Viterbi Decoding for OFDM systems operating in narrow band interference

by

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I hereby declare that I am the sole author of this thesis. This is a true copy of the thesis, including any required final revisions, as accepted by my examiners.

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Abstract

Our main objective in this thesis is to study the effect of narrow band interference on OFDM systems operating in the 2.4 Ghz ISM band and identify ways to improve upon existing techniques to deal with them. We first consider how narrow band signals interfere with OFDM systems. Various noise variance estimation and signal to noise ratio estimation techniques for OFDM systems are then discussed. We also study the conventional Viterbi Algorithm that is used in OFDM wireless systems and the proposed modifications to it in the literature. Our main contribution is a detailed experimental analysis of a modified Viterbi Algorithm that outperforms the conventional one in the presence of narrow band interference. Interference samples captured using a wireless hardware platform were used in simulation to test this modified algorithm. From our analysis we realize that in the presence of narrow band frequency selective interference (such as Bluetooth), the conventional Viterbi Algorithm can be modified to improve the performance of OFDM systems.

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Chapter 1

Introduction

In todays world people heavily rely on mobile access to information. Work is information based and requires access to several technologies such as email, text messaging and Internet connectivity. This dependence on information technology has given rise to an explosive growth of portable, low cost wireless devices such as smart phones and PDAs. The number of portable computing devices such as laptops and the hand held PDAs have outnumbered the number of desktop PCs in the world. On the other hand applications that embrace wireless capabilities of a network are growing by leaps and bounds.

The manufacturers of these devices are subjected to many constraints such as low power consumption, proficiency in frequently making and breaking connections and the capability to identify and use available resources. These devices have created a need for wireless personal access networks that typically span short 10 meter radii. Wireless Local Area Networks, on the other hand could span over a hundred meters and their main purpose is to augment traditional wired local area networks such as Ethernet LANs. This can be looked as a last mile connectivity technology, that enables more devices to participate in a network due to its seamless wireless nature.

Emerging wireless technologies promise high bandwidth, wireless communication capabilities and easy access to the Internet. They therefore require access to radio spectrum in order to transmit information at high data rates. Due to the direct relationship between higher data rates and required bandwidth, more and more radio spectrum bandwidth is required for faster connections. On the other hand the radio spectrum being a natural resource, has limited availability and so a high demand means higher costs. This has led to the emergence of the globally available unlicensed radio spectrum known as the industrial, scientific and medical (ISM) radio bands.

Unfortunately the problem does not end there, as the lack of licensing brings with it a new set of problems. Low cost wireless devices have perforated into the global wireless markets, and many of them use the ISM band at the same time thus creating severe interference. Performance of these devices can be severely degraded depending on the amount of interference in the wireless channel. This brings us to our present research problem.

1.1 Interference in the ISM band

One of the main problems in the 2.4 GHz ISM band is interference. Orthogonal Frequency Division Multiplexing (OFDM) technology is used by many devices operating in this band. Although unlicensed use of this band is allowed, users must follow rules defined in the Code for Federal Regulations of the Federal Communications Commission (FCC), relating to total radiated power and the use of the spread spectrum modulation schemes. Apart from these regulations, not much is done to restrict the interference among the various devices using different technologies such as OFDM and Bluetooth. Thus, the major drawback of the unlicensed ISM band is that frequencies must be shared and potential interference tolerated. While the spread spectrum and power rules are fairly effective in dealing with multiple users

in the band, provided the radios are physically separated, the same is not true for close proximity radios. Multiple users sharing the same frequency spectrum cause interference to each other, thus raising the effective noise floor and resulting in performance degradation. The impact of interference may be even more severe when radios of different applications use the same band while located in close proximity.
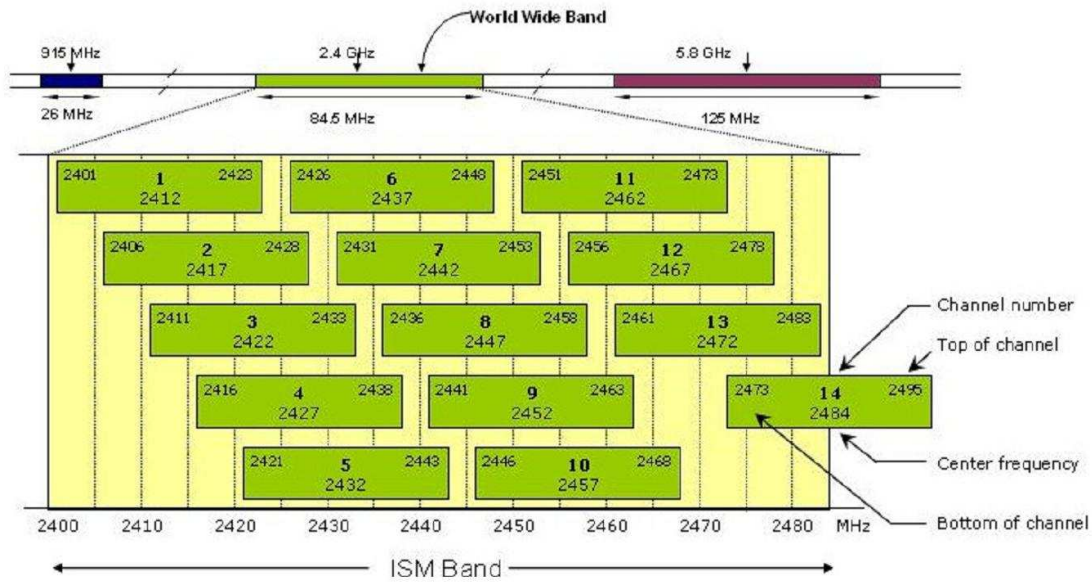
Figure 1.1: The ISM band layout obtained from http://www.experts-exchange.com on Sept. 2, 2009.

There are a number of industry led activities focused on coexistence in the 2.4 GHz band. The IEEE 802.15.2 Coexistence Task Group was formed in order to evaluate the performance of Bluetooth devices interfering with WLAN devices and develop a model for coexistence which will consist of a set of recommended practices and possible modifications to the Bluetooth and the IEEE 802.11 standard specifications that allow the proper operation of these protocols in a cooperative way. At the same time, the Bluetooth Special Interest Group

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formed its own task group on Coexistence. Both the Bluetooth and the IEEE working groups maintain liaison relations and are looking at similar techniques for alleviating the impact of interference. The proposals considered by the groups range from collaborative schemes intended for Bluetooth and IEEE 802.11 protocols to be implemented in the same device to fully independent solutions that rely on interference detection and estimation.

According to [3], collaborative scheme mechanisms have been proposed to the IEEE 802.15 Workshop on Services and Applications in the Wireless Public Infrastructure Existence Task Group and are based on a MAC time domain solution that alternates the transmission of Bluetooth and WLAN packets (assuming both protocols are implemented in the same device and use a common transmitter). A priority of access is given to Bluetooth for transmitting voice packets, while WLAN is given priority for transmitting data.

Non-collaborative mechanisms that have been considered range from physical layer solutions such as adaptive frequency hopping, to those based on network layer protocols such as packet scheduling and traffic control. They all use similar techniques for detecting the presence of other devices in the band such as measuring the bit or frame error rate, the the signal to interference ratio or signal strength(often implemented as the Received Signal Strength Indicator (RSSI)). For example, each device can maintain a bit error rate measurement per frequency used. Frequency hopping devices can then know which frequencies are occupied by other users of the band and thus modify their frequency hopping pattern. They can even choose not to transmit on a certain frequency if that frequency is occupied. The first technique is known as adaptive frequency hopping, while the second technique is known as MAC scheduling. Each technique has advantages and disadvantages. One of the advantages in using a scheduling policy is that it does not require any changes in the FCC rules. In fact, title 47, part 15 of the FCC rules on radio frequency devices, allows a frequency hopping system to recognize the presence of other users within the same spectrum band so

that it adapts its hop sets to avoid hopping on occupied channels. However scheduling in the Bluetooth specifications is vendor implementation specific. Therefore, one can easily implement a scheduling policy with the currently available Bluetooth chip set. On the other hand, adaptive frequency hopping requires changes to the Bluetooth hopping pattern and therefore a new Bluetooth chip set design. While both collaborative and non-collaborative techniques can reduce the Bluetooth packet loss and the impact of interference on the other system, only the adaptive frequency hopping technique can increase the Bluetooth through-put by maximizing the spectrum usage. As the number of interfering devices increase, each system is forced to transmit less often in order to avoid collisions. Thus, as the band occupancy increases, the duty cycle is reduced imposing time domain solutions. Frequency domain solutions such as adaptive frequency hopping can only be effective when the band occupancy is low.

1.2   Contributions of the thesis

Our main objective in this thesis is to study the effect of narrow band interference on OFDM systems operating in the 2.4 Ghz ISM band and identify ways to improve upon existing techniques to deal with them. We first consider how narrow band signals interfere with OFDM systems. Various noise variance estimation and signal to noise ratio estimation techniques for OFDM systems are then discussed. We also study the conventional Viterbi Algorithm that is used in OFDM wireless systems and the proposed modifications to it in the literature. Our main contribution is a detailed experimental analysis of a modified Viterbi Algorithm that outperforms the conventional one in the presence of narrow band interference. Interference samples captured using a wireless hardware platform were used in simulation to test this modified algorithm. From our analysis we realize that in the presence

of narrow band frequency selective interference (such as Bluetooth), the conventional Viterbi Algorithm can be modified to improve the performance of OFDM systems.

1.3   Organization of the thesis

Chapter two deals with wireless technologies in the ISM band with specific emphasis to Bluetooth devices. In Chapter three we give a short overview of the Viterbi Algorithm and how it has been used for OFDM systems. Chapter four presents a detailed review of how signal to noise ratio and noise variance are estimated in an OFDM systems. Chapter five talks about the hardware platform that was used to capture interference samples from the ISM band. Chapter six deals with our simulation and results. Chapter seven has the Conclusion.

Chapter 2

Wireless Technologies in the 2.4 GHz Band

Due to it's worldwide availability, the Industrial Scientific and Medical (ISM) radio spectrum is suitable for popular low cost wireless devices. These devices form Wireless Personal Access Networks (WPAN) and Wireless Local Area Networks (WLAN). The networks sharing this radio frequency among various wireless devices lead to severe interference and performance degradation. Here is an overview of the various radio access technologies operating in the 2.4 GHz unlicensed ISM band.

Orthogonal Frequency Division Multiplexing is a very popular technique to overcome frequency selectivity of the channel. This technique is widely used by many devices operating in this frequency band. In this technique, the entire available bandwidth are subdivided into a number of narrower orthogonal sub channels which are then used to transmit data. Some of its applications in Wireless Communications include digital radio and Wireless Local Area Network (WLAN). Due to its efficiency, it has been adopted as a modulation scheme in IEEE standards such as 802.11 and 802.22. The systems model typically used by an OFDM system

is shown in fig.2.1. It can be broadly classified into the following blocks: 1. Interleaver 2. Convolutional Encoder 3. QPSK Modulator 4. OFDM Modulator 5. Transmission Channel 6. OFDM Demodulator 7. Demodulator and Decoder 8. Deinterleaver.
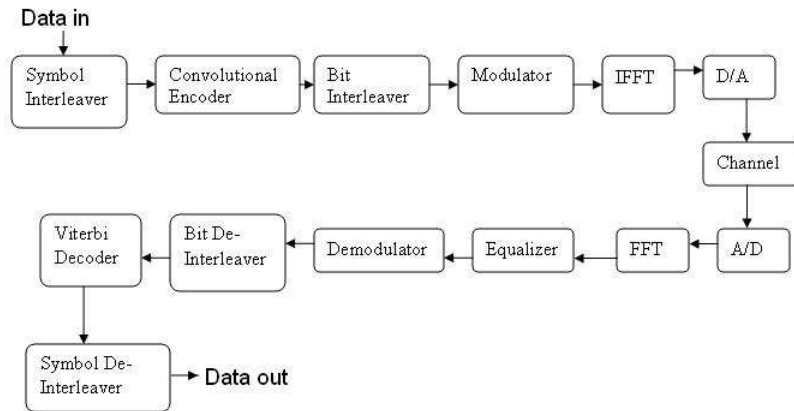
Figure 2.1: Block Diagram of the Physical Layer of a typical wireless OFDM system

A Convolutional code is used in an OFDM system as an inner code to correct the error events which consist of few bit errors. If a burst of bit errors occur at the channel output, the Convolutional code cannot be decoded correctly. Therefore bit interleaving is used to spread the bit error over the entire OFDM symbol. However there will still be error events which the decoder of the Convolutional code is unable to correct. In this case a burst of errors always will occur at the decoder output. Reed-Solomon (RS)Coder can be used to correct these burst errors. Because there is more than a single RS codeword in each OFDM symbol, an additional interleaving of the RS code symbol can be implemented.

The WPAN Technology, based on the Bluetooth Specification is now a part of the IEEE

standard 802.15. It is aimed at replacing non-interoperable proprietary cables that connect phones, laptops, PDAs and other portable devices together. Bluetooth operates in the ISM frequency band starting at 2.402 GHz and ending at 2.483 GHz in the USA, and Europe. 79 RF channels of 1 MHz width are used. The air interface is based on an antenna power of 1 mW. The signal is modulated using binary Gaussian Frequency Shift Keying (GFSK) scheme and the raw data rate is defined at 1 Mbits/s. A Time Division Multiplexing (TDM) technique divides the channel into $625\mu s$ slots. Transmission occurs in packets that occupy an odd number of slots (up to 5). Each packet is transmitted on a different hop frequency with a maximum frequency hopping rate of 1600 hops/s. Two or more units communicating on the same channel form a piconet, where one unit operates as a master and the others (a maximum of seven active at the same time) act as slaves. A channel is designed as a unique pseudo-random frequency hopping sequence derived from the master devices 48-bit address and its Bluetooth clock value. Slaves in the piconet synchronize their timing and frequency hopping to the master upon connection establishment. In the connection mode, the master controls the access to the channel using a polling scheme where master and slave transmissions alternate. A slave packet always follows a master packet transmission.

Previous work such as [4] and [5] have studied coexistence techniques of Bluetooth and OFDM systems and the performance degradation of frequency hopping systems resulting from band or multi tone jamming. Narrow band interference to OFDM systems caused by Bluetooth devices in the ISM band are studied in [1]. In this paper, the performance degradation of OFDM systems from Bluetooth interference is modeled and numerically analyzed. Bit error probability performance based on the effective signal to noise power ratio (SNR) is also provided for OFDM systems in additive white Gaussian noise. The power spectral density of the OFDM signal in [1] is expressed as:

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$$G_S(f) = \sum_{k=-N/2(k\neq 0)}^{N/2} \frac{P_{sub}}{W_{sub}} sinc^2(f/W_{sub} - k) \tag{2.1}$$

where N is the number of subcarriers, P_{sub} is the power of one OFDM subcarrier and W_{sub} is the subcarrier spacing. A Bluetooth signal is characterized by a frequency hopping or Gaussian frequency shift keying modulation scheme with respect to the useful signal [6]. Because it is too difficult to obtain the exact PSD expression for the Bluetooth signal, it is assumed that the PSD of the Bluetooth signal is rectangular in the 3dB bandwidth,

$$G_B(f) = \frac{P_B}{W_B} rect\Big((f - f_d)/W_B\Big) \tag{2.2}$$
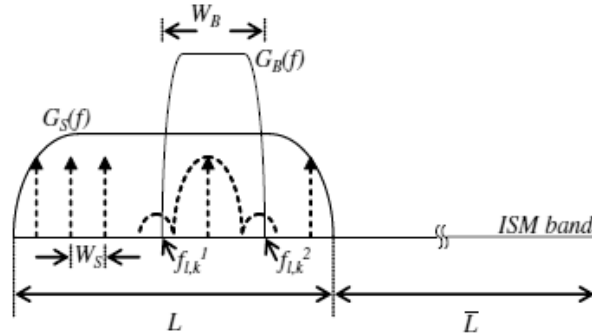
Figure 2.2: PSD of Bluetooth overlapping that of OFDM in ISM band. This image was obtained from [1]

where P_B and W_B are the power and the bandwidth of Bluetooth signal. The distance of center frequency between OFDM signal and Bluetooth is f_d, which is determined by the channel location Bluetooth occupies. Bluetooth signals unintentionally interfere with OFDM signals thus causing partial band jamming as shown in 2.2. The subcarrier spacing is less than the bandwidth of Bluetooth $W_{sub} < W_B$. The OFDM system in [1] was simulated using 52 subcarriers and no coexistence techniques were used for Bluetooth. In AWGN, the bit error probability was shown to decrease as SNR increases up to 9 dB. After 9 dB SNR,

the performance was shown to be limited by the the ratio between the signal power to the bluetooth interference power.

Chapter 3

Viterbi Algorithm for OFDM systems

The Viterbi Algorithm was proposed in 1967 [7] by Andrew J. Viterbi, as a method decoding Convolutional codes, a popular forward error correction code used in many digital communication systems. This algorithm is highly popular due to its simplicity and is one of the most highly used algorithms in today's wireless devices.

The Viterbi decoder is based on the principles of Maximum Likelihood sequence Estimation. It typically uses Euclidean (soft decisions) or Hamming (hard decisions) distances as a decision metric. The soft decision technique provides about 2 dB gain over the hard decision decoding. While these techniques may provide satisfactory performance in AWGN channels, they may be improved upon in the case of channels with narrow band interference. In [8] it is shown that the maximum likelihood sequence estimator should perform the following minimization:

$$arg \min_{X} \frac{(X - \hat{X})^2}{\sigma^2} \tag{3.1}$$

Where X is the transmitted bit sequence and $\hat{X}$ is the soft bit sequence at the output of the demodulator. While the conventional Viterbi decoder performs a similar operation in estimating the received sequence X, it ignores the noise variance σ^2 term in its computation.

In this case, the metric looks like the following:

$$arg \min_{X} (X - \hat{X})^2 \tag{3.2}$$

In [2], soft metrics for Viterbi decoding were determined from noise variance estimates of a 802.11g WLAN receiver. Using noise variance estimates for Convolutional decoding showed substantial improvements in 802.11g system performance. These systems were described as performing bit interleaved coded modulation (BICM) in conjunction with OFDM. According to the authors, the noise in these systems are usually assumed to be white, i.e. σ^2 is usually assumed to be constant for all the OFDM subcarriers and is therefore ignored in the Viterbi metric. However, if σ^2 does vary from from one subcarrier to another, as would be the case if some of the frequency bins had Bluetooth interference while others did not, neglecting the noise variance term in the calculation of the soft metric could lead to severe degradation in performance. One thousand packets of 802.11g packet format (Fig. 3.1) were used in simulation. In addition to AWGN of fixed variance, simulated Bluetooth interference was added to 3 contiguous subcarriers in each OFDM symbol. It was reported here that if a Bluetooth device transmitting at 1 mW and an 802.11g device transmitting at 50 mW over 20 MHz (2.5 mW per 1 MHz) were placed at equal distances from a receiving 802.11g device, the signal to interference (or Jamming [1]) ratio at the receiver would be about 4 dB. Noise variance estimates were averaged over 10 and 60 OFDM frames and these averaged noise variances were used in the Viterbi decoding metric. The performance using both these cases were 4 dB worse at packet error rate of 10^{-2}. In comparison to the conventional decoding which ignores the variable noise variance in the decoding process, the modified algorithm performs a lot better. However one limiting factor of this work is that it does not deal with changing interference characteristics over the duration of a packet by identifying the location of the interference within the frequency band. The method of adding simulated Bluetooth

interference in an artificial fashion makes the simulation unrealistic.

PLCP Preamble 12 symbols	Signal 1 OFDM Frame	Data Variable No. of OFDM Frames

Figure 3.1: 802.11g PHY packet structure obtained from in [2]

A modified Viterbi metric for frequency selective multipath fading channels is presented in [9]. In this paper simulation of an OFDM system used in IEEE 802.11a is presented. A modification technique similar to the technique we just studied was used here. It is reported that although variations of this technique have occurred occasionally in industry, the technique has not received much attention in the literature. Two modified metrics were proposed in this study. They were

$$\min_{X} q|X - \hat{X}|^2 \tag{3.3}$$

where

$$q = \frac{|\hat{H}|^2}{\sigma^2} \tag{3.4}$$

or

$$q = \frac{|\hat{H}|^2}{\gamma} \tag{3.5}$$

where γ was an experimentally obtained scaling factor and $\hat{H}$ is the channel estimate. The first metric corresponds to a measurement of the signal to noise ratio. In their simulation this metric was scaled and clipped so as to force those sections of the decoder trellis for which SNR is small to have a lower branch metric and contribute less to the path metric. As a result branches whose channel quality are higher were emphasized in the decision making process. Three different channel models were studied here. The Additive White Gaussian Noise, Flat Fading and an Exponential Channel. The Exponential channel was defined as

a frequency selective multipath channel with AWGN and the channel transfer function was that of an FIR filter with root mean square (r.m.s) delay spread 25 or 75 ns. This channel was considered similar to the ones over which IEEE802.11 compatible devices operate. A 10 dB improvement in the performance of the Viterbi Decoder was reported for the Exponential channel. It was also mentioned that using only the magnitude of the channel estimate lead to a substantial improvement in performance. One limitation, or rather shortcoming of this work is that while it considers frequency selective multipath fading environments, it only studies simulated channels. The other aspect which was not looked upon in particular was narrow band Bluetooth interference.

Chapter 4

SNR and Noise Variance Estimation in OFDM systems

This chapter presents a detailed literature review of different signal to noise and noise variance estimation techniques presented in the literature over the years. These techniques give us insight into how our understanding of the OFDM technology has evolved over the years. The techniques presented in this section can be used in systems estimated noise variance and SNR to improve the performance of the Viterbi decoder.

Signal-to-Noise Ratio(SNR), has long been used as the standard measure of quality of analog signals in noisy environments. SNR estimation algorithms for Orthogonal Frequency Division Multiplexing(OFDM) systems, can be classified into two categories: The data-aided estimator, for which known (or pilot) data is transmitted and used in the receiver, and the non-data-aided estimator.

The SNR estimation technique presented in [10], which falls in the data-aided estimator category, is based on tracking the delay-subspace using the estimated channel correlation matrix. However, since it only uses the pilot settled for channel estimation, it has no more

capacity loss compared to the general OFDM system. This technique can also be applied to MIMO-OFDM systems.

The system model for this scenario is interesting, and it is advisable to take a look at it before the estimator can be discussed in more detail. It is assumed that the signal is transmitted over a multipath Rayleigh fading channel characterized by

$$h(t,\tau) = \sum_{l=1}^{L} h_l(t)\delta(\tau - \tau_l), \qquad (4.1)$$

where $h_l(t)$ are the different path complex gains, τ_l are different path time delays, and L is the number of paths. $h_l(t)$ are wide-sense stationary (WSS) narrow-band complex Gaussian processes and the different path gains are uncorrelated with respect to each other where the average energy of the total channel energy is normalized to one.

At the receiver side, with the assumptions that the guard interval duration is longer than the channel maximum excess delay and the channel is quasi-stationary (i.e. the channel does not change within one OFDM symbol duration), the nth subcarrier output during the ith OFDM symbol can be represented as

$$Y_{i,n} = X_{i,n}.H_{i,n} + n_{i,n}. \qquad (4.2)$$

Here $n_{i,n}$ is a white complex Gaussian noise with variance σ_N^2, $H_{i,n}$ is the channel frequency response given by

$$H_{i,n} = \sum_{l=0}^{L} h_l(iT_s)e^{-j2\pi\frac{n\tau_l}{NT}} \qquad (4.3)$$

where $h_l(iT_s)$ denotes the channel l^{th} path gain during the ith OFDM symbol and T is the sampling time interval of the OFDM signal.

In this paper the SNR during the ith OFDM symbol is defined as

$$\rho = \frac{\sum_{l=1}^{L} |h_l(i.T_s)|^2}{\sigma_N^2}. \qquad (4.4)$$

The method in [10] required M pilot subcarriers to be inserted into every OFDM symbol. It was assumed that $M > L$. The channel frequency response for each pilot was found out by dividing the received pilot with the original. The correlation matrix R of these M pilot tone channel estimates was computed and the eigenvalues of R were computed through eigenvalue decomposition. L, the number of paths, was estimated by the well-known Minimum Descriptive Length technique presented in [11]. The estimated eigenvalues and number of paths L was then used to compute the SNR. The correlation matrix R was obtained, based on a moving average of a certain number of observation vectors. This is justified as in a mobile communications the multipath time delays are slowly varying in time. On the other hand, the amplitude and relative phase of each path vary faster [12]. It was also shown by simulation results, that this estimator is able to estimate the true SNR accurately after an observation interval of about 20 OFDM symbols for various fading channels. In practical wireless systems however, the assumption that $M > L$ made in [10] may not apply. For instance in IEEE802.11a devices, the number of pilot tones M is equal to four. In this case it may be expected that the number of paths L will be greater than four.

In a recent work by Socheleau et al [13] a non data aided SNR estimation technique was proposed. Such schemes are targeted towards applications such as cognitive radio, where terminals might need to sense the link quality with all the surrounding networks in order to find the most suitable link for communication. This method does not require the receiver to know the locations of the pilots. Instead the cyclostationarity induced by the cyclic-prefix is used to determine the SNR. The cyclic-prefix is an integral part of any OFDM signal and it is used to overcome inter-symbol-interference at the receiver. Assuming that the channel impulse response is not longer than L, the last L samples of the OFDM symbol

are appended to the beginning as the prefix of that symbol. This redundancy introduced by the cyclic-prefix is used in [13] to estimate the noise variance. Perfect non-data-aided synchronization is assumed, algorithms for which can be found in [14] and [15]. It can be theoretically proved that the estimator with cyclic prefix length equal to the length of the channel impulse response L has the smallest noise variance. In order to achieve the minimum variance, a method based on maximum likelihood principle for estimating L was also derived. The cyclostationary statistics introduced by the cyclic-prefix [16] is used to estimate the signal power. The estimator is actually based on the cyclic autocorrelation defined in [17]. This estimator was simulated with fixed WiMax signals with 256 subcarriers per symbol and a cyclic prefix length of 32. Results show that at low SNR values (i.e and without perfect knowledge of L (i.e. when it needs to estimate L using the technique described above), the normalized mean square error (NMSE) between the predicted and actual noise variance is higher. The same is true for the NMSE of the signal power estimator. However the performance is greatly improved at high SNR, and if a larger number of OFDM symbols are taken into account.

In a paper by Cui et al[18], a noise variance estimation technique, similar to the one in [13] is described. Here also it is assumed that the guard interval is larger than the delay spread of the channel, i.e. the length of the channel impulse response. The main difference here is in the assumptions about L, the number of multipaths. The results in [18] are derived for large values of L. For the noise variance calculation, an arbitrary threshold value α is used to determine L. Simulation was done using 64 subcarrier OFDM, QPSK modulation at baseband, and cyclic-prefix length 16. A channel model with L=6 was used. Results show

that probability of correct detection of number of paths using $\alpha = 0.01$ was low at low SNR.

In comparing the above two methods, we see that the estimator in [13] outperforms that in [18]. This is because of the negative impact of the arbitrary threshold used by the latter in order to determine L. This in turn negatively effects the signal power and noise variance estimation. Since the signal power and noise variance are not independent, the SNR estimation gets deteriorated at low as well as high SNR. However the advantage of the estimator in [18] is its simplicity which would reduce the complexity of its design comparatively.

Traditional maximum likelihood (ML) and minimum mean square error (MMSE) algorithms for SNR estimation only applies if the transmitter knows the channel estimates. The performance of the SNR estimator is dependent on the chosen channel estimation technique [19]. In [20] Pauluzzi derived an ML SNR estimator for $M - ary$ PSK signals in a complex AWGN channel. Assuming that in the jth symbol period, the ith pilot subcarrier is modulated with a complex value $a(i, j)$. Also assuming that the same pilot signal is sent on the same pilot subcarrier in different OFDM symbol periods, which means $a(i, j) = a(i, l)$ for any i, j and l. Then the complex baseband system model for the ith pilot subcarrier can be formulated as

$$y(i, j) = \sqrt{S}h(i, j)a(i, j) + \sqrt{N}n(i, j) \tag{4.5}$$

where $n(i, j)$ is complex, zero-mean AWGN and $h(i, j)$ is the complex channel factor. For convenience, the variance of $h(i, j), n(i, j)$, and $a(i, j)$ are assumed to be normalized to unity. S is a signal power scale factor, and N is a noise power scale factor. In this way OFDM

converts a multipath channel into a set of parallel time-variant linear channels.

We will now look at a one more algorithm that does not require the channel estimates in order to estimate the SNR. The M_2M_4 algorithm and Boumard's Algorithm [21] fall in this category. The M_2M_4 algorithm for moment-based SNR estimation of real AWGN channels was derived in [22], which was later extended for complex channels in [20] and can be applied to time variant linear channels according to [19].

Compared to the ML and MMSE estimators, the noise variance estimation part of Boumard's algorithm [21], does not depend on prior knowledge of channel estimates. Consequently the performance of the algorithm deteriorates with faster channel fading as Doppler frequency increases. This algorithm uses two OFDM training symbols from each of two MIMO transmitting antennas in order to compute the noise variance estimate. Then using the channel coefficient estimates given by a channel estimator and the estimated noise variance, the SNR is computed. This algorithm is based on the assumption that the channel varies slowly in time as well as frequency. In other words, two consecutive time-domain channel estimates for any antenna pair are considered identical. Also the channel degradation for adjacent OFDM subcarriers is considered the same.

Chapter 5

Wireless Open Access Research Platform

The experimental setup consisted of a Wireless Open Access Research Platform from Rice University (Fig. 5.1). It provides a unique platform to develop, implement and test advanced wireless algorithms. This platform can be used to implement various Digital Communications algorithms in hardware and their performance can be tested in the real world scenario. This platform contains a Virtex 2 pro FPGA for Baseband signal processing and Medium Access Control. It contains Radio Boards (Fig. 5.2) that are used to receive interference data. This board has a 14-bit analog to digital converter for Rx I/Q samples and can operate in both 2.4 and 5GHz radio frequencies This board has Ethernet ports that can be used to communicate with computers and it can be controlled through MATLAB. Figure 5.3 shows the radio board, the on-board memory buffer and the PC which reads data off the buffer. The radio board is controlled by the FPGA and when it is given the read signal, it begins sampling the desired channel of the ISM band. These In-phase(I) and Quadrature(Q) samples are stored in their respective buffers. Once the buffers are full, the information from the buffers are

sent to the PC through a UDP link over Ethernet. This action had to be repeated as many times as required for the necessary number of data samples to be captured.

This work started as an endeavor to understand the kind of interference that limits the performance of 802.11 devices. The Primary interfering signals of interest was Bluetooth. The interference samples used for our simulation were captured in an environment where Bluetooth devices were operating as shown in 5.4. Some of these devices were nearby and some were further away thus creating a realistic scenario.
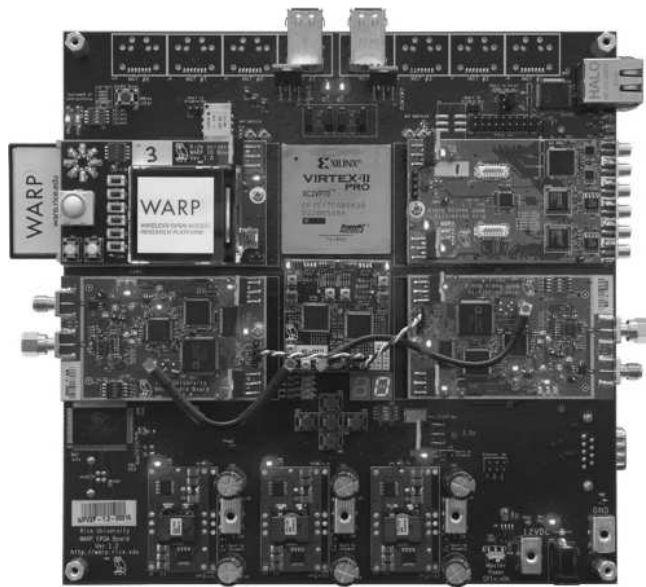

Figure 5.1: Wireless Open Access Research Platform

A very important part of this thesis dealt with capturing narrow band interference signals from the real world. Much time and effort therefore went into setting up a realistic test bed for capturing Bluetooth signals. A Bluetooth adapter for wireless data transfer and a wireless computer mouse were used to generate Bluetooth traffic.

A brief overview of the Bluetooth standard has been given in the Wireless Technologies in the 2.4 GHz Band chapter. There we have seen that 79 radio frequency channels of 1 MHz width are used by Bluetooth. These channels overlap ISM band channels 1 to 6 of
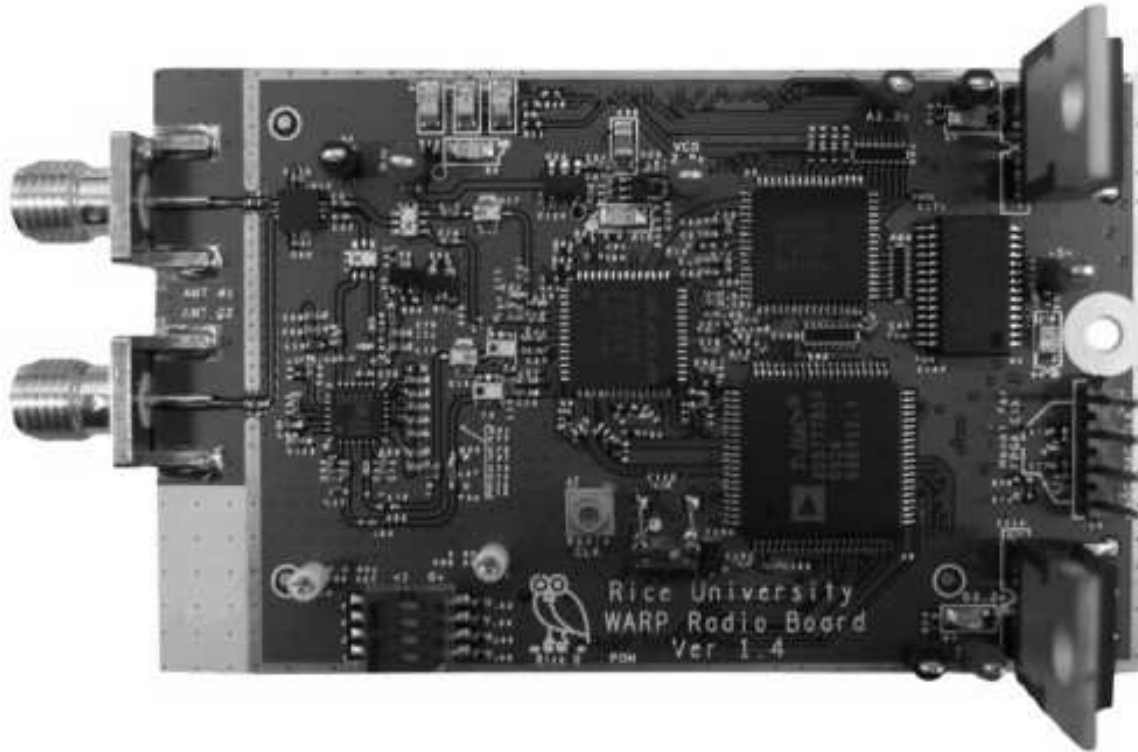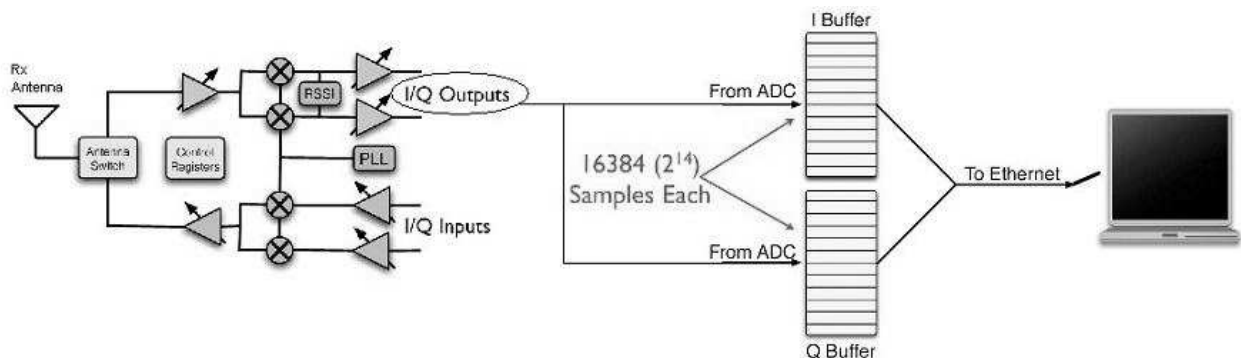
Figure 5.2: The Radio Board in WARP

Figure 5.3: The Flow diagram showing how the interference samples were captured

North America. The air interface is based on an antenna power of 1 mW. The Bluetooth signal is modulated using binary Gaussian Frequency Shift Keying (GFSK) scheme. Also a Time Division Multiplexing (TDM) technique divides the channel into $625\mu s$ slots. The ISM band in North America on the other hand, can be divided into three main channels.

These are channels 1, 6 and 11. Channel 1 ranges from 2.401 GHz to 2.423 GHz, channel 6 from 2.426 GHz to 2.448 GHz and channel 11 from 2.451 GHz to 2.473 GHz. The rest of the channels are known as the sub-channels, and overlap with one or more of these main channels. Bluetooth interference signals pseudo-randomly hop the 79 channels each of width 1 MHz thus interfering with the OFDM systems operating in those channels.

In fig. 5.4 we can see the experimental test bed used for capturing the Bluetooth interference samples which are later used in our simulation. As shown in the figure, this test bed was comprised of a PC running MATLAB, a WARP board set up to capture Bluetooth interference from channel 1 of ISM band and two Bluetooth devices operating in the neighborhood. During the experiments Bluetooth device 1 and 2 in fig. 5.4 were moved from time to time within operating range to the PC and the WARP board so as to create a realistic environment.
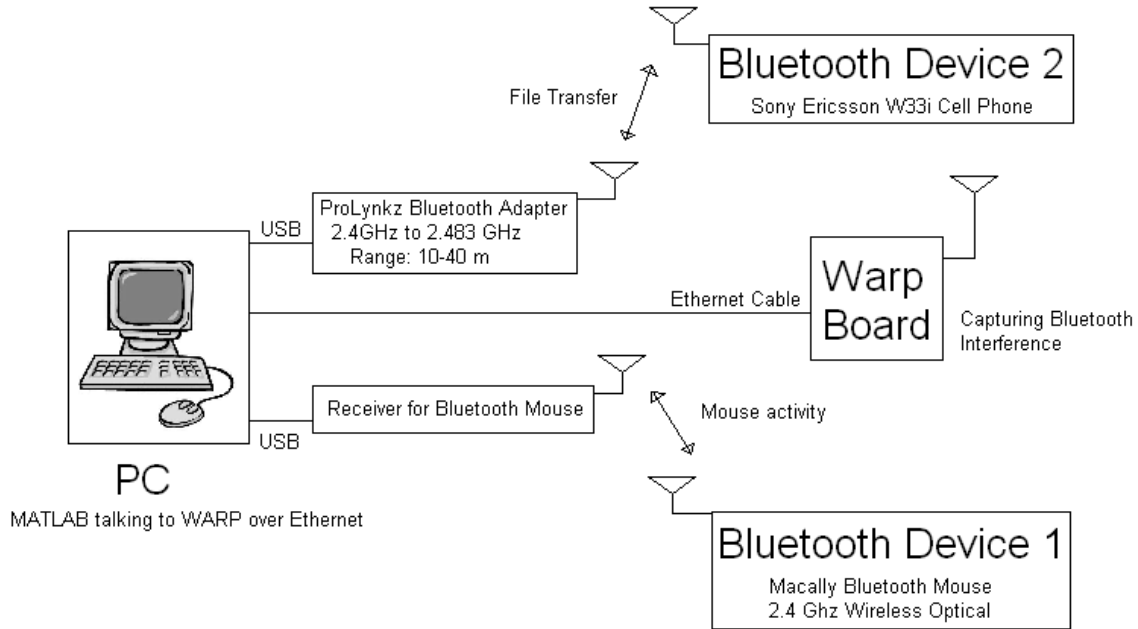

Figure 5.4: Test bed for capturing Bluetooth signals

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5.1 Initial Experiments

This section deals with the experimental work performed with the WARP platform. The setup that was used for the experiments comprised of a PC with MATLAB running in it and a WARP board that was connected to the laptop by Ethernet cable. We performed numerous experiments to test our equipment. Different channels of the ISM band were sensed using the radio board that samples the data in the channel.

One of the test experiments we performed involved the use of the Agilent Signal Generator and a WARP board. Using the Signal generator, a constant multicarrier signal was generated and transmitted. Then the WARP board was used to sense the channel and record samples. On computing a Fourier transform on the received samples we were able to reconstruct the signal that was being transmitted. It was also a good way for us to test proper functioning of the WARP board. Later when we performed this same experiment without the signal generator operating, it gave us an idea of how frequency selective the interference really was. We will eventually see that it is this frequency selective nature of the interference that we will mitigate by modifying the Viterbi algorithm. The narrow band interference is time varying as well and that is easy to understand as its primary sources, Bluetooth devices, operate using frequency hopping schemes. We therefore had to spend a lot of time on these experiments to get a good feel of how the interference behaves.

5.2 Limitations of the Experimental Setup

In order to capture enough interference samples we had to keep reading and erasing the buffer in the WARP board shown in fig. 5.3. Once the buffer was full, the sampling of the channel had to be stopped temporarily in order to create and send UDP (User Datagram Protocol) packets from the WARP board to MATLAB in the PC. Based on our experimental observations, it took 0.7568 seconds on average to sample and read 16384 (2^{14} samples into the buffer. This number of samples corresponds to 128 OFDM symbols at our sampling rate of 40MHz and assuming a 3.2 μs OFDM symbol duration. This can be considered among the limitations of this experimental setup as in between consecutive sets of buffering and reading , there was a significant delay. However we can still use these samples in our simulations because we are treating it as interference that is added to our transmitted signal. It would have been more realistic if there was no such delay introduced and we had uninterrupted access to the interference samples.

Another factor limiting the scope of this study is that we have to use a finite number of bits in our simulation. Since we are trying to study the performance of our algorithm in the presence of narrow band interference, our simulation was limited by the number of bits we could process in a reasonable amount of time. It is good to note here that if we ran the simulation for longer with more and more bits and interference samples, the performance improvement would be expected to continually increase instead of stagnating around a certain value as we will see. However, in this study we have limited the number of interference samples to 1628288 interference samples.

Chapter 6

Simulation and Results

6.1 Simulation

So far we have considered the experimental setup used to capture narrow band interference samples from the ISM band. We now look at the simulation of an OFDM system under the influence of this interference. Figure 6.1, shows the PHY layer model used in our simulation. Although this model does not do all the typical PHY layer signal processing, it is used as a model in our simulation because of its simplicity. We tried to specifically study the effect of the interference samples we captured on the OFDM symbols. As a result, we did not consider any fading in our channel. As a result it should be noted that we do not do equalization, interleaving and deinterleaving-interleaving. In our channel the interference samples captured from the WARP board and AWGN noise are added to the transmitted OFDM signal.

We have considered two cases in our simulations, the first case deals with a hypothetical situation where we know the position of the Bluetooth interference in the OFDM symbol. By using a modified Viterbi metric, we reduce the magnitudes of the soft bits belonging to the

affected OFDM subcarriers appropriately. Thus these soft bits are classified as less reliable in the subsequent channel decoding process. The idea here is to compare the performance of the conventional and modified Viterbi algorithms in the presence of this narrow band interference. The second case is that of a simple narrowband interference estimator that we propose. Using this estimated interference in the modified Viterbi decoder, we again compare its performance with the conventional Viterbi decoder.

The [171 133] Convolutional Code was used for our simulation. The simulation tries to bring out the improvement in performance of the modified Viterbi decoder over the conventional one. Our simulation technique, is similar but more realistic compared to [2] and [9]. We study the performance of a Viterbi decoder that makes use of the noise variance σ^2 in its decision metric. It should be pointed out here that when we say noise variance we mean the variance of noise plus interference. The Viterbi decoder used in our simulation performs maximum likelihood sequence estimation based on the following minimization:

$$arg \min_X \frac{(X - \hat{X})^2}{\sigma^2} \tag{6.1}$$

Where X is the transmitted bit sequence and $\hat{X}$ is the soft bit at the output of the demodulator. This is the metric used in the channel decoding process.

The Viterbi Decoder function in MATLAB had to be modified to include the MATLAB implements Viterbi decoding in the C programming language. So we had to get access to the C file and modify it according to our needs. As shown in 6.1, the modified decoder function takes the soft bits form the output of the QPSK demodulator and the interference variance over each QPSK symbol as inputs. Bluetooth interference samples were captured from the real world using a wireless hardware platform as discussed earlier.

We have tried to simulate a channel with narrow band interference in out simulation. For this purpose we focused on the 2.4 Ghz ISM band spectrum. This spectrum contains

signals from IEEE 802.11 compatible Wireless Access Points, and Bluetooth devices. In our lab environment both these kinds of signals were present as discussed earlier. We also introduced additional Bluetooth devices to add narrow band interference to the channel. In the following section we will see how the above modified decoder outperforms the conventional Viterbi decoder for a channel with narrow band interference. By using the channel noise and interference variance estimates in the decoding process we try to reduce the bit error rate of the system. The results described in this section show that this is possible using the modified decoding algorithm used in our simulation. It is then compared to the performance of the conventional Viterbi decoder which does not take the narrow band interference variance into account.

Figure 6.1, shows the PHY layer model used in our simulation. Although this model does not do all the typical PHY layer signal processing, it is used as a model in our simulation because of its simplicity. We tried to specifically study the effect of the interference samples we captured on the OFDM symbols. As a result, we did not consider any fading in our channel. As a result it should be noted that we do not do equalization, interleaving and deinterleaving. In our channel the interference samples captured from the WARP board and AWGN noise are added to the transmitted OFDM signal.

We used 1628288 interference samples (Fig 6.2) captured by the WARP boards, to simulate our channel. Complex additive white Gaussian noise (AWGN) was later added to the signal at the receiver. The SNR of the AWGN and the power level of the interfering signal were varied in the simulations. The results show that the modified Viterbi algorithm outperforms the conventional one for any given interference power. Furthermore this improvement increases with the SNR of AWGN.
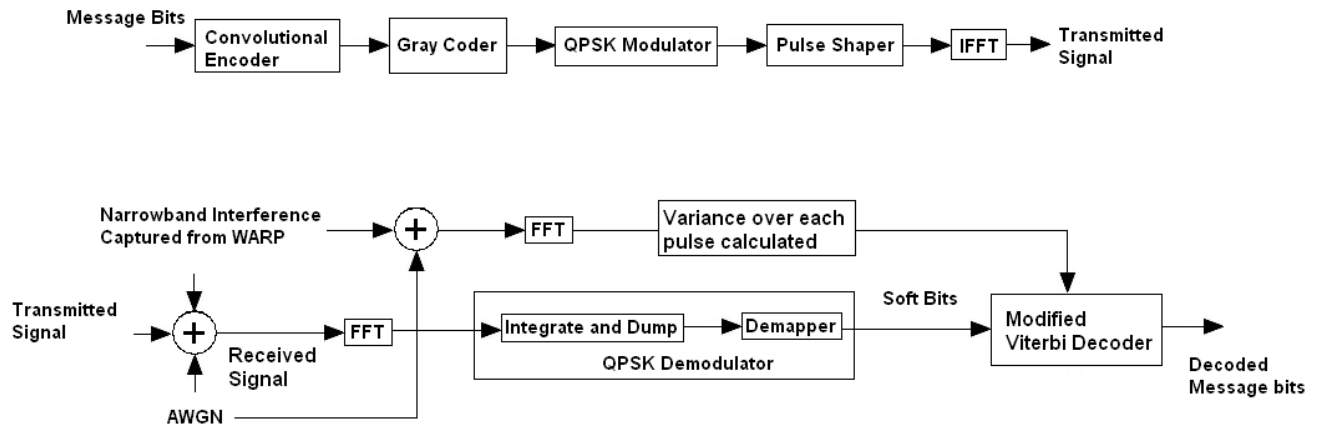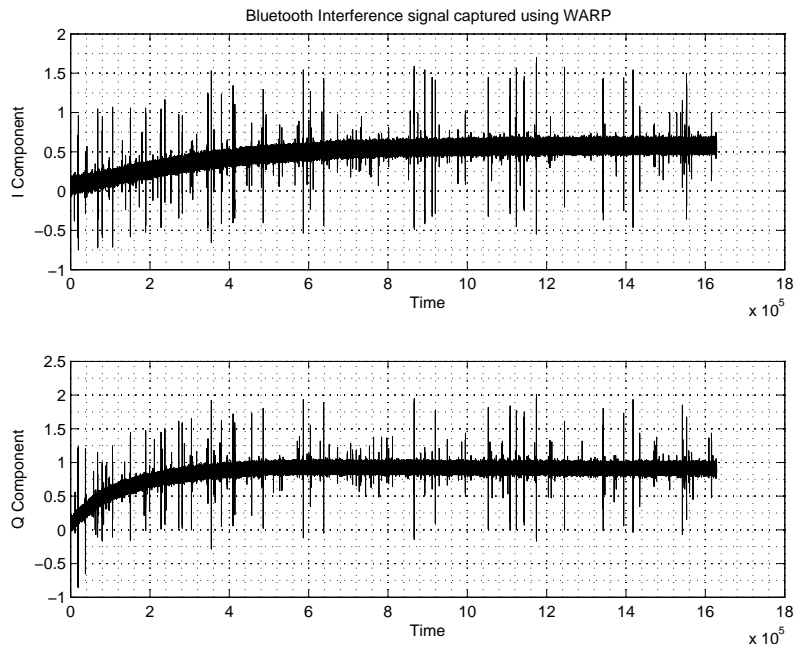
Figure 6.1: Simulation model

Figure 6.2: The interference signal used for simulation

6.2 Power adjustments for Simulation signals

In order to use the interference samples captured by the WARP hardware platform, we needed to adjust the power of the different signals appropriately. The three signals under consideration here are the transmitted OFDM signal, the narrow band interference signal captured by WARP and the AWGN signal. The AWGN noise was generated and added in MATLAB, while the interference was captured using WARP and added in simulation.

In the simulations of this chapter, the WARP signal (fig. 6.2) is considered only as interference . AWGN, that was added to the transmitted signal during simulation was the only source of noise. This AWGN with different SNR values with respect to the transmitted signal power, was generated using MATLAB. Then the average power of the WARP interference data was adjusted with respect to the transmitted signal power. We have used the term Signal to Jamming Ratio (SJR) to describe the ratio of the signal power and the average interference power. Our simulation results later in this chapter will show two cases of SJR values; the first one with SJR equal to 5 dB and the second one with SJR 0 dB.

In Appendix A, we show a way of computing the AWGN power from the WARP signal. This is a more realistic scenario because in a practical system there is always some amount of noise added by the receiving antenna. By using the WARP signal to determine the noise power we try to identify the noise that was added by the WARP hardware. The simulation discussed in appendix A does not assume the entire WARP signal (fig. 6.2) as interference. It uses the WARP signal samples in between bursts of interference, to model the AWGN noise. However we shall see that the result of Appendix A differs with that of figure 6.4 only at lower SNR values.

6.3 Results

In this section we present the simulation results. It should be noted that the graphs presented in this section show Bit Error Rate on the Y axis and AWGN SNR on the X axis. For each graph the SJR is constant. In the next section we consider two cases. The first case shows the performance with SJR was 5 dB and the second case had SJR was 0 dB (Meaning the average signal power was equal to the average narrow band interference power).

Figure 6.3 shows the BER performance for the case where the avenge interference power is 5 dB lower than the signal power. We can see from this figure that the modified Viterbi decoder performs a lot better than the conventional decoder. The Improvement becomes more prominent as SNR increases.

In the next simulation the power of the interference was the same as that of the actual OFDM signal. In this case, as shown in Fig. 6.4 the modified Viterbi algorithm performs much better than the conventional one. Such a situation could arise in a scenario where a Bluetooth device and a WLAN receiver are operating in close proximity. In this case, as can be seen from the simulation result the modified algorithm outperforms the conventional algorithm by a much bigger margin. It should be noted here, that the performance of the conventional Viterbi decoder is similar to the BER performance reported in [1]. There also the BER curve approaches 10^{-2} for the case with signal to Jamming ratio SJR=0dB.
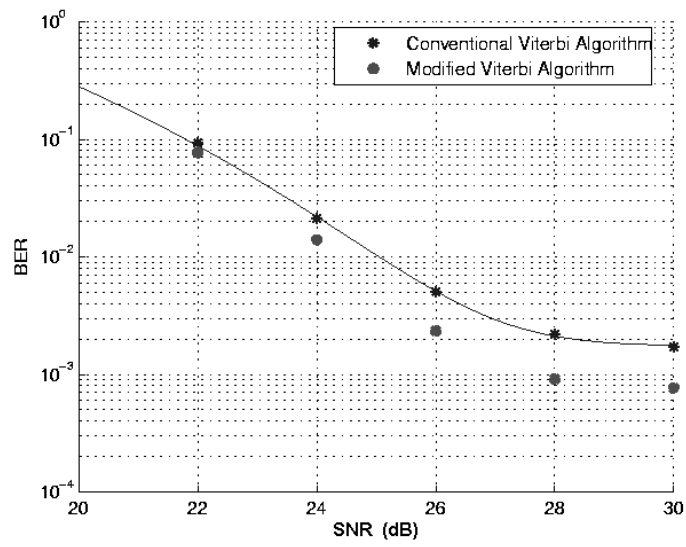
Figure 6.3: BER vs SNR. Here the average interference Power is 5 dB less than the average Signal power
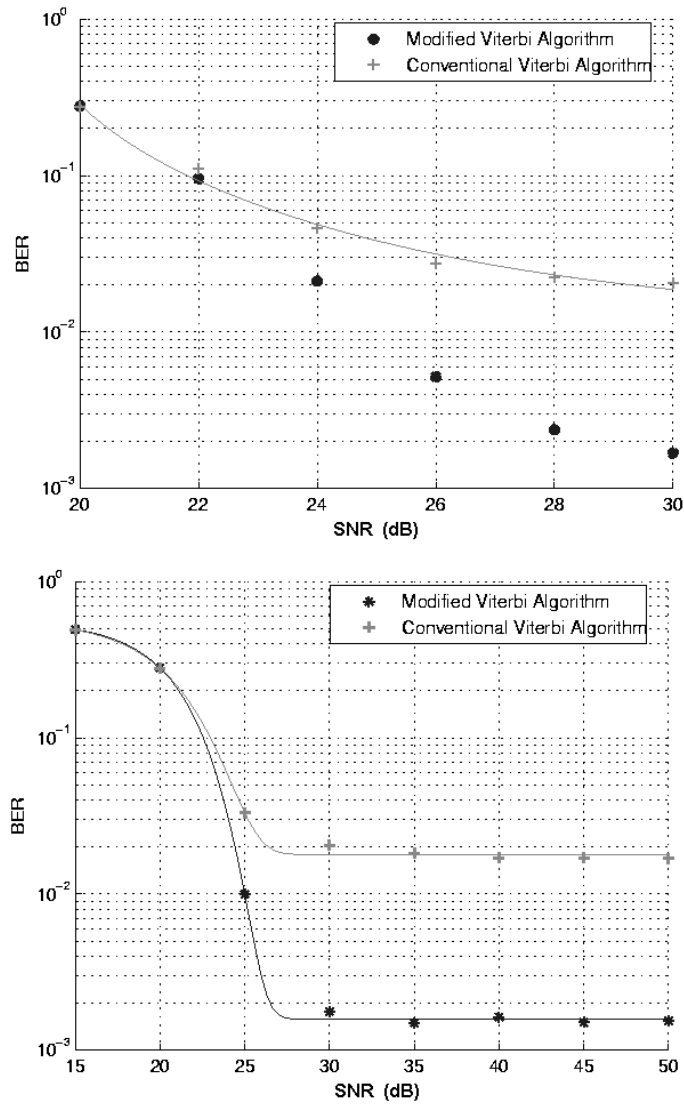
Figure 6.4: BER vs SNR. Here the average interference Power is equal to the average Signal power

6.3.1 A simple Noise Variance Estimation Technique

In a practical wireless communication system, the interference needs to be estimated from the received signal in order to use the decoding technique presented in this thesis. In this section we propose a simple technique to do so. The receiver in our simulation model is therefore modified as shown in Fig. 6.5. Although the technique presented in this section is neither the best nor optimal, it can be thought as the first step to understanding the performance of the modified Viterbi decoding technique in a practical situation.
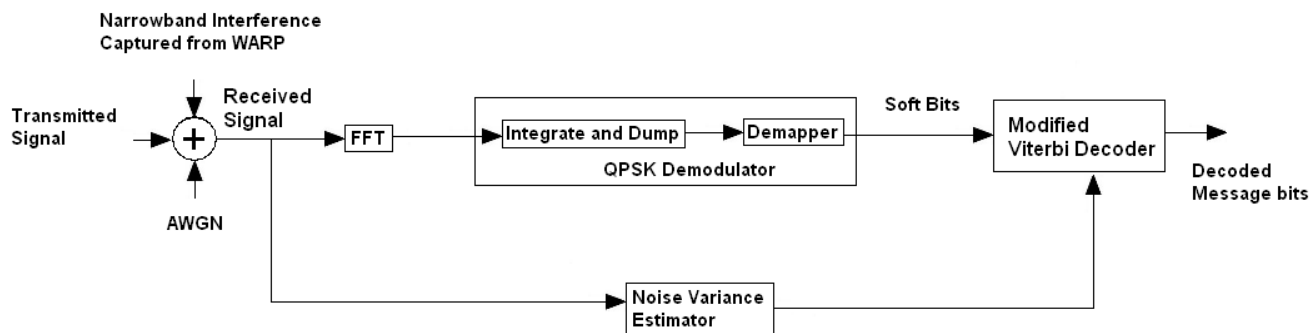

Figure 6.5: Modified simulation model

In order to identify the interference in the received signal, we take the received signal and break it into windows of 100 samples each. The variance of these windows are calculated individually and stored. This gives us an array of noise variance to work with as shown by the topmost plot in figure 6.6. The variances values in this array that exceed a suitably defined threshold, are used to identify the interference. The idea was to isolate these spiked regions which represent bursts of Bluetooth interference. This was based on the assumption that the narrow band interference power is higher than the intended received signal. In reality Bluetooth devices operate over much shorter distances compared to OFDM systems such as WLANs, and their received power can indeed be much higher than the received OFDM signal. Also, in this simulation the signal to Jamming ratio was kept at 0 dB. In

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other words the average narrow band interference power was equal to the average signal power. Therefore most of the narrow band interference bursts captured using WARP, had higher power than that of the intended signal. We then select the corresponding values of the original received signal as our interference. This process is illustrated in Fig. 6.6. The last plot in this figure shows the signal that was used to compute the noise variance. It was used the same way as the narrow band interference in Fig. 6.1 to obtain the noise variance σ^2 used by the decoder.

The main reason a window size of 100 was chosen was to keep the simulation as realistic as possible. Channel 1 of the ISM band of approximate bandwidth of 20 MHz was sampled by the WARP board. The highest baseband frequency component is 10 MHz and so according to Nyquist criterion the sampling rate was 20MHz and the time period was therefore 50ns. On the other hand, we know that in the IEEE802.11 standard the OFDM symbol time period is 3.2 μs plus 0.8 μs for cyclic prefix., i.e. 4 μs in total. Therefore the number of samples per OFDM symbol captured by the WARP board was 80. A smaller window size of 100 is more realistic than a larger window size. If a window size of 1000 was used, it would span 12.5 OFDM symbols. In that case the noise variance estimation technique would slow down a real system significantly and It would only be able to decode bits after every 12.5 OFDM symbols had been received.

The performance of the modified Viterbi algorithm using the above estimated noise variance can we seen in Fig.6.7. Comparing this curve to the conventional Viterbi decoder we can see the performance improvement as shown in Fig.6.8
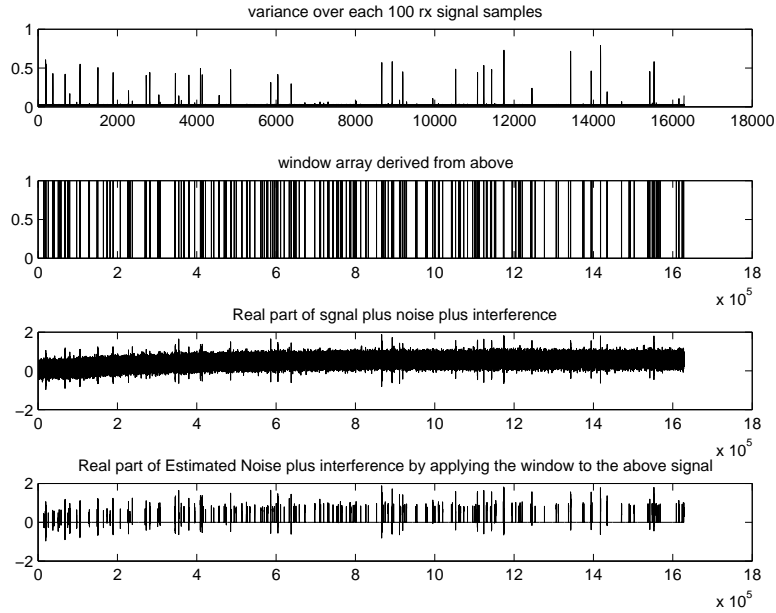
Figure 6.6: The steps followed for Noise Variance Estimation. The threshold used was arbitrarily decided by inspection.
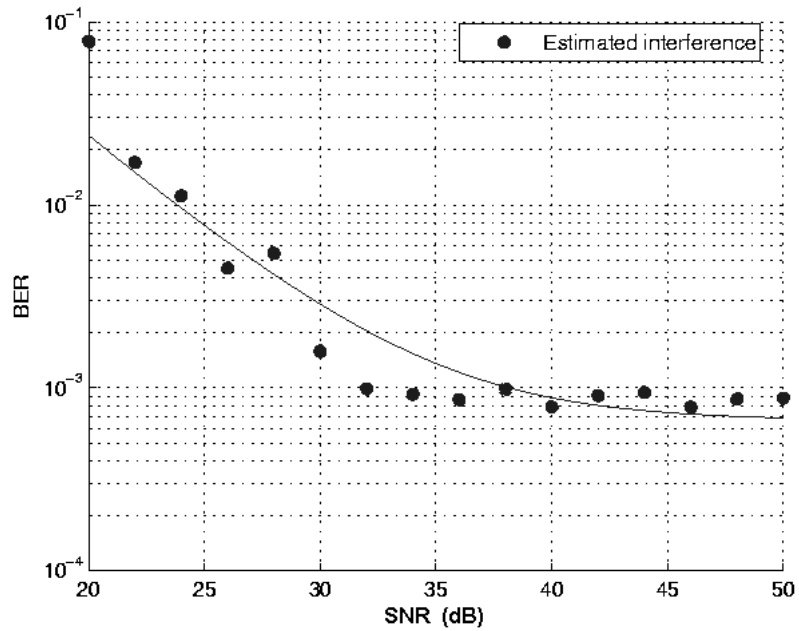

Figure 6.7: BER vs SNR. Here also the average interference Power is equal to the Signal power. Knowledge of the interference has not been assumed but it has been estimated from the received signal.
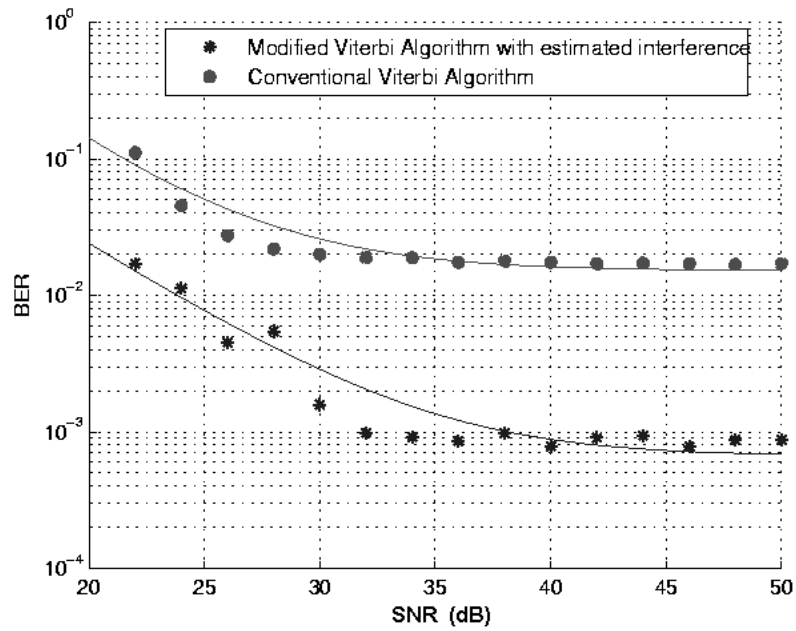
Figure 6.8: BER vs SNR. Here also the average interference Power is equal to the Signal power. Knowledge of the interference has not been assumed but it has been estimated from the received signal.

6.4 Comparing the performances shown in Fig. 6.4 and Fig. 6.8

Comparing Fig. 6.4 and Fig. 6.8 one may notice that the modified Viterbi algorithm seems to perform slightly better at higher AWGN SNR in the latter. This is because in the noise estimation technique presented in the previous section, we have isolated the narrow band interference from the received signal. Only those parts of the received signal were considered interference affected which had sudden bursts of significantly higher variance. On the other hand for the curve in Fig. 6.4 we took into account the AWGN noise as well as the interference samples captured using WARP, in calculating the noise variance over each QPSK symbol.

Chapter 7

Conclusion

In this thesis we considered a modification to the conventional metric in the Viterbi decoder used in the OFDM systems. This metric takes into account frequency selective narrow band interference introduced by various devices in the 2.4 Ghz ISM band. Bluetooth devices, which also operate in this frequency band, are a threat to the smooth performance of OFDM systems. It has been shown in this thesis, that if an OFDM system has a way of identifying narrow band interference in the received signal, there is a way to improve the bit error rate at the receiver using a modified Viterbi decoder. In other words, if we know the interference that degrades the transmitted signal, then we can use this knowledge in order to reduce its effect on the received bits. Practical systems that are able to estimate narrow band interference can use this technique to its advantage. Noise variance estimation techniques reviewed in this thesis such as [10], [18] and [21], could also be used together with the modified Viterbi decoder to make OFDM systems more proficient in handling narrow band interference.

Appendix A

A technique to determine the AWGN noise from the WARP signal

As discussed before, the three signals we consider in our simulation are the transmitted signal, the narrow band interference signal captured by WARP and the AWGN signal. Here we consider a different way of adjusting the the power of these signals, than our previous simulation in chapter 6. Using this technique we again did the the simulation as shown in figure 6.1.

We first identified regions in the captured interference data that are in between the bursts of higher interference power as shown in fig. A.1. The samples of these regions of the signal were considered noise samples. Additive White Gaussian noise of zero mean and unit variance (from MATLAB) was scaled by the variance of these regions of the signal, to obtain the AWGN signal. We then adjusted the power of the transmitted message signal to obtain different SNR's with respect to this AWGN. Finally the average power of the WARP interference data was adjusted to make it equal to the transmitted signal power. In figure A.2 we present the simulation results with SJR of 0 dB. As we can see, this result differs

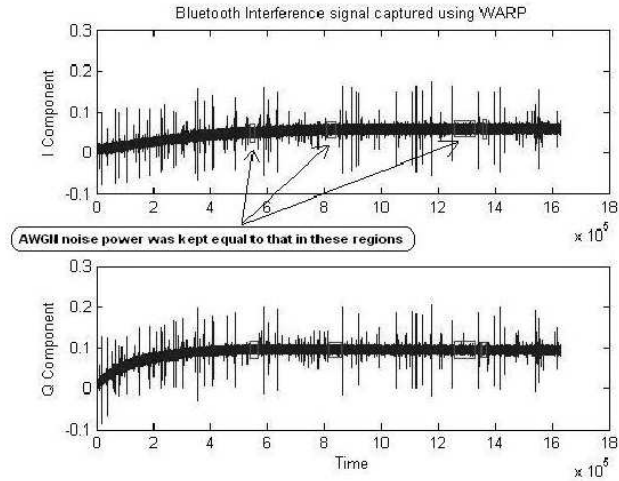with that of figure 6.4 only at lower SNR values such as 20 dB. At higher SNR values they are the same.

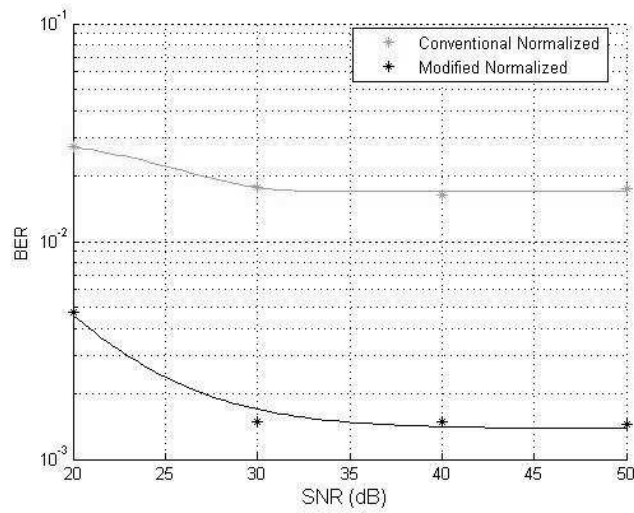Figure A.1: The interference used to adjust the AWGN signal power

Figure A.2: BER vs SNR. Here the average interference Power is equal to the average Signal power

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