each of the service reliability measures above has been tested using regression analysis. The second reliability measure mentioned above provided the best relationship with GC_R model. Thus, the service reliability is identified by the difference between the 90^{th} and 50^{th} percentile of the difference in travel time (DTT) between actual (ATT) and scheduled travel time (STT). For a given trip, as the transit service becomes less reliable, the difference between DDT 90^{th} and 50^{th} percentile increases.

The difference between the DTT 90^{th} and 50^{th} percentile can be estimated as follows

1. Sort the difference in travel time data in ascending order, N is the number of records.
2. Specify the appropriate percentile p^{th},
3. Calculate the rank (n) of the value corresponding to the p^{th} percentile

$$n = \frac{N}{100}p + \frac{1}{2} \tag{5.1}$$

4. Take the value corresponds to rank n.

5.4 Reliability Impact on User's GC

It is desirable to understand the relationship between transit users' generalized cost and the reliability of transit service. The knowledge of this relationship permits the evaluation of initiatives or policies that impact service reliability. GC_R is influenced by service reliability as well as trip specific characteristics. Therefore, the nature of the relationship between service reliability and GC_R is first examined separately for each O-D pair.

Field data for the iXpress service provides only a single level of service reliability for each O-D pair. Therefore, a simulation model was developed to estimate users' GC_R corresponding to different service reliability levels. The service reliability levels of each O-D pair are controlled by multiplying the empirical data of the difference in travel time (DTT) by a factor. For example, to simulate a service that is more reliable than the field observed level, the field data is multiplied by a factor smaller than one. On the other hand, the data is multiplied by a factor greater than one to make the service less reliable than the field observed level.

For each segment (O-D pair stations), 31 different reliability levels were simulated by multiplying the field DTT data by factors ranging from 0 to 3.0 in increments of 0.1. The difference between the 90^{th} and 50^{th} percentile of the difference in travel time was calculated as a reliability measure. At each reliability level, 5000 passengers were simulated to estimate the average of GC_R.

A regression analysis was conducted with the average of GC_R as the dependent variable and the reliability measure as the independent variable. Consider the results for trips from Conestoga Mall to Uptown Waterloo in Figure 5.5. The data are best represented by an exponential relationship in the following form

$$GC_R = a\, e^{b\,(RM)} \tag{5.2}$$

where

RM	The reliability measure ($90^{th} - 50^{th}$ percentile of DTT)
a	Constant
b	Reliability measure coefficient

The coefficients of all three regressions are statistically significant and the regressions explain more than 98% of the variation observed in the data (Table 5.2). As expected, risk-averse passengers have a higher GC_R than other passengers for all levels of service reliability. Figure 5.5 also shows the conventional generalized cost (GC_C) for this trip which is constat for all levels of service reliability. At the observed service reliability obtained from the empirical data, it was found that the GC_C underestimates the GC_R cost for all passengers types (38% for risk-averse; 20% for risk-moderate; and less than 1% for risk-neutral).

A similar analysis was done for a longer trip, from Conestoga Mall to Ainslie Terminal (Figure 5.6 and Table 5.3). The expected travel time for this trip is 78 min. From these results, it can be observed that the GC_C cost underestimates the GC_R cost for both risk-averse and risk-moderate passengers by around 12% and 3.5%, respectively. However, for risk-neutral passengers, the GC_C cost overestimates the GC_R by approximately 5%. These results show that the level of over or underestimation of user costs (relative to GC_R) depends on the level of service reliability and passenger class.

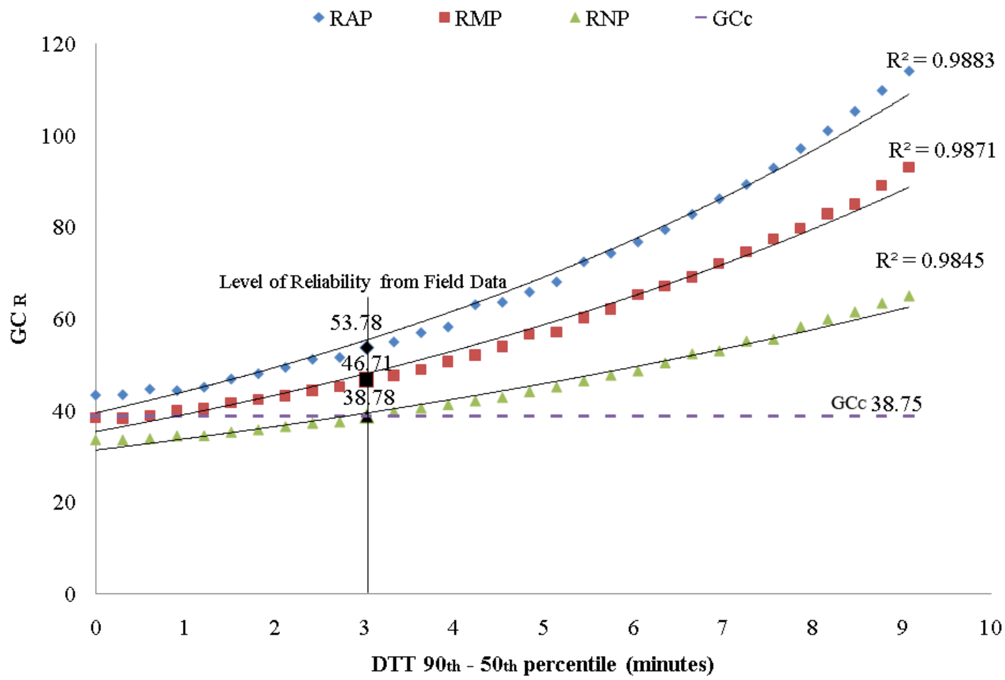


Figure 5.5: The Average of GC_R with Different Reliability Level for the trip From Conestoga Mall to Uptown Waterloo for the each Passenger Class

Table 5.2: Regression Analysis Coefficients for the Trip from Conestoga Mall to Uptown Waterloo

Passenger Class		Coefficients	t-value	p-value
RAP	Constant (a)	39.49	83.821	0.000
	Reliability Measure (b)	0.112	49.552	0.000
RMP	Constant (a)	35.4	88.157	0.000
	Reliability Measure (b)	0.101	47.158	0.000
RNP	Constant (a)	31.36	106.703	0.000
	Reliability Measure (b)	0.076	42.901	0.000

Table 5.3: Regression Analysis Coefficients for the Trip from Conestoga Mall to Ainslie

Passenger Class		Coefficients	t-value	p-value
RAP	Constant (a)	80.274	61.923	0.000
	Reliability Measure (b)	0.041	33.076	0.000
RMP	Constant (a)	77.592	73.903	0.000
	Reliability Measure (b)	0.034	32.942	0.000
RNP	Constant (a)	75.822	115.363	0.000
	Reliability Measure (b)	0.022	33.118	0.000

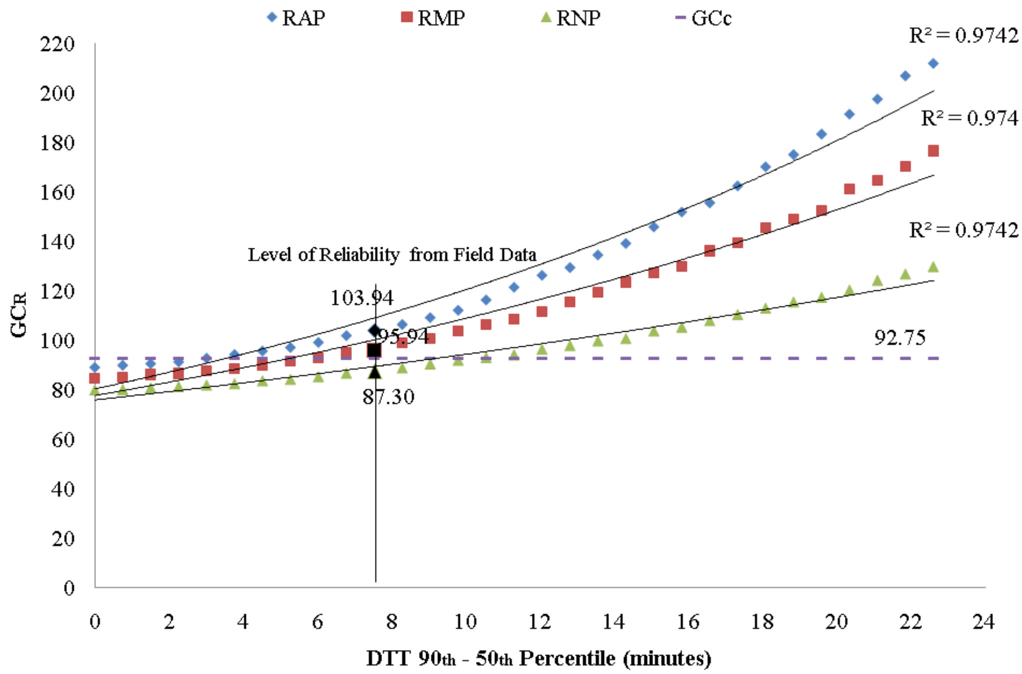


Figure 5.6: The Average of GC_R with Different Reliability Level for the trip From Conestoga Mall to Ainslie Terminal for the each Passenger Class

It is observed from the results above that passengers are sensitive to service reliability according to their characteristics. In addition, GC_R varies as a function of service reliability, passengers characteristics, and trip length. Therefore, a multiple linear regression analysis was implemented of each passenger class using the results from the simulation to model the relationship between the GC_R, reliability level, and trip length as follows

$$GC_R = a + b\,(RM) + c\,(SIVT) \tag{5.3}$$

where

RM	The reliability measure ($90^{th} - 50^{th}$ percentile of DTT)
$SIVT$	Scheduled in-vehicle time
a	Intercept
b	Reliability measure coefficient
c	Scheduled in-vehicle time coefficient

The regression analysis results are provided in Table 5.4. The three models obtained were statically significant and the signs of coefficients were logical. As observed, the risk-averse passengers are the most sensitive to service reliability because of the high weighting penalties incurred as shown in the last two figures. From Table 5.4, it is found that they are more sensitive to service reliability than risk-moderate passengers by around 31%. In contrast, the risk-neutral passengers are the least sensitive passengers to service reliability.

Table 5.4: The GC_R Models' coefficients as a Function of Reliability Level and Trip length

Passenger Class		**Coefficients**	**t-value**	**p-value**
RAP	Intercept (a)	37.90	47.20	0.000
	Reliability Level (b)	1.80	44.63	0.000
	SIVT (c)	0.32	12.14	0.000
RMP	Intercept (a)	30.37	50.14	0.000
	Reliability Level (b)	1.37	45.11	0.000
	SIVT (c)	0.44	22.30	0.000
RNP	Intercept (a)	20.99	63.57	0.000
	Reliability Level (b)	0.79	47.52	0.000
	SIVT (c)	0.63	57.68	0.000

In conclusion, the usage of conventional generalized cost GC_C function in mode split process is likely to be underestimating the generalized cost of transit mode. From the results obtained, the GC_C underestimates GC_R for 100% of the risk-averse passengers, around 75% of the risk-moderate passengers, and around 25% of the risk-neutral passengers. These underestimations mislead planners to make effective and right decisions. In contrast, the usage of the proposed GC_R formulation reflects the actual cost of the unreliable transit service. Unlike GC_C, the GC_R formulation has a significant relationship with service reliability as shown in the results that can be used to evaluate the benefits from the enhancement of service reliability versus the infrastructure investments.

Chapter 6

Conclusion and Future Research

6.1 Conclusion

This research has focused on quantifying and understanding the impact of service reliability on transit user cost. A review of previous studies that incorporated the service reliability effects on passenger's generalized cost models was conducted and limitations were found in these models. Based on the seminal work of Small [44], Noland and Small [32], and Bates [2], a generalized cost model was proposed by extending the existing models to address the limitations identified in the existing models.

Based on the assumed generalized cost model (GC_R) in this research, a Mont-Carlo simulation model has been developed to quantify the impact of service reliability on the user cost. The simulation model was developed at a passenger level by considering the distribution of bus departure time from origin, actual arrival time at destination, and actual in-vehicle travel time between O-D pair stations. These distributions are based on data obtained from AVLS in the iXpress service at the Regional Municipality of Waterloo. All passengers were assumed to fall in one of three passenger classes according to their assumed risk tolerance characteristics namely risk-averse, risk-moderate, or risk-neutral.

After running the simulation, for each given O-D stations pair, the generalized cost calculated by the proposed generalized cost formulation (GC_R) varies from one passenger to another due to the variation in travel time. With regard to the three passenger classes, the results showed that the risk-averse passengers are the most sensitive to service reliability. They were found to experience the highest (GC_R)

among the three classes. In contrast, the risk-neutral passengers were the least sensitive to variation in service reliability.

The variation in passenger waiting time and in-vehicle time were not captured by the conventional generalized cost model which assumes constant values for these two parameters for all passengers. Therefore, the generalized cost was the same for all passengers regardless of their characteristics.

The relationship between the generalized cost including both GC_C and GC_R with different reliability levels for two different trips (short and long) has been studied. This relationship for GC_C is known to be linear with a slope of zero between GC_C and the reliability. The regression analysis results between the GC_R and the reliability level showed a statistically significant exponential relationship which indicates that the GC_R was impacted by the variation in service reliability. The GC_R was increasing as the service unreliability increases.

Generally, it appears that the GC_C is more likely to underestimate the generalized cost. These magnitudes of underestimations of user cost relative to GC_R depends on the level of service reliability and passenger class. The GC_R provides the actual user cost that reflects the impact of the service unreliability based on the research assumptions.

It is recommended that planning studies for the purpose of evaluating the impact of transit service changes have on ridership should make use of GC that considers service reliability in order to predict more accurate estimates of ridership. More accurate ridership estimates help to more effectively evaluate the infrastructure investments in the transportation system.

6.2 Future Research

While this research sought to analyze comprehensively and synthesize the impact of service reliability on users cost, further research in this area is needed. Regarding the proposed generalized cost model, the parameters coefficients have to be calibrated to field data. In addition, further investigation and more attention have to be given to test and validate the technique used in valuing the anxiety term included in the model.

It was assumed in this research that the passengers are fall in one of three different classes with same fraction of total passengers. However, this assumption has to be studied and investigated in addition to calibrating the specific passenger class coefficients of each parameter within the GC_R model. Moreover, the peak of passenger arrival time distribution of different passenger classes needs to be considered as a function of the variation of service departure time. Finally, a more robust measure of reliability has to be investigated.

APPENDICES

Appendix A

Data Analysis Results

Table A.1: Number of Passengers Who rid TU-1 and arrived either Earlier or Later than their NAT

Station		**TU-1 (ontime)**			**TU-1 (Late)**		
From	**To**	**RAP**	**RMP**	**RNP**	**RAP**	**RMP**	**RNP**
1	2	404	219	0	0	0	0
1	3	393	270	43	0	0	1
1	4	186	54	0	0	0	0
1	5	748	673	481	0	0	0
1	6	1025	946	689	2	0	3
1	7	962	902	759	2	0	1
1	8	1427	1314	1073	1	2	0
1	9	1771	1595	1461	0	5	3
1	10	1874	1756	1591	5	3	3
1	11	2122	1749	1394	3	8	3
1	12	2307	2076	1836	6	5	6
1	13	3322	3290	2833	7	4	4
2	1	526	97	0	2	3	0
2	3	179	0	0	0	0	0
2	4	0	0	0	0	0	0
2	5	479	350	93	0	0	0
2	6	692	564	241	1	0	1
2	7	710	599	321	1	1	0
2	8	1106	972	680	2	1	4
2	9	1374	1324	1118	1	1	4
2	10	1552	1446	1181	4	1	4
2	11	1692	1523	1071	5	6	6
2	12	1965	1739	1661	3	5	6

Station		TU-1 (ontime)			TU-1 (Late)		
From	**To**	**RAP**	**RMP**	**RNP**	**RAP**	**RMP**	**RNP**
2	13	3341	3144	2605	4	10	4
3	1	465	109	0	6	0	0
3	2	455	0	0	7	0	0
3	4	0	0	0	0	0	0
3	5	483	350	82	1	1	1
3	6	788	586	242	2	1	1
3	7	755	615	352	1	3	2
3	8	1154	972	773	1	2	3
3	9	1535	1363	1160	3	5	3
3	10	1710	1472	1268	2	6	10
3	11	1675	1554	1148	3	2	2
3	12	2054	1890	1666	6	6	5
3	13	3277	3293	2496	6	9	9
4	1	558	212	0	7	0	0
4	2	418	0	0	5	0	0
4	3	474	0	0	1	0	0
4	5	357	263	0	4	1	0
4	6	602	429	104	2	1	2
4	7	736	552	287	2	2	3
4	8	1060	955	650	3	6	4
4	9	1372	1325	1145	5	3	4
4	10	1586	1411	1191	4	6	9
4	11	1778	1429	1083	10	8	6
4	12	2107	1809	1558	7	8	11
4	13	3364	3097	2299	8	9	11
5	1	710	426	23	3	7	0
5	2	464	221	0	1	3	0
5	3	511	339	32	5	3	0
5	4	627	292	0	3	6	0
5	6	242	64	0	0	0	0
5	7	308	170	0	0	0	0
5	8	691	446	160	3	2	1
5	9	1014	884	661	1	5	2
5	10	1137	1014	793	3	2	5

Station		TU-1 (ontime)			TU-1 (Late)		
From	**To**	**RAP**	**RMP**	**RNP**	**RAP**	**RMP**	**RNP**
5	11	1158	997	674	2	8	1
5	12	1565	1342	1207	2	6	2
5	13	3114	2702	1915	3	4	7
6	1	775	393	0	7	0	0
6	2	467	231	0	4	3	0
6	3	633	286	0	5	2	0
6	4	661	279	0	6	3	0
6	5	477	72	0	5	0	0
6	7	179	0	0	0	0	0
6	8	331	244	0	0	2	0
6	9	623	558	343	2	4	4
6	10	831	710	423	1	1	2
6	11	1176	985	443	7	2	2
6	12	2059	1298	952	2	3	7
6	13	2795	2675	1635	6	9	4
7	1	894	515	43	8	5	0
7	2	608	330	5	8	4	0
7	3	731	443	78	12	6	0
7	4	631	364	32	5	6	0
7	5	430	165	0	6	4	0
7	6	668	235	0	6	6	0
7	8	357	212	0	0	3	0
7	9	792	626	406	1	9	2
7	10	839	790	562	5	10	3
7	11	959	796	461	5	12	7
7	12	1290	1167	921	6	7	11
7	13	2688	2292	1724	13	11	18
8	1	1381	1120	692	10	11	2
8	2	1051	839	562	6	7	6
8	3	1111	897	637	7	9	6
8	4	1120	895	572	12	9	8
8	5	742	551	286	5	5	2
8	6	822	644	320	9	8	4
8	7	676	536	77	6	9	2

Station		TU-1 (ontime)			TU-1 (Late)		
From	**To**	**RAP**	**RMP**	**RNP**	**RAP**	**RMP**	**RNP**
8	9	265	191	0	0	0	0
8	10	395	299	97	0	1	1
8	11	710	443	139	3	2	1
8	12	978	799	653	4	2	3
8	13	2387	2037	1207	5	5	8
9	1	1625	1168	741	15	15	9
9	2	1140	899	584	9	8	9
9	3	1208	986	598	10	17	7
9	4	1215	913	637	13	16	15
9	5	833	698	305	11	9	2
9	6	924	723	394	9	9	10
9	7	753	612	331	6	12	7
9	8	393	156	0	5	2	0
9	10	49	0	0	0	0	0
9	11	399	172	0	3	1	0
9	12	715	557	416	2	2	2
9	13	2049	1562	876	6	6	5
10	1	1906	1425	1013	18	9	8
10	2	1349	1129	858	18	9	16
10	3	1459	1240	928	15	13	21
10	4	1364	1188	925	22	17	23
10	5	1110	845	554	11	10	10
10	6	1060	948	714	16	12	11
10	7	982	829	639	8	19	10
10	8	622	412	181	3	3	2
10	9	517	304	70	2	3	1
10	11	179	0	0	0	0	0
10	12	611	468	263	1	1	0
10	13	1907	1345	762	8	6	4
11	1	2935	2400	1772	26	25	18
11	2	2232	1853	1609	34	25	22
11	3	2501	2011	1680	32	38	36
11	4	2276	2047	1523	30	21	27
11	5	1824	1599	1291	25	27	23

Station		TU-1 (ontime)			TU-1 (Late)		
From	To	RAP	RMP	RNP	RAP	RMP	RNP
11	6	1873	1659	1379	37	27	40
11	7	1818	1521	1180	35	23	38
11	8	1765	1254	799	24	19	24
11	9	1329	1105	726	15	18	14
11	10	1259	825	163	15	13	4
11	12	77	0	0	1	0	0
11	13	1201	768	244	11	13	7
12	1	2801	2616	1877	16	20	19
12	2	2648	2114	1613	16	23	14
12	3	2503	2325	1678	18	19	15
12	4	2430	2159	1618	26	23	17
12	5	2144	1827	1226	15	9	5
12	6	2157	1799	1363	16	24	23
12	7	2066	1678	1305	16	18	20
12	8	2053	1630	916	19	12	9
12	9	1818	1262	725	12	14	10
12	10	1605	1122	477	17	11	10
12	11	342	112	0	1	0	0
12	13	632	293	0	6	5	0
13	1	3231	2861	2387	11	15	18
13	2	2723	2586	2009	10	17	16
13	3	2917	2544	2091	19	15	22
13	4	2922	2654	1946	17	18	19
13	5	2560	2189	1624	16	20	21
13	6	2593	2309	1849	23	25	22
13	7	2636	2194	1618	32	28	24
13	8	2740	2183	1372	12	20	14
13	9	2528	1890	1189	9	13	17
13	10	2381	1687	1060	14	10	9
13	11	704	432	122	2	0	0
13	12	467	179	0	2	0	0

Table A.2: Number of Passengers Who rid TU and arrived either Earlier or Later than their NAT

Station		**TU (ontime)**			**TU (Late)**		
From	**To**	**RAP**	**RMP**	**RNP**	**RAP**	**RMP**	**RNP**
1	2	2942	2852	3006	89	163	
1	3	2847	2959	3072	79	123	
1	4	3131	3203	3014	54	84	
1	5	2577	2586	2784	36	45	
1	6	2330	2327	2580	28	20	
1	7	2350	2408	2546	16	19	
1	8	1946	1987	2201	15	9	
1	9	1548	1705	1860	16	15	
1	10	1388	1578	1762	12	9	
1	11	1197	1594	1882	11	10	
1	12	1026	1251	1451	10	9	
1	13	0	0	537	0	0	
2	1	2639	2987	2689	152	282	
2	3	2991	3079	2541	105	249	
2	4	3087	3017	2783	62	79	
2	5	2789	2858	3159	43	64	
2	6	2665	2687	2951	44	56	
2	7	2563	2717	2964	28	39	
2	8	2175	2328	2626	27	31	
2	9	1992	1960	2159	13	23	
2	10	1816	1842	2084	21	20	
2	11	1561	1797	2270	15	15	
2	12	1365	1488	1726	8	14	

Station		TU (ontime)			TU (Late)		
From	To	RAP	RMP	RNP	RAP	RMP	RNP
2	13	0	0	886	0	0	6
3	1	2657	2970	2754	162	248	331
3	2	2764	2873	2492	164	340	323
3	4	3073	2925	2770	103	84	111
3	5	2854	2850	3032	68	90	188
3	6	2484	2619	3015	37	66	159
3	7	2572	2656	2858	37	49	100
3	8	2123	2309	2550	17	41	55
3	9	1790	1924	2125	16	25	51
3	10	1594	1880	1976	17	24	41
3	11	1566	1788	2161	22	22	57
3	12	1236	1393	1681	16	19	28
3	13	0	80	816	0	1	13
4	1	2614	2972	2727	125	193	285
4	2	2761	2947	2682	132	295	347
4	3	2617	2862	2689	211	375	435
4	5	2834	2860	2289	158	235	439
4	6	2633	2792	2870	91	150	324
4	7	2577	2666	2868	64	72	171
4	8	2202	2321	2577	49	57	116
4	9	1903	1994	2078	35	55	81
4	10	1743	1861	2062	22	35	70
4	11	1498	1849	2218	22	33	66
4	12	1205	1511	1698	18	31	37
4	13	0	193	1001	0	1	17
5	1	2436	2853	2940	114	171	317
5	2	2691	2912	2864	166	257	397
5	3	2673	2722	2716	197	285	517
5	4	2542	2617	2423	190	385	618
5	6	3002	3035	2733	123	198	260
5	7	2875	3004	3046	101	144	175
5	8	2637	2749	2964	59	93	195
5	9	2277	2398	2551	53	71	83
5	10	2144	2217	2539	40	46	60

Station		TU (ontime)			TU (Late)		
From	**To**	**RAP**	**RMP**	**RNP**	**RAP**	**RMP**	**RNP**
5	11	2108	2358	2530	40	39	85
5	12	1709	1974	2101	23	39	30
5	13	293	615	1322	1	8	16
6	1	2483	2771	3103	85	142	202
6	2	2758	2909	3028	111	189	238
6	3	2619	2816	2923	135	211	358
6	4	2517	2851	2794	130	267	459
6	5	2736	2868	2780	135	270	346
6	7	3108	3049	2516	119	168	161
6	8	2842	2916	3067	109	162	294
6	9	2646	2559	2810	106	128	217
6	10	2408	2623	2720	60	70	151
6	11	2060	2335	2791	34	55	110
6	12	1276	2013	2272	13	29	76
6	13	524	685	1619	2	7	39
7	1	2377	2630	3108	82	123	215
7	2	2618	2874	3021	103	160	269
7	3	2483	2752	2891	108	184	312
7	4	2561	2706	2914	144	225	412
7	5	2757	2987	2718	130	234	305
7	6	2489	2732	2556	150	344	550
7	8	2844	2854	2775	144	241	380
7	9	2395	2515	2746	113	153	242
7	10	2345	2564	2521	71	96	194
7	11	2342	2409	2663	71	103	172
7	12	1957	2092	2308	56	77	108
7	13	540	1002	1628	5	24	55
8	1	1947	2071	2485	58	91	132
8	2	2199	2404	2607	80	99	140
8	3	2123	2274	2565	83	117	171
8	4	2084	2376	2559	93	99	173
8	5	2542	2599	2802	107	130	229
8	6	2489	2435	2656	123	166	324
8	7	2496	2602	2632	160	203	601

Station		TU (ontime)			TU (Late)		
From	**To**	**RAP**	**RMP**	**RNP**	**RAP**	**RMP**	**RNP**
8	9	2889	2957	2396	141	214	347
8	10	2889	2945	2988	67	103	215
8	11	2505	2837	3099	45	77	139
8	12	2263	2422	2722	42	48	64
8	13	903	1316	2057	7	17	51
9	1	1634	2101	2354	69	98	171
9	2	2164	2271	2517	106	101	192
9	3	2006	2221	2456	123	141	227
9	4	2071	2136	2427	116	162	279
9	5	2340	2539	2634	128	184	317
9	6	2213	2381	2423	216	244	454
9	7	2280	2395	2465	273	335	531
9	8	2692	2825	2591	248	365	473
9	10	3167	3039	2914	144	190	210
9	11	2842	3055	3049	67	109	149
9	12	2627	2670	2818	52	66	73
9	13	1296	1731	2378	9	16	66
10	1	1457	1756	2192	39	69	108
10	2	1878	2061	2345	81	92	164
10	3	1758	1988	2252	82	90	154
10	4	1842	2002	2194	99	121	203
10	5	2247	2176	2566	104	149	218
10	6	2128	2150	2365	151	186	259
10	7	2155	2225	2383	197	236	317
10	8	2538	2742	2755	179	224	339
10	9	2559	2760	2909	191	263	421
10	11	3061	3060	2856	105	171	156
10	12	2657	2790	2992	60	68	89
10	13	1358	1971	2503	15	42	79
11	1	369	900	1426	12	29	88
11	2	984	1342	1674	41	69	115
11	3	753	1163	1551	40	69	126
11	4	933	1317	1570	40	83	133
11	5	1395	1539	1889	93	117	178

Station		TU (ontime)			TU (Late)		
From	**To**	**RAP**	**RMP**	**RNP**	**RAP**	**RMP**	**RNP**
11	6	1252	1491	1801	92	144	205
11	7	1354	1617	1888	105	173	248
11	8	1450	1927	2203	89	160	286
11	9	1819	2034	2283	138	216	303
11	10	1949	2189	2728	140	236	479
11	12	3028	2992	2819	216	288	329
11	13	1976	2354	2820	103	198	305
12	1	513	664	1374	13	16	71
12	2	686	1117	1632	18	39	80
12	3	755	1000	1570	17	39	61
12	4	845	1081	1630	31	44	96
12	5	1159	1433	1946	36	75	125
12	6	1083	1540	1711	38	87	159
12	7	1171	1515	1838	73	119	181
12	8	1154	1649	2258	37	79	184
12	9	1436	2014	2327	60	117	205
12	10	1693	2030	2561	78	154	242
12	11	2872	3037	2884	104	178	237
12	13	2551	2778	2810	159	228	327
13	1	122	388	930	2	10	25
13	2	450	848	1274	10	15	42
13	3	381	736	1205	6	17	47
13	4	335	693	1293	15	34	54
13	5	827	1038	1560	21	41	83
13	6	627	929	1482	24	39	78
13	7	630	991	1684	22	42	99
13	8	555	1097	1864	15	30	98
13	9	788	1290	2101	20	51	104
13	10	947	1583	2116	22	62	109
13	11	2580	2818	2963	76	126	177
13	12	2761	2941	2838	107	234	257

Table A.3: Number of Passengers Who rid TU+1 and arrived either Earlier or Later than their NAT

Station		**TU+1 (ontime)**			**TU+1 (Late)**		
From	**To**	**RAP**	**RMP**	**RNP**	**RAP**	**RMP**	**RNP**
1	2	0	0	1	0	0	11
1	3	0	0	0	0	0	0
1	4	0	0	110	0	0	47
1	5	0	0	0	0	0	0
1	6	0	0	0	0	0	0
1	7	0	0	0	0	0	0
1	8	0	0	0	0	0	0
1	9	0	0	0	0	0	0
1	10	0	0	0	0	0	0
1	11	0	0	0	0	0	0
1	12	0	0	0	0	0	0
1	13	0	0	0	0	0	0
2	1	0	0	110	0	0	175
2	3	0	18	72	0	36	513
2	4	90	188	253	24	96	234
2	5	0	0	0	0	0	0
2	6	0	0	0	0	0	0
2	7	0	0	0	0	0	0
2	8	0	0	0	0	0	0
2	9	0	0	0	0	0	0
2	10	0	0	0	0	0	0
2	11	0	0	0	0	0	0
2	12	0	0	0	0	0	0

Station		TU+1 (ontime)			TU+1 (Late)		
From	**To**	**RAP**	**RMP**	**RNP**	**RAP**	**RMP**	**RNP**
2	13	0	0	0	0	0	0
3	1	0	0	147	0	0	151
3	2	0	40	85	0	71	386
3	4	90	168	230	33	109	304
3	5	0	0	0	0	0	0
3	6	0	0	0	0	0	0
3	7	0	0	0	0	0	0
3	8	0	0	0	0	0	0
3	9	0	0	0	0	0	0
3	10	0	0	0	0	0	0
3	11	0	0	0	0	0	0
3	12	0	0	0	0	0	0
3	13	0	0	0	0	0	0
4	1	0	0	151	0	0	156
4	2	0	50	90	0	54	219
4	3	0	0	58	0	5	273
4	5	0	0	12	0	0	548
4	6	0	0	0	0	0	0
4	7	0	0	0	0	0	0
4	8	0	0	0	0	0	0
4	9	0	0	0	0	0	0
4	10	0	0	0	0	0	0
4	11	0	0	0	0	0	0
4	12	0	0	0	0	0	0
4	13	0	0	0	0	0	0
5	1	0	0	0	0	0	0
5	2	0	0	5	0	0	19
5	3	0	0	0	0	0	0
5	4	0	0	16	0	0	281
5	6	0	0	75	0	0	268
5	7	0	0	87	0	0	90
5	8	0	0	0	0	0	0
5	9	0	0	0	0	0	0
5	10	0	0	0	0	0	0

Station		TU+1 (ontime)			TU+1 (Late)		
From	To	RAP	RMP	RNP	RAP	RMP	RNP
5	11	0	0	0	0	0	0
5	12	0	0	0	0	0	0
5	13	0	0	0	0	0	0
6	1	0	0	22	0	0	17
6	2	0	0	39	0	0	23
6	3	0	0	5	0	0	7
6	4	0	0	8	0	0	25
6	5	0	0	67	0	0	244
6	7	0	21	112	0	42	525
6	8	0	0	19	0	0	14
6	9	0	0	0	0	0	0
6	10	0	0	0	0	0	0
6	11	0	0	0	0	0	0
6	12	0	0	0	0	0	0
6	13	0	0	0	0	0	0
7	1	0	0	0	0	0	0
7	2	0	0	0	0	0	0
7	3	0	0	0	0	0	0
7	4	0	0	0	0	0	0
7	5	0	0	72	0	0	192
7	6	0	0	23	0	0	241
7	8	0	0	60	0	0	130
7	9	0	0	0	0	0	0
7	10	0	0	0	0	0	0
7	11	0	0	0	0	0	0
7	12	0	0	0	0	0	0
7	13	0	0	0	0	0	0
8	1	0	0	0	0	0	0
8	2	0	0	0	0	0	0
8	3	0	0	0	0	0	0
8	4	0	0	0	0	0	0
8	5	0	0	0	0	0	0
8	6	0	0	0	0	0	0
8	7	0	0	0	0	0	0

Station		TU+1 (ontime)			TU+1 (Late)		
From	**To**	**RAP**	**RMP**	**RNP**	**RAP**	**RMP**	**RNP**
8	9	0	0	19	0	0	581
8	10	0	0	0	0	0	0
8	11	0	0	0	0	0	0
8	12	0	0	0	0	0	0
8	13	0	0	0	0	0	0
9	1	0	0	0	0	0	0
9	2	0	0	0	0	0	0
9	3	0	0	0	0	0	0
9	4	0	0	0	0	0	0
9	5	0	0	0	0	0	0
9	6	0	0	0	0	0	0
9	7	0	0	0	0	0	0
9	8	0	0	26	0	0	224
9	10	0	16	75	0	38	158
9	11	0	0	109	0	0	45
9	12	0	0	0	0	0	0
9	13	0	0	0	0	0	0
10	1	0	0	0	0	0	0
10	2	0	0	0	0	0	0
10	3	0	0	0	0	0	0
10	4	0	0	0	0	0	0
10	5	0	0	0	0	0	0
10	6	0	0	0	0	0	0
10	7	0	0	0	0	0	0
10	8	0	0	0	0	0	0
10	9	0	0	0	0	0	0
10	11	0	48	202	0	18	144
10	12	0	0	0	0	0	0
10	13	0	0	0	0	0	0
11	1	0	0	0	0	0	0
11	2	0	0	0	0	0	0
11	3	0	0	0	0	0	0
11	4	0	0	0	0	0	0
11	5	0	0	0	0	0	0

Station		TU+1 (ontime)			TU+1 (Late)		
From	**To**	**RAP**	**RMP**	**RNP**	**RAP**	**RMP**	**RNP**
11	6	0	0	0	0	0	0
11	7	0	0	0	0	0	0
11	8	0	0	0	0	0	0
11	9	0	0	0	0	0	0
11	10	0	0	0	0	0	0
11	12	0	14	109	0	7	120
11	13	0	0	0	0	0	0
12	1	0	0	0	0	0	0
12	2	0	0	0	0	0	0
12	3	0	0	0	0	0	0
12	4	0	0	0	0	0	0
12	5	0	0	0	0	0	0
12	6	0	0	0	0	0	0
12	7	0	0	0	0	0	0
12	8	0	0	0	0	0	0
12	9	0	0	0	0	0	0
12	10	0	0	0	0	0	0
12	11	0	0	90	0	0	143
12	13	0	0	140	0	0	71
13	1	0	0	0	0	0	0
13	2	0	0	0	0	0	0
13	3	0	0	0	0	0	0
13	4	0	0	0	0	0	0
13	5	0	0	0	0	0	0
13	6	0	0	0	0	0	0
13	7	0	0	0	0	0	0
13	8	0	0	0	0	0	0
13	9	0	0	0	0	0	0
13	10	0	0	0	0	0	0
13	11	0	0	0	0	0	0
13	12	0	0	86	0	0	128

Table A.4: Average GC_R and GC_C of each stations Pair for each Passenger Class

Station		GC_R **(minutes)**			GC_C **(minutes)**
From	**To**	**RAP**	**RMP**	**RNP**	
1	2	39.83	33.07	24.79	24.75
1	3	41.78	35.46	27.03	27.75
1	4	41.71	35.52	28.22	31.75
1	5	48.75	42.00	34.83	34.75
1	6	52.75	46.15	38.28	38.75
1	7	55.98	48.65	41.25	41.75
1	8	61.13	54.51	46.66	46.75
1	9	62.88	56.57	49.52	53.75
1	10	66.26	60.63	54.65	60.75
1	11	78.66	72.44	66.26	75.75
1	12	84.60	79.34	72.76	81.75
1	13	103.98	96.59	87.54	92.75
2	1	48.06	40.23	28.75	25.75
2	3	35.90	28.77	20.53	21.75
2	4	35.47	29.15	21.76	25.75
2	5	42.21	36.27	28.06	28.75
2	6	46.69	40.30	31.99	32.75
2	7	50.29	43.00	34.69	35.75
2	8	55.49	48.71	40.84	40.75
2	9	57.18	50.52	42.90	47.75
2	10	60.06	54.97	48.65	54.75
2	11	72.16	66.70	60.26	69.75

Station		GC_R (minutes)			GC_C (minutes)
From	**To**	**RAP**	**RMP**	**RNP**	
2	12	79.00	73.26	66.88	75.75
2	13	98.27	91.13	81.58	86.75
3	1	51.11	42.23	31.58	28.75
3	2	42.46	34.90	23.66	21.75
3	4	32.67	26.28	19.04	22.75
3	5	39.88	33.47	25.45	25.75
3	6	43.10	37.33	29.75	29.75
3	7	46.69	40.27	32.01	32.75
3	8	52.72	45.83	37.94	37.75
3	9	54.04	47.66	40.74	44.75
3	10	57.59	51.81	46.01	51.75
3	11	68.67	63.95	57.56	66.75
3	12	76.14	70.58	64.13	72.75
3	13	94.90	87.62	78.95	83.75
4	1	54.57	45.81	33.79	31.75
4	2	46.16	37.82	26.99	24.75
4	3	44.60	36.08	24.58	21.75
4	5	37.43	30.82	23.24	21.75
4	6	41.05	34.28	26.75	25.75
4	7	44.39	37.05	29.33	28.75
4	8	50.00	43.31	35.20	33.75
4	9	51.06	44.88	37.71	40.75
4	10	54.50	48.82	43.24	47.75
4	11	66.81	61.33	54.73	62.75
4	12	72.48	68.00	61.53	68.75
4	13	92.09	84.69	75.91	79.75
5	1	57.45	49.65	37.91	34.75
5	2	50.23	41.57	30.97	27.75
5	3	49.14	39.76	29.32	24.75
5	4	47.18	37.98	27.37	21.75
5	6	37.10	30.05	22.21	22.75
5	7	40.59	33.43	24.78	25.75
5	8	45.00	38.75	30.57	30.75
5	9	47.03	40.47	32.84	38.75

Station		GC_R (minutes)			GC_C (minutes)
From	**To**	**RAP**	**RMP**	**RNP**	
5	10	50.59	44.92	38.96	44.75
5	11	63.09	56.41	50.65	59.75
5	12	68.79	63.31	56.64	65.75
5	13	88.54	81.26	71.23	76.75
6	1	62.28	53.42	41.89	39.75
6	2	54.62	45.81	34.94	32.75
6	3	53.52	44.17	33.43	29.75
6	4	52.02	42.33	31.27	26.75
6	5	46.34	36.56	25.83	23.75
6	7	35.62	28.84	20.54	21.75
6	8	41.56	34.73	27.12	26.75
6	9	43.26	36.99	29.07	33.75
6	10	46.54	40.94	34.95	40.75
6	11	58.56	53.07	46.55	55.75
6	12	65.10	59.58	53.16	61.75
6	13	84.78	77.26	67.59	72.75
7	1	66.69	56.76	45.50	42.75
7	2	58.07	49.52	38.77	35.75
7	3	57.29	47.96	36.68	32.75
7	4	56.02	46.33	34.65	29.75
7	5	49.31	40.10	29.33	26.75
7	6	46.81	37.62	25.36	21.75
7	8	39.39	32.65	24.99	23.75
7	9	40.32	34.46	27.90	30.75
7	10	43.95	39.29	32.86	37.75
7	11	55.38	51.36	45.34	52.75
7	12	62.64	57.76	51.49	58.75
7	13	82.11	75.67	65.88	69.75
8	1	68.57	60.06	49.94	46.75
8	2	60.92	52.77	42.28	39.75
8	3	59.23	51.34	40.85	36.75
8	4	58.47	49.71	38.56	33.75
8	5	51.80	43.64	33.76	30.75
8	6	49.56	41.01	30.40	25.75

Station		GC_R (minutes)			GC_C (minutes)
From	To	RAP	RMP	RNP	
8	7	46.17	37.27	27.70	22.75
8	9	35.77	28.63	21.52	21.75
8	10	38.05	32.81	26.73	28.75
8	11	51.04	44.37	38.37	43.75
8	12	57.25	50.73	45.10	49.75
8	13	76.24	68.83	59.30	60.75
9	1	74.72	65.50	54.11	54.75
9	2	67.72	58.81	48.00	47.75
9	3	66.30	57.56	46.09	44.75
9	4	64.40	55.29	43.61	41.75
9	5	58.64	49.44	38.96	38.75
9	6	56.73	45.90	35.49	33.75
9	7	53.41	43.09	32.22	30.75
9	8	46.97	37.39	26.44	22.75
9	10	34.84	29.83	22.74	25.75
9	11	46.98	41.64	34.77	40.75
9	12	54.01	47.79	41.32	46.75
9	13	73.10	65.26	55.94	57.75
10	1	80.76	72.23	61.71	61.75
10	2	73.31	65.03	54.72	54.75
10	3	71.90	63.05	52.84	51.75
10	4	70.73	62.54	50.76	48.75
10	5	64.80	56.32	45.42	45.75
10	6	61.94	53.40	42.54	40.75
10	7	60.63	50.25	39.55	37.75
10	8	53.48	45.22	34.29	29.75
10	9	49.50	41.21	30.30	25.75
10	11	40.95	35.15	28.60	31.75
10	12	46.97	41.67	35.00	37.75
10	13	67.11	59.10	49.45	48.75
11	1	95.69	87.85	76.94	77.75
11	2	89.42	80.84	70.31	70.75
11	3	86.60	79.03	68.07	67.75
11	4	86.29	76.99	66.61	64.75

Station		GC_R (minutes)			GC_C (minutes)
From	To	RAP	RMP	RNP	
11	5	81.66	71.89	61.37	61.75
11	6	78.15	69.57	58.31	56.75
11	7	74.88	66.62	55.11	53.75
11	8	69.26	60.61	50.25	45.75
11	9	66.58	58.43	46.42	41.75
11	10	57.30	48.51	38.08	32.75
11	12	37.47	33.03	24.77	24.75
11	13	57.16	50.13	40.75	35.75
12	1	100.42	92.47	82.50	84.75
12	2	93.13	85.31	75.25	77.75
12	3	92.06	83.08	73.81	74.75
12	4	90.73	82.51	71.49	71.75
12	5	84.90	76.82	66.31	68.75
12	6	81.87	73.80	63.74	63.75
12	7	78.57	70.90	60.53	60.75
12	8	73.40	65.67	55.34	52.75
12	9	70.23	61.69	51.36	48.75
12	10	61.12	53.18	43.50	39.75
12	11	44.06	35.80	26.37	25.75
12	13	49.55	42.74	31.92	29.75
13	1	109.93	101.58	91.66	94.75
13	2	101.98	94.27	84.47	87.75
13	3	101.00	92.72	82.65	84.75
13	4	98.94	91.16	80.66	81.75
13	5	93.92	85.65	75.89	78.75
13	6	91.95	83.07	72.50	73.75
13	7	88.31	80.24	69.72	70.75
13	8	83.32	74.77	64.85	62.75
13	9	79.50	71.84	60.86	58.75
13	10	70.90	62.75	52.97	49.75
13	11	53.78	45.89	36.34	35.75
13	12	46.56	38.87	29.23	28.75

Table A.5: Statistical Dispersion Measures of *DTT* data of each Stations Pair

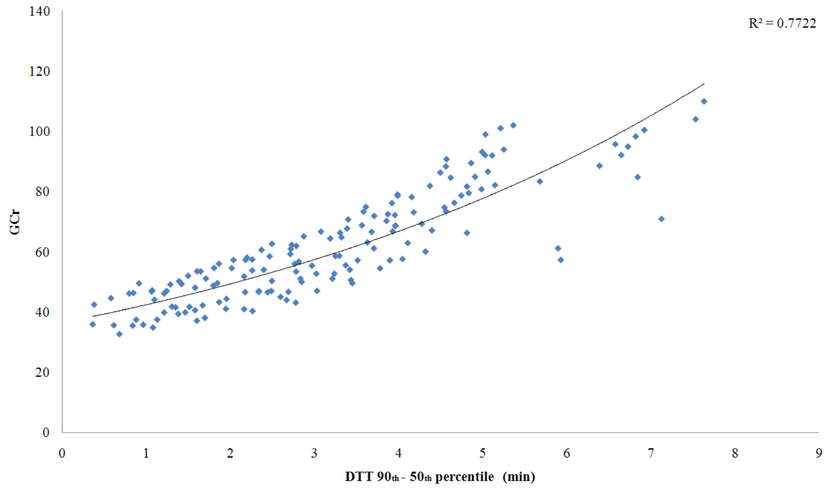
Station		**Data Dispersion Measures (minute)**					
From	**To**	**Mean**	**St Deviation**	**COV**	$90^{th}\%$	$50^{th}\%$	$90^{th} - 50^{th}$
1	2	0.57	1.10	1.92	1.70	0.48	1.22
1	3	0.33	1.20	3.68	1.58	0.27	1.31
1	4	-1.06	1.74	-1.64	0.42	-1.10	1.52
1	5	1.35	2.29	1.69	3.17	1.37	1.80
1	6	1.50	3.35	2.23	4.25	1.23	3.03
1	7	1.43	2.77	1.94	4.07	1.30	2.77
1	8	2.09	3.62	1.73	5.68	1.97	3.71
1	9	3.09	4.49	1.45	6.81	2.69	4.11
1	10	2.89	4.64	1.61	7.15	2.33	4.82
1	11	1.80	4.88	2.71	6.26	1.51	4.75
1	12	3.46	5.87	1.69	8.12	3.50	4.62
1	13	4.04	7.27	1.80	10.90	3.37	7.54
2	1	-0.88	2.08	-2.37	0.38	-1.20	1.58
2	3	-0.59	0.67	-1.13	-0.23	-0.60	0.37
2	4	-1.97	2.08	-1.05	-1.20	-2.04	0.84
2	5	0.49	1.35	2.76	2.11	0.43	1.67
2	6	0.65	2.45	3.77	2.96	0.27	2.69
2	7	0.55	1.96	3.54	2.96	0.47	2.50
2	8	1.24	2.74	2.22	4.38	1.01	3.37
2	9	2.26	3.62	1.60	5.77	2.25	3.52
2	10	2.09	3.81	1.82	6.11	1.78	4.32
2	11	1.09	4.29	3.92	4.98	1.02	3.96

Station		Data Dispersion Measures (minute)					
From	**To**	**Mean**	**St Deviation**	**COV**	$90^{th}\%$	$50^{th}\%$	$90^{th} - 50^{th}$
2	12	2.81	5.16	1.84	7.09	3.10	3.99
2	13	3.41	6.65	1.95	9.94	3.12	6.82
3	1	-1.33	2.40	-1.80	0.07	-1.64	1.71
3	2	-0.80	0.87	-1.08	-0.43	-0.82	0.38
3	4	-1.54	1.63	-1.06	-0.95	-1.63	0.68
3	5	0.95	1.46	1.54	2.33	0.87	1.47
3	6	1.09	2.55	2.35	3.63	0.85	2.78
3	7	1.02	2.05	2.01	3.26	0.92	2.34
3	8	1.70	2.91	1.71	4.71	1.47	3.24
3	9	2.73	3.87	1.42	6.01	2.58	3.42
3	10	2.57	4.03	1.57	6.32	2.27	4.05
3	11	1.59	4.37	2.75	5.45	1.48	3.97
3	12	3.29	5.37	1.63	7.43	3.51	3.92
3	13	3.90	6.86	1.76	10.38	3.65	6.73
4	1	-1.86	2.90	-1.56	-0.18	-2.20	2.02
4	2	-1.33	1.68	-1.26	-0.65	-1.45	0.80
4	3	-0.67	1.14	-1.69	-0.18	-0.77	0.58
4	5	1.26	1.43	1.14	2.08	1.20	0.88
4	6	1.32	2.45	1.85	3.07	1.12	1.95
4	7	1.25	1.98	1.59	3.11	1.15	1.96
4	8	1.85	2.89	1.56	4.60	1.75	2.85
4	9	2.76	3.84	1.39	5.82	2.60	3.22
4	10	2.55	4.02	1.58	6.12	2.33	3.78
4	11	1.50	4.37	2.91	5.37	1.43	3.93
4	12	3.02	5.27	1.74	7.20	3.33	3.88
4	13	3.48	6.72	1.93	9.89	3.24	6.65
5	1	-0.66	2.33	-3.51	1.40	-0.87	2.26
5	2	-0.08	1.01	-12.99	1.23	-0.17	1.39
5	3	0.59	1.06	1.79	1.79	0.50	1.29
5	4	0.79	1.05	1.33	1.74	0.67	1.07
5	6	-0.33	1.66	-5.04	0.96	-0.65	1.61
5	7	-0.29	1.11	-3.83	1.15	-0.43	1.58
5	8	0.31	1.92	6.20	2.80	0.20	2.60
5	9	1.23	2.70	2.20	4.20	1.17	3.03

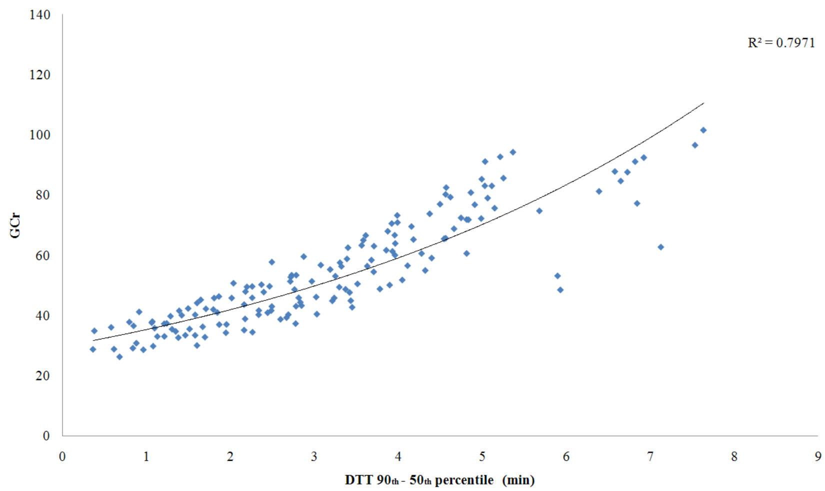
Station		Data Dispersion Measures (minute)					
From	**To**	**Mean**	**St Deviation**	**COV**	$90^{th}\%$	$50^{th}\%$	$90^{th} - 50^{th}$
5	10	1.05	3.03	2.89	4.44	1.00	3.44
5	11	0.02	3.88	172.37	3.58	-0.05	3.63
5	12	1.61	4.61	2.86	5.45	1.88	3.57
5	13	2.11	6.15	2.92	8.41	2.02	6.39
6	1	-1.57	2.97	-1.89	0.85	-1.88	2.73
6	2	-0.98	1.66	-1.69	0.68	-1.13	1.81
6	3	-0.37	1.36	-3.66	1.12	-0.48	1.61
6	4	-0.18	1.11	-6.24	1.18	-0.32	1.50
6	5	-1.29	1.49	-1.15	-0.52	-1.37	0.85
6	7	-0.55	0.76	-1.38	0.07	-0.55	0.62
6	8	0.10	1.34	12.74	1.38	0.03	1.35
6	9	0.73	2.08	2.87	2.72	0.85	1.87
6	10	0.64	2.34	3.67	3.31	0.87	2.45
6	11	0.16	4.52	27.76	3.04	-0.22	3.25
6	12	2.00	6.19	3.09	4.79	1.92	2.88
6	13	2.19	6.95	3.18	8.76	1.92	6.85
7	1	-1.52	3.11	-2.05	1.25	-1.83	3.08
7	2	-0.99	1.89	-1.90	1.08	-1.12	2.20
7	3	-0.39	1.65	-4.24	1.60	-0.58	2.18
7	4	-0.21	1.40	-6.67	1.52	-0.35	1.87
7	5	-1.36	1.75	-1.28	-0.04	-1.47	1.42
7	6	-0.37	0.75	-2.03	0.57	-0.50	1.07
7	8	0.34	1.21	3.57	1.62	0.23	1.38
7	9	1.19	2.27	1.91	3.53	1.27	2.27
7	10	1.03	2.57	2.50	3.76	1.08	2.67
7	11	0.00	3.51	1063.06	2.86	-0.12	2.97
7	12	1.54	4.21	2.74	4.55	2.05	2.50
7	13	2.08	5.63	2.70	7.48	2.33	5.15
8	1	0.64	3.13	4.93	4.38	0.42	3.96
8	2	1.16	2.39	2.06	3.79	1.07	2.72
8	3	1.75	2.74	1.56	4.38	1.67	2.72
8	4	1.93	2.73	1.42	4.34	1.87	2.47
8	5	0.81	1.88	2.33	2.92	0.75	2.17
8	6	1.80	2.26	1.26	3.55	1.70	1.85

Station		Data Dispersion Measures (minute)					
From	**To**	**Mean**	**St Deviation**	**COV**	$90^{th}\%$	$50^{th}\%$	$90^{th} - 50^{th}$
8	7	1.85	2.08	1.13	3.03	1.82	1.21
8	9	0.76	1.09	1.42	1.65	0.68	0.97
8	10	0.61	1.60	2.64	2.18	0.48	1.70
8	11	-0.38	2.95	-7.65	1.94	-0.90	2.84
8	12	1.10	3.34	3.05	3.46	1.42	2.04
8	13	1.74	4.75	2.73	6.27	1.60	4.67
9	1	-0.54	3.83	-7.15	3.85	-0.70	4.55
9	2	-0.06	2.86	-47.68	3.50	0.11	3.39
9	3	0.51	2.95	5.75	4.12	0.81	3.31
9	4	0.70	2.81	4.04	4.05	0.86	3.19
9	5	-0.46	2.59	-5.67	2.80	-0.50	3.30
9	6	0.53	2.34	4.38	3.28	0.47	2.82
9	7	0.60	2.05	3.43	3.27	0.48	2.79
9	8	-0.16	0.95	-5.77	0.93	-0.32	1.25
9	10	-0.51	1.26	-2.45	0.42	-0.67	1.08
9	11	-1.51	3.26	-2.16	0.64	-1.85	2.49
9	12	-0.04	3.08	-78.56	2.38	-0.02	2.40
9	13	0.61	4.29	7.05	4.67	0.48	4.18
10	1	0.16	4.00	24.79	5.10	0.11	4.99
10	2	0.64	3.13	4.85	4.39	0.80	3.59
10	3	1.21	3.33	2.74	5.15	1.43	3.71
10	4	1.39	3.22	2.32	5.05	1.65	3.40
10	5	0.24	2.71	11.41	3.68	0.36	3.32
10	6	1.21	2.73	2.26	4.22	1.43	2.79
10	7	1.27	2.45	1.92	3.82	1.44	2.37
10	8	0.51	1.39	2.74	2.07	0.42	1.65
10	9	0.51	0.87	1.72	1.40	0.48	0.92
10	11	-1.34	2.65	-1.98	0.35	-1.82	2.17
10	12	0.12	2.89	23.88	2.36	0.02	2.34
10	13	0.68	4.20	6.16	4.77	0.37	4.40
11	1	0.87	5.69	6.52	7.50	0.92	6.58
11	2	1.54	5.01	3.26	6.92	2.05	4.87
11	3	2.09	5.25	2.51	7.65	2.58	5.07
11	4	2.23	5.10	2.29	7.32	2.82	4.50

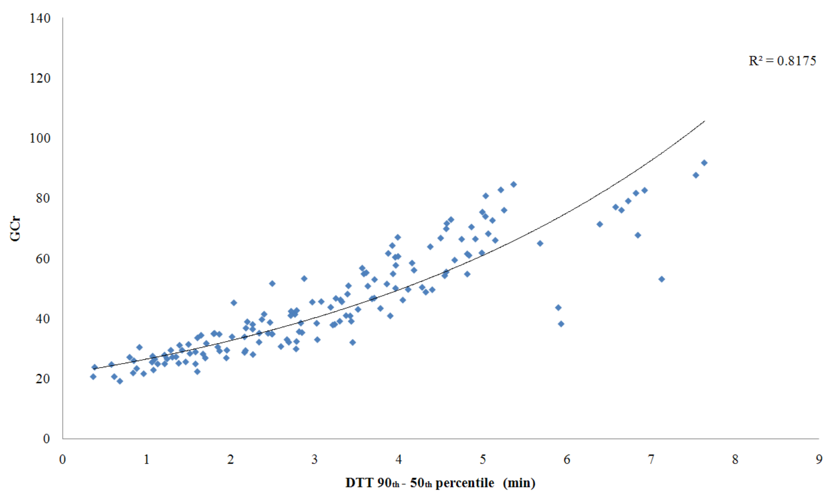
Station		Data Dispersion Measures (minute)					
From	**To**	**Mean**	**St Deviation**	**COV**	$90^{th}\%$	$50^{th}\%$	$90^{th} - 50^{th}$
11	5	1.02	4.58	4.48	6.20	1.38	4.82
11	6	2.21	4.93	2.24	6.78	2.62	4.16
11	7	2.23	4.66	2.09	6.23	2.62	3.61
11	8	1.33	4.16	3.14	5.43	1.15	4.28
11	9	1.41	3.81	2.70	4.73	1.05	3.68
11	10	0.38	3.93	10.33	4.90	-1.03	5.93
11	12	-0.33	0.98	-2.97	0.73	-0.40	1.13
11	13	0.17	3.06	17.80	3.50	-0.40	3.90
12	1	0.80	5.81	7.23	7.73	0.80	6.93
12	2	1.52	5.07	3.34	6.91	1.92	5.00
12	3	2.07	5.30	2.57	7.60	2.57	5.03
12	4	2.19	5.14	2.34	7.22	2.65	4.57
12	5	0.89	4.64	5.19	5.88	0.97	4.91
12	6	2.11	5.04	2.39	6.61	2.23	4.38
12	7	2.13	4.76	2.24	6.14	2.15	3.99
12	8	1.11	4.41	3.96	5.42	0.85	4.57
12	9	1.16	4.02	3.46	4.69	0.83	3.86
12	10	0.18	4.31	23.76	5.02	-0.88	5.90
12	11	-0.88	1.43	-1.62	0.10	-1.00	1.10
12	13	-0.94	2.78	-2.96	1.87	-1.58	3.45
13	1	1.73	6.67	3.86	9.07	1.43	7.64
13	2	2.60	5.63	2.16	8.04	2.68	5.37
13	3	3.17	5.92	1.87	8.53	3.32	5.22
13	4	3.30	5.83	1.76	8.45	3.42	5.04
13	5	1.94	5.16	2.66	6.90	1.64	5.26
13	6	3.15	5.75	1.83	7.95	2.83	5.12
13	7	3.16	5.53	1.75	7.30	2.73	4.57
13	8	2.00	5.20	2.60	7.22	1.53	5.68
13	9	2.00	4.81	2.40	6.13	1.29	4.84
13	10	1.00	5.02	5.03	6.96	-0.17	7.13
13	11	-0.19	2.38	-12.40	1.81	-0.45	2.26
13	12	-0.84	2.29	-2.73	0.99	-1.18	2.18



(a) Case1: Risk-Averse Passengers



(b) Case1: Risk-Prone Passengers



(c) Case2: Risk-Neutral Passengers

Figure A.1: The Relationship Between Estimated GC_R for Each Passengers class and DTT $90^{th} - 50^{th}$ percentile

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