Transmission Strategies for the Gaussian Parallel Relay Channel

by

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AUTHOR'S DECLARATION

I hereby declare that I am the sole author of this thesis. This is a true copy of the thesis, including any required final revisions, as accepted by my examiners.

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Abstract

Cooperative wireless communication has received significant attention during recent years due to several reasons. First, since the received power decreases rapidly with distance, the idea of multi-hopping is becoming of particular importance. In multi-hopped communication, the source exploits some intermediate nodes as relays. Then the source sends its message via those relays to the destination. Second, relays can emulate some kind of distributed transmit antennas to form spatial diversity and combat multi-path fading effect of the wireless channel.

Parallel Relay Channel is an information theoretical model for a communication system whereby a sender aims to communicate to a receiver with the help of relay nodes. It represents the simplest model for a multihop wireless network and a full understanding of the limits of communication over such a channel can potentially shed light on the design of more efficient wireless networks. However, the capacity of the relay channel has been established only for few special cases and little progress has been made toward solving the general case since the early 1980s.

In this dissertation, motivated by practical constraints, we study the information theoretical limits of the half-duplex Gaussian Parallel Relay channel , as well as, the transmission strategies for the parallel relay channel with bandwidth mismatch between the first and the second hops.

Chapter 2 investigates the problem of communication for a network composed of two half-duplex parallel relays with additive white Gaussian noise (AWGN). There is no direct link between the source and the destination. However, the relays can communicate with each other through the channel between them. Two protocols, i.e., *Simultaneous* and *Successive* relaying, associated with two possible relay scheduling are proposed. The simultaneous relaying protocol is based on *Broadcast-multiaccess with Common Message (BCM)* scheme. For the successive relaying protocol: (i) a *Non-Cooperative* scheme based on the *Dirty Paper Coding (DPC)*, and (ii) a *Cooperative* scheme based on the *Block Markov Encoding (BME)* are considered. The composite scheme of employing BME in *at most* one relay and DPC in *at least* another one is shown to achieve at least the same rate when compared to the *Cooperative* and *Non-Cooperative* schemes. A *"Simultaneous-Successive Relaying based on*

Dirty paper coding scheme" (SSRD) is also proposed. The optimum scheduling of the relays and hence the capacity of the half-duplex Gaussian parallel relay channel in the low and high signal-to-noise ratio (SNR) scenarios is derived. In the low SNR scenario, it is revealed that under certain conditions for the channel coefficients, the ratio of the achievable rate of the simultaneous relaying based on BCM to the cut-set bound tends to be 1. On the other hand, as SNR goes to infinity, it is proved that successive relaying, based on the DPC, asymptotically achieves the capacity of the network.

Schein and Gallager introduced the Gaussian parallel relay channel in 2000. They proposed the Amplify-and-Forward (AF) and the Decode-and-Forward (DF) strategies for this channel. For a long time, the best known achievable rate for this channel was based on the AF and DF with time sharing (AF-DF). Recently, a Rematch-and-Forward (RF) scheme for the scenario in which different amounts of bandwidth can be assigned to the first and second hops were proposed. In chapter 3, we propose a *Combined Amplify-and-Decode Forward (CADF)* scheme for the Gaussian parallel relay channel. We prove that the CADF scheme always gives a better achievable rate compared to the RF scheme, when there is a bandwidth mismatch between the first hop and the second hop. Furthermore, for the equal bandwidth case (Schein's setup), we show that the time sharing between the CADF and the DF schemes (CADF-DF) leads to a better achievable rate compared to the time sharing between the RF and the DF schemes (RF-DF) as well as the AF-DF.

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During the last one year an half of my PhD studies, I was so lucky and fortunate to know and fall in love with my beloved Elnaz, and finally marry her. I would really appreciate her incredible continual support and encouragement during this short period.

To my parents,
Fatemeh Roshanravan and Seyed Mohsen Changiz Rezaei
and
To my beloved wife,
Elnaz Rezaei

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List of Abbreviations

AF	Amplify and Forward
DF	Decode and Forward
CF	Compress and Forward
RF	Rematch and Forward
MLM	Modulo Lattice Modulation
JSCC	Joint Source Channel Coding
DPC	Dirty Paper Coding
BME	Block Markov Encoding
BCM	Broadcast-multiaccess with Common Message
SF	Scale-Forward
SSRD	Simultaneous-Successive Relaying Protocol based on Dirty paper coding
CADF	Combined Amplify and Decode Forward
MAC	Multiple Access Channel
BC	Broadcast Channel
MIMO	Multiple-Input Multiple-Output
MISO	Multiple-Input Single-Output
SIMO	Single-Input Multiple-Output
SNR	Signal to Noise Ratio
AWGN	Additive White Gaussian Noise

Notation

Regular Letters	Scalars
Boldface Lower-Case Letters	Vectors
$f(n) = O(g(n))$	If and only if there exist positive constants c, and n_0 such that $\|f(n)\| < \|cg(n)\|$ for all $n \geq n_0$.
$f(n) = \Theta(g(n))$	If and only if there exist positive constants c_1, c_2, and n_0 such that $\|c_1 g(n)\| < \|f(n)\| < \|c_2 g(n)\|$ for all $n \geq n_0$.
$\\|.\\|$	The Euclidean norm of a vector.
$< .,. >$	Inner product two vectors in the Euclidean space.
$C(x)$	$\frac{1}{2}\log_2(1+x)$
$\bar{x}$	$1 - x$
$A_\epsilon^{(n)}$	Denotes the set of weakly jointly typical sequences for any intended set of random variables.

Chapter 1

Introduction

The continuous growth in wireless communication has motivated information theoretists to extend Shannon's information theoretic arguments for a single user channel to the scenarios that involve communication among multiple users. In this regard, cooperative communication in which a source exploits some intermediate nodes as relays, to transmit its data to an intended destination has received significant attention during recent years. Relays can emulate distributed transmit antennas to combat the multi-path fading effect and increase the physical coverage area.

Relay channel is a three terminal network which was introduced for the first time by Van der Meulen in 1971 [1]. The most important capacity results of the relay channel were reported by Cover and El Gamal [2]. Two relaying strategies are proposed in [2]. In one strategy, the relay decodes the transmitted message and forwards the re-encoded version to the destination, while in another one the relay does not decode the message, but sends the quantized received values to the destination (Compress-and-Forward (CF) scheme). Zahedi and El Gamal considered two different cases of the frequency division Gaussian relay channel. They derived lower and upper bounds on the capacity of this channel, which in turn translates to upper and lower bounds on the minimum required energy per bit for the reliable transmission [17]. The authors also derived a single letter characterization of the capacity of the frequency division Additive White Gaussian Noise (AWGN) relay channel with simple linear relaying scheme [18][19]. Recently, Cover and Young-Han Kim in [44] studied a class of deterministic relay channel and derived its capacity with the hash-and-forward and CF schemes. Marko Aleksic, Peyman Razaghi, and Wei Yu in [45] derived the capacity of a class of modulo-sum relay channels using the CF scheme of [2]. They showed that the capacity of this channel is strictly below the cut-set bound.

Moreover, several works study transmission strategies for multi-relay channels (See [3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 28, 25, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43]). Schein in [3, 4] establishes upper and lower bounds on the capacity of a full-duplex parallel relay channel. The parallel relay channel in [3, 4] consists of a source, two relays and a destination, where there is no direct link between the source and the destination, and also between the two relays. Generally, the best rate reported for the full-duplex Gaussian parallel relay channel is based on the time sharing between the combination of the Amplify-Forward (AF) and Decode-Forward (DF) schemes (CADF scheme) and the Decode-Forward (DF) scheme (See [5],[6]).

Parallel relay network is also considered in [12]. However, in the setup of [12], unlike the setup in [3, 4, 5, 6], it is assumed that more than two relays exist in the network, and there is also a direct link between the source and the destination. Motivated by applications in sensor networks, the authors assume large bandwidth resources allowing orthogonal transmissions at different nodes. They characterize optimum resource allocation for AF and DF and show that the wide-band regime minimizes the energy cost per information bit in DF, while AF should work in the band-limited regime to achieve the best rate.

Xie and Kumar generalized the block Markov encoding scheme of [2] for a network of multiple relays [7]. Furthermore, Gastpar, Kramer, and Gupta extended the CF scheme in [2] to a multiple relay channel by introducing the concept of antenna polling in [8] and [9]. They showed that when the relays are close to the destination, this strategy achieves the antenna-clustering capacity. On the other hand, when relays are close to the source, the DF strategy can achieve the capacity in a wireless relay network [10]. In [11], Amichai, Shamai, Steinberg and Kramer considered the problem of a nomadic terminal sending information to a remote destination via agents with lossless connections. They investigated the case that these agents do not have any decoding capability, so they must compress what is received. This case is also fully characterized for the Gaussian channel. Razaghi and Yu in [13] proposed a parity-forwarding scheme for full-duplex multiple relay networks. They showed that relay networks can be degraded in several ways, and parity-forwarding achieves capacity for a new degraded form. Recently, Salman Avestimehr, Suhas Diggavi and David Tse in [26, 27, 28] further studied the capacity of wireless relay networks. The authors in [26][27], proposed a deterministic model for a multiuser communication channel and generalized the max-flow min-cut theorem from the wire-line to the wireless networks. In [28], they proposed an achievable rate for the Gaussian relay networks and showed that their achievable rate is within a constant bit (determined by the graph topology of the network) from the cut-set bound.

1.1 Half-Duplex Relaying and Parallel Relay Channel

Recently, half-duplex relaying has drawn a great deal of attention (See [17, 18, 19, 15, 16, 20, 21], [25], [34, 35, 36, 37, 41, 42, 43]). The problem of time division relaying is considered by Host-Madsen and Zhang [20]. By considering fading scenarios, and assuming channel state information (CSI), they study upper and lower bounds on the outage capacity and the Ergodic capacity.

Half-duplex relaying, in multiple relay networks, is studied in [25, 34, 35, 36, 37, 41, 42, 43]. Gastpar in [25] shows that in a Gaussian parallel relay channel with infinite number of relays, the optimum relaying scheme is AF. Rankov and Wittneben in [34, 35] further study the problem of half-duplex relaying in a two-hop communication scenario. In their study, they also consider a parallel relay setup with two relays where there is no direct link between the source and the destination, while there exists a link between the relays. Their relaying protocols are based on either AF or DF, in which the relays successively forward their messages from the source to the destination. We call this protocol "*Successive Relaying*" in the sequel.

Xue and Sandhu in [36] also study different half-duplex relaying protocols and schemes for the Gaussian parallel relay channel with two relays. Unlike our model in Fig. 1.1, the inter-relay channel does not exist between two relays in [36]. They propose two time sharing patterns. In time sharing pattern I, total available time is divided into two stages. In the first stage, the source transmits its signal to both relays, and both relays receive it. Having received the transmitted signal from the source during the first stage, the relays transmit their signal coherently to the destination in the second stage. On the other hand, in time sharing pattern II, although total available time is again divided into two stages, in each stage the source and only one relay are in transmit mode, while the other relay and the destination are in receive mode. For the time sharing pattern I, which we call *"Simultaneous Relaying"* protocol in the sequel, they propose "Scale-Forward (SF)", "Broadcast-multiaccess with Common Message (BCM)", "Compress-Forward (CF)", and two hybrid schemes, i.e., "Decode-Forward via one link while Scale-Forward via other (Hybrid DF-SF)", and "Decode-Forward via one link while Compress-Forward via other (Hybrid DF-CF)". For the time sharing pattern II, they propose "Decode-Forward (DF)" scheme. They prove that the DF scheme is the best under time sharing pattern II. They also prove that this scheme achieves the capacity in certain symmetric case. However, it should be noted that the capacity result of [36] is based on the peak power constraint. This means that each node is assumed to transmit with a fixed power independent from the portion of the time it is in the transmit mode.

1.2 Summary of Dissertation and Main Contributions

Since constructing a large-scale wireless network is very expensive, it is important to understand how to efficiently utilize the available power and bandwidth resources. The Gaussian parallel relay channel with two relays which was introduced for the first time by Schein and Gallager is one of the basic building blocks of a general network (See [3, 4]). Furthermore, motivated by practical constraints, half-duplex relays which cannot transmit and receive at the same time and in the same frequency band are of great importance. Hence, as one of our goals in this thesis, we study and analyze the performance limits of a half-duplex Gaussian parallel relay channel. Moreover, by proposing a new coding scheme we improve the achievable rate of the Schein's Gaussian parallel relay channel. A chapter is dedicated to each of these topics. A summary of the contributions of this dissertation is as follows.

Chapter 2: Half-Duplex Gaussian Parallel Relay Channel

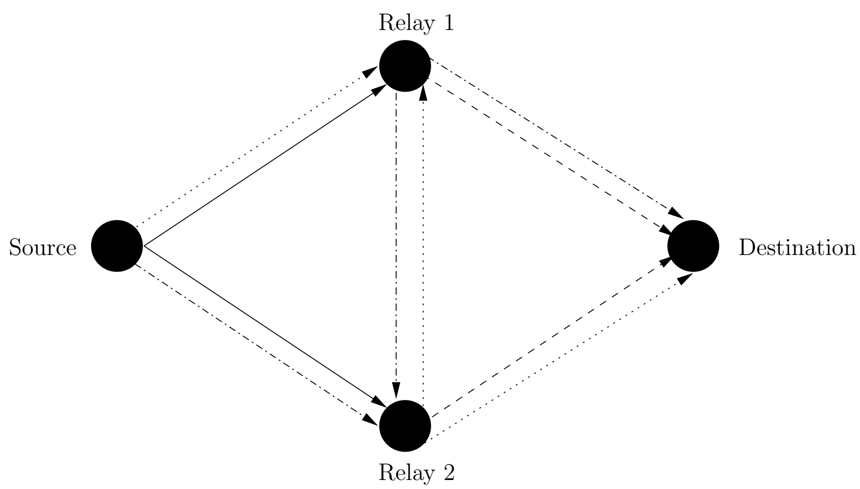


Figure 1.1: Half-Duplex Gaussian Parallel Relay Channel (Solid, dotted, dashed, and dash-dotted lines represent channels that are orthogonal to each other in the time domain).

In Chapter 2, different transmission strategies for the half-duplex Gaussian parallel relay channel with two relays are proposed and their optimalities are investigated (See Fig. 1.1). The summary of the contribution of this chapter is as follows.

- **Scheduling Protocols**

  Simultaneous and successive relaying protocols are proposed. The simultaneous relaying protocol is based on the "Broadcast-multiaccess with Common Message (BCM)" scheme of [36]. For the successive relaying protocol, a *Non-Cooperative* scheme based on "Dirty Paper Coding (DPC)" and also a *Cooperative* scheme based on "Block Markov Encoding (BME)" are proposed. Furthermore, simultaneous and successive relaying protocols are combined and a "Simultaneous-Successive Relaying based on Dirty paper coding" (SSRD) scheme with a new achievable rate is proposed.

It is shown that in the low SNR scenario and under certain channel conditions, SSRD scheme is converted to simultaneous relaying based on BCM, while in the high SNR scenarios, it becomes successive relaying based on DPC (to achieve the capacity).

- **Capacity for symmetric scenarios**

  We show that in the symmetric case, the DPC scheme achieves the successive cut-set bound.

- **Different Types of Decoding**

  Two different types of decoding, i.e., *successive* and *backward* decoding, at the destination for the BME scheme are proposed. We prove that the achievable rate of BME with backward decoding is greater than or equal to that of BME with successive decoding, i.e., $R_{BME_{back}} \geq R_{BME_{succ}}$.

- **Composite BME-DPC scheme**

  It is proved that BME with backward decoding leads to a simple strategy in which *at most* one of the relays is required to cooperate with the other relay in sending the bin index of the other relay's message. Accordingly, in the Gaussian case, the combination of BME in *at most* one relay and DPC in *at least* the other relay always achieves a rate greater than or equal to that achieved by the simple BME or DPC schemes.

Chapter 3: A New Achievable Rate for the Gaussian Parallel Relay Channel

In chapter 3, we consider the Gaussian parallel relay channel with a source, a destination, and a set of relays. There is no direct link from the source to the destination. This parallel relay channel is a special case of a multiple relay network in which the source broadcasts its data to all the relays, and the relays transmit their data coherently to the destination.

Summary of the contributions of this chapter is as follows.

- **The Bandwidth Mismatch Case**

  A combined Amplify and Decode Forward (CADF) scheme for the bandwidth mismatch case, where the bandwidth associated with different hops is different, is proposed. The superiority of this scheme compared with the Rematch and Forward scheme of [48][49] is proved.

- **A New Achievable Rate for the Schein and Gallager's Set up**

  We show that time sharing between the CADF and DF schemes (CADF-DF) always outperforms the RF-DF and the AF-DF. Hence, a new achievable rate for the Gaussian parallel relay channel with two relays is obtained.

Chapter 2

Half-Duplex Gaussian Parallel Relay Channel

In this chapter[1], we study transmission strategies for a network with a source, a destination, and two half-duplex relays with additive white Gaussian noise which cooperate with each other to facilitate data transmission from the source to the destination. Furthermore, it is assumed that no direct link exists between the source and the destination. Therefore, this channel is similar to the one considered in [3, 4] with two differences: First, the relays in [3, 4] are full-duplex nodes, and second, unlike in our work, the relays in [3, 4] are not allowed to communicate with each other (See Fig. 1.1).

Our primary objective is to find the best scheduling of the relays in the intended setup. We consider two relaying protocols, i.e., simultaneous relaying versus successive relaying, associated with two possible relay schedulings.

For simultaneous relaying, each relay exploits "Broadcast-multiaccess with Common Message (BCM)" scheme of [36]. Therefore, similar to time sharing pattern I of [36], in a fixed pre-assigned portion of the time, the relays receive the signal transmitted from the source, and in the remaining time slot they transmit the re-encoded version of the decoded message together.

The proposed successive relaying protocol is similar to the time sharing pattern II of [36]. However, since unlike [36], we assume a channel between two relays, the DF scheme of [36] is not applicable here. Indeed, due to the presence of the inter-relay channel in our model, transmitting relay produces interference on the receiving relay. Therefore, we consider two

[1]Portions reprinted, with permission, from (Seyed Saeed Changiz Rezaei, Shahab Oveis Gharan, and Amir K. Khandani, "Relay Scheduling in the Half-Duplex Gaussian Parallel Channel", IEEE Transaction on Information Theory, Volume 56, Issue 6, pp. 2668 - 2687, June 2010). © [2010] IEEE.

approaches to deal with this interference. We propose a *Non-Cooperative* scheme based on "Dirty Paper Coding (DPC)" and also a *Cooperative* scheme based on "Block Markov Encoding (BME)". In the Non-Cooperative scheme, since the source knows the interference due to the transmitting relay on the receiving relay non-causally, it effectively remove the inter-relay channel by exploiting DPC scheme. On the other hand, in the Cooperative scheme, we allow the receiving relay to decode not only the signal transmitted by the source, but also the signal transmitted by the transmitting relay. Knowing the message of each other, the relays cooperate together to facilitate data transmission from the source to the destination.

Furthermore, simultaneous and successive relaying protocols are combined and a "Simultaneous Successive Relaying based on Dirty paper coding" (SSRD) scheme with a new achievable rate is proposed.

Since in simultaneous relaying the source transmits and the destination receives only in a portion of the time, simultaneous relaying is not spectrally efficient. However, simultaneous relaying does not suffer from the inter-relay interference. On the other hand, although successive relaying is spectrally efficient, the inter-relay interference can degrade the performance. Hence, a natural question of optimum scheduling arises.

As the main result of this chapter, we derive the optimum relay scheduling in low and high SNR scenarios. In low SNR scenarios and under certain channel conditions, we show that the ratio of the achievable rate of BCM for simultaneous relaying to the cut-set bound tends to one. On the other hand, in high SNR scenarios, we prove that the gap between the achievable rate of the proposed DPC for successive relaying and the cut-set bound tends to zero as $O\left(\frac{1}{\log SNR}\right)$. In other words, it is shown that in the low SNR scenario and under certain channel conditions, SSRD scheme is converted to simultaneous relaying based on BCM, while in the high SNR scenarios, it becomes successive relaying based on DPC (to achieve the capacity). Besides this main result, the following results are also obtained in this chapter:

1. It is proved that BME with backward decoding leads to a simple strategy in which *at most* one of the relays is required to cooperate with the other relay in sending the bin index of the other relay's message. Accordingly, in the Gaussian case, the combination of BME in *at most* one relay and DPC in *at least* the other relay always achieves a rate greater than or equal to that achieved by the simple BME or DPC schemes.

2. Two different types of decoding, i.e., *successive* and *backward* decoding, at the destination for the BME scheme are proposed. We prove that the achievable rate of BME with

backward decoding is greater than or equal to that of BME with successive decoding, i.e., $R_{BME_{back}} \geq R_{BME_{succ}}$.

3. In the degraded case, where the destination receives a degraded version of the received signals at the relays, BME with backward decoding achieves the successive cut-set bound.

4. In the symmetric case, the DPC scheme achieves the successive cut-set bound.

It is worth noting that in our work, unlike [36], we assume average power constraint for the nodes over all transmit and receive modes.

After this work was completed, we became aware of [37] which has independently proposed an achievable rate based on the combination of superposition coding, BME and DPC. In their scheme, the intended message "w" is split into a message which is transmitted to the destination by exploiting cooperation between the relays "w_r" and a message which is transmitted to the destination without using any cooperation between the relays "w_d". Hence, the signal associated with "w_d", transmitted by one relay, can be considered as interference on the other relay. "w_r" is transmitted by using BME and "w_d" is transmitted by employing DPC. Therefore, in their general scheme, the associated signals with these two messages are superimposed and transmitted. As the channel between the two relays becomes strong, their proposed scheme is converted to BME. On the other hand, as the channel becomes weak, their proposed scheme becomes DPC. The assumption of [37] for node power consumption is similar to ours.

The approach of this work is different from [37], in the sense that we compare the successive and simultaneous relaying protocols. We show that each one achieves the capacity in certain scenarios. Specifically, unlike [37], we also propose BME based on backward decoding, and consequently, establish the mentioned results 1 to 4 above.

This chapter is organized as follows. In section 2.1, the system model is explained. In section 2.2, the coding schemes for the half-duplex Gaussian parallel relay channel are proposed and their associated rates are derived. Section 2.3 is devoted to optimality results, and finally simulation results are discussed in section 2.4.

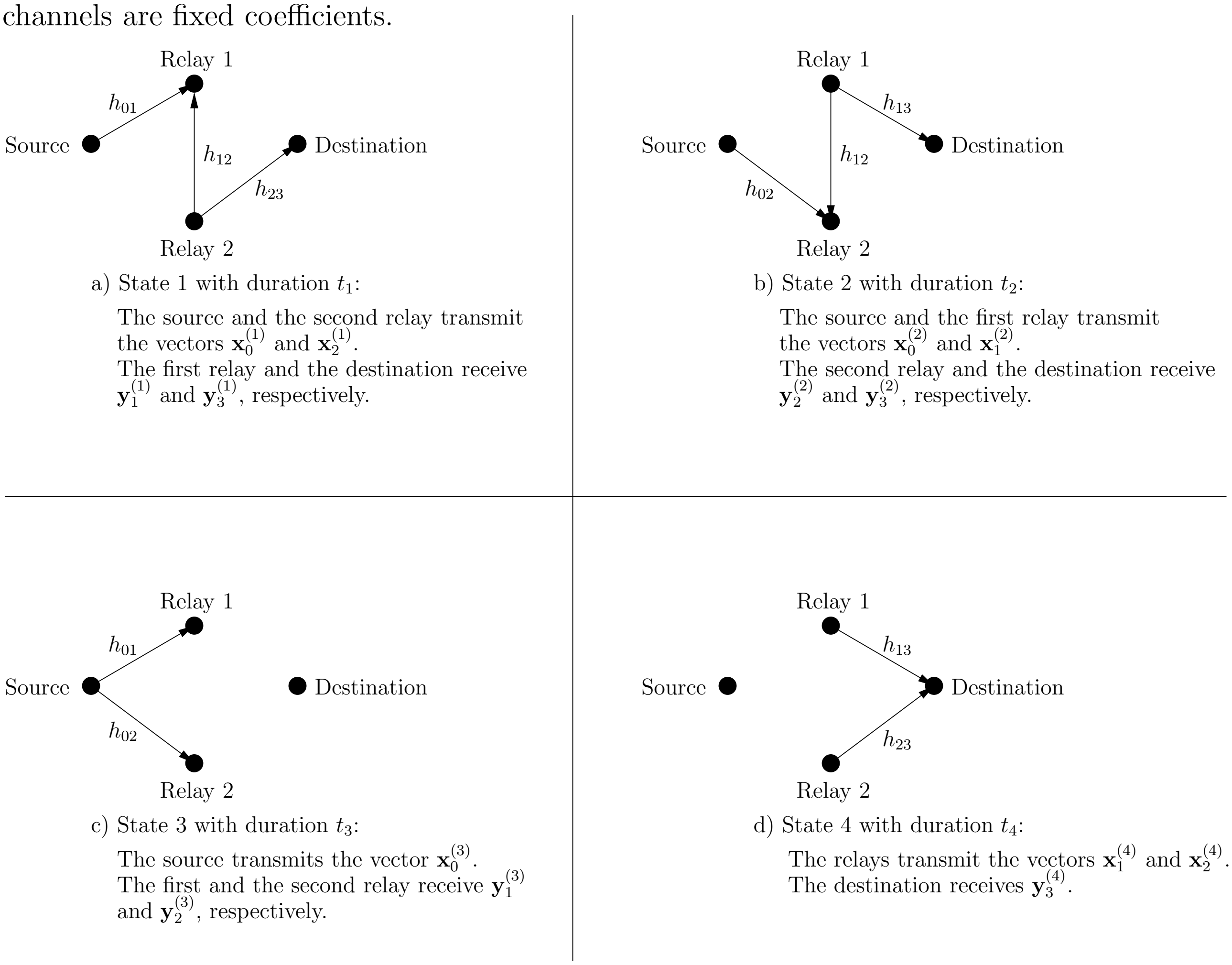
2.1 The System Model

We consider a Gaussian network which consists of a source, two half-duplex relays, and a destination, and there is no direct link between the source and the destination. Here we define four states according to the transmitting and receiving mode of each relay (See

Fig. 2.1). Assuming n uses of the network, n_b denotes the number of network use when the network is operating in state b. Hence, denoting the portion of the total network use that the network is in state b as t_b, we have $t_b = \frac{n_b}{n}$, and therefore $\sum_{b=1}^{4} t_b = 1$. Nodes 0, 1, 2, and 3 represent the source, relay 1, relay 2, and the destination, respectively. Moreover, the transmitting and receiving signals at node a during state b are represented by $\mathbf{x}_a^{(b)}$ and $\mathbf{y}_a^{(b)}$, respectively. Hence, at each node $c \in \{1, 2, 3\}$, we have

$$\mathbf{y}_c^{(b)} = \sum_{a\in\{0,1,2\}} h_{ac}\mathbf{x}_a^{(b)} + \mathbf{z}_c^{(b)}. \tag{2.1}$$

where h_{ac}'s denote channel coefficients from node a to node c, and $\mathbf{z}_c^{(b)}$ is the AWGN term with zero mean and variance of "1" per dimension. It is worth mentioning that noises at the relays and destination at each state of transmission are independent from each other and channels are fixed coefficients.



a) State 1 with duration t_1:
The source and the second relay transmit the vectors $\mathbf{x}_0^{(1)}$ and $\mathbf{x}_2^{(1)}$.
The first relay and the destination receive $\mathbf{y}_1^{(1)}$ and $\mathbf{y}_3^{(1)}$, respectively.

b) State 2 with duration t_2:
The source and the first relay transmit the vectors $\mathbf{x}_0^{(2)}$ and $\mathbf{x}_1^{(2)}$.
The second relay and the destination receive $\mathbf{y}_2^{(2)}$ and $\mathbf{y}_3^{(2)}$, respectively.

c) State 3 with duration t_3:
The source transmits the vector $\mathbf{x}_0^{(3)}$.
The first and the second relay receive $\mathbf{y}_1^{(3)}$ and $\mathbf{y}_2^{(3)}$, respectively.

d) State 4 with duration t_4:
The relays transmit the vectors $\mathbf{x}_1^{(4)}$ and $\mathbf{x}_2^{(4)}$.
The destination receives $\mathbf{y}_3^{(4)}$.

Figure 2.1: System Model.

Noting the transmission strategies in Fig. 2.1, we have

$$\mathbf{y}_1^{(1)} = h_{01}\mathbf{x}_0^{(1)} + h_{21}\mathbf{x}_2^{(1)} + \mathbf{z}_1^{(1)}, \tag{2.2}$$
$$\mathbf{y}_3^{(1)} = h_{23}\mathbf{x}_2^{(1)} + \mathbf{z}_3^{(1)}, \tag{2.3}$$
$$\mathbf{y}_2^{(2)} = h_{02}\mathbf{x}_0^{(2)} + h_{12}\mathbf{x}_1^{(2)} + \mathbf{z}_2^{(2)}, \tag{2.4}$$
$$\mathbf{y}_3^{(2)} = h_{13}\mathbf{x}_1^{(2)} + \mathbf{z}_3^{(2)}, \tag{2.5}$$
$$\mathbf{y}_k^{(3)} = h_{0k}\mathbf{x}_0^{(3)} + \mathbf{z}_k^{(3)}, k \in \{1, 2\}, \tag{2.6}$$
$$\mathbf{y}_3^{(4)} = \sum_{k=1}^{2} h_{k3}\mathbf{x}_k^{(4)} + \mathbf{z}_3^{(4)}. \tag{2.7}$$

Throughout the paper, we assume that $h_{01} \geq h_{02}$ unless specified otherwise, and from reciprocity assumption, we have $h_{12} = h_{21}$. Furthermore, the power constraints P_0, P_1, and P_2 should be satisfied for the source, the first relay, and the second relay, respectively. Hence, denoting the power consumption of node a at state b by $P_a^{(b)} = \frac{\left\|\mathbf{x}_a^{(b)}\right\|^2}{n}$, we have

$$\begin{aligned} P_0^{(1)} + P_0^{(2)} + P_0^{(3)} &= P_0, \\ P_1^{(2)} + P_1^{(4)} &= P_1, \\ P_2^{(1)} + P_2^{(4)} &= P_2. \end{aligned} \tag{2.8}$$

2.2 Achievable Rates and Coding Schemes

In this section, we propose two cooperative protocols, i.e., *Successive* and *Simultaneous* relaying protocols, for a half-duplex Gaussian parallel relay channel.

Since we will propose *achievable schemes* for parallel relay channel in this chapter and the next chapter, let us recall the definition of achievability:

Definition 2.2.1. *Assume message $w \in [1, M]$. The rate R of an (M, n) code is defined by $R = \frac{\log M}{n}$ bits per transmission. The rate R is said to be* achievable *by a relay channel if, for any $\epsilon > 0$ and for all n sufficiently large, there exists an (M, n) code with $M \geq 2^{nR}$ such that $\lambda_n < \epsilon$, where λ_n is the maximal probability of error. The* capacity *of the relay channel is the supremum of the set of achievable rates.*

2.2.1 Successive Relaying Protocol

In *Successive* relaying protocol, relay one and relay two are not allowed simultaneously to transmit, or receive, i.e. $t_3 = t_4 = 0$, and the relations between the transmitted and the received signals at the relays and at the destination follow from (2.2)-(2.5). For the successive relaying protocol, we propose a *Non-Cooperative* and a *Cooperative Coding* scheme in the sequel. In the proposed schemes, the time is divided into odd and even time slots with the duration t_1 and t_2, respectively. Accordingly, at each odd and even time slots, the source transmits a new message to one of the relays, and the destination receives a new message from the other relay, successively (See Fig. 2.2).

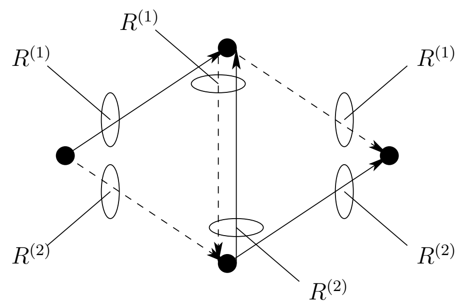


Figure 2.2: Information flow transfer for successive relaying protocol for two relays.

Non-Cooperative Coding

In the Non-Cooperative Coding scheme, each relay considers the other relay's signal as interference. Since the source knows each relay's message, it can apply the Gelfand-Pinsker's coding scheme to transmit its message to the other relay. For a review of Gelfand-Pinsker's result and Dirty Paper Coding see Appendix A. The following Theorem gives the achievable rate of this scheme.

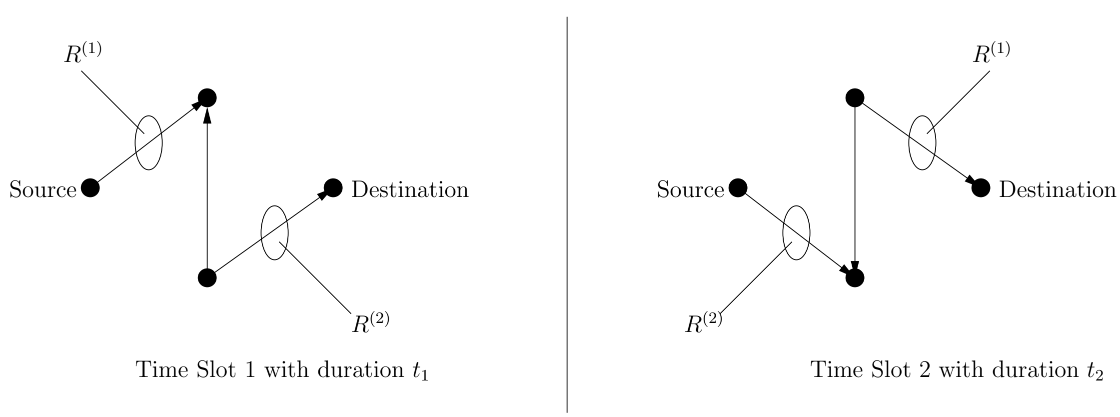


Figure 2.3: Successive relaying protocol based on Non-Cooperative Coding.

Theorem 2.2.1. *For the half-duplex parallel relay channel, assuming successive relaying, the following rate R_{DPC} is achievable:*

$$R_{DPC} = \max_{0 \leq t_1, t_2, t_1+t_2=1} R^{(1)} + R^{(2)}, \tag{2.9}$$

subject to:

$$R^{(1)} \leq \min\left(t_1(I(U_0^{(1)}; Y_1^{(1)}) - I(U_0^{(1)}; X_2^{(1)})), t_2 I(X_1^{(2)}; Y_3^{(2)})\right), \tag{2.10}$$

$$R^{(2)} \leq \min\left(t_2(I(U_0^{(2)}; Y_2^{(2)}) - I(U_0^{(2)}; X_1^{(2)})), t_1 I(X_2^{(1)}; Y_3^{(1)})\right). \tag{2.11}$$

with probabilities:

$$p(x_2^{(1)}, u_0^{(1)}, x_0^{(1)}) = p(x_2^{(1)})p(u_0^{(1)}|x_2^{(1)})p(x_0^{(1)}|u_0^{(1)}, x_2^{(1)}),$$
$$p(x_1^{(2)}, u_0^{(2)}, x_0^{(2)}) = p(x_1^{(2)})p(u_0^{(2)}|x_1^{(2)})p(x_0^{(2)}|u_0^{(2)}, x_1^{(2)}).$$

where $|\mathcal{U}_0^{(1)}| \leq \min\{|\mathcal{X}_0^{(1)}|, |\mathcal{Y}_1^{(1)}|\} + |\mathcal{X}_2^{(1)}| - 1$ *and* $|\mathcal{U}_0^{(2)}| \leq \min\{|\mathcal{X}_0^{(2)}|, |\mathcal{Y}_2^{(2)}|\} + |\mathcal{X}_1^{(2)}| - 1.$

Proof. See Appendix C. □

From Theorem 2.2.1, the achievable rate of the proposed scheme for the Gaussian case can be obtained as follows.

Corollary 2.2.1. *For the half-duplex Gaussian parallel relay channel, assuming successive relaying protocol with power constraints at the source and at each relay, DPC achieves the following rate:*

$$R_{DPC} = \max \ R^{(1)} + R^{(2)}, \tag{2.12}$$

where the maximization (2.12) is over parameters t_1, t_2, $P_0^{(1)}$, and $P_0^{(2)}$ subject to the following constraints:

$$\begin{aligned}
&R^{(1)} \leq \min\left(t_1 C\left(\frac{h_{01}^2 P_0^{(1)}}{t_1}\right), t_2 C\left(\frac{h_{13}^2 P_1}{t_2}\right)\right),\\
&R^{(2)} \leq \min\left(t_2 C\left(\frac{h_{02}^2 P_0^{(2)}}{t_2}\right), t_1 C\left(\frac{h_{23}^2 P_2}{t_1}\right)\right),\\
&P_0^{(1)} + P_0^{(2)} = P_0,\\
&t_1 + t_2 = 1,\\
&0 \leq t_1, t_2, P_0^{(1)}, P_0^{(2)}.
\end{aligned}$$

Proof. See Appendix D. □

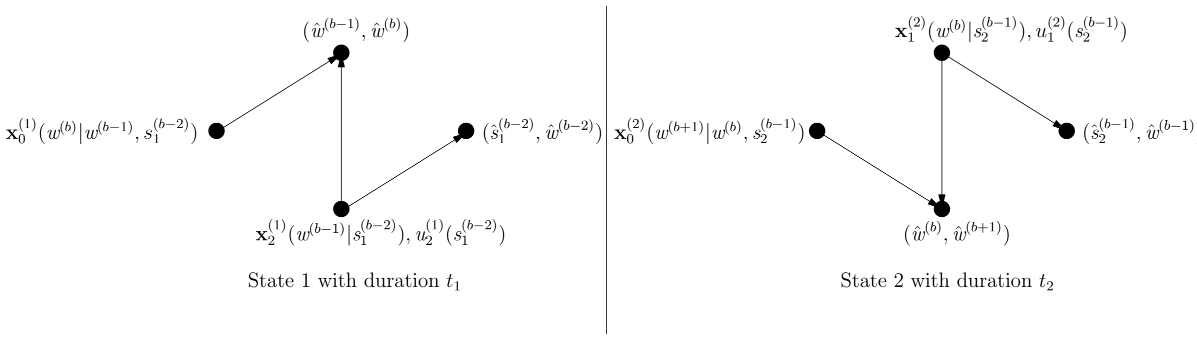


Figure 2.4: Successive relaying protocol based on Cooperative Coding.

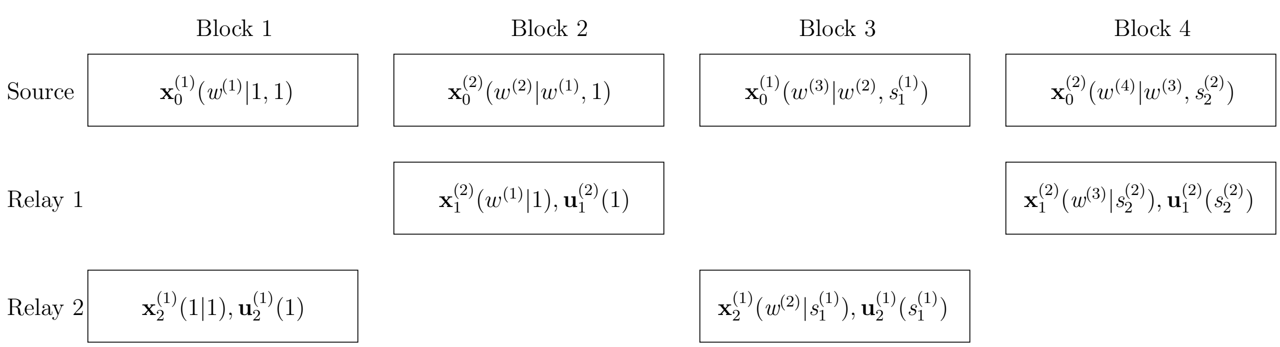


Figure 2.5: Decode-and-forward for successive relaying protocol.

Cooperative Coding

In this type of coding scheme, we assume that, at each block, the receiving relay decodes not only the new transmitted message from the source, but also the previous message transmitted from the transmitting relay (See Figs. 2.2 and 2.4). Our proposed coding scheme is based on binning, superposition coding, and Block Markov Encoding. The source sends B messages $w^{(1)}, w^{(2)}, \cdots, w^{(B)}$ in $B+2$ blocks. For a review of Block Markov Encoding scheme see Appendix B.

Generally, this scheme can be described as follows (See Figs. 2.4 and 2.5). In block b, the relay $(b+1)$ mod 2+1 decodes the transmitted messages $w^{(b)}$ and $w^{(b-1)}$ from the source and the other relay, respectively. In block $b+1$, it broadcasts $w^{(b)}$ and the bin index of $w^{(b-1)}$, $s^{(b-1)}_{(b+2) \bmod 2+1}$, to the destination using the binning function defined next.

Definition (The Binning Function): The binning function $f_{Bin}^{((b+1) \bmod 2+1)}(w^{(b-2)}) : W = \{1, 2, \cdots, 2^{nR^{((b+1) \bmod 2+1)}}\}$ $\longrightarrow \{1, 2, \ldots, 2^{nr_{Bin}^{((b+1) \bmod 2+1)}}\}$ is defined by $f_{Bin}^{((b+1) \bmod 2+1)}(w^{(b-2)}) = s^{(b-2)}_{(b+1) \bmod 2+1}$, where $f_{Bin}^{((b+1) \bmod 2+1)}(.)$ assigns a randomly uniform distributed integer between 1 and $2^{nr_{Bin}^{((b+1) \bmod 2+1)}}$ independently to each member of W.

As indicated in Fig. 2.5, in the first block, the source transmits the codeword $\mathbf{x}_0^{(1)}(w^{(1)}|1,1)$, with i.i.d entries and distribution $p(x_0^{(1)}|x_2^{(1)}, u_2^{(1)})$, to the first relay, while the second relay transmits a doubly indexed codeword $\mathbf{x}_2^{(1)}(1|1)$ and the codeword $\mathbf{u}_2^{(1)}(1)$, with i.i.d entries and distributions $p(x_2^{(1)}|u_2^{(1)})$ and $p(u_2^{(1)})$, to the first relay and destination. In the second block, the source transmits the codeword $\mathbf{x}_0^{(2)}(w^{(2)}|w^{(1)}, 1)$, with i.i.d entries and distribution $p(x_0^{(2)}|x_1^{(2)}, u_1^{(2)})$, to the second relay, and having decoded the message $w^{(1)}$, the first relay broadcasts the codewords $\mathbf{x}_1^{(2)}(w^{(1)}|1)$ and $\mathbf{u}_1^{(2)}(1)$, with i.i.d entries and distributions $p(x_1^{(2)}|u_1^{(2)})$ and $p(u_1^{(2)})$, to the second relay and destination. It should be noted that the destination cannot decode the message $w^{(1)}$ at the end of this block; however, the second relay decodes $w^{(1)}$ and $w^{(2)}$ messages. Using the binning function, it finds the bin index of $w^{(1)}$ according to $s_1^{(1)} = f_{Bin}^{(1)}(w^{(1)})$. In the third block, the source transmits the codeword $\mathbf{x}_0^{(1)}(w^{(3)}|w^{(2)}, s_1^{(1)})$, with i.i.d entries and distribution $p(x_0^{(1)}|x_2^{(1)}, u_2^{(1)})$, to the first relay, and the second relay broadcasts the codewords $\mathbf{x}_2^{(1)}(w^{(2)}|s_1^{(1)})$ and $\mathbf{u}_2^{(1)}(s_1^{(1)})$, with i.i.d entries and distributions $p(x_2^{(1)}|u_2^{(1)})$ and $p(u_2^{(1)})$, to the first relay and destination (for the detailed description of the codebook constructions see Appendix E).

Two types of decoding can be used at the destination: successive decoding and backward decoding. Successive decoding at the destination can be described as follows. At the end of the bth block, the destination cannot decode the message $w^{(b-1)}$; however, having decoded the bin index $s^{(b-2)}_{(b+1) \bmod 2+1}$ from the received vector of the bth block, it can decode the message $w^{(b-2)}$ from $s^{(b-2)}_{(b+1) \bmod 2+1}$ and the received vector of the $(b-1)$th block. On the other hand, backward decoding can be explained as follows. Having received the sequence of the $B+2$th block, the destination starts decoding the intended messages. In the $B+2$th block, one of the relays transmits the dummy message "1" along with the bin index of the message $w^{(B)}$ to the destination. Having received this bin index, the destination decodes it, and then backwardly decodes messages $w^{(b)},\ b = B, B-1, \cdots, 1$ and their bin indices. The following theorem gives the achievable rate of the proposed scheme.

Theorem 2.2.2. *For the half-duplex parallel relay channel, assuming successive relaying, the BME scheme achieves the rates $R_{BME_{succ}}$ and $R_{BME_{back}}$ using successive and backward decoding, respectively:*

$$\begin{aligned}
R_{BME_{succ}} = &\max_{0\le t_1,t_2,t_1+t_2=1} R^{(1)} + R^{(2)} \le \max_{0\le t_1,t_2,t_1+t_2=1} \min \Big(\\
&\min\left(t_1 I\left(X_0^{(1)};Y_1^{(1)} \mid X_2^{(1)},U_2^{(1)}\right), t_2 I\left(X_1^{(2)};Y_3^{(2)} \mid U_1^{(2)}\right) + t_1 I\left(U_2^{(1)};Y_3^{(1)}\right)\right) + \\
&\min\left(t_1 I\left(X_2^{(1)};Y_3^{(1)} \mid U_2^{(1)}\right) + t_2 I\left(U_1^{(2)};Y_3^{(2)}\right), t_2 I\left(X_0^{(2)};Y_2^{(2)} \mid X_1^{(2)},U_1^{(2)}\right)\right), \\
&t_1 I\left(X_0^{(1)},X_2^{(1)};Y_1^{(1)} \mid U_2^{(1)}\right), t_2 I\left(X_0^{(2)},X_1^{(2)};Y_2^{(2)} \mid U_1^{(2)}\right)\Big). \qquad (2.13)
\end{aligned}$$

with probabilities

$$\begin{aligned}
&p(x_0^{(1)},x_2^{(1)},u_2^{(1)}) = p(u_2^{(1)})p(x_2^{(1)}|u_2^{(1)})p(x_0^{(1)}|x_2^{(1)},u_2^{(1)}),\\
&p(x_0^{(2)},x_1^{(2)},u_1^{(2)}) = p(u_1^{(2)})p(x_1^{(2)}|u_1^{(2)})p(x_0^{(2)}|x_1^{(2)},u_1^{(2)}),\\
&p(x_2^{(1)},u_2^{(1)}) = p(u_2^{(1)})p(x_2^{(1)}|u_2^{(1)}),\\
&p(x_1^{(2)},u_1^{(2)}) = p(u_1^{(2)})p(x_1^{(2)}|u_1^{(2)}).
\end{aligned}$$

$$\begin{aligned}
R_{BME_{back}} = &\max_{0\le t_1,t_2,t_1+t_2=1} R^{(1)} + R^{(2)} \le \\
&\max_{0\le t_1,t_2,t_1+t_2=1} \min\left(t_1 I\left(X_0^{(1)},X_2^{(1)};Y_1^{(1)}\right), t_2 I\left(X_0^{(2)},X_1^{(2)};Y_2^{(2)}\right),\right. \\
&t_1 I\left(X_0^{(1)};Y_1^{(1)} \mid X_2^{(1)}\right) + t_2 I\left(X_0^{(2)};Y_2^{(2)} \mid X_1^{(2)}\right), \\
&\left. t_1 I\left(X_2^{(1)};Y_3^{(1)}\right) + t_2 I\left(X_1^{(2)};Y_3^{(2)}\right)\right). \qquad (2.14)
\end{aligned}$$

with probabilities

$$\begin{aligned}
&p(x_0^{(1)},x_2^{(1)}) = p(x_2^{(1)})p(x_0^{(1)}|x_2^{(1)}),\\
&p(x_0^{(2)},x_1^{(2)}) = p(x_1^{(2)})p(x_0^{(2)}|x_1^{(2)}).
\end{aligned}$$

Proof. See Appendix E. □

Remark 2.2.1. *According to the discussion in Appendix E,* $r_{Bin}^{(1)} = 0$ *or* $r_{Bin}^{(2)} = 0$*. In other words, in the Cooperative BME scheme based on backward decoding, at most one relay is necessary to use binning function for the message it receives from another, and the other relay is not necessary to cooperate with this relay. Therefore, we propose a composite BME-DPC scheme for the Gaussian case. In this scheme, at most one of the relays decodes the other relay's message. Having decoded that, it then uses the binning function to cooperate with the other relay. On the other hand, using the DPC scheme, the source cancels the interference due to one relay on the other. In cases that both* $r_{Bin}^{(1)} = 0$ *and* $r_{Bin}^{(2)} = 0$*, the DPC scheme is applied.*

Proposition 2.2.1. *The BME with backward decoding achieves at least the same rate as the one with successive decoding, i.e.,* $R_{BME_{back}} \geq R_{BME_{succ}}$*.*

Proof. For the first term of minimization (2.13), we have

$$\begin{aligned}
&\min\left(t_1 I\left(X_0^{(1)}; Y_1^{(1)} \mid X_2^{(1)}, U_2^{(1)}\right), t_2 I\left(X_1^{(2)}; Y_3^{(2)} \mid U_1^{(2)}\right) + t_1 I\left(U_2^{(1)}; Y_3^{(1)}\right)\right) + \\
&\min\left(t_1 I\left(X_2^{(1)}; Y_3^{(1)} \mid U_2^{(1)}\right) + t_2 I\left(U_1^{(2)}; Y_3^{(2)}\right), t_2 I\left(X_0^{(2)}; Y_2^{(2)} \mid X_1^{(2)}, U_1^{(2)}\right)\right) \leq \\
&\min\left(t_1 I\left(X_0^{(1)}; Y_1^{(1)} \mid X_2^{(1)}, U_2^{(1)}\right) + t_2 I\left(X_0^{(2)}; Y_2^{(2)} \mid X_1^{(2)}, U_1^{(2)}\right),\right. \\
&\left. t_1 I\left(X_2^{(1)}, U_2^{(1)}; Y_3^{(1)}\right) + t_2 I\left(X_1^{(2)}, U_1^{(2)}; Y_3^{(2)}\right)\right). \qquad (2.15)
\end{aligned}$$

Let us focus on $t_1 I\left(X_0^{(1)}; Y_1^{(1)} \mid X_2^{(1)}, U_2^{(1)}\right) + t_2 I\left(X_0^{(2)}; Y_2^{(2)} \mid X_1^{(2)}, U_1^{(2)}\right)$:

$$\begin{aligned}
&t_1 I\left(X_0^{(1)}; Y_1^{(1)} \mid X_2^{(1)}, U_2^{(1)}\right) + t_2 I\left(X_0^{(2)}; Y_2^{(2)} \mid X_1^{(2)}, U_1^{(2)}\right) \overset{(a)}{=} \\
&t_1 H\left(Y_1^{(1)} \mid X_2^{(1)}, U_2^{(1)}\right) - t_1 H\left(Y_1^{(1)} \mid X_0^{(1)}, X_2^{(1)}\right) + \\
&t_2 H\left(Y_2^{(2)} \mid X_1^{(2)}, U_1^{(2)}\right) - t_2 H\left(Y_2^{(2)} \mid X_0^{(2)}, X_1^{(2)}\right) \overset{(b)}{\leq} \\
&t_1 H\left(Y_1^{(1)} \mid X_2^{(1)}\right) - t_1 H\left(Y_1^{(1)} \mid X_0^{(1)}, X_2^{(1)}\right) + \\
&t_2 H\left(Y_2^{(2)} \mid X_1^{(2)}\right) - t_2 H\left(Y_2^{(2)} \mid X_0^{(2)}, X_1^{(2)}\right) \overset{(c)}{=} \\
&t_1 I\left(X_0^{(1)}; Y_1^{(1)} \mid X_2^{(1)}\right) + t_2 I\left(X_0^{(2)}; Y_2^{(2)} \mid X_1^{(2)}\right). \qquad (2.16)
\end{aligned}$$

(a) and (c) follow from the definition of mutual information, the fact that $U_2^{(1)} \longrightarrow \left(X_0^{(1)}, X_2^{(1)}\right) \longrightarrow Y_1^{(1)}$ and $U_1^{(2)} \longrightarrow \left(X_0^{(2)}, X_1^{(2)}\right) \longrightarrow Y_2^{(2)}$ form Markov chain, and (b) follows from the fact that conditioning reduces entropy. Inequality (b) becomes equality if $p(x_0^{(1)}, x_2^{(1)}, u_2^{(1)}) = p(u_2^{(1)})p(x_2^{(1)})p(x_0^{(1)}|x_2^{(1)})$ and $p(x_0^{(2)}, x_1^{(2)}, u_1^{(2)}) = p(u_1^{(2)})p(x_1^{(2)})p(x_0^{(2)}|x_1^{(2)})$. Using the similar argument for $t_1 I\left(X_2^{(1)}, U_2^{(1)}; Y_3^{(1)}\right) + t_2 I\left(X_1^{(2)}, U_1^{(2)}; Y_3^{(2)}\right)$,

$t_1 I\left(X_0^{(1)}, X_2^{(1)}; Y_1^{(1)} \mid U_2^{(1)}\right)$, and $t_2 I\left(X_0^{(2)}, X_1^{(2)}; Y_2^{(2)} \mid U_1^{(2)}\right)$ in (2.13) and (2.15), and the fact $U_2^{(1)} \longrightarrow X_2^{(1)} \longrightarrow Y_3^{(1)}$, $U_1^{(2)} \longrightarrow X_1^{(2)} \longrightarrow Y_3^{(2)}$, $U_2^{(1)} \longrightarrow \left(X_0^{(1)}, X_2^{(1)}\right) \longrightarrow Y_1^{(1)}$, $U_1^{(2)} \longrightarrow \left(X_0^{(2)}, X_1^{(2)}\right) \longrightarrow Y_2^{(2)}$ form Markov chain, and Appendix E, along with comparing $R_{BME_{succ}}$ and $R_{BME_{back}}$ in Theorem 2.2.2, we have $R_{BME_{back}} \geq R_{BME_{succ}}$. $\square$

Hence, from the discussion in Remark 2.2.1 and Proposition 2.2.1, we have the following theorem.

Theorem 2.2.3. *For the Gaussian case, the composite BME-DPC scheme achieves the following rate $R_{BME-DPC}$. Furthermore, $R_{BME-DPC} \geq \max(R_{BME_{back}}, R_{DPC})$. In other words, the composite BME-DPC scheme always achieves a rate greater than or equal to that of the BME and DPC schemes for the Gaussian case.*

$$R_{BME-DPC} = \max\left(R_{BME-DPC_1}, R_{BME-DPC_2}, R_{DPC}\right), \tag{2.17}$$

where for the $R_{BME-DPC_1}$ and $R_{BME-DPC_2}$, we have

$$\begin{aligned} R_{BME-DPC_1} =& \max \ R^{(1)} + R^{(2)} \\ =& \max \ \min \left(t_1 C\left(\frac{h_{01}^2 P_0^{(1)} + h_{12}^2 P_2 + 2h_{01}h_{12}\sqrt{\bar{\alpha} P_0^{(1)} P_2}}{t_1}\right), \right. \\ & t_1 C\left(\frac{h_{01}^2 \alpha P_0^{(1)}}{t_1}\right) + t_2 C\left(\frac{h_{02}^2 P_0^{(2)}}{t_2}\right), \\ & \left. t_1 C\left(\frac{h_{23}^2 P_2}{t_1}\right) + t_2 C\left(\frac{h_{13}^2 P_1}{t_2}\right), t_2 C\left(\frac{h_{02}^2 P_0^{(2)}}{t_2}\right) + t_2 C\left(\frac{h_{13}^2 P_1}{t_2}\right)\right), \\ & \textit{subject to:} \\ & \quad r_{Bin}^{(1)} = 0. \end{aligned} \tag{2.18}$$

$$\begin{aligned} R_{BME-DPC_2} =& \max \ R^{(1)} + R^{(2)} \\ =& \max \ \min \left(t_2 C\left(\frac{h_{02}^2 P_0^{(2)} + h_{12}^2 P_1 + 2h_{02}h_{12}\sqrt{\bar{\beta} P_0^{(2)} P_1}}{t_2}\right), \right. \\ & t_2 C\left(\frac{h_{02}^2 \beta P_0^{(2)}}{t_2}\right) + t_1 C\left(\frac{h_{01}^2 P_0^{(1)}}{t_1}\right), \\ & \left. t_2 C\left(\frac{h_{13}^2 P_1}{t_2}\right) + t_1 C\left(\frac{h_{23}^2 P_2}{t_1}\right), t_1 C\left(\frac{h_{01}^2 P_0^{(1)}}{t_1}\right) + t_1 C\left(\frac{h_{23}^2 P_2}{t_1}\right)\right), \\ & \textit{subject to:} \\ & \quad r_{Bin}^{(2)} = 0. \end{aligned} \tag{2.19}$$

where the maximization in (2.17) is over parameters t_1, t_2, $P_0^{(1)}$, $P_0^{(2)}$, α, and β, subject to the following constraints:

$$
\begin{aligned}
&P_0^{(1)} + P_0^{(2)} = P_0,\\
&t_1 + t_2 = 1,\\
&0 \le t_1, t_2, P_0^{(1)}, P_0^{(2)},\\
&0 \le \alpha, \beta \le 1.
\end{aligned}
$$

Proof. First, let us assume that $r_{Bin}^{(1)} = 0$. Now, we show that every rate pairs $\left(R^{(1)}, R^{(2)}\right)$ satisfying (4.42)-(4.48) satisfy (2.18). After specializing (4.42)-(4.48) for the Gaussian case and comparing with (2.18), one observes that the second term in minimization (4.42) does not exist. Substituting $r_{Bin}^{(1)} = 0$ in (4.43)-(4.48), one can obtain the other three corresponding terms. Comparing those terms with (2.18), it can be readily seen that $R_{BME-DPC_1} \geq R_{BME_{back}}$. Now, assuming $r_{Bin}^{(2)} = 0$, and using the similar argument, one can easily prove that $R_{BME-DPC_2} \geq R_{BME_{back}}$. Furthermore, by the definition of the composite BME-DPC scheme in Remark 1, we should have $R_{BME-DPC} = \max\left(R_{BME-DPC_1}, R_{BME-DPC_2}, R_{DPC}\right)$. Therefore, $R_{BME-DPC} \geq \max(R_{BME_{back}}, R_{DPC})$, and the theorem is proved. □

Remark 2.2.2. *Assuming $r_{Bin}^{(1)} = 0$, and $r_{Bin}^{(2)} \neq 0$ ($r_{Bin}^{(1)} \neq 0$, and $r_{Bin}^{(2)} = 0$), the destination jointly decodes the current message and the bin index of the next message at the end of even (odd) blocks and then it can decode the next message at the end of odd (even) blocks. Therefore, using backward decoding is not necessary in the BME-DPC scheme.*

From Theorem 2.2.2, we have the following corollary for the Gaussian case.

Corollary 2.2.2. *For the half-duplex Gaussian parallel relay channel, assuming successive relaying protocol with power constraints at the source and each relay, BME achieves the* *following rates*

$$
\begin{aligned}
R_{BME_{succ}} = \max\ \ \min\Bigg(& R_{BME_1} + R_{BME_2}, \\
& t_1 C\left(\frac{h_{01}^2 P_0^{(1)} + h_{12}^2\theta_2 P_2 + 2h_{01}h_{12}\sqrt{\bar{\alpha}_1\theta_2 P_0^{(1)} P_2}}{t_1}\right), \\
& t_2 C\left(\frac{h_{02}^2 P_0^{(2)} + h_{12}^2\theta_1 P_1 + 2h_{02}h_{12}\sqrt{\bar{\alpha}_2\theta_1 P_0^{(2)} P_1}}{t_2}\right)\Bigg),
\end{aligned} \tag{2.20}
$$

$$
\begin{aligned}
R_{BME_{back}} = \max\ \ \min\Bigg(& t_1 C\left(\frac{h_{01}^2 P_0^{(1)} + h_{12}^2 P_2 + 2h_{01}h_{12}\sqrt{\bar{\beta}_1 P_0^{(1)} P_2}}{t_1}\right), \\
& t_2 C\left(\frac{h_{02}^2 P_0^{(2)} + h_{12}^2 P_1 + 2h_{02}h_{12}\sqrt{\bar{\beta}_2 P_0^{(2)} P_1}}{t_2}\right), \\
& t_1 C\left(\frac{h_{01}^2\beta_1 P_0^{(1)}}{t_1}\right) + t_2 C\left(\frac{h_{02}^2\beta_2 P_0^{(2)}}{t_2}\right), t_1 C\left(\frac{h_{23}^2 P_2}{t_1}\right) + t_2 C\left(\frac{h_{13}^2 P_1}{t_2}\right)\Bigg).
\end{aligned} \tag{2.21}
$$

where the maximization in (2.20) and (2.21) is over parameters $t_1,\ t_2,\ P_0^{(1)},\ P_0^{(2)},\ \alpha_1,\ \alpha_2,\ \beta_1,\ \beta_2,\ \theta_1,$ *and* θ_2 *subject to the following constraints:*

$$
R_{BME_1} = \min\left(t_1 C\left(\frac{h_{01}^2\alpha_1 P_0^{(1)}}{t_1}\right), t_1 C\left(\frac{h_{23}^2\bar{\theta}_2 P_2}{h_{23}^2\theta_2 P_2 + t_1}\right) + t_2 C\left(\frac{h_{13}^2\theta_1 P_1}{t_2}\right)\right), \tag{2.22}
$$

$$
R_{BME_2} = \min\left(t_2 C\left(\frac{h_{02}^2\alpha_2 P_0^{(2)}}{t_2}\right), t_2 C\left(\frac{h_{13}^2\bar{\theta}_1 P_1}{h_{13}^2\theta_1 P_1 + t_2}\right) + t_1 C\left(\frac{h_{23}^2\theta_2 P_2}{t_1}\right)\right), \tag{2.23}
$$

$$
\begin{aligned}
& P_0^{(1)} + P_0^{(2)} = P_0, \\
& t_1 + t_2 = 1, \\
& 0 \le t_1, t_2, P_0^{(1)}, P_0^{(2)}, \\
& 0 \le \alpha_1, \alpha_2, \beta_1, \beta_2, \theta_1, \theta_2 \le 1.
\end{aligned}
$$

Proof. See Appendix F. □

2.2.2 Simultaneous Relaying Protocol

Figure 2.6 shows simultaneous relaying protocol. In simultaneous relaying, in state 3 with duration t_3 the source transmits its signal simultaneously to the two relays. Following state 3, in state 4 with duration t_4, two relays transmit their signal coherently to the destination. Hence, in this protocol, $t_1 = t_2 = 0$ and our system model follows from (2.6) and (2.7).

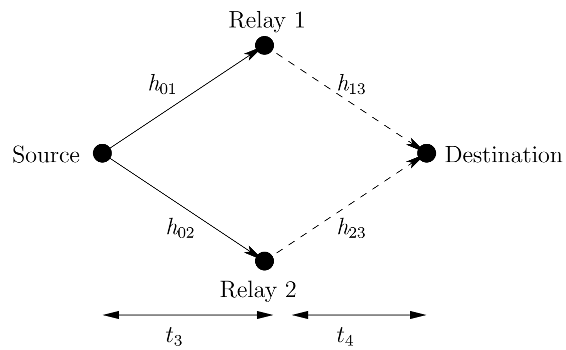


Figure 2.6: Simultaneous relaying protocol for two relays.

Broadcast-multiacess with Common Message (BCM)

In the BCM scheme each relay decodes the transmitted message from the source in state 3 (Broadcast (BC) State), and forwards its re-encoded version in state 4 (Multiple Access (MAC) State). It is worth noting that this scheme was previously proposed in [4] and considered in [36]. The following theorem gives the achievable rate of the BCM scheme for the Gaussian case.

Theorem 2.2.4. *For the half-duplex Gaussian parallel relay channel, assuming simultaneous relaying protocol with power constraints at the source and at each relay, BCM achieves the following rate*

$$R_{BCM} = \max \ R_p + R_c, \tag{2.24}$$

where the maximization (2.24) is over parameters $t_3,\ t_4,\ P_{0,p}^{(3)},\ P_{0,c}^{(3)},\ P_{1,p}^{(4)},$ *and* $P_{1,c}^{(4)}$ *subject to the following constraints:*

$$\begin{aligned}
&R_p \leq \min\left(t_3 C\left(\frac{h_{01}^2 P_{0,p}^{(3)}}{t_3}\right), t_4 C\left(\frac{h_{13}^2 P_{1,p}^{(4)}}{t_4}\right)\right), R_c \leq t_3 C\left(\frac{h_{02}^2 P_{0,c}^{(3)}}{t_3 + h_{02}^2 P_{0,p}^{(3)}}\right),\\
&R_p + R_c \leq t_4 C\left(\frac{h_{13}^2 P_{1,p}^{(4)} + \left(h_{13}\sqrt{P_{1,c}^{(4)}} + h_{23}\sqrt{P_2}\right)^2}{t_4}\right),\\
&P_{0,p}^{(3)} + P_{0,c}^{(3)} = P_0,\ P_{1,p}^{(4)} + P_{1,c}^{(4)} = P_1,\ t_3 + t_4 = 1,\\
&0 \leq t_3,\ t_4,\ P_{0,p}^{(3)},\ P_{0,c}^{(3)},\ P_{1,p}^{(4)},\ P_{1,c}^{(4)}.
\end{aligned}$$

$P_{0,p}^{(3)}$ *and* $P_{0,c}^{(3)}$ *are portions of the source total power* $P_0^{(3)}$ *associated with the private and common messages at the source, and* $P_{1,p}^{(4)}$ *and* $P_{1,c}^{(4)}$ *are portions of relay 1 total power* $P_1^{(4)}$ *associated with the private and common messages at relay 1, respectively.*

Proof. See [4], and [36]. □

Interestingly, successive decoding at the destination does not degrade the performance of the BCM scheme in the Gaussian case as shown in the following proposition.

Proposition 2.2.2. *The rate of the BCM scheme is achievable by successive decoding of the common and private messages at the destination.*

Proof. See Appendix G. □

2.2.3 Simultaneous-Successive Relaying Protocol based on Dirty paper coding (SSRD)

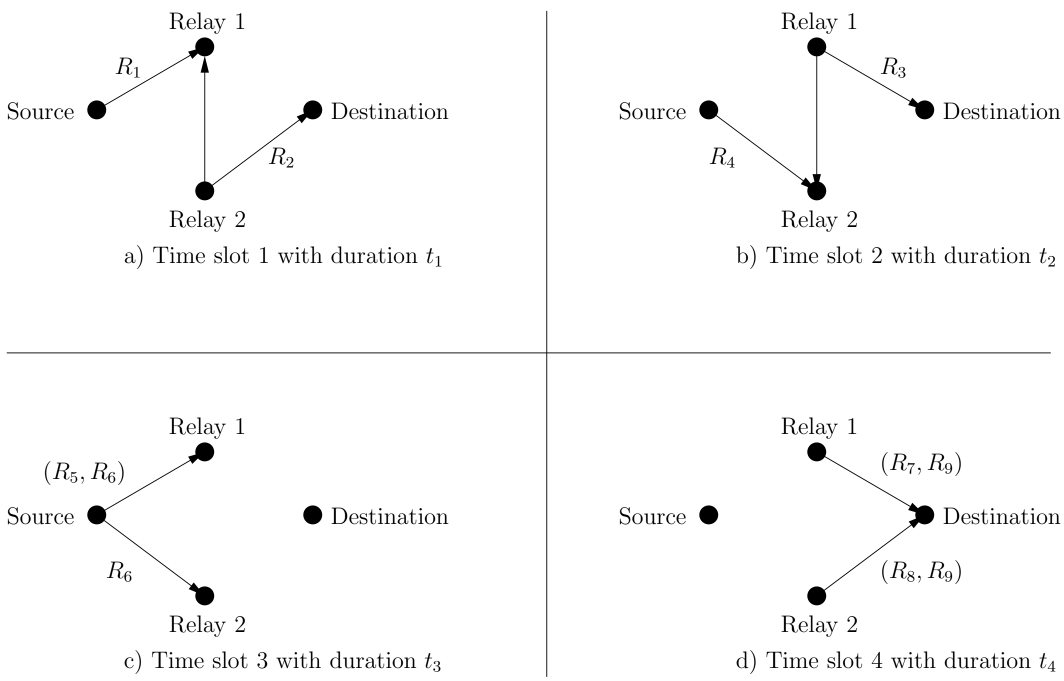


Figure 2.7: SSRD Scheme for the Half-Duplex Parallel Relay Channel.

In this section, we propose an achievable rate for the half-duplex parallel relay channel. Our achievable scheme is based on the combination of the successive relaying protocol based on the DPC scheme and simultaneous relaying protocol based on the BCM scheme (SSRD scheme). Hence, we have the following theorem.

Theorem 2.2.5. *Considering Fig. 2.7, for the half-duplex parallel relay channel, SSRD scheme achieves the following rate R_{SSRD}*

$$R_{SSRD} = \max \ \min \left(R_1 + R_4 + R_5 + R_6, R_2 + R_3 + R_7 + R_8 + R_9 \right), \tag{2.25}$$

where maximization (2.25) is over parameters $t_1,\ t_2,\ t_3,\ t_4,\ P_0^{(1)},\ P_0^{(2)},\ P_{0,p}^{(3)},\ P_{0,c}^{(3)},\ P_1^{(2)},\ P_{1,p}^{(4)}$ $,\ P_{1,c}^{(4)},\ P_2^{(1)},\ P_{2,p}^{(4)},$ *and* $P_{2,c}^{(4)}$ *subject to the following constraints:*

$$\begin{aligned}
&R_9 \leq R_6, R_1 + R_5 \leq R_3 + R_7, R_4 \leq R_2 + R_8, \\
&P_0^{(1)} + P_0^{(2)} + P_{0,p}^{(3)} + P_{0,c}^{(3)} = P_0, \\
&P_1^{(2)} + P_{1,p}^{(4)} + P_{1,c}^{(4)} = P_1,\ P_2^{(1)} + P_{2,p}^{(4)} + P_{2,c}^{(4)} = P_2,
\end{aligned} \tag{2.26}$$

$$\begin{aligned}
&t_1 + t_2 + t_3 + t_4 = 1, \\
&0 \leq t_1,\ t_2,\ t_3,\ t_4,\ P_0^{(1)},\ P_0^{(2)},\ P_{0,p}^{(3)},\ P_{0,c}^{(3)},\ P_1^{(2)},\ P_{1,p}^{(4)},\ P_{1,c}^{(4)},\ P_2^{(1)},\ P_{2,p}^{(4)},\ P_{2,c}^{(4)}.
\end{aligned}$$

where

$$R_1 = t_1 C\left(\frac{h_{01}^2 P_0^{(1)}}{t_1}\right), R_2 = t_1 C\left(\frac{h_{23}^2 P_2^{(1)}}{t_1}\right), R_3 = t_2 C\left(\frac{h_{13}^2 P_1^{(2)}}{t_2}\right), R_4 = t_2 C\left(\frac{h_{02}^2 P_0^{(2)}}{t_2}\right),$$
$$R_5 = t_3 C\left(\frac{h_{01}^2 P_{0,p}^{(3)}}{t_3}\right), R_6 = t_3 C\left(\frac{h_{02}^2 P_{0,c}^{(3)}}{t_3 + h_{02}^2 P_{0,p}^{(3)}}\right), R_7 = t_4 C\left(\frac{h_{13}^2 P_{1,p}^{(4)}}{t_4}\right), R_8 = t_4 C\left(\frac{h_{23}^2 P_{2,p}^{(4)}}{t_4}\right),$$
$$R_9 = t_4 C\left(\frac{\left(h_{13}\sqrt{P_{1,c}^{(4)}} + h_{23}\sqrt{P_{2,c}^{(4)}}\right)^2}{t_4 + h_{13}^2 P_{1,p}^{(4)} + h_{23}^2 P_{2,p}^{(4)}}\right).$$

Proof. The SSRD scheme is illustrated in Fig. 2.7. As indicated in the figure, transmission is performed in 4 states. Relay 1 transmits its private message which was received in state 1 (with rate R_1) and state 3 (with rate R_5) in state 2 (with rate R_3) and state 4 (with rate R_7). On the other hand, relay 2 transmits its private message which was received in state 2 (with rate R_4) in state 1 (with rate R_2) and state 4 (with rate R_8). Furthermore, the two relays send the common message they have already received in state 3 (with rate R_6) coherently in state 4 (with rate R_9). As observed, here we consider the private rate for both relays in the MAC state, i.e., state 4. This is due to the reason that relay 2 also receives the private message in state 2. Hence, from the above description, Fig. 2.7, and using corollary 2.2.1, Theorem 2.2.4, and Proposition 2.2.2, the theorem is proved. □

Remark 2.2.3. *According to Theorem 2.2.3, another combined simultaneous-successive relaying protocol based on BME is not necessary. However, a "Simultaneous-Successive Relaying protocol based on BME-DPC", can be easily derived. Assuming the first relay decodes the second one's message, the achievable rate of this new scheme would be the same as* R_{SSRD}*. However, since the messages for the second relay are common,* R_8 *in the expression of the* *achievable rate is zero. Furthermore, the following constraints instead of (2.26) should be satisfied:*

$$R_9 \le R_4 + R_6,\ R_1 + R_5 \le R_3 + R_7,\ R_1 + R_4 \le t_1 I\left(X_0^{(1)}, X_2^{(1)}; Y_1^{(1)}\right). \tag{2.27}$$

2.3 Optimality Results

In this section, the optimality of the proposed achievable schemes in the previous sections is investigated.

The authors in [32] proposed some upper bounds on the achievable rate for general half-duplex multi-terminal networks. From the discussion in [32], the cut-set upper bound C^{up} is

$$\begin{aligned} C^{up} \triangleq \max_{0\le \hat{t}_1, \hat{t}_2, \hat{t}_1+\hat{t}_2=1} \min\Big(& \hat{t}_1 I\left(X_0^{(1)}; Y_1^{(1)} \mid X_2^{(1)}\right) + \hat{t}_2 I\left(X_0^{(2)}; Y_2^{(2)} \mid X_1^{(2)}\right) + \hat{t}_3 I\left(X_0^{(3)}; Y_1^{(3)}, Y_2^{(3)}\right), \\ & \hat{t}_2 I\left(X_0^{(2)}, X_1^{(2)}; Y_2^{(2)}, Y_3^{(2)}\right) + \hat{t}_3 I\left(X_0^{(3)}; Y_2^{(3)}\right) + \hat{t}_4 I\left(X_1^{(4)}; Y_3^{(4)} \mid X_2^{(4)}\right), \\ & \hat{t}_1 I\left(X_0^{(1)}, X_2^{(1)}; Y_1^{(1)}, Y_3^{(1)}\right) + \hat{t}_3 I\left(X_0^{(3)}; Y_1^{(3)}\right) + \hat{t}_4 I\left(X_2^{(4)}; Y_3^{(4)} \mid X_1^{(4)}\right), \\ & \hat{t}_1 I\left(X_2^{(1)}; Y_3^{(1)}\right) + \hat{t}_2 I\left(X_1^{(2)}; Y_3^{(2)}\right) + \hat{t}_4 I\left(X_1^{(4)}, X_2^{(4)}; Y_3^{(4)}\right)\Big). \end{aligned} \tag{2.28}$$

In fact, (2.28) is a special case of the cut-set bound in [14].

By setting $\hat{t}_3 = \hat{t}_4 = 0$ in (2.28), we obtain an upper bound on the successive relaying protocol which we call it *successive cut-set bound* in the sequel.

Theorem 2.3.1. *In a degraded half-duplex parallel relay channel where the destination receives a degraded version of the received signals at relays, i.e.* $X_2^{(1)} \longrightarrow Y_1^{(1)} \longrightarrow Y_3^{(1)}$ *and* $X_1^{(2)} \longrightarrow Y_2^{(2)} \longrightarrow Y_3^{(2)}$*, BME based on backward decoding achieves the successive cut-set bound.*

Proof. Setting $\hat{t}_3 = \hat{t}_4 = 0$ in (2.28) and comparing the result with (2.14) the theorem is proved. □

Theorem 2.3.2. *In symmetric scenarios, where* $h_{01} = h_{02}$*,* $h_{13} = h_{23}$*, and* $P_1 = P_2$*, Non-Cooperative DPC scheme achieves the successive cut-set bound.*

Proof. Due to the symmetric assumption and using the successive relaying protocol, we have $t_1 = t_2 = \frac{1}{2}$, $P_0^{(1)} = P_0^{(2)} = \frac{P_0}{2}$. Hence, R_{DPC} in (2.12) becomes

$$\begin{aligned} R_{DPC} &= \min\left(C\left(h_{01}^2 P_0\right), \frac{1}{2} C\left(h_{01}^2 P_0\right) + \frac{1}{2} C\left(2h_{13}^2 P_1\right), C\left(2h_{13}^2 P_1\right)\right), \\ &= \min\left(C\left(h_{01}^2 P_0\right), C\left(2h_{13}^2 P_1\right)\right). \end{aligned}$$

On the other hand, from the symmetric assumption, the optimum value for $\hat{t}_1$ and $\hat{t}_2$ in the successive cut-set bound is equal to .5, and the optimum values for the source power in states 1 and 2 are the same. Therefore, the successive cut-set bound can be upper-bounded by $\min\left(C\left(h_{01}^2 P_0\right), C\left(2h_{13}^2 P_1\right)\right)$ which is equal to R_{DPC}, and the theorem is proved. □

In high SNR scenarios, we have the following theorem.

Theorem 2.3.3. *In high SNR scenarios, assuming non-zero source-relay and relay-destination links, when power available for the source and each relay tends to infinity as $P_1 = \gamma_1 P_0,\ P_2 = \gamma_2 P_0$ with γ_1, γ_2 constants independent of the SNR, time slots $\hat{t}_3$ and $\hat{t}_4$ in (2.28) tend to zero as $O\left(\frac{1}{\log P_0}\right)$. Furthermore, the upper bound on the capacity of the half-duplex parallel relay channel in high SNR scenarios is*

$$C^{up} = R_{DPC} + O\left(\frac{1}{\log P_0}\right).$$

In other words, DPC achieves the capacity of a half-duplex Gaussian parallel relay channel as SNR goes to infinity.

Proof. Substituting $X_0^{(1)} \sim \mathcal{N}(0, \hat{P}_0^{(1)}),\ X_0^{(2)} \sim \mathcal{N}(0, \hat{P}_0^{(2)}),\ X_0^{(3)} \sim \mathcal{N}(0, \hat{P}_0^{(3)}),\ X_1^{(2)} \sim \mathcal{N}(0, \hat{P}_1^{(2)}),\ X_1^{(4)} \sim \mathcal{N}(0, \hat{P}_1^{(4)}),\ X_2^{(1)} \sim \mathcal{N}(0, \hat{P}_2^{(1)})$, and $X_2^{(4)} \sim \mathcal{N}(0, \hat{P}_2^{(4)})$ in (2.28), and assuming complete cooperation between the transmitting and receiving nodes for each cut in (2.28), we have

$$\begin{aligned}
C^{up} \leq \max\ \min\Bigg(& \hat{t}_1 C\left(\frac{h_{01}^2 \hat{P}_0^{(1)}}{\hat{t}_1}\right) + \hat{t}_2 C\left(\frac{h_{02}^2 \hat{P}_0^{(2)}}{\hat{t}_2}\right) + \hat{t}_3 C\left(\frac{(h_{01}^2 + h_{02}^2)\hat{P}_0^{(3)}}{\hat{t}_3}\right), \\
& \hat{t}_2 C\left(\frac{h_{02}^2 \hat{P}_0^{(2)}}{\hat{t}_2} + \frac{(h_{12}^2 + h_{13}^2)\hat{P}_1^{(2)}}{\hat{t}_2} + \frac{2h_{02}h_{12}\sqrt{\hat{P}_0^{(2)}\hat{P}_1^{(2)}}}{\hat{t}_2} + \frac{h_{02}^2 h_{13}^2 \hat{P}_0^{(2)}\hat{P}_1^{(2)}}{\hat{t}_2^2}\right) + \\
& \hat{t}_3 C\left(\frac{h_{02}^2 \hat{P}_0^{(3)}}{\hat{t}_3}\right) + \hat{t}_4 C\left(\frac{h_{13}^2 \hat{P}_1^{(4)}}{\hat{t}_4}\right), \\
& \hat{t}_1 C\left(\frac{h_{01}^2 \hat{P}_0^{(1)}}{\hat{t}_1} + \frac{(h_{12}^2 + h_{23}^2)\hat{P}_2^{(1)}}{\hat{t}_1} + \frac{2h_{01}h_{12}\sqrt{\hat{P}_0^{(1)}\hat{P}_2^{(1)}}}{\hat{t}_1} + \frac{h_{01}^2 h_{23}^2 \hat{P}_0^{(1)}\hat{P}_2^{(1)}}{\hat{t}_1^2}\right) + \\
& \hat{t}_3 C\left(\frac{h_{01}^2 \hat{P}_0^{(3)}}{\hat{t}_3}\right) + \hat{t}_4 C\left(\frac{h_{23}^2 \hat{P}_2^{(4)}}{\hat{t}_4}\right), \\
& \hat{t}_1 C\left(\frac{h_{23}^2 \hat{P}_2^{(1)}}{\hat{t}_1}\right) + \hat{t}_2 C\left(\frac{h_{13}^2 \hat{P}_1^{(2)}}{\hat{t}_2}\right) + \\
& \hat{t}_4 C\left(\frac{h_{13}^2 \hat{P}_1^{(4)} + h_{23}^2 \hat{P}_2^{(4)} + 2h_{13}h_{23}\sqrt{\hat{P}_1^{(4)}\hat{P}_2^{(4)}}}{\hat{t}_4}\right)\Bigg).
\end{aligned} \tag{2.29}$$

subject to:

$$\hat{P}_0^{(1)} + \hat{P}_0^{(2)} + \hat{P}_0^{(3)} = P_0,$$
$$\hat{P}_1^{(2)} + \hat{P}_1^{(4)} = P_1,$$
$$\hat{P}_2^{(1)} + \hat{P}_2^{(4)} = P_2,$$
$$\hat{t}_1 + \hat{t}_2 + \hat{t}_3 + \hat{t}_4 = 1,$$
$$0 \le \hat{t}_1,\ \hat{t}_2,\ \hat{t}_3,\ \hat{t}_4,\ \hat{P}_0^{(1)},\ \hat{P}_0^{(2)},\ \hat{P}_0^{(3)},\ \hat{P}_1^{(2)},\ \hat{P}_1^{(4)},\ \hat{P}_2^{(1)},\ \hat{P}_2^{(4)}.$$

Furthermore, from corollary 2.2.1, the achievable rate of the DPC scheme can be expressed as

$$\begin{aligned} R_{DPC} = \min\Bigg(& t_1 C\left(\frac{h_{01}^2 P_0^{(1)}}{t_1}\right) + t_2 C\left(\frac{h_{02}^2 P_0^{(2)}}{t_2}\right), \\ & t_2 C\left(\frac{h_{02}^2 P_0^{(2)}}{t_2}\right) + t_2 C\left(\frac{h_{13}^2 P_1}{t_2}\right), \\ & t_1 C\left(\frac{h_{01}^2 P_0^{(1)}}{t_1}\right) + t_1 C\left(\frac{h_{23}^2 P_2}{t_1}\right), \\ & t_1 C\left(\frac{h_{23}^2 P_2}{t_1}\right) + t_2 C\left(\frac{h_{13}^2 P_1}{t_2}\right)\Bigg). \end{aligned} \tag{2.30}$$

By setting $P_0^{(1)} = P_0^{(2)} = \frac{P_0}{2}$ and $t_1 = t_2 = 0.5$ in (2.30), expression (2.30) can be simplified as

$$R_{DPC} \ge \frac{1}{2} \ln P_0 + c. \tag{2.31}$$

where c is some constant which depends on channel coefficients. Knowing that the term corresponding to each cut-set in (2.29) for the optimum values of $\hat{t}_1, \cdots, \hat{t}_4$ is indeed an upper-bound for R_{DPC}, and by setting $\hat{P}_0^{(1)} = \hat{P}_0^{(2)} = \hat{P}_0^{(3)} = P_0$ in (2.29), we have the following inequality between (2.31) and the first cut of (2.29).

$$\begin{aligned} \frac{1}{2}\ln P_0 + c \le & \frac{\hat{t}_1}{2}\ln\left(\frac{h_{01}^2 P_0}{\hat{t}_1}\right) + \frac{\hat{t}_2}{2}\ln\left(\frac{h_{02}^2 P_0}{\hat{t}_2}\right) + \frac{\hat{t}_3}{2}\ln\left(\frac{(h_{01}^2 + h_{02}^2) P_0}{\hat{t}_3}\right) + \\ & \frac{\hat{t}_1^2}{2h_{01}^2 P_0} + \frac{\hat{t}_2^2}{2h_{02}^2 P_0} + \frac{\hat{t}_3^2}{2(h_{01}^2 + h_{02}^2) P_0} \\ = & \frac{(1 - \hat{t}_4)}{2}\ln P_0 + \frac{\hat{t}_1}{2}\ln h_{01}^2 + \frac{\hat{t}_2}{2}\ln h_{02}^2 + \frac{\hat{t}_3}{2}\ln\left(h_{01}^2 + h_{02}^2\right) \\ & - \frac{\hat{t}_1}{2}\ln \hat{t}_1 - \frac{\hat{t}_2}{2}\ln \hat{t}_2 - \frac{\hat{t}_3}{2}\ln \hat{t}_3 + \frac{\hat{t}_1^2}{2h_{01}^2 P_0} + \frac{\hat{t}_2^2}{2h_{02}^2 P_0} + \frac{\hat{t}_3^2}{2\left(h_{01}^2 + h_{02}^2\right) P_0}. \end{aligned} \tag{2.32}$$

Note that in deriving (2.31) and (2.32), the following inequality is applied to lower/upper-bound the corresponding terms:

$$\ln(x) \leq \ln(1+x) \leq \ln(x) + \frac{1}{x}, \forall x > 0. \tag{2.33}$$

Consequently, we have

$$\begin{aligned}
\hat{t}_4 \quad &\leq \quad \frac{1}{\ln P_0}\left(2c + \hat{t}_1 \ln h_{01}^2 + \hat{t}_2 \ln h_{02}^2 + \hat{t}_3 \ln\left(h_{01}^2 + h_{02}^2\right) - \hat{t}_1 \ln \hat{t}_1 - \hat{t}_2 \ln \hat{t}_2 - \hat{t}_3 \ln \hat{t}_3\right) \\
&+ \quad \frac{1}{\ln P_0}\left(\frac{\hat{t}_1^2}{h_{01}^2 P_0} + \frac{\hat{t}_2^2}{h_{02}^2 P_0} + \frac{\hat{t}_3^2}{\left(h_{01}^2 + h_{02}^2\right) P_0}\right).
\end{aligned}$$

Hence, we can bound the optimum value of $\hat{t}_4$ in (2.29) as

$$0 \leq \hat{t}_4 \lesssim O\left(\frac{1}{\log P_0}\right). \tag{2.34}$$

Similarly, by considering the fourth cut in (2.29), we can derive another bound on the optimum value of $\hat{t}_3$ as follows:

$$0 \leq \hat{t}_3 \lesssim O\left(\frac{1}{\log P_0}\right). \tag{2.35}$$

Applying the inequality between (2.31) and the term corresponding to the second cut in (2.29), knowing (from (2.34) and (2.35)) the fact that $\hat{t}_3 \leq \frac{c_3}{\ln P_0}$, and $\hat{t}_4 \leq \frac{c_4}{\ln P_0}$ (where c_3 and c_4 are constants), and using inequalities (2.33), and

$$\ln(1+x) \leq x, \forall x \geq 0, \tag{2.36}$$

we obtain

$$
\begin{aligned}
&\frac{1}{2}\ln P_0 + c \leq \\
&\frac{\hat{t}_2}{2}\ln\left(\frac{h_{02}^2 h_{13}^2 \gamma_1 P_0^2}{\hat{t}_2^2}\left(1+\frac{\hat{t}_2}{\gamma_1 h_{13}^2 P_0}+\frac{\hat{t}_2\left(h_{12}^2+h_{13}^2\right)}{h_{02}^2 h_{13}^2 P_0}+\frac{\hat{t}_2 h_{12}}{h_{13}^2 h_{02}\sqrt{\gamma_1}P_0}\right)\right)+ \\
&\frac{\hat{t}_3}{2}\ln\left(\frac{h_{02}^2 P_0}{\hat{t}_3}\right)+\frac{\hat{t}_4}{2}\ln\left(\frac{h_{13}^2\gamma_1 P_0}{\hat{t}_4}\right)+ \\
&\frac{\hat{t}_2^3}{2\left(\hat{t}_2 h_{02}^2 P_0+\hat{t}_2\gamma_1\left(h_{12}^2+h_{13}^2\right)P_0+2\hat{t}_2 h_{02}h_{12}\sqrt{\gamma_1}P_0+h_{02}^2 h_{13}^2\gamma_1 P_0^2\right)}+ \\
&\frac{\hat{t}_3^2}{2h_{02}^2 P_0}+\frac{\hat{t}_4^2}{2\gamma_1 h_{13}^2 P_0} \\
&\leq \hat{t}_2\ln P_0+\frac{\hat{t}_2}{2}\ln\left(\frac{h_{02}^2 h_{13}^2\gamma_1}{\hat{t}_2^2}\right)+\frac{\hat{t}_2^2}{2\gamma_1 h_{13}^2 P_0}+\frac{\hat{t}_2^2\left(h_{12}^2+h_{13}^2\right)}{2h_{02}^2 h_{13}^2 P_0}+\frac{\hat{t}_2^2 h_{12}}{2h_{13}^2 h_{02}\sqrt{\gamma_1}P_0}+ \\
&\frac{c_3}{2\ln P_0}\ln h_{02}^2-\frac{c_3}{2\ln P_0}\ln\hat{t}_3+\frac{c_3}{2}+\frac{c_4}{2\ln P_0}\ln\gamma_1 h_{13}^2-\frac{c_4}{2\ln P_0}\ln\hat{t}_4+\frac{c_4}{2}+ \\
&\frac{\hat{t}_2^3}{2\left(\hat{t}_2 h_{02}^2 P_0+\hat{t}_2\gamma_1\left(h_{12}^2+h_{13}^2\right)P_0+2\hat{t}_2 h_{02}h_{12}\sqrt{\gamma_1}P_0+h_{02}^2 h_{13}^2\gamma_1 P_0^2\right)}+ \\
&\frac{\hat{t}_3^2}{2h_{02}^2 P_0}+\frac{\hat{t}_4^2}{2\gamma_1 h_{13}^2 P_0}
\end{aligned}
$$

Therefore, we have

$$
\begin{aligned}
\frac{1}{2}\ln P_0 + c \lesssim{}& \hat{t}_2\ln P_0+\acute{c} \\
&+O\left(\frac{1}{\ln P_0}\right)+O\left(\frac{1}{P_0}\right).
\end{aligned}
$$

Hence,

$$
\frac{1}{2}-\frac{c_2}{\log P_0}\leq\hat{t}_2. \tag{2.37}
$$

Similarly, from the third cut of (2.29), for $\hat{t}_1$ we have

$$
\frac{1}{2}-\frac{c_1}{\log P_0}\leq\hat{t}_1. \tag{2.38}
$$

From (2.37) and (2.38), and also the fact that $\hat{t}_1+\hat{t}_2+\hat{t}_3+\hat{t}_4=1$, we obtain

$$
\frac{1}{2}-\frac{c_2}{\log P_0}\leq\hat{t}_2\leq\frac{1}{2}+\frac{c_1}{\log P_0}, \tag{2.39}
$$

$$
\frac{1}{2}-\frac{c_1}{\log P_0}\leq\hat{t}_1\leq\frac{1}{2}+\frac{c_2}{\log P_0}. \tag{2.40}
$$

Hence, from (2.34), (2.35), (2.39), and (2.40) as $P_0 \to \infty$, $\hat{t}_3$, $\hat{t}_4 \to 0$ and $\hat{t}_1$, $\hat{t}_2 \to 0.5$. This proves the first part of the theorem.

Moreover, knowing that each term corresponding to the four cuts in (2.29) is greater than $0.5\ln(P_0) + c$ and as $\hat{t}_1, \hat{t}_2$ are strictly above zero (approaching 0.5), we can easily conclude that

$$\hat{P}_0^{(1)}, \hat{P}_0^{(2)}, \hat{P}_1^{(2)}, \hat{P}_2^{(1)} = \Theta\left(P_0\right). \tag{2.41}$$

Now, we prove that the DPC scheme with the parameters $t_1 = \hat{t}_1 + \frac{\hat{t}_3+\hat{t}_4}{2}$, $t_2 = \hat{t}_2 + \frac{\hat{t}_3+\hat{t}_4}{2}$, $P_0^{(1)} = \hat{P}_0^{(1)}$ and $P_0^{(2)} = \hat{P}_0^{(2)}$, where $\hat{t}_1, \cdots, \hat{t}_4, \hat{P}_0^{(1)}, \hat{P}_0^{(2)}$ are the parameters corresponding to the maximum value of (2.29), achieves the capacity with a gap no more than $O\left(\frac{1}{\log P_0}\right)$. To prove this, we show that each of the four terms in (2.30) is no more than $O\left(\frac{1}{\log P_0}\right)$ below the corresponding term (from the same cut) in (2.29). To show this, for the first cut we have

$$\begin{aligned}
&\hat{t}_1 C\left(\frac{h_{01}^2 \hat{P}_0^{(1)}}{\hat{t}_1}\right) + \hat{t}_2 C\left(\frac{h_{02}^2 \hat{P}_0^{(2)}}{\hat{t}_2}\right) + \hat{t}_3 C\left(\frac{(h_{01}^2 + h_{02}^2)\hat{P}_0^{(3)}}{\hat{t}_3}\right) - t_1 C\left(\frac{h_{01}^2 P_0^{(1)}}{t_1}\right)\\
&-t_2 C\left(\frac{h_{02}^2 P_0^{(2)}}{t_2}\right) \overset{(a)}{\leq} \frac{\hat{t}_1}{2}\ln\left(\frac{h_{01}^2 \hat{P}_0^{(1)}}{\hat{t}_1}\right) + \frac{\hat{t}_2}{2}\ln\left(\frac{h_{02}^2 \hat{P}_0^{(2)}}{\hat{t}_2}\right) + \hat{t}_3 C\left(\frac{(h_{01}^2 + h_{02}^2)\hat{P}_0^{(3)}}{\hat{t}_3}\right)\\
&-\left(\frac{\hat{t}_1}{2} + \frac{\hat{t}_3 + \hat{t}_4}{4}\right)\ln\left(\frac{h_{01}^2 \hat{P}_0^{(1)}}{t_1}\right) - \left(\frac{\hat{t}_2}{2} + \frac{\hat{t}_3 + \hat{t}_4}{4}\right)\ln\left(\frac{h_{02}^2 \hat{P}_0^{(2)}}{t_2}\right) + \frac{\hat{t}_1^2}{2h_{01}^2 \hat{P}_0^{(1)}} + \frac{\hat{t}_2^2}{2h_{02}^2 \hat{P}_0^{(2)}} \overset{(b)}{\lesssim}\\
&\frac{\hat{t}_1}{2}\ln\left(\frac{h_{01}^2 \hat{P}_0^{(1)}}{\hat{t}_1}\right) + \frac{\hat{t}_2}{2}\ln\left(\frac{h_{02}^2 \hat{P}_0^{(2)}}{\hat{t}_2}\right) + \frac{\hat{t}_3}{2}\ln\left(\frac{(h_{01}^2 + h_{02}^2)P_0}{\hat{t}_3 + \hat{t}_1}\right) - \left(\frac{\hat{t}_1}{2} + \frac{\hat{t}_3 + \hat{t}_4}{4}\right)\ln\left(\frac{h_{01}^2 \hat{P}_0^{(1)}}{t_1}\right)\\
&-\left(\frac{\hat{t}_2}{2} + \frac{\hat{t}_3 + \hat{t}_4}{4}\right)\ln\left(\frac{h_{02}^2 \hat{P}_0^{(2)}}{t_2}\right) + O\left(\frac{1}{\log P_0}\right) \overset{(c)}{\lesssim}\\
&\frac{\hat{t}_3}{2}\ln\left(\frac{P_0}{\sqrt{\hat{P}_0^{(1)}\hat{P}_0^{(2)}}}\right) - \frac{\hat{t}_4}{4}\ln\left(\hat{P}_0^{(1)}\hat{P}_0^{(2)}\right)\\
&+O\left(\frac{1}{\log P_0}\right) \overset{(d)}{\lesssim} O\left(\frac{1}{\log P_0}\right).
\end{aligned} \tag{2.42}$$

Here, (a) follows from (2.33), noting the function $\hat{t}_1\ln(P_0 - x - y) + \hat{t}_2\ln(y) + \hat{t}_3\ln\left(\hat{t}_3 + (h_{01}^2 + h_{02}^2)\,x\right)$ takes its maximum value at $x \leq \frac{\hat{t}_3}{\hat{t}_3+\hat{t}_1}P_0$ and hence substituting $\hat{P}_0^{(3)} = \frac{\hat{t}_3}{\hat{t}_3+\hat{t}_1}P_0$ and finally noting $\hat{P}_0^{(1)}, \hat{P}_0^{(2)} = \Theta(P_0)$ result in (b) , (c) follows from $\hat{t}_3, \hat{t}_4 = O\left(\frac{1}{\log P_0}\right)$ and $\ln\left(\frac{t_1}{\hat{t}_1}\right) = O\left(\frac{1}{\log P_0}\right)$, and finally (d) follows from $\hat{P}_0^{(1)}, \hat{P}_0^{(2)} = \Theta(P_0)$.

Next, we bound the difference between the terms in the fourth cut of (2.29) and the fourth term in R_{DPC}

$$\begin{aligned}
&\hat{t}_1 C\left(\frac{h_{23}^2 \hat{P}_2^{(1)}}{\hat{t}_1}\right) + \hat{t}_2 C\left(\frac{h_{13}^2 \hat{P}_1^{(2)}}{\hat{t}_2}\right) + \hat{t}_4 C\left(\frac{h_{13}^2 \hat{P}_1^{(4)} + h_{23}^2 \hat{P}_2^{(4)} + 2h_{13}h_{23}\sqrt{\hat{P}_1^{(4)}\hat{P}_2^{(4)}}}{\hat{t}_4}\right) \\
&-t_1 C\left(\frac{h_{23}^2 P_2}{t_1}\right) - t_2 C\left(\frac{h_{13}^2 P_1}{t_2}\right) \overset{(a)}{\lesssim} \\
&\frac{\hat{t}_1}{2}\ln\left(\frac{h_{23}^2 \hat{P}_2^{(1)}}{\hat{t}_1}\right) + \frac{\hat{t}_2}{2}\ln\left(\frac{h_{13}^2 \hat{P}_1^{(2)}}{\hat{t}_2}\right) + \hat{t}_4 C\left(\frac{h_{13}^2 \hat{P}_1^{(4)} + h_{23}^2 \hat{P}_2^{(4)} + 2h_{13}h_{23}\sqrt{\hat{P}_1^{(4)}\hat{P}_2^{(4)}}}{\hat{t}_4}\right) \\
&-\left(\frac{\hat{t}_1}{2} + \frac{\hat{t}_3 + \hat{t}_4}{4}\right)\ln\left(\frac{h_{23}^2 P_2}{t_1}\right) - \left(\frac{\hat{t}_2}{2} + \frac{\hat{t}_3 + \hat{t}_4}{4}\right)\ln\left(\frac{h_{13}^2 P_1}{t_2}\right) + O\left(\frac{1}{P_0}\right) \overset{(b)}{\lesssim} \\
&\frac{\hat{t}_1}{2}\ln\left(\frac{h_{23}^2 P_2}{\hat{t}_1}\right) + \frac{\hat{t}_2}{2}\ln\left(\frac{h_{13}^2 P_1}{\hat{t}_2}\right) + \hat{t}_4 \ln\left(h_{13}\sqrt{\frac{P_1}{\hat{t}_2 + \hat{t}_4}} + h_{23}\sqrt{\frac{P_2}{\hat{t}_1 + \hat{t}_4}}\right) \\
&-\left(\frac{\hat{t}_1}{2} + \frac{\hat{t}_3 + \hat{t}_4}{4}\right)\ln\left(\frac{h_{23}^2 P_2}{t_1}\right) - \left(\frac{\hat{t}_2}{2} + \frac{\hat{t}_3 + \hat{t}_4}{4}\right)\ln\left(\frac{h_{13}^2 P_1}{t_2}\right) + O\left(\frac{1}{P_0}\right) \overset{(c)}{\lesssim} \\
&\frac{\hat{t}_4}{2}\ln\left(\frac{2}{\sqrt{(\hat{t}_1 + \hat{t}_4)(\hat{t}_2 + \hat{t}_4)}} + \frac{h_{13}}{h_{23}(\hat{t}_2 + \hat{t}_4)}\sqrt{\frac{P_1}{P_2}} + \frac{h_{23}}{(\hat{t}_1 + \hat{t}_4)h_{13}}\sqrt{\frac{P_2}{P_1}}\right) - \frac{\hat{t}_3}{4}\ln(P_1 P_2) \\
&+O\left(\frac{1}{\log P_0}\right) \overset{(d)}{\lesssim} O\left(\frac{1}{\log P_0}\right).
\end{aligned} \tag{2.43}$$

Here, (a) follows from (2.33) and noting $\hat{P}_1^{(2)}, \hat{P}_2^{(1)} = \Theta(P_0)$, noting the function $\hat{t}_1 \ln(P_2 - y) + \hat{t}_2 \ln(P_1 - x) + \hat{t}_4 \ln\left(\hat{t}_4 + \left(h_{13}\sqrt{x} + h_{23}\sqrt{y}\right)^2\right)$ takes its maximum value at $x \leq \frac{\hat{t}_4}{\hat{t}_4 + \hat{t}_2} P_1, y \leq \frac{\hat{t}_4}{\hat{t}_4 + \hat{t}_1} P_2$ and hence substituting $\hat{P}_1^{(4)} = \frac{\hat{t}_4}{\hat{t}_4 + \hat{t}_2} P_1$ and $\hat{P}_2^{(4)} = \frac{\hat{t}_4}{\hat{t}_4 + \hat{t}_1} P_2$ result in (b), (c) follows from $\hat{t}_3, \hat{t}_4 = O\left(\frac{1}{\log P_0}\right)$ and $\hat{t}_1, \hat{t}_2 = 0.5 + O\left(\frac{1}{\log P_0}\right)$, and finally (d) follows from the facts that $\frac{P_1}{P_2} = \Theta(1)$, $\hat{t}_1 + \hat{t}_4, \hat{t}_2 + \hat{t}_4 = \Theta(1)$, and $\hat{t}_4 = O(\frac{1}{\log P_0})$.

Next, we bound the difference between the terms in the second cut of (2.29) and the second term in R_{DPC}

$$\hat{t}_2 C\left(\frac{h_{02}^2\hat{P}_0^{(2)}}{\hat{t}_2}+\frac{(h_{12}^2+h_{13}^2)\hat{P}_1^{(2)}}{\hat{t}_2}+\frac{2h_{02}h_{12}\sqrt{\hat{P}_0^{(2)}\hat{P}_1^{(2)}}}{\hat{t}_2}+\frac{h_{02}^2h_{13}^2\hat{P}_0^{(2)}\hat{P}_1^{(2)}}{\hat{t}_2^2}\right)+\hat{t}_3C\left(\frac{h_{02}^2\hat{P}_0^{(3)}}{\hat{t}_3}\right)$$
$$+\hat{t}_4C\left(\frac{h_{13}^2\hat{P}_1^{(4)}}{\hat{t}_4}\right)-t_2C\left(\frac{h_{02}^2P_0^{(2)}}{t_2}\right)-t_2C\left(\frac{h_{13}^2P_1}{t_2}\right)\overset{(a)}{\lesssim}$$
$$\frac{\hat{t}_2}{2}\ln\left(\frac{h_{02}^2h_{13}^2\hat{P}_0^{(2)}\hat{P}_1^{(2)}}{\hat{t}_2^2}\right)+\hat{t}_3C\left(\frac{h_{02}^2\hat{P}_0^{(3)}}{\hat{t}_3}\right)+\hat{t}_4C\left(\frac{h_{13}^2\hat{P}_1^{(4)}}{\hat{t}_4}\right)-\left(\frac{\hat{t}_2}{2}+\frac{\hat{t}_3+\hat{t}_4}{4}\right)\ln\left(\frac{h_{02}^2h_{13}^2\hat{P}_0^{(2)}P_1}{t_2^2}\right)$$
$$+O\left(\frac{1}{P_0}\right)\overset{(b)}{\lesssim}\frac{\hat{t}_2}{2}\ln\left(\frac{h_{02}^2h_{13}^2\hat{P}_0^{(2)}P_1}{\hat{t}_2^2}\right)+\frac{\hat{t}_3}{2}\ln\left(\frac{h_{02}^2P_0}{\hat{t}_3+\hat{t}_2}\right)+\frac{\hat{t}_4}{2}\ln\left(\frac{h_{13}^2P_1}{\hat{t}_4+\hat{t}_2}\right)$$
$$-\left(\frac{\hat{t}_2}{2}+\frac{\hat{t}_3+\hat{t}_4}{4}\right)\ln\left(\frac{h_{02}^2\hat{P}_0^{(2)}}{t_2}\right)-\left(\frac{\hat{t}_2}{2}+\frac{\hat{t}_3+\hat{t}_4}{4}\right)\ln\left(\frac{h_{13}^2P_1}{t_2}\right)+O\left(\frac{1}{P_0}\right)\overset{(c)}{\lesssim}$$
$$\frac{\hat{t}_3}{4}\ln\left(\frac{P_0^2}{\hat{P}_0^{(2)}P_1}\right)+\frac{\hat{t}_4}{4}\ln\left(\frac{P_1}{\hat{P}_0^{(2)}}\right)+O\left(\frac{1}{\log P_0}\right)\overset{(d)}{\lesssim}O\left(\frac{1}{\log P_0}\right). \tag{2.44}$$

Here, (a) follows from (2.33), the fact that $P_0^{(2)}=\hat{P}_0^{(2)}=\Theta(P_0)$ and upper-bounding $\hat{P}_0^{(3)}\leq P_0$, $\hat{P}_1^{(4)}\leq P_1$, noting the facts that $\hat{P}_0^{(2)}+\hat{P}_0^{(3)}\leq P_0$ and $\hat{P}_1^{(2)}+\hat{P}_1^{(4)}=P_1$, the functions $\hat{t}_2\ln(P_0-x)+\hat{t}_3\ln\left(\hat{t}_3+h_{02}^2x\right)$ and $\hat{t}_2\ln(P_1-y)+\hat{t}_4\ln\left(\hat{t}_4+h_{13}^2y\right)$ are maximized at $x\leq\frac{\hat{t}_3}{\hat{t}_2+\hat{t}_3}P_0$ and $y\leq\frac{\hat{t}_4}{\hat{t}_2+\hat{t}_4}P_1$, hence, substituting $\hat{P}_0^{(3)}=\frac{\hat{t}_3}{\hat{t}_2+\hat{t}_3}P_0$ and $\hat{P}_1^{(4)}=\frac{\hat{t}_4}{\hat{t}_2+\hat{t}_4}P_1$ upper-bounds the expression which results in (b), (c) follows from $\hat{t}_3,\hat{t}_4=O\left(\frac{1}{\log P_0}\right),\hat{t}_1,\hat{t}_2=0.5+O\left(\frac{1}{\log P_0}\right)$, and finally (d) follows from the fact that $\hat{P}_0^{(2)},P_1=\Theta(P_0)$ and also $\hat{t}_3,\hat{t}_4=O\left(\frac{1}{\log P_0}\right)$.

Noting that the second and the third cuts are the same, and using the same argument as in (2.44), we can bound the difference between the terms in the third cut of (2.29) and the third term in R_{DPC} as

$$\hat{t}_1C\left(\frac{h_{01}^2\hat{P}_0^{(1)}}{\hat{t}_1}+\frac{(h_{12}^2+h_{23}^2)\hat{P}_2^{(1)}}{\hat{t}_1}+\frac{2h_{01}h_{12}\sqrt{\hat{P}_0^{(1)}\hat{P}_2^{(1)}}}{\hat{t}_1}+\frac{h_{01}^2h_{23}^2\hat{P}_0^{(1)}\hat{P}_2^{(1)}}{\hat{t}_1^2}\right) \tag{2.45}$$
$$+\hat{t}_3C\left(\frac{h_{01}^2\hat{P}_0^{(3)}}{\hat{t}_3}\right)+\hat{t}_4C\left(\frac{h_{23}^2\hat{P}_2^{(4)}}{\hat{t}_4}\right)-t_1C\left(\frac{h_{01}^2P_0^{(1)}}{t_1}\right)-t_1C\left(\frac{h_{23}^2P_2}{t_1}\right)\lesssim O\left(\frac{1}{\log P_0}\right).$$

Observing (2.42), (2.43), (2.44) and (2.45), completes the proof of the theorem. □

Theorem 2.3.4. *In low SNR scenarios, assuming $P_1=\gamma_1P_0$, $P_2=\gamma_2P_0$ with γ_1,γ_2 constants independent of the SNR, when the power available for the source and each relay tends to zero and $\left(h_{13}\sqrt{\gamma_1}+h_{23}\sqrt{\gamma_2}\right)^2\leq\min\left(h_{01}^2,h_{02}^2\right)$, the ratio of the achievable rate of the simultaneous relaying protocol based on BCM to cut-set upper bound goes to 1. In this scenario $t_3=t_4=\frac{1}{2}$, and no private messages should be transmitted.*

Proof. By the same argument as in Theorem 2.3.3 and considering only the fourth cut, we obtain another upper bound on the capacity. By the following inequality

$$\ln(1+x) \leq x. \tag{2.46}$$

we can bound the upper bound on the capacity as

$$C^{up} \quad \leq \quad \frac{\left(h_{13}\sqrt{\gamma_1}+h_{23}\sqrt{\gamma_2}\right)^2 P_0}{2\ln 2}. \tag{2.47}$$

Now, assuming $t_1 = t_2 = 0, t_3 = t_4 = \frac{1}{2}$, and transmitting just the common message, we can achieve the following rate R_{BCM}:

$$R_{BCM} = \min\left(\frac{1}{2}C\left(2h_{02}^2P_0\right), \frac{1}{2}C\left(2\left(h_{13}\sqrt{\gamma_1}+h_{23}\sqrt{\gamma_2}\right)^2 P_0\right)\right). \tag{2.48}$$

According to the Taylor expansion of $\ln(1+x)$ at $x = 0$, we have

$$x - \frac{x^2}{2} \leq \ln\left(1+x\right), \tag{2.49}$$

Hence,

$$\begin{aligned} &\frac{1}{\ln 2}\min\left(\frac{h_{02}^2P_0}{2} - \frac{h_{02}^4P_0^2}{2}, \frac{\left(h_{13}\sqrt{\gamma_1}+h_{23}\sqrt{\gamma_2}\right)^2 P_0}{2} - \frac{\left(h_{13}\sqrt{\gamma_1}+h_{23}\sqrt{\gamma_2}\right)^4 P_0^2}{2}\right) \\ &\leq R_{BCM}. \end{aligned} \tag{2.50}$$

By (2.47), (2.50), and $\left(h_{13}\sqrt{\gamma_1}+h_{23}\sqrt{\gamma_2}\right)^2 \leq \min\left(h_{01}^2, h_{02}^2\right)$, we have

$$\lim_{P_0 \to 0}\frac{R_{BCM}}{C^{up}} \to 1. \tag{2.51}$$

□

2.4 Simulation Result

In this section, the achievable rate of different proposed schemes, i.e., SSRD, DPC, BME, the proposed composite BME-DPC, combined BME DPC of [37], the proposed schemes in [36], i.e., BCM, SF, Hybrid DF-CF, Hybrid DF-SF, and CF are compared with each other and with the upper bound in different channel conditions. However, since in [36] unlike in our work, the nodes cannot accumulate their powers in idle modes to consume in transmitting modes, we apply our power constraint assumption to the schemes of [36].

Figures 2.8 and 2.9 compare the achievable rate of the SSRD scheme with that of the DPC scheme for successive relaying and the BCM scheme along with schemes of [36] for simultaneous relaying protocols. Here the symmetric scenario in which $P_1 = P_2$ and $h_{01} = h_{02} = h_{12} = h_{13} = h_{23} = 1$ is considered. The upper bound is also included in the figure.

In order to satisfy the condition in Theorem 2.3.4, i.e., $\left(h_{13}\sqrt{\gamma_1} + h_{23}\sqrt{\gamma_2}\right)^2 \leq \min\left(h_{01}^2, h_{02}^2\right)$, in Fig. 2.8 we also assume $P_0 = P_1 + 5(dB) = P_2 + 5(dB)$. As Fig. 2.8 shows, SSRD achievable rate almost coincides with the upper bound over all ranges of SNR. As proved in the previous section, in high SNR scenario, the SSRD scheme coincides with DPC and the successive relaying protocol becomes optimum, while in low SNR scenario it coincides with BCM and the simultaneous relaying protocol is optimum.

On the other hand, in Fig. 2.9 we assume that $P_0 = P_1 = P_2$. In this situation, the condition in Theorem 2.3.4 is no longer satisfied. Therefore, as this figure show, the ratio of the achievable rate of the SSRD scheme to the cut-set bound, i.e., $\frac{R_{SSRD}}{C^{up}}$ does not tend to one. Furthermore, the achievable rates of the SSRD, DPC, BCM, and Hybrid DF-CF schemes coincide with each other.

As Figs. 2.8 and 2.9 show, the proposed scheme of this paper, i.e. SSRD, DPC, and BCM outperform the ones proposed in [36].

Figure 2.10 compares the achievable rate of different successive relaying schemes, i.e., DPC, BME schemes with successive and backward decoding, proposed composite BME-DPC and the combined BME DPC of [37] with each other and the successive cut-set bound for the asymmetric scenario. It shows as the inter-relay channel becomes stronger, BME scheme can achieve the successive cut-set bound, while the achievable rate of the DPC is independent of that channel. Furthermore, this figure indicates BME with backward decoding gives a better achievable rate with respect to BME with successive decoding which was proposed in [37]. Moreover, it can be seen from this figure that the achievable rate of our proposed composite BME-DPC scheme coincides with that of the combined BME DPC scheme of [37].

Remark 2.4.1. *We did not study the CF based strategies here. While DF based strategies achieve the capacity of the single relay channel or general relay networks in many certain scenarios, CF based strategies are known to achieve rates with at least a constant gap from the cut-set upper bound (See [28]). Indeed, the only examples for the optimality of CF based strategies that have been discovered most recently, are based on a very special case of the discrete single relay channel (See [44, 45]). It is worth mentioning that in [45], the capacity of the relay channel which is obtained by the CF strategy is strictly below the cut-set upper bound.*

In our setup, we study the possible scenarios in which the capacity can be achieved by our proposed schemes (asymptotically high SNR, asymptotically low SNR, successive degraded and

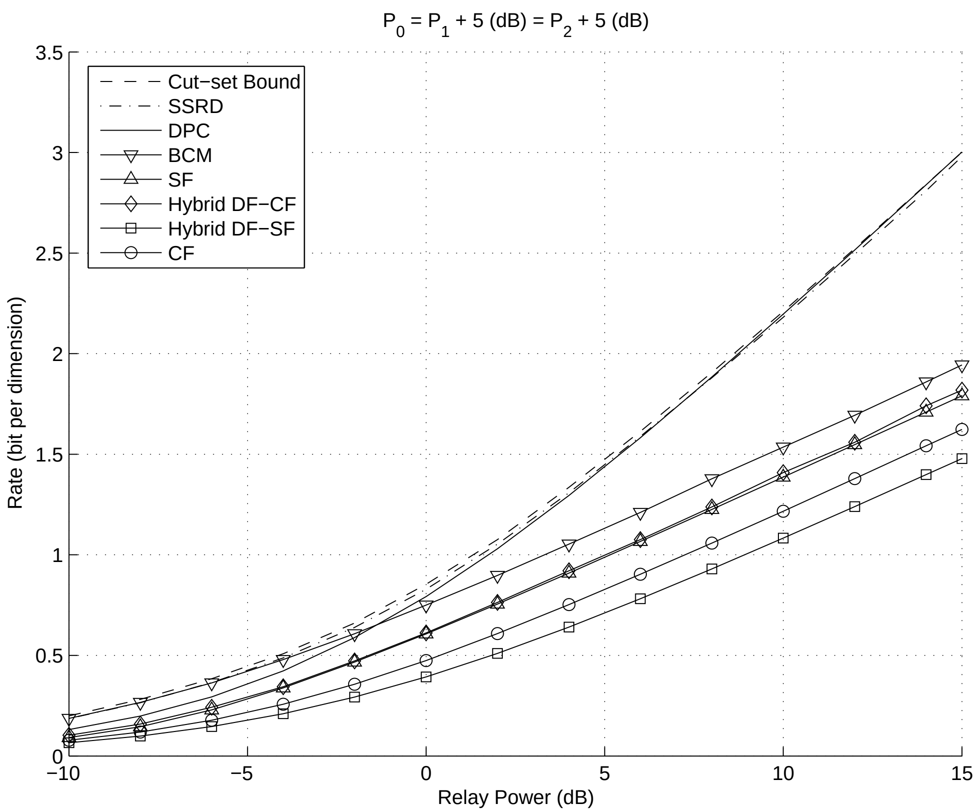


Figure 2.8: Rate versus relay power for the symmetric scenario.

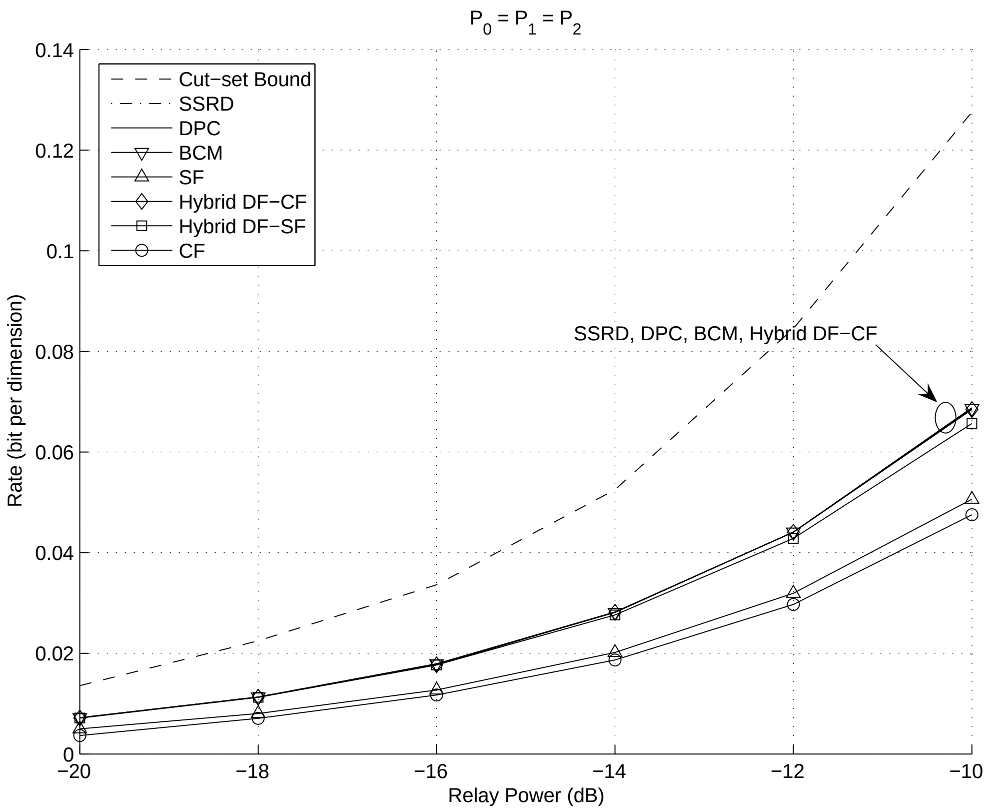


Figure 2.9: Rate versus relay power for the symmetric scenario.

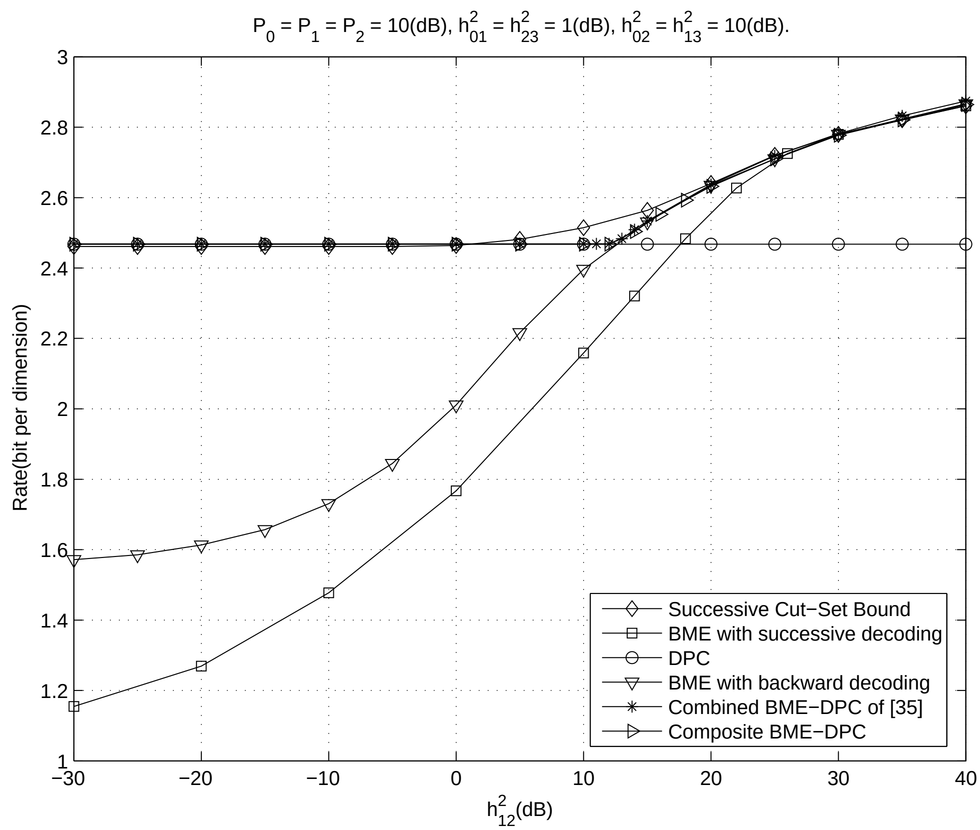


Figure 2.10: Rate versus inter relay gain.

symmetric cases). However, in the CF strategy, since none of the relays decode the source's message, they have to quantize the source's message with a quantization noise whose power is in the same order as the power of the noise at the destination (assuming the signal powers are the same in the network and the noise powers are different). In this way, the relays are sure that the destination can decode their quantized signal. However, by imposing this quantization-level noise, the rate achieved by the CF strategy would differ from the cut-set upper bound with an additive gap (in high SNR regime) or a multiplicative gap (in low SNR regime).

The CF based strategies also suffer from the following problem in the successive protocol. The source is not aware of the quantized signal of each relay. Hence, it can neither cooperate with the transmitting relay at the receiver side of the receiving relay (i.e. the extended MAC would be reduced to the simple MAC) nor perform the DPC scheme to cancel the interference of each relay's signal imposed at the receiver side of the other relay.

Chapter 3

Combined Amplify and Decode Forward

For many years, Schein and Gallager's achievable rate based on the time sharing between the AF and DF schemes (AF-DF) was the best known achievable scheme for the Gaussian parallel relay channel with two relays. Since then there was no reported improvement in the literature. However, more recently, Yuval Kochman, Anatoly Khina, Uri Erez, Ram Zamir in [48][49], proposed the Rematch-and-Forward (RF) scheme for this channel. This scheme is based on the use of analog modulo-lattice modulation (See [47]) and hybrid digital and analog coding for joint source channel coding (See [50]). The RF scheme is used for the scenarios in which there is a bandwidth mismatch between the source-relays and relays-destination channels. Furthermore, the authors showed that the time sharing between the RF and DF scheme (RF-DF), in certain scenarios, achieves a better rate than the Schein and Gallager's scheme.

In this chapter[1], we propose a Combined Amplify-and-Decode (CADF) scheme, when there is a bandwidth mismatch between the source-relays (Broadcast: BC) and relays-destination (Multiple Access: MAC) channels. We prove that this scheme always achieves a better rate than the RF scheme. Furthermore, we show that time sharing between the CADF and DF schemes (CADF-DF) always outperforms the RF-DF and the AF-DF.

This chapter is organized as follows. The system model is explained in section 3.1. The CADF scheme for the bandwidth mismatch case is proposed and discussed in section 3.2. And finally simulation results are discussed in section 3.3.

[1]Portions reprinted, with permission, from (Seyed Saeed Changiz Rezaei, Shahab Oveis Gharan, and Amir K. Khandani, "A New Achievable Rate for the Gaussian Parallel Relay Channel", Proc. Of IEEE International Symposium on Information Theory 2009 (ISIT 2009), pp. 194 - 198). © [2010] IEEE.

3.1 The System Model

The setup of the system model considered in this paper is similar to [48]. Here, we consider a Gaussian network which consists of a source, M relays, and a destination with no direct link between the source and the destination.

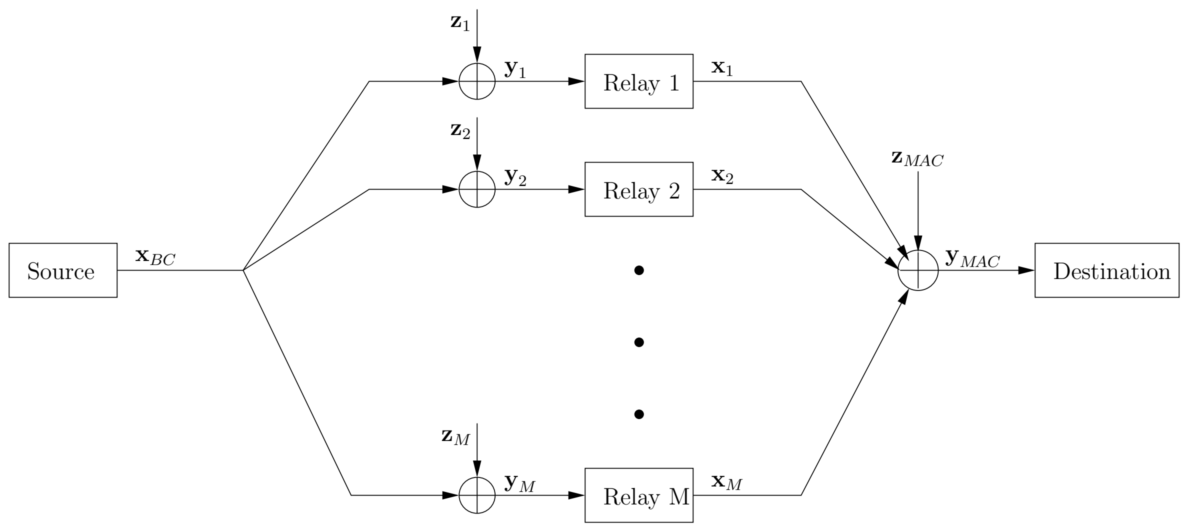


Figure 3.1: The Gaussian Parallel Relay Channel.

Nodes $1, \cdots, M$ represent relay 1 , $\cdots$, relay M, respectively. The transmitted vectors from the source and the relays, and the received vectors at the relays and the destination are denoted by $\mathbf{x}_{BC}$, $\mathbf{x}_m (m = 1, \cdots, M)$ and $\mathbf{y}_m (m = 1, \cdots, M)$, and $\mathbf{y}_{MAC}$, respectively. Hence, we have

$$\mathbf{y}_m = \mathbf{x}_{BC} + \mathbf{z}_m, \quad m \in \{1, \cdots, M\}, \tag{3.1}$$

$$\mathbf{y}_{MAC} = \sum_{m=1}^{M} \mathbf{x}_m + \mathbf{z}_{MAC}. \tag{3.2}$$

where $\mathbf{z}_m$ and $\mathbf{z}_{MAC}$ are the AWGN terms. Throughout the paper, for the sake of simplicity, we consider the symmetric case in which all the AWGN terms have zero mean and the variance "1" per dimension. It is worth mentioning that noises at the relays and destination are independent from each other.

Furthermore, the average power constraints P_s, P_m $(m \in \{1, \cdots, M\})$ should be satisfied for the source and relay nodes:

$$\frac{1}{n} E \parallel \mathbf{x}_{BC} \parallel^2 \leq P_s, \tag{3.3}$$

$$\frac{1}{n} E \parallel \mathbf{x}_m \parallel^2 \leq P_m, \quad m \in \{1, \cdots, M\}. \tag{3.4}$$

where n denotes the corresponding vector length.

Due to the symmetry assumption, we have

$$P_1 = P_2 = \cdots = P_M = P_r. \tag{3.5}$$

It should be noted that for the bandwidth mismatch case P_s and P_r are the power constraints per unit of bandwidth.

3.2 The Bandwidth Mismatch Case

In this section, we study the problem of bandwidth mismatch between the first and second hop. This problem may arise in many practical situations. For instance, the available bandwidth for the source and the relays to transmit their signals may not be equal. As another example, consider a half-duplex parallel relay channel, assuming a constant bandwidth from the source to the destination, the optimum amount of bandwidth for the first and second hops is not necessarily the same. Hence, the *Combined Amplify-and-Decode Forward (CADF)* scheme is proposed for these types of situations in the sequel.

Here we assume that for each ρ uses of BC channel, one use of the MAC channel is allowed. ρ can be either less or greater than "1". Practically, this means that one can allocate less or more resources in terms of time or frequency to the first hop compared to the second hop. On the other hand, from the mathematical point of view, this means that the length of the codeword used at the source side is ρ times the length of the codewords used at the relays side.

According to the cut-set bound Theorem (See [14]), on the cuts corresponding to the first and second hop, the upper bound, C_{up}, on the capacity of this channel, C_s, is (See [48]):

$$C_s \leq C_{up} \triangleq \min\left(\rho C\left(MP_s\right), C\left(M^2P_r\right)\right). \tag{3.6}$$

3.2.1 The Combined Amplify-and-Decode Forward (CADF)

In this section, CADF scheme is studied. This scheme is illustrated in Figs. 3.2. In this strategy, the intended message is split into AF and DF messages. The AF message itself is split into L AF sub-messages. Each AF sub-message is transmitted in $2\alpha_l (l = 1, \cdots, L)$ fraction of the available bandwidth from the source to the destination. The DF message is superimposed on the AF message and transmitted from the source to the relays in $\sum_{l=1}^{L} \alpha_l + \beta_1$ dimensions. Having decoded the DF message, each relay transmits the re-encoded version on top of the AF message in $\sum_{l=1}^{L} \alpha_l + \beta_2$ dimensions. Due to the water-filling result of the

DF message on the AF message and from (3.3) and (3.4), in α_l band from the source to each relay, we have

$$P_{s,AF_l} + P_{s,DF_l} = P_s, \quad l = 1, \cdots, L. \tag{3.7}$$

Similarly, for the relay side we have

$$P_{r,AF_l} + P_{r,DF_l} = P_r, \quad l = 1, \cdots, L. \tag{3.8}$$

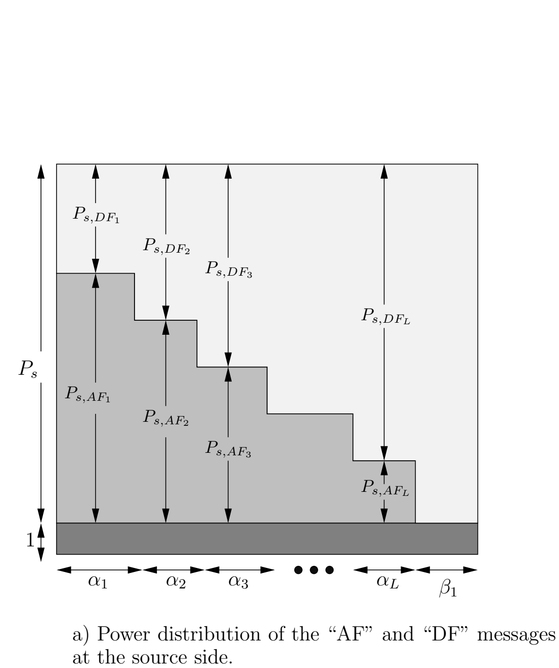


Figure 3.2: Power distribution of the "AF" and "DF" messages at the source and relay sides.

Furthermore, due to the bandwidth constraint for the BC and MAC channel, we have

$$\sum_{l=1}^{L} \alpha_l + \beta_1 = \rho, \tag{3.9}$$

$$\sum_{l=1}^{L} \alpha_l + \beta_2 = 1. \tag{3.10}$$

The above discussions result in the following Theorem.

Theorem 3.2.1. *For the Gaussian parallel relay channel, the CADF achieves the following* *rate:*

$$\begin{aligned}
R_{CADF} \leq & \max \sum_{l=1}^{L} R_{AF_l} + R_{DF_l} + R_{DF}, \qquad (3.11)\\
& \textit{subject to:}\\
& \quad R_{DF_l} \leq \alpha_l C\left(\frac{P_{s,DF_l}}{P_{s,AF_l}+1}\right),\\
& \quad R_{AF_l} \leq \alpha_l C\left(\frac{M^2 P_{r,AF_l} P_{s,AF_l}}{M P_{r,AF_l} + P_{s,AF_l} + 1}\right),\\
& \quad R_{DF_l} \leq \alpha_l C\left(\frac{M^2 P_{r,DF_l}\left(P_{s,AF_l}+1\right)}{M P_{r,AF_l} + P_{s,AF_l} + 1}\right),\\
& \quad R_{AF_l} + R_{DF_l} \leq \alpha_l C\left(\frac{M^2 P_r P_{s,AF_l} + M^2 P_{r,DF_l}}{M P_{r,AF_l} + P_{s,AF_l} + 1}\right),\\
& \quad R_{DF} \leq \beta_1 C\left(P_s\right),\\
& \quad R_{DF} \leq \beta_2 C\left(M^2 P_r\right),\\
& \quad \sum_{l=1}^{L} \alpha_l + \beta_1 = \rho,\\
& \quad \sum_{l=1}^{L} \alpha_l + \beta_2 = 1,\\
& \quad P_{s,AF_l} + P_{s,DF_l} = P_s,\\
& \quad P_{r,AF_l} + P_{r,DF_l} = P_r,\\
& \quad 0 \leq \alpha_l, \beta_1, \beta_2,\\
& \quad 0 \leq P_{s,AF_l}, P_{s,DF_l} \leq P_s, 0 \leq P_{r,AF_l}, P_{r,DF_l} \leq P_r, l = 1, \cdots, L.
\end{aligned}$$

Proof. See Appendix H. □

Remark 3.2.1. *For the half-duplex scenarios, instead of the constraints* $\sum_{l=1}^{L} \alpha_l + \beta_1 = \rho$ *and* $\sum_{l=1}^{L} \alpha_l + \beta_2 = 1$ *for the bandwidths of the first and second hops separately, we assume a constant bandwidth from the source to the destination, i.e.,* $2\sum_{l=1}^{L} \alpha_l + \beta_1 + \beta_2 = 1$.

Proposition 3.2.1. *The CADF scheme achieves the same rate* R_{CADF}*, assuming successive decoding of the DF and AF messages at the receiver side. Moreover, the achievable rate can* *be simplified as*

$$R_{CADF} \leq \max \sum_{l=1}^{L} \alpha_l C\left(\frac{M^2 P_{r,AF_l} P_{s,AF_l}}{M P_{r,AF_l} + P_{s,AF_l} + 1}\right) + \min\left(\sum_{l=1}^{L} \alpha_l C\left(\frac{P_{s,DF_l}}{P_{s,AF_l}+1}\right) + \beta_1 C\left(P_s\right),\right.$$
$$\left.\sum_{l=1}^{L} \alpha_l C\left(\frac{M^2 P_{r,DF_l}(P_{s,AF_l}+1)}{M^2 P_{r,AF_l} P_{s,AF_l} + M P_{r,AF_l} + P_{s,AF_l} + 1}\right) + \beta_2 C\left(M^2 P_r\right)\right), \quad (3.12)$$

subject to:

$$\sum_{l=1}^{L} \alpha_l + \beta_1 = \rho,$$
$$\sum_{l=1}^{L} \alpha_l + \beta_2 = 1,$$
$$P_{s,AF_l} + P_{s,DF_l} = P_s,$$
$$P_{r,AF_l} + P_{r,DF_l} = P_r,$$
$$0 \leq \alpha_l, \beta_1, \beta_2,$$
$$0 \leq P_{s,AF_l}, P_{s,DF_l} \leq P_s,\ 0 \leq P_{r,AF_l}, P_{r,DF_l} \leq P_r,\ l = 1, \cdots, L.$$

Proof. From (3.11) and the discussion in Appendix H, we can consider the AF and the DF messages at band α_l in the second hop as the messages of a MAC with the following inequalities

$$R_{AF_l} \leq \alpha_l C\left(\frac{M^2 P_{r,AF_l} P_{s,AF_l}}{M P_{r,AF_l} + P_{s,AF_l} + 1}\right), \quad (3.13)$$
$$R_{DF_l} \leq \alpha_l C\left(\frac{M^2 P_{r,DF_l}(P_{s,AF_l}+1)}{M P_{r,AF_l} + P_{s,AF_l} + 1}\right), \quad (3.14)$$
$$R_{AF_l} + R_{DF_l} \leq \alpha_l C\left(\frac{M^2 P_r P_{s,AF_l} + M^2 P_{r,DF_l}}{M P_{r,AF_l} + P_{s,AF_l} + 1}\right). \quad (3.15)$$

It can be readily verified that subject to the constraint $P_{r,AF_l} + P_{r,DF_l} = P_r$, the right-hand side of (3.15) is a decreasing function of P_{r,AF_l} or equivalently an increasing function of P_{r,DF_l}. Now, let us equate R_{AF_l} in (3.15) with the AF rate $\acute{R}_{AF_l}$ of another MAC which is achieved by successive decoding of the DF and AF messages. Therefore, we have

$$R_{AF_l} = \acute{R}_{AF_l} = \alpha_l C\left(\frac{M^2 \acute{P}_{r,AF_l} P_{s,AF_l}}{M \acute{P}_{r,AF_l} + P_{s,AF_l} + 1}\right) \leq \alpha_l C\left(\frac{M^2 P_{r,AF_l} P_{s,AF_l}}{M P_{r,AF_l} + P_{s,AF_l} + 1}\right). \quad (3.16)$$

According to (3.16), we have

$$\begin{aligned} \acute{P}_{r,AF} &\leq P_{r,AF} \Longrightarrow \\ R_{AF_l} + R_{DF_l} &\leq \acute{R}_{AF_l} + \acute{R}_{DF_l}, \\ R_{DF_l} &\leq \acute{R}_{DF_l}. \end{aligned}$$

Hence, (R_{AF_l}, R_{DF_l}) lies in the corner point of the MAC with parameters $(\acute{R}_{AF_l}, \acute{R}_{DF_l})$, i.e. successive decoding of the DF and AF messages achieves R_{CADF}.

Hence, we can assume

$$R_{AF_l} = \alpha_l C\left(\frac{M^2 P_{r,AF_l} P_{s,AF_l}}{M P_{r,AF_l} + P_{s,AF_l} + 1}\right). \tag{3.17}$$

Now, from (3.15) and (3.17) inequality (3.14) is concluded. Hence, inequality (3.14) is extra. Therefore, by considering the appropriate order of decoding for the DF and the AF messages at the destination, the proposition is proved. □

Remark 3.2.2. *Due to the complicated nature of the formula for R_{CADF}, it cannot be verified that R_{CADF} is a convex or concave function of P_{s,AF_l} and consequently claimed $L > 1$ or $L = 1$, respectively. However, the following proposition gives an upper bound on the number of bands L.*

Proposition 3.2.2. *The optimum number of bands L in the CADF scheme is at most equal to two. Furthermore, for the half-duplex scenarios assuming one of the α_l's is non-zero, depending on $\rho < 1$ or $\rho > 1$, either $\beta_1 = 0$ and $\beta_2 \neq 0$ or $\beta_1 \neq 0$ and $\beta_2 = 0$.*

Proof. Assuming variables P_{s,AF_l}, P_{s,DF_l}, P_{r,AF_l}, and P_{r,DF_l} in (3.11) as constant parameters, one can cast the optimization problem (3.11) in a linear form with variables α_l, β_1, and β_2 as the optimization parameters. In order to do that, we introduce a parameter $\lambda \in \mathbb{R}$ to (3.11), and assume that the difference between the two terms in the minimization (3.11) is λ. Hence, we have the following linear optimization problem which is equivalent to (3.11):

$$R_{CADF} \leq \max_{\lambda \in \mathbb{R}} \ (\min(-\lambda, 0) + f(\lambda)), \tag{3.18}$$

where

$$f(\lambda) = \max \sum_{l=1}^{L} \alpha_l \left(C\left(\frac{M^2 P_{r,AF_l} P_{s,AF_l}}{M P_{r,AF_l} + P_{s,AF_l} + 1}\right) + C\left(\frac{P_{s,DF_l}}{P_{s,AF_l} + 1}\right)\right) + \beta_1 C(P_s), \tag{3.19}$$

subject to:

$$\sum_{l=1}^{L} \alpha_l \left(C\left(\frac{M^2 P_{r,AF_l} P_{s,AF_l}}{M P_{r,AF_l} + P_{s,AF_l} + 1}\right) + C\left(\frac{P_{s,DF_l}}{P_{s,AF_l} + 1}\right) \right.$$
$$\left. - C\left(\frac{M^2 P_r P_{s,AF_l} + M^2 P_{r,DF_l}}{M P_{r,AF_l} + P_{s,AF_l} + 1}\right)\right) + \beta_1 C(P_s) - \beta_2 C\left(M^2 P_r\right) = \lambda, \tag{3.20}$$

$$\sum_{l=1}^{L} \alpha_l + \beta_1 = \rho, \tag{3.21}$$

$$\sum_{l=1}^{L} \alpha_l + \beta_2 = 1, \tag{3.22}$$

$$0 \leq \alpha_l, \beta_1, \beta_2, l = 1, \cdots, L. \tag{3.23}$$

For $\rho < 1$, from (3.21), (3.22), and knowing $\beta_1 \geq 0$, $\beta_2 > 0$ can be concluded. Hence, substituting β_2 from (3.22) into (3.19) and (3.20), (3.19)-(3.23) becomes

$$f(\lambda) = \max\ \mathbf{c}^T\mathbf{y}, \tag{3.24}$$

$$\text{subject to:}$$

$$\mathbf{A}\mathbf{y} = \mathbf{b}, \tag{3.25}$$

$$\mathbf{y} \succeq 0. \tag{3.26}$$

where

$$\begin{aligned}
&\mathbf{y} = [\alpha_1, \alpha_2, \alpha_3, \cdots, \alpha_L, \beta_1]^T, \\
&c_l = C\left(\frac{M^2 P_{r,AF_l} P_{s,AF_l}}{M P_{r,AF_l} + P_{s,AF_l} + 1}\right) + C\left(\frac{P_{s,DF_l}}{P_{s,AF_l} + 1}\right),\ l = 1, \cdots, L, \\
&c_{L+1} = C\left(P_s\right),
\end{aligned}$$

$$\begin{aligned}
A_{1l} &= C\left(\frac{M^2 P_{r,AF_l} P_{s,AF_l}}{M P_{r,AF_l} + P_{s,AF_l} + 1}\right) + C\left(\frac{P_{s,DF_l}}{P_{s,AF_l} + 1}\right) \\
&\quad - C\left(\frac{M^2 P_r P_{s,AF_l} + M^2 P_{r,DF_l}}{M P_{r,AF_l} + P_{s,AF_l} + 1}\right) + C\left(M^2 P_r\right),\ l = 1, \cdots, L, \\
A_{1L+1} &= C\left(P_s\right),\ A_{2l} = 1,\ l = 1, \cdots, L+1, \\
\mathbf{b} &= \left[\lambda + C\left(M^2 P_r\right), \rho\right]^T.
\end{aligned}$$

The optimum solution of (3.24), $\mathbf{y}_{opt}$, is an extreme point of the region $\mathcal{F} = \{\mathbf{A}\mathbf{y} = \mathbf{b}, \mathbf{y} \succeq 0\}$. On the other hand, $\mathbf{y}_{opt}$ is an extreme point of $\mathcal{F}$ if and only if it is a basic feasible solution of (3.24). Since the rank of matrix $\mathbf{A}$ is at most 2, the basic feasible solution of $\mathcal{F}$ has at most 2 non-zero entries (See [52]). Therefore, the only possible cases are $\alpha_i \neq 0$, $\alpha_j \neq 0$ (where $i \neq j$), and $\beta_2 \neq 0$ or $\alpha_i \neq 0$, $\beta_1 \neq 0$, and $\beta_2 \neq 0$.

Having the similar argument for $\rho > 1$, we can easily prove that the only possible cases are $\alpha_i \neq 0$, $\alpha_j \neq 0$ (where $i \neq j$), and $\beta_1 \neq 0$ or $\alpha_i \neq 0$, $\beta_1 \neq 0$, and $\beta_2 \neq 0$. Hence, the optimum number of bands L is at most equal to two.

For the half-duplex scenarios, from Remark 1, the optimization problem (3.19) becomes a linear optimization problem with two constraints. Using the similar argument as in the bandwidth mismatch case, only two optimization parameters would be non-zero. Hence, assuming one of the α_l's is non-zero and $\rho \neq 1$, depending on $\rho < 1$ or $\rho > 1$, either $\beta_1 = 0$ and $\beta_2 \neq 0$ or $\beta_1 \neq 0$ and $\beta_2 = 0$. Therefore, from the above argument, for the half-duplex scenarios the optimum number of bands L is at most equal to one. $\square$

By considering the appropriate order of decoding for the DF message and the AF message at the destination and from Proposition 3.2.2, the achievable rate can be simplified as

$$R_{CADF} \leq \max \sum_{l=1}^{2} \alpha_l C\left(\frac{M^2 P_{r,AF_l} P_{s,AF_l}}{M P_{r,AF_l} + P_{s,AF_l} + 1}\right) + \min\left(\sum_{l=1}^{2} \alpha_l C\left(\frac{P_{s,DF_l}}{P_{s,AF_l} + 1}\right) + \beta_1 C\left(P_s\right),\right.$$

$$\left.\sum_{l=1}^{2} \alpha_l C\left(\frac{M^2 P_{r,DF_l}(P_{s,AF_l}+1)}{M^2 P_{r,AF_l} P_{s,AF_l} + M P_{r,AF_l} + P_{s,AF_l} + 1}\right) + \beta_2 C\left(M^2 P_r\right)\right), \quad (3.27)$$

subject to:

$$\sum_{l=1}^{2} \alpha_l + \beta_1 = \rho, \quad (3.28)$$

$$\sum_{l=1}^{2} \alpha_l + \beta_2 = 1, \quad (3.29)$$

$$P_{s,AF_l} + P_{s,DF_l} = P_s, \quad (3.30)$$

$$P_{r,AF_l} + P_{r,DF_l} = P_r, \quad (3.31)$$

$$0 \leq \alpha_l, \beta_1, \beta_2, \quad (3.32)$$

$$0 \leq P_{s,AF_l}, P_{s,DF_l} \leq P_s, 0 \leq P_{r,AF_l}, P_{r,DF_l} \leq P_r,\ l = 1, 2. \quad (3.33)$$

3.2.2 The Traditional Coding Schemes

The achievable rates for the traditional coding schemes such as the Decode-and-Forward (DF), the Amplify-and-Forward (AF), and the Compress-and-Forward (CF) are derived in [48]. These are highlighted for comparison purposes:

Decode-and-Forward (DF)

In this scheme, the codeword $\mathbf{x}_m$ in (3.2) is a re-encoded version of the decoded message at relay m. Hence, the source transmits its message such that each relay can decode it. Hence, the DF scheme achieves

$$R_{DF} = \min\left(\rho C\left(P_s\right), C\left(M^2 P_r\right)\right). \quad (3.34)$$

Amplify-and-Forward (AF)

In the AF scheme, the relay m transmits a re-scaled version of the signal received from the BC channel. Hence, the AF scheme achieves

$$R_{AF} = \gamma C\left(\frac{M^2 P_r P_s}{M P_r + P_s + 1}\right). \quad (3.35)$$

where $\gamma = \min(\rho, 1)$.

Compress-and-Forward (CF)

In the CF scheme, the relay m estimates the transmitted codeword and digitally compresses its estimation. Then, it encodes the compressed value to an appropriate channel codeword and sends it over the MAC channel [48]. Hence, the CF scheme achieves

$$\begin{aligned} R_{CF} &= \rho C\left(P_{CF}\right), \\ &\text{subject to:} \\ &\quad (1+MP_r)^{\frac{1}{\rho}} = 1 + P_{CF}\left(\frac{MP_s}{MP_s - P_{CF} + 1}\right)^M . \end{aligned} \tag{3.36}$$

3.2.3 The Rematch-and-Forward (RF) scheme

The RF scheme based on two different approaches was proposed in [48] and [49]. The RF scheme in [48] can be briefly explained as follows. Depending on $\rho > 1$ or $\rho < 1$, the source conducts the up-sampling or down-sampling operation, and the relays do the reverse operation and then estimate the transmitted signal. Indeed, this scheme matches a colored source to a channel and is implemented using the modulo lattice modulation. For further details see [46][47][48]. The following Theorem is proved in [48].

Theorem 3.2.2. *For the Gaussian parallel relay channel with expansion factor ρ, assuming $P_s > 1$, the RF scheme based on Modulo Lattice Modulation (MLM) achieves the following rate*

$$R_{RF_{MLM}} = C\left(\frac{M^2P_r(P_s^\rho - 1)}{(P_s^\rho + MP_r)^\gamma(P_s^\rho + M^2P_r)^{1-\gamma}}\right). \tag{3.37}$$

On the other hand, the RF scheme in [49] can be considered as a joint source channel coding scheme and also a variant of the Hybride Digital Analog (HDA) coding approaches which were previously proposed in [50]. In the RF scheme based on joint source channel coding, a random white Gaussian codebook with the bandwidth of the MAC section is used. An appropriate joint source channel coding scheme is exploited in the BC section while analog transmission is used over the MAC section. The following theorem is proved in [49].

Theorem 3.2.3. *For the Gaussian parallel relay channel with expansion factor ρ, assuming $P_s > 1$, the RF scheme based on Joint Source Channel coding approaches (JSCC) achieves* *the following rates for $\rho > 1$ and $\rho <= 1$, respectively*

$$\begin{aligned} &\textit{For } \rho > 1\textit{:} \\ &R_{RF_{JSCC}} = C\left(M^2 \frac{\acute{P}_s P_r}{\acute{P}_s + MP_r + 1}\right), \qquad (3.38) \\ &\textit{For } \rho <= 1\textit{:} \\ &R_{RF_{JSCC}} = \rho C\left(M^2 \frac{\tilde{P}_s P_r}{\tilde{P}_s + MP_r + 1}\right) + (1-\rho) C\left(M^2 \frac{\tilde{P}_s P_r}{\tilde{P}_s + M^2 P_r + 1}\right), \qquad (3.39) \end{aligned}$$

where

$$\begin{aligned} C\left(\tilde{P}_s\right) &= \rho C\left(P_s\right), \\ \acute{P}_s &= \frac{P_s \tilde{P}_s - \rho}{P_s + \rho}. \end{aligned}$$

Theorem 3.2.4. *The CADF scheme achieves a better rate than the RF scheme based on Modulo Lattice Modulation and the RF scheme based on Joint Source Channel Coding, i.e.,* $R_{CADF} \geq \max\left(R_{RF_{MLM}}, R_{RF_{JSCC}}\right)$.

Proof. Throughout the proof, we assume that $L = 1$ and depending on $\rho < 1$ or $\rho > 1$, either $\beta_1 = 0$ and $\beta_2 \neq 0$ or $\beta_1 \neq 0$ and $\beta_2 = 0$.

RF scheme based on Modulo Lattice Modulation:

Case 1 : $\rho \leq 1$

Consider the proposed scheme with $P_{s,AF} = P_s^\rho - 1$, $P_{s,DF} = P_s - P_s^\rho + 1$, and assume that no DF message is superimposed on the AF message at the relay, i.e. $P_{r,AF} = P_r$ and $P_{r,DF} = 0$. Hence, the achievable rate of the CADF scheme can be simplified to

$$R_{CADF} = \rho C\left(\frac{M^2 P_r\left(P_s^\rho - 1\right)}{MP_r + P_s^\rho}\right) + \min\left\{\rho C\left(\frac{P_s - P_s^\rho + 1}{P_s^\rho}\right), (1-\rho) C(M^2 P_r)\right\} \qquad (3.40)$$

Now, let us define $SNR_{AF} \triangleq \frac{M^2 P_r\left(P_s^\rho - 1\right)}{MP_r + P_s^\rho}$ and $SNR_{KF} \triangleq \frac{M^2 P_r\left(P_s^\rho - 1\right)}{P_s^\rho + M^2 P_r}$. It is easy to show that

$$R_{CADF} \geq \rho C(SNR_{AF}) + (1-\rho) C(SNR_{KF}). \qquad (3.41)$$

To prove this, consider the fact that $SNR_{KF} \leq M^2 P_r$ and on the other hand, since $P_s > 1$ as in [48], we have $\left(\frac{P_s+1}{P_s}\right)^\rho \left(\frac{P_s^\rho + M^2 P_r}{1 + M^2 P_r}\right)^{1-\rho} \geq 1$ which results in $(1-\rho)\log\left(\frac{P_s^\rho\left(1+M^2 P_r\right)}{P_s^\rho + M^2 P_r}\right) \leq \rho \log\left(\frac{P_s+1}{P_s^\rho}\right)$ or equivalently $(1-\rho) C(SNR_{KF}) \leq \rho C\left(\frac{P_s - P_s^\rho + 1}{P_s^\rho}\right)$. Now, we can lower-bound the right-hand-side of (3.41) as follows

$$\begin{aligned}\rho C(SNR_{AF})+(1-\rho)C(SNR_{KF}) &= \rho\log(1+SNR_{AF})+(1-\rho)\log(1+SNR_{KF})\\ &= \log\left((1+SNR_{AF})^{\rho}(1+SNR_{KF})^{1-\rho}\right)\\ &\overset{(a)}{\geq} \log\left(1+SNR_{AF}^{\rho}SNR_{KF}^{1-\rho}\right)\\ &= R_{RF_{MLM}}. \end{aligned} \tag{3.42}$$

Here, (a) follows from applying Holder's inequality with $p=\frac{1}{\rho}$ and $q=\frac{1}{1-\rho}$ (See [51]). Comparing (3.41) and (3.42) completes the proof.

Case 2 : $\rho > 1$

For the sake of simplicity we assume that no DF message is superimposed on the AF message at the source, i.e. $P_{s,AF}=P_s$ and $P_{s,DF}=0$. Here two cases are considered:

i) $(\rho-1)C(P_s) > C(M^2P_r)$. In this case, we have $R_{CADF}=R_{DF}=C(M^2P_r)$ which is obviously greater than $R_{RF_{MLM}}$. In fact, R_{CADF} is also equal to the capacity of the channel.

ii) otherwise, we have

$$R_{CADF}=C\left(\frac{M^2\left(P_{r,AF}+P_{r,DF}\right)P_s}{MP_{r,AF}+P_s}\right), \tag{3.43}$$

where re-scaling the AF portion of the received signal at the relay with $\sqrt{\frac{P_{r,AF}}{P_s}}$, we have $P_{r,AF}+P_{r,DF}+\frac{P_{r,AF}}{P_s}=P_r$. Simplifying (3.43), we have

$$R_{CADF}=C\left(\frac{MP_s\left(1+MP_r\right)}{MP_{r,AF}+P_s}-M\right), \tag{3.44}$$

On the other hand, knowing

$$(\rho-1)C(P_s)=C\left(\frac{M^2P_{r,DF}}{M^2P_{r,AF}+\frac{MP_{r,AF}}{P_s}+1}\right), \tag{3.45}$$

we can derive $P_{r,AF}$ as

$$MP_{r,AF}=\frac{M^2P_sP_r-P_s^{\rho}}{MP_s^{\rho}+P_s^{\rho-1}+MP_s+M}. \tag{3.46}$$

From (3.46), one can easily verify that $MP_{r,AF}<\frac{MP_r}{P_s^{\rho-1}}$. Substituting $MP_{r,AF}$ with $\frac{MP_r}{P_s^{\rho-1}}$ in (3.44), we conclude that $R_{CADF}>R_{RF_{MLM}}$.

RF scheme based on Joint Source Channel Coding:

Case 1 : $\rho \leq 1$

Consider the proposed scheme with $P_{s,AF}=(1+P_s)^{\rho}-1$, $P_{s,DF}=P_s-(1+P_s)^{\rho}+1$, and assume that no DF message is superimposed on the AF message at the relay, i.e. $P_{r,AF} = P_r$ and $P_{r,DF} = 0$. Hence, the achievable rate of the CADF scheme can be simplified to

$$R_{CADF} = \rho C\left(\frac{M^2 P_r\left((1+P_s)^\rho - 1\right)}{MP_r + (1+P_s)^\rho}\right) + \min\left\{\rho C\left(\frac{P_s + 1 - (1+P_s)^\rho}{(1+P_s)^\rho}\right), (1-\rho)C(M^2 P_r)\right\} \tag{3.47}$$

Now, let us define $SNR_{AF} \triangleq \frac{M^2 P_r((1+P_s)^\rho - 1)}{MP_r + P_s^\rho}$ and $SNR_{KF} \triangleq \frac{M^2 P_r((1+P_s)^\rho - 1)}{(1+P_s)^\rho + M^2 P_r}$. It is easy to show that

$$R_{CADF} \geq R_{RF_{JSCC}} = \rho C(SNR_{AF}) + (1-\rho)C(SNR_{KF}). \tag{3.48}$$

To prove this, consider the fact that $SNR_{KF} \leq M^2 P_r$ and on the other hand, since $P_s > 1$ as in [48], we have $\left(\frac{(1+P_s)^\rho + M^2 P_r}{1 + M^2 P_r}\right)^{1-\rho} \geq 1$ which results in $(1-\rho)\log\left(\frac{(1+P_s)^\rho\left(1+M^2 P_r\right)}{(1+P_s)^\rho + M^2 P_r}\right) \leq \rho\log\left(\frac{P_s + 1}{(1+P_s)^\rho}\right)$ or equivalently $(1-\rho)C(SNR_{KF}) \leq \rho C\left(\frac{P_s - (1+P_s)^\rho + 1}{(1+P_s)^\rho}\right)$.

Case 2 : $\rho > 1$

For the sake of simplicity we assume that no DF message is superimposed on the AF message at the source, i.e. $P_{s,AF} = P_s$ and $P_{s,DF} = 0$. Here two cases are considered:

i) $(\rho - 1)C(P_s) > C(M^2 P_r)$. In this case, we have $R_{CADF} = R_{DF} = C(M^2 P_r)$ which is obviously greater than $R_{RF_{JSCC}}$. In fact, R_{CADF} is also equal to the capacity of the channel.

ii) otherwise, we have

$$R_{CADF} = C\left(\frac{M^2 P_r (P_s + 1)}{MP_{r,AF} + P_s + 1}\right), \tag{3.49}$$

where re-scaling the AF portion of the received signal at the relay with $\sqrt{\frac{P_{r,AF}}{P_s+1}}$, we have $P_{r,AF} + P_{r,DF} = P_r$. On the other hand, knowing

$$(\rho - 1)C(P_s) = C\left(\frac{M^2 P_{r,DF}}{M^2 P_{r,AF} + \frac{MP_{r,AF}}{P_s+1} + 1}\right), \tag{3.50}$$

we can derive $P_{r,AF}$ as

$$MP_{r,AF} = \frac{MP_r(P_s+\rho)(1+P_s)}{P_s\left((1+P_s)^\rho - 1\right) - \rho} \times \frac{\frac{1}{MP_r(P_s+\rho)} + \frac{M}{P_s+\rho} - \frac{(1+P_s)^{\rho-1}}{MP_r(P_s+\rho)}}{\frac{M(1+P_s)^\rho}{P_s((1+P_s)^\rho - 1) - \rho} + \frac{(1+P_s)^{\rho-1}}{P_s((1+P_s)^\rho - 1) - \rho} - \frac{1}{P_s((1+P_s)^\rho - 1) - \rho}}. \tag{3.51}$$

From (3.51), one can easily verify that $MP_{r,AF} < \frac{MP_r(P_s+\rho)(1+P_s)}{P_s((1+P_s)^\rho - 1) - \rho}$. Substituting $MP_{r,AF}$ with $\frac{MP_r(P_s+\rho)(1+P_s)}{P_s((1+P_s)^\rho - 1) - \rho}$ in (3.49), we conclude that $R_{CADF} > R_{RF_{JSCC}}$. $\square$

Remark 3.2.3. *As proved in Theorem 3.2.4, the achievable rate of the CADF scheme is always better than that of the RF scheme. Consequently, one can conclude that the achievable rate of the time sharing of the CADF scheme with the DF or AF schemes always leads to a better rate than the achievable rate of the time sharing of the RF scheme with the DF or AF schemes. This fact is justified in the simulation result section.*

3.3 Simulation Results

In this section, the achievable rates of the proposed CADF scheme with that of the traditional coding schemes and the upper bound are compared. We noticed in the simulation results that the RF scheme based on Modulo Lattice Modulation and Joint Source Channel Coding approaches leads to the same achievable rates. Hence, the curves associated with the rate of the RF scheme are indicated with R_{RF} in the sequel.

Fig. 3.3 compares the achievable rates of different schemes when $\rho = 0.5 < 1$. On the other hand, Fig. 3.4 compares the achievable rates of different schemes when $\rho = 2 > 1$. As we proved in the previous sections and, from these figures, as the number of relays increases, the CADF scheme always outperforms the RF scheme.

Figs. 3.5 compare the achievable rate of the CADF scheme with that of other schemes for the half-duplex scenarios. Assuming a constant bandwidth from the source to the destination, the optimum bandwidths for the first and second hops are obtained. Fig. 3.5 show that, as the number of relays increases, the CADF scheme outperforms the other schemes considerably.

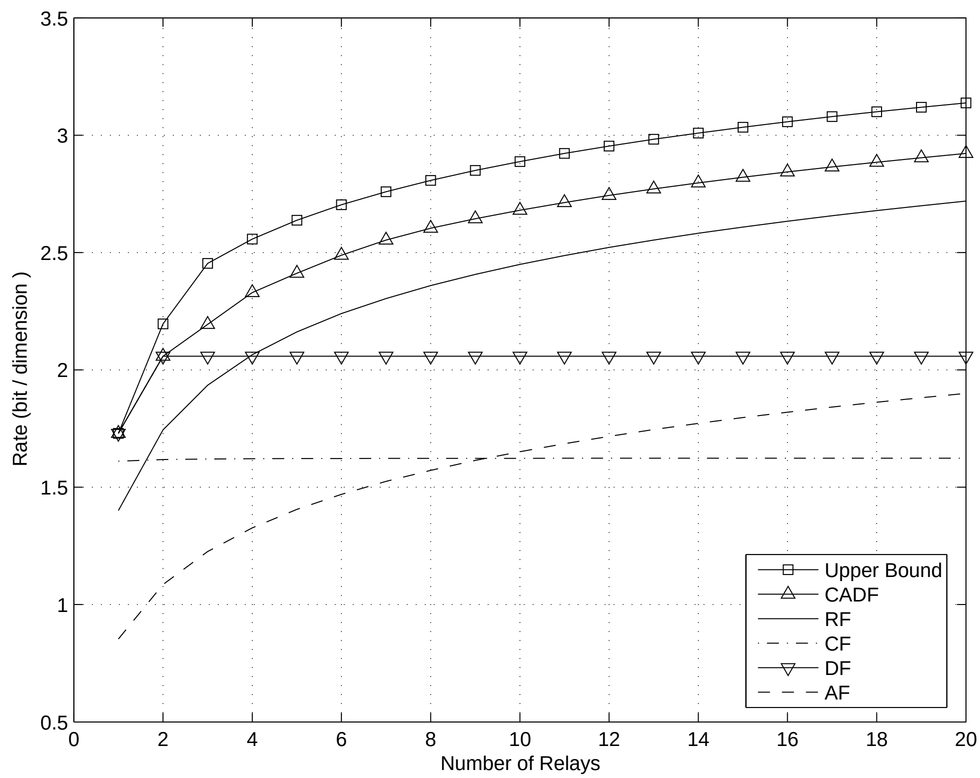


Figure 3.3: Rate versus number of relays ($\rho = 0.5$, $P_s = 300$, $MP_r = 10$).

Fig. 3.6 compares the achievable rate of the CADF-DF with that of the RF-DF in [48], and the AF-DF of [3] [4] in Schein's parallel relay setup (i.e. parallel relay with two relays

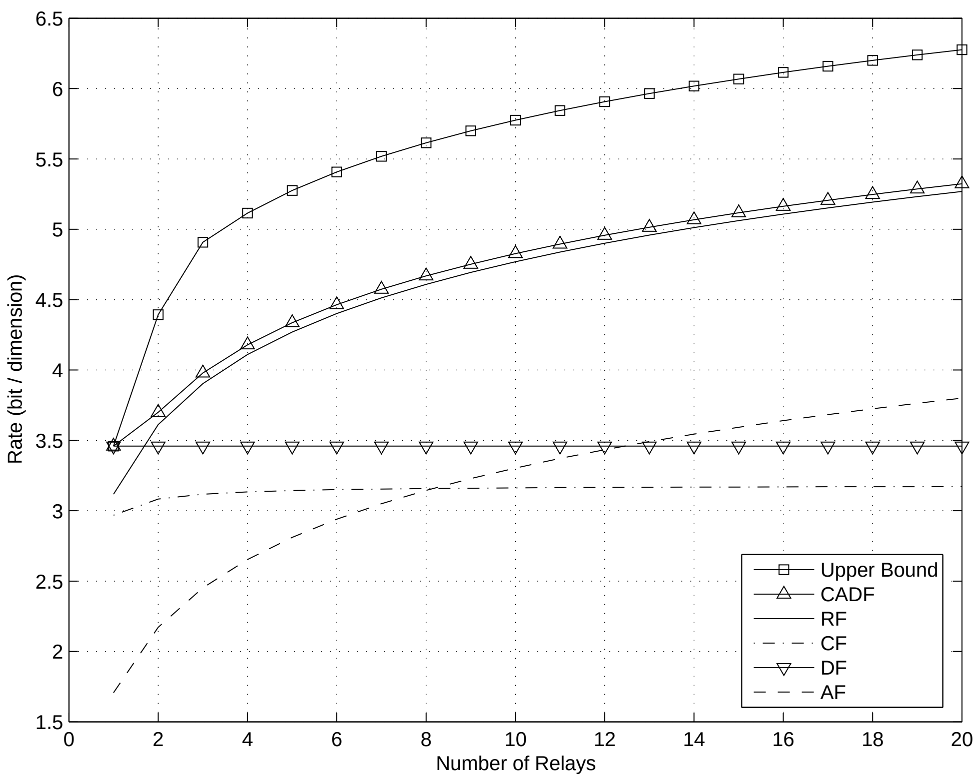


Figure 3.4: Rate versus number of relays ($\rho = 2$, $P_s = 10$, $MP_r = 300$).

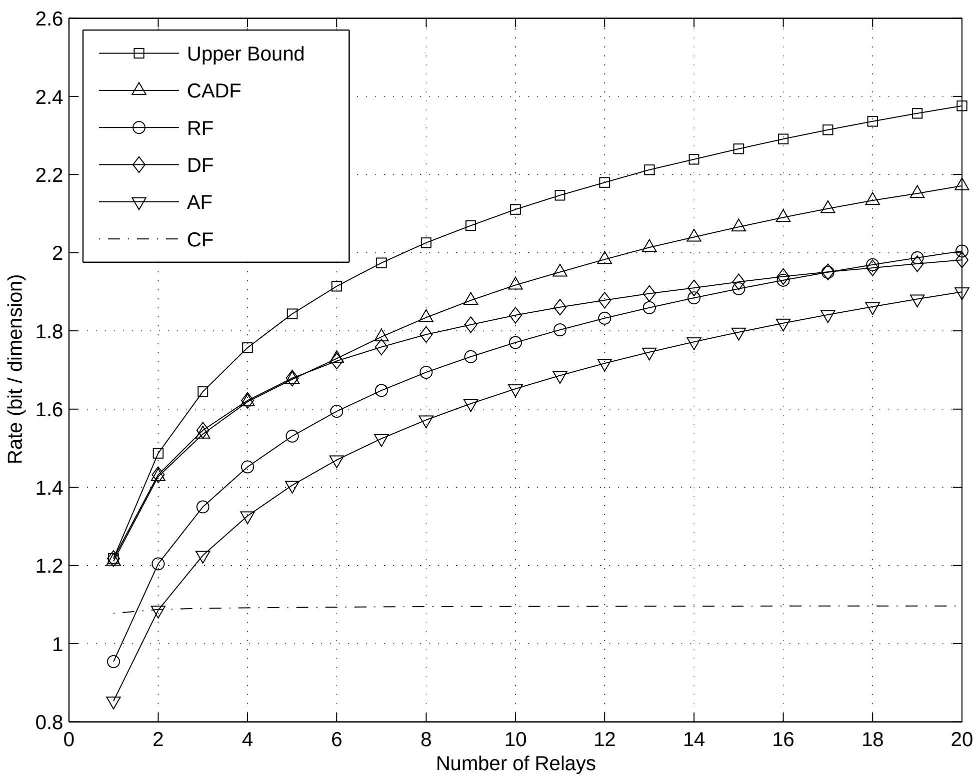


Figure 3.5: Rate versus number of relays for the half-duplex scenario ($P_s = 300$, $MP_r = 10$).

and no bandwidth mismatch). Here, we assume that $P_s = 20(dB)$. In this figure, we assume that the total dimensions from the source to the destination is "2". The assigned dimension to the BC channel is equal to the one assigned to the MAC channel. In the time sharing between the CADF and DF schemes, $t_1 + t_2$ dimensions are assigned to the CADF scheme (t_1 dimensions for the BC channel, and t_2 dimensions for the MAC channel) while $2 - t_1 - t_2$ is assigned to the DF scheme ($1 - t_1$ dimensions for the BC channel, and $1 - t_2$ dimensions for the MAC channel) with different peak powers. The same time sharing pattern is used for the time sharing between the RF and the DF schemes [48] [49].

As Fig. 3.6 shows, the CADF-DF considerably outperforms the RF-DF and AF-DF. It is worth noting that as the Schein's AF-DF can be considered as a special case of the CADF-DF, we can expect that the achievable rate of the CADF-DF is always better than the AF-DF. On the other hand, from the result of Theorem 3.2.4, the CADF-DF always outperforms the RF-DF in the Schein's setup.

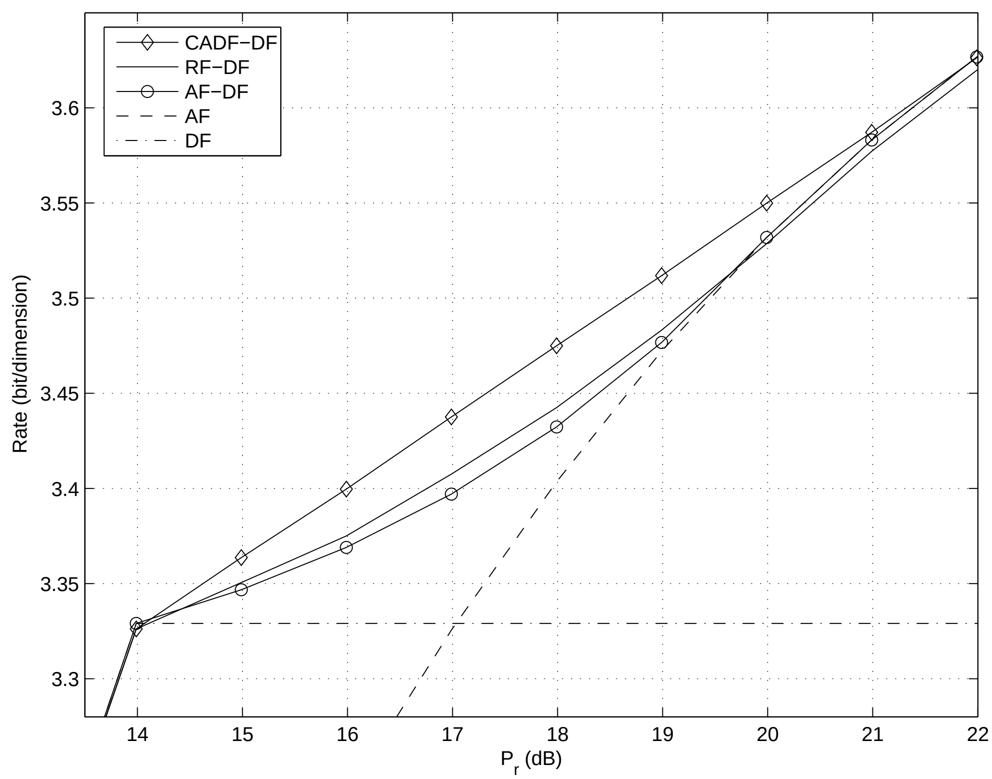


Figure 3.6: Achievable Rates by Time Sharing.

Chapter 4

Conclusion and Future Research Direction

4.1 Conclusion

In this dissertation, we investigated the problem of cooperative strategies for a half-duplex parallel relay channel with two relays. We derived the optimum relay scheduling and hence the asymptotic capacity of the half-duplex Gaussian parallel relay channel in low and high SNR scenarios.

Simultaneous and *Successive* relaying protocols, associated with two possible relay schedulings were proposed. For simultaneous relaying, each relay employs *BCM*. On the other hand, for successive relaying, we proposed a *Non-Cooperative Coding* scheme based on DPC and a *Cooperative Coding* scheme based on BME. Moreover, a coding scheme based on the combination of DPC and BME, in which at least one of the relays uses DPC while at most the other one employs BME was proposed. We showed that this composite scheme achieves at least the same rate as the cooperative coding based on BME with backward or successive decoding as well as the DPC scheme in the Gaussian case.

We also proposed the SSRD scheme as a combination of the simultaneous and successive protocols based on DPC. In high SNR scenarios, we proved that our *Non-Cooperative Coding* scheme based on *DPC* asymptotically achieves the capacity. Hence, in the high SNR scenario, the optimum relay scheduling is *Successive*. On the other hand, in low SNR where $(h_{13}\gamma_1 + h_{23}\gamma_2)^2 \leq \min(h_{01}^2, h_{02}^2)$, *BCM* achieves the capacity. Hence, in low SNR scenario and under the condition specified above for the channel coefficients, the optimum relay scheduling is *Simultaneous*.

Furthermore, we considered the problem of data transmission for the Gaussian parallel relay channel when there is a bandwidth mismatch between the BC channel and the MAC channel. A *Combined Amplify-and-Decode Forward* (CADF) scheme was proposed and it was proved that the CADF always outperforms the RF scheme presented in [48] [49]. It was also shown that the CADF scheme always outperforms other traditional coding schemes, i.e., AF, DF, and CF. For the case in which there exists no bandwidth mismatch between the BC and the MAC channels, using the time sharing between the CADF and DF schemes (CADF-DF) always outperforms the RF-DF in [48] [49], and the AF-DF in [3] [4].

4.2 Future Research Direction

Now, future research direction is explained. Two different directions associated with chapter 2 and 3 can be followed. Associated with chapter 2 and 3, one can consider half-duplex relay-interference network with two sources, two relays, and two destinations, and also proposing new coding schemes for the parallel relay channel.

4.2.1 Half-Duplex Relay-Interference Network

Here we consider a half-duplex relay-interference network which consists of two sources, two relays, and two destinations (See Figs. 4.1 and 4.2). Here similar to chapter 2, simultaneous and successive relaying protocols is proposed.

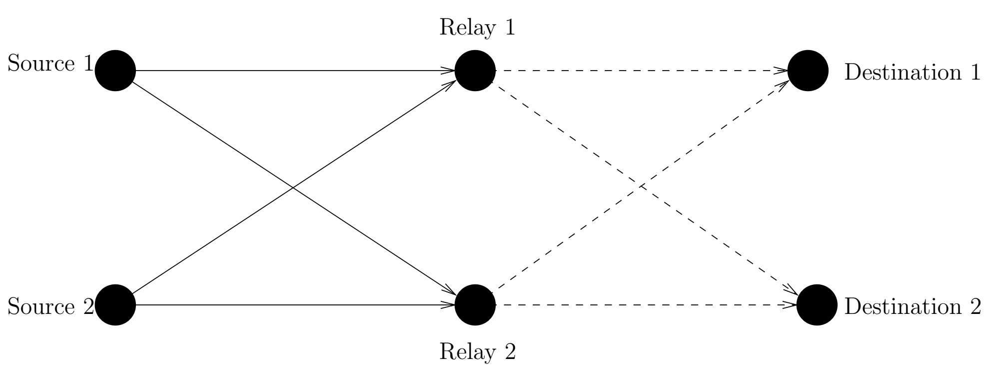


Figure 4.1: Simultaneous Relaying Protocol for Half-duplex Relay-Interference Network.

In simultaneous relaying, in one time slot two sources send their messages simultaneously to the relays. Having decoded their messages, the relays transmits their respective messages to the destinations. Obviously, this protocol removes the inter-relay interference. However, two X channels associated with time slot one and two is produced. Furthermore, this protocol is not spectrally efficient (See Fig. 4.1).

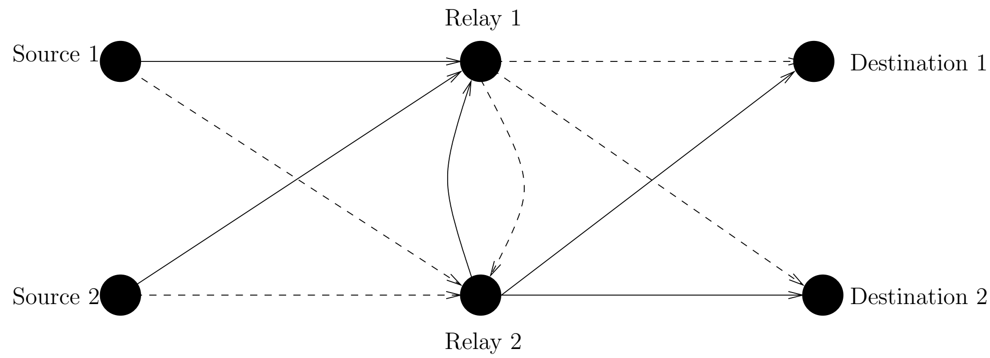


Figure 4.2: Successive Relaying Protocol for Half-duplex Relay-Interference Network.

On the other hand, in successive relaying, in the one time slot, two sources send their messages to the first relay, while the second relay transmits the messages it has previously received from the sources and the other relay to the destinations and the first relay. This protocol, unlike the simultaneous one, is spectrally efficient. We believe that some combination of Interference Alignment and Dirty Paper Coding can effectively remove the interference due to one relay on the other one. Therefore, it seems that successive using relaying protocol the optimum degrees of freedom of the network can be achieved (See Fig. 4.2). In order to explain the achievable scheme for the successive relaying protocol let us consider the following example.

Motivating Example

As an example we consider the system in 4.3. As indicated in the figure, nodes 1, 2, 3, and 4 equipped with two antennas while each of nodes 5 and 6 have 1 antenna. Node 1 tries to send data to node 5, and node 2 tries to send data to node 6.

Our proposed scheme is based on the combination of successive-relaying and alignment. Transmission is performed in two phases. In odd time slots, we have a MAC at node 3 and a BC at node 4. On the other hand, in even time slots, we have a MAC at node 4 and a BC at node 3.

The bottleneck in this system is the interference of node 4 on node 3 in odd time slots and the interference of node 3 on node 4 at even time slots. We manage this interference by using alignment. Let us assume that in odd time slots, node 1 and 2 use the direction vectors $\mathbf{u}_{31}$ and $\mathbf{u}_{32}$ to send their data to node 3. Moreover, node 4 uses the vectors $\mathbf{u}_{54}$ and $\mathbf{u}_{64}$ to send the corresponding data to nodes 5 and 6, respectively. We choose $\mathbf{u}_{54}$ and $\mathbf{u}_{64}$ such that (See Fig. 4.4):

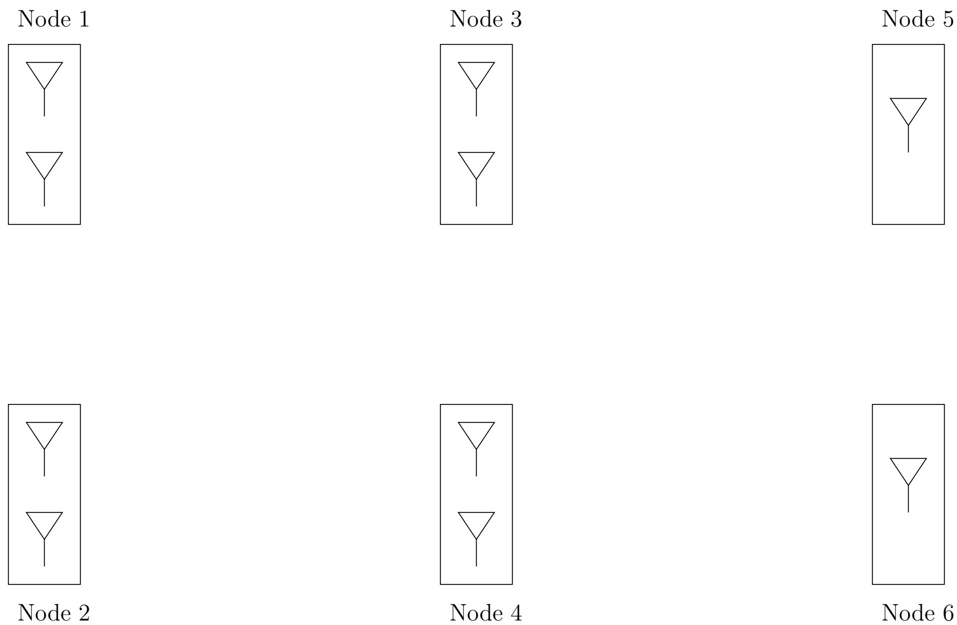


Figure 4.3: Relay-Interference Network.

$$\mathbf{u}_{54} \perp \mathbf{H}_{64},$$
$$\mathbf{u}_{64} \perp \mathbf{H}_{54}.$$

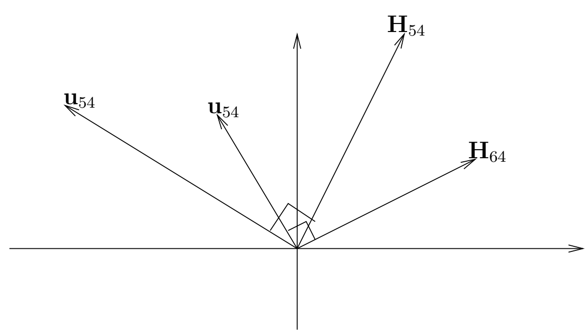


Figure 4.4: Interference Alignment.

Then we choose $\mathbf{u}_{31}$ and $\mathbf{u}_{32}$ such that:

$$\mathbf{H}_{31}\mathbf{u}_{31} || \mathbf{H}_{34}\mathbf{u}_{54},$$
$$\mathbf{H}_{32}\mathbf{u}_{32} || \mathbf{H}_{34}\mathbf{u}_{64},$$

Therefore, $\mathbf{u}_{31}$ and $\mathbf{u}_{32}$ are determined. We can use the similar approach for the even time slots and compute $\mathbf{u}_{41}$, $\mathbf{u}_{42}$, $\mathbf{u}_{63}$, and $\mathbf{u}_{53}$. Having received $\mathbf{y}_3^{(1)}$ at node node 3, it computes

$$r_{31} \triangleq < \mathbf{y}_3^{(1)}, \phi_{31} >, \; r_{32} \triangleq < \mathbf{y}_3^{(1)}, \phi_{32} >,$$

where ϕ_{31} is a unit vector orthogonal to $\mathbf{H}_{34}\mathbf{u}_{64}$ and $\mathbf{H}_{32}\mathbf{u}_{32}$, i.e., $\phi_{31} \perp \mathbf{H}_{34}\mathbf{u}_{64}, \; \mathbf{H}_{32}\mathbf{u}_{32}$. Similarly, $\phi_{32} \perp \mathbf{H}_{34}\mathbf{u}_{54}, \; \mathbf{H}_{31}\mathbf{u}_{31}$. We note that r_{31} has no interference from data of node 2. Similarly, r_{32} has not interference from data of node 1. Now, in the next even time slot, node 3 transmits

$$\gamma \left(r_{31}\mathbf{u}_{53} + r_{32}\mathbf{u}_{63} \right),$$

and node 5 receives

$$\mathbf{y}_5^{(2)} = \gamma < \mathbf{H}_{53}, \mathbf{u}_{53} > .r_{31},$$
$$\mathbf{y}_6^{(2)} = \gamma < \mathbf{H}_{63}, \mathbf{u}_{63} > .r_{32},$$

where γ is a constant.

It is worth mentioning that with this approach, we decompose the channel to two channels as in Fig. 4.5. These two channels have no interference on each other. Consequently, we can apply any non-cooperative and cooperative coding schemes for each of these channels separately.

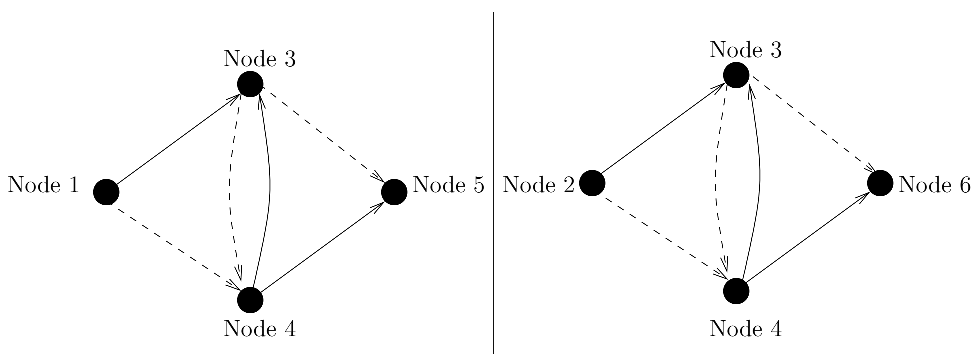


Figure 4.5: Channel Decomposition through Interference Alignment.

4.2.2 New Coding Schemes for Parallel Relay Channel

Proposing new coding schemes for Parallel Relay channel is another direction for future research.

Novel Compress and Forward Schemes

Salman Avestimehr, Suhas Diggavi and David Tse in [28] proposed a new compress and forward scheme for the Gaussian relay network. They proved that their scheme has a constant gap from the cut set bound. However, the gap from the cut set bound is a function of the network size. Furthermore, their scheme destroys the correlation in the codeword level which can be beneficial in some scenarios. As an example consider a parallel relay channel with two relays with ternary input alphabet. Assume that the noises at relays are reversely correlated. It can be readily verified that simple relay forwarding, benefitting from the correlation of the received signals, can achieve the capacity of this channel. This is an example that shows us the correlation in the codeword level which is completely destroyed in [28] can be even capacity achieving in some scenarios.

Bursty Schemes in the Discrete Memoryless Cases

It is shown in [3, 4] that bursty AF scheme achieves the cut set bound in very low SNR scenarios. Proposing corresponding schemes for very noisy discrete memoryless parallel relay channel would help to understand more comprehensively the nature of the relay problem.

Appendix A

Gelfand-Pinsker and Dirty Paper Coding

Theorem A. 1. *The capacity of a Discrete Memoryless Channel with i.i.d. state S with probability $p(s)$, where $\mathbf{s}$ is available to the encoder only, is given by*

$$C = max_{p(u,x|s)}(I(U;Y) - I(U:S)), \tag{4.1}$$

where $|\mathcal{U}| \leq \min\{|\mathcal{X}|, |\mathcal{Y}|\} + |\mathcal{S}| - 1$.

Proof. Codebook Generation: Fix $p(u|s)p(x|u,s)$. Randomly generate $2^{n(I(U;Y)-\delta(\epsilon))}$ i.i.d. $\mathbf{u} \in A_\epsilon^{(n)}(U)$ sequences according to a uniform distribution over $A_\epsilon^{(n)}(U)$ and partition them into 2^{nR} equal size bins (so sequences $\mathbf{u}(1), \cdots, \mathbf{u}\left(2^{n(I(U;Y)-R-\delta(\epsilon))}\right)$ are in Bin 1, . . .). We would like the U^ns in each bin to cover $A_\epsilon^{(n)}(U)$. This requires that $I(U;Y) - R > I(U;S)$, or $R < I(U;Y) - I(U;S)$.

Encoding: To send the message $w \in [1, 2^{nR}]$, the sender chooses any $\mathbf{u}(k)$ in bin w such that, $(\mathbf{u}(k), \mathbf{s}) \in A_\epsilon^{(n)}$. The sender then chooses an $\mathbf{x} \in A_\epsilon^{(n)}(X|\mathbf{u}(k), \mathbf{s})$ and sends it. If $\mathbf{s} \notin A_\epsilon^{(n)}(S)$ or no such $\mathbf{u}(k)$ exists, then w is assigned an arbitrary $\mathbf{x}$ sequence.

Decoding: Upon receiving $\mathbf{y}$, the decoder looks for a unique $\mathbf{u}(k)$ such that $(\mathbf{u}(k), \mathbf{y}) \in A_\epsilon^{(n)}$ and declares the index of the bin containing $\mathbf{u}(k)$ as $\hat{w}$, otherwise an error is declared.

Probability of Error: Without loss of generality, assume $w = 1$ and $k = 1$ and define the events

$$\begin{aligned}
E_0 &= \{\mathbf{s} \in A_\epsilon^{(n)}\}, \\
E_1 &= \{\exists \mathbf{u} : (\mathbf{u}, \mathbf{s}) \in A_\epsilon^{(n)}, \mathbf{u} \in Bin1\}, \\
E_{2k} &= \{(\mathbf{u}(k), \mathbf{y}) \in A_\epsilon^{(n)}\}.
\end{aligned}$$

Hence:

$$\begin{aligned}
P_e^{(n)} &= P\left(E_0^c \bigcup E_1^c \bigcup E_{21}^c \bigcup \left(\bigcup_{k\neq 1} E_{2k}\right)\right) \\
&\leq P\left(E_0^c\right) + P\left(E_1^c \bigcap E_0\right) + \sum_{k=2}^{2^{n(I(U;Y)-\delta(\epsilon))}} P\left(E_{2k}\right),
\end{aligned}$$

We now bound the probability of each event:

1. $P\left(E_0^c\right) \to 0$ as $n \to \inf$.

2. The probability of the second event

$$
\begin{aligned}
P\left(E_1^c \bigcap E_0\right) &= P\left\{(\mathbf{u},\mathbf{s}) \notin A_\epsilon^{(n)} \text{for all} \mathbf{u} \in \text{Bin } 1\right\} \\
&= \sum_{\mathbf{s}\in A_\epsilon^{(n)}} p(\mathbf{s}) P\left\{(\mathbf{u},\mathbf{s}) \notin A_\epsilon^{(n)} \text{for all } \mathbf{u} \in \text{Bin } 1\right\} \\
&= \sum_{\mathbf{s}\in A_\epsilon^{(n)}} p(\mathbf{s}) \left(1 - P\left\{(\mathbf{u},\mathbf{s}) \in A_\epsilon^{(n)}\right\}\right)^{2^{n(I(U;Y)-R-\delta(\epsilon))}} \\
&\leq e^{-2^{n(I(U;Y)-R-\delta(\epsilon))}2^{-n(I(U;S)+\acute{\delta}(\epsilon))}},
\end{aligned}
$$

which approaches 0 as $n \to \inf$, if

$$
R < I(U;Y) - I(U;S) - \acute{\delta}(\epsilon) - \delta(\epsilon),
$$

3. By construction $(\mathbf{u}(1), \mathbf{s}, \mathbf{x}) \in A_\epsilon^{(n)}$. Now, since $\mathbf{y}$ has probability $\prod_{i=1}^n p(y_i|x_i, s_i)$, then by the Markov lemma, for n sufficiently large

$$
P\left\{(\mathbf{u}(1), \mathbf{s}, \mathbf{x}, \mathbf{y}) \in A_\epsilon^{(n)}\right\},
$$

which implies that $(\mathbf{u}(1), \mathbf{y}) \in A_\epsilon^{(n)}$.

4. Note that for $k \neq 1$

$$
P\left\{(\mathbf{u}(k), \mathbf{y}) \in A_\epsilon^{(n)}\right\} \leq 2^{-n(I(U;Y)-3\epsilon)},
$$

Therefore,

$$
\sum_{k=2}^{2^{n(I(U;Y)-\delta(\epsilon))}} \leq 2^{n(I(U;Y)-\delta(\epsilon))}2^{-n(I(U;Y)-3\epsilon)},
$$

which by selecting $\delta(\epsilon) > 3\epsilon$, goes to 0 as $n \to \infty$.

Now, we prove the converse using the inequality $H\left(W|Y^n\right) \leq nRP_e^{(n)} + 1 = n\epsilon_n$ which is called the Fano inequality. Hence, consider

$$
\begin{aligned}
nR &\leq I\left(W;Y^n\right) + n\epsilon_n \\
&= \sum_{i=1}^{n} I\left(W;Y_i|Y^{i-1}\right) + n\epsilon_n \\
&\leq \sum_{i=1}^{n} I\left(W,Y^{i-1};Y_i\right) + n\epsilon_n \\
&= \sum_{i=1}^{n} I\left(W,Y^{i-1},S_{i+1}^n;Y_i\right) - \sum_{i=1}^{n} I\left(Y_i;S_{i+1}^n|W,Y^{i-1}\right) + n\epsilon_n \\
&\overset{(a)}{=} \sum_{i=1}^{n} I\left(W,Y^{i-1},S_{i+1}^n;Y_i\right) - \sum_{i=1}^{n} I\left(Y^{i-1};S_i|W,S_{i+1}^n\right) + n\epsilon_n \\
&\overset{(b)}{=} \sum_{i=1}^{n} I\left(W,Y^{i-1},S_{i+1}^n;Y_i\right) - \sum_{i=1}^{n} I\left(W,Y^{i-1},S_{i+1}^n;S_i\right) + n\epsilon_n
\end{aligned}
$$

where (a) follows from the Csiszar sum Lemma and (b) follows from the fact that $\left(W,S_{i+1}^n\right)$ is independent of S_i. Now, define $U_i = \left(W,Y^{i-1},S_{i+1}^n\right)$.Note that $\left(W,Y^{i-1},S_{i+1}^n\right) \to (X_i,S_i) \to Y_i$ form a Markov chain, thus

$$
\begin{aligned}
nR &\leq \sum_{i=1}^{n} \left(I\left(U_i;Y_i\right) - I\left(U_i;S_i\right)\right) + n\epsilon_n \\
&\leq n \max_{p(u,x|s)} \left(I\left(U;Y\right) - I\left(U;S\right)\right) + n\epsilon_n \qquad (4.2)
\end{aligned}
$$

which completes the proof of the converse. □

Dirty Paper Coding

Theorem A. 2. *Consider the AWGN channel with additive white Gaussian state, i.e., At time i, $Y_i = X_i + S_i + Z_i$, where the input X has average power constraint P, the states S_i are i.i.d.$\sim \mathcal{N}(0;Q)$ and the noise Z_i are i.i.d.$\sim \mathcal{N}(0;N)$, the noise and state are independent. Assume that only the encoder knows the state vector $\mathbf{s}$. The capacity is $C\left(\frac{P}{N}\right)$.*

Proof. We know the capacity expression, so we need to find the best distribution on U and X given S subject to the power constraint. Let us try $U = X + \alpha S$, where $X \sim \mathcal{N}(0,P)$ independent of S. With this choice, we have

$$
\begin{aligned}
I\left(U;Y\right) &= h\left(X+S+Z\right) - h\left(X+S+Z|X+\alpha S\right) \\
&= h\left(X+S+Z\right) + h\left(X+\alpha S\right) - h\left(X+S+Z, X+\alpha S\right) \\
&= \frac{1}{2}\log\left(\frac{(P+Q+N)(P+\alpha^2 Q)}{PQ(1-\alpha)^2 + N(P+\alpha^2 Q)}\right),
\end{aligned}
$$

and $I(U;S) = \frac{1}{2}\log\left(\frac{P+\alpha^2 Q}{P}\right)$. Thus,

$$\begin{aligned} R(\alpha) &= I(U;Y) - I(U;S) \\ &= \frac{1}{2}\log\left(\frac{P(P+Q+N)}{PQ(1-\alpha)^2 + N(P+\alpha^2 Q)}\right) \end{aligned}$$

Maximizing with respect to α, we find that $\alpha^* = \frac{P}{P+N}$. Substituting we obtain

$$R\left(\frac{P}{P+N}\right) = C\left(\frac{P}{N}\right). \tag{4.3}$$

$\square$

Appendix B

Block Markov Encoding

Since Block Markov Encoding (BME) is used extensively in this chapter, here we explain this type of encoding by deriving the achievable rate of the DF scheme for the single relay channel.

A single relay channel consists of four finite sets $\mathcal{X}$, $\mathcal{X}_1$, $\mathcal{Y}$, and $\mathcal{Y}_1$ and a collection of probability mass functions $p(y, y_1|x, x_1)$ on $\mathcal{Y} \times \mathcal{Y}_1$, one for each $(X, X_1) \in \mathcal{X} \times \mathcal{X}_1$. The interpretation is that x is the input to the channel and y is the output of the channel, y_1 is the relay's observation, and x_1 is the input symbol chosen by the relay, as shown in Fig. 4.6. The problem is to find the capacity of the channel between the source X and the destination Y. The relay channel combines a broadcast channel (X to Y and Y_1) and a multiple-access channel (X and X_1 to Y).

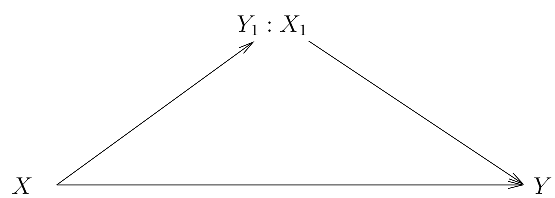


Figure 4.6: Relay Channel.

Theorem B. 1. *The following rate R is achievable for the relay channel:*

$$R = \sup_{p(x,x_1)} \min\{I\left(X, X_1; Y\right), I\left(X; Y_1|X_1\right)\},$$

where the supremum is over all joint distributions on $\mathcal{X} \times \mathcal{X}_1$.

Proof. Achievability: We use the following simple block Markov coding scheme. Consider B blocks of transmission, each of n symbols. A sequence of $B-1$ messages, $w_i \in \mathcal{W}, i = 1, 2, \cdots, B-1$, each selected independently and uniformly over $\{1, 2, \cdots, 2^{nR}\}$ is to be sent over the channel in nB transmissions, so the average rate will be $R(B-1)/B$ (Note that as $B \to$ inf for a fixed n, the rate $R(B-1)/B$ is arbitrarily close to R.) We define a doubly-indexed set of codewords:

$\mathcal{C} = \{\mathbf{x}(w|s), \mathbf{x}_1(s)\} : w \in \{1, \cdots, 2^{nR}\}, s \in \{1, \cdots, 2^{nR_0}\}, \mathbf{x} \in \mathcal{X}^n, \mathbf{x}_1 \in \mathcal{X}_1^n.$

We will also need a partition $\mathcal{S} = \{S_1, S_2, \cdots, S_{2^{nR_0}}\}$ of $\mathcal{W} = \{1, 2, \cdots, 2^{nR}\}$ into 2^{nR_0} cells, with $S_i \bigcap S_j = \emptyset$, $i \neq j$, and $\bigcup S_i = \mathcal{W}$. The partition will enable us to send side information to the receiver in the manner of Slepian and Wolf ([14]).

Generation of random code: Fix $p(x_1)p(x|x_1)$.

First generate at random 2^{nR_0} i.i.d n-sequences in $\mathcal{X}_1^n$, each drawn according to $p(\mathbf{x}_1) = \prod_{i=1}^n p(x_{1i})$. Index them as $\mathbf{x}_1(s),\ s \in \{1, 2, \cdots, 2^{nR_0}\}$. For each $\mathbf{x}_1(s)$, generate 2^{nR} conditionally independent n-sequences $\mathbf{x}(w|s),\ w \in \{1, 2, \cdots, 2^{nR}\}$, drawn independently according to $p(\mathbf{x}|\mathbf{x}_1(s)) = \prod_{i=1}^n p(x_i|x_{1i}(s))$. This defines the random codebook $\mathcal{C} = \{\mathbf{x}(w|s), \mathbf{x}_1(s)\}$. The random partition $\mathcal{S} = \{S_1, S_2, \cdots, S_{2^{nR_0}}\}$ of $\{1, 2, \cdots, 2^{nR}\}$ is defined as follows. Let each integer $w \in \{1, 2, \cdots, 2^{nR}\}$ be assigned independently , according to a uniform distribution over the indices $s = 1, 2, \cdots,\ 2^{nR_0}$, to cells S_s.

Encoding: Let $w_i \in \{1,\ 2, \cdots,\ 2^{nR}\}$ be the new index to be sent in block i, and let s_i be defined as the partition corresponding to w_{i-1} (i.e., $w_{i-1} \in S_{s_i}$). The encoder sends $\mathbf{x}(w_i|s_i)$. The relay has an estimate $\hat{\hat{w}}_{i-1}$ of the previous index w_{i-1}. (This will be made precise in the decoding section.) Assume that $\hat{\hat{w}}_{i-1} \in S_{\hat{\hat{s}}_i}$. The relay encoder sends $\mathbf{x}_1(\hat{\hat{s}}_i)$ in block i.

Decoding: We assume that at the end of block $i-1$, the receiver knows $(w_1, w_2, \cdots, w_{i-2})$ and $(s_1, s_2, \cdots, s_{i-1})$ and the relay knows $(w_1, w_2, \cdots, w_{i-1})$ and consequently, $(s_1, s_2, \cdots, s_i)$. The decoding procedures at the end of block i are as follows:

1. Knowing s_i and upon receiving $\mathbf{y}_1(i)$, the relay receiver estimates the message of the transmitter $\hat{\hat{w}}_i = w$ if and only if there exists a unique w such that $(\mathbf{x}(w|s_i), \mathbf{x}_1(s_i), \mathbf{y}_1(i))$ are jointly typical. It can be shown that $\hat{\hat{w}}_i = w_i$ with an arbitrarily small probability of error if

$$R < I(X; Y_1|X_1) \tag{4.4}$$

and n is sufficiently large.

2. The receiver declares that $\hat{s}_i = s$ was sent iff there exists one and only one s such that $(\mathbf{x}_1(s), \mathbf{y}(i))$ are jointly typical. s_i can be decoded with arbitrarily small probability of error if

$$R_0 < I(X_1; Y) \tag{4.5}$$

and n is sufficiently large.

3. Assuming that s_i is decoded correctly at the receiver, the receiver constructs a list $L(\mathbf{y}(i-1))$ of indices that the receiver considers to be jointly typical with $\mathbf{y}(i-1)$ in the $(i-1)$th block. The receiver then declares $\hat{w}_{i-1} = w$ as the index sent in block $i-1$ if there is a unique w in $S_{s_i} \bigcap L(\mathbf{y}(i-1))$. If n is sufficiently large and if

$$R < I(X; Y|X_1) + R_0, \tag{4.6}$$

then $\hat{w}_{i-1} = w_{i-1}$ with arbitrarily small probability of error. Combining the two constraints (4.16) and (4.6), R_0 drops out, leaving

$$R < I(X;Y|X_1) + I(X_1;Y) = I(X,X_1;Y). \tag{4.7}$$

$\square$

Appendix C

Proof of Theorem 2.2.1

<u>*Codebook Construction:*</u>

Let us divide blocks $1, 2, \cdots, B+1$ into odd and even numbers. At odd and even blocks, source generates $2^{nr^{(1)}_{AUX}}$ and $2^{nr^{(2)}_{AUX}}$ sequences $\mathbf{u}_0^{(1)}(q_1)$ and $\mathbf{u}_0^{(2)}(q_2)$ according to $\prod_{i=1}^{t_1 n} p(u_{0,i}^{(1)})$ and $\prod_{i=1}^{t_2 n} p(u_{0,i}^{(2)})$, respectively. Then, source throws $\mathbf{u}_0^{(1)}$ and $\mathbf{u}_0^{(2)}$ sequences uniformly into $2^{nR^{(1)}}$ and $2^{nR^{(2)}}$ bins, respectively. Let us denote $\mathcal{B}_1(w^{(b)})$ and $\mathcal{B}_2(w^{(b)})$ as the set of sequences at the odd or even block that belong to the $w^{(b)}$'th bin, respectively (for odd blocks, $w^{(b)} \leq 2^{nR^{(1)}}$, and for the even blocks, $w^{(b)} \leq 2^{nR^{(2)}}$).

Relay 1 and relay 2 generate $2^{nR^{(1)}}$ and $2^{nR^{(2)}}$ i.i.d $\mathbf{x}_1^{(2)}$ and $\mathbf{x}_2^{(1)}$ sequences according to probabilities $\prod_{i=1}^{t_2 n} p\left(x_{1,i}^{(2)}\right)$ and $\prod_{i=1}^{t_1 n} p\left(x_{2,i}^{(1)}\right)$. Furthermore, for all q_1 and q_2, the source generates double indexed codebooks $\mathbf{x}_0^{(1)}\left(w^{(b)}|w^{(b-1)}, q_1\right)$ and $\mathbf{x}_0^{(2)}\left(w^{(b)}|w^{(b-1)}, q_2\right)$ according to $\prod_{i=1}^{t_1 n} p(x_{0,i}^{(1)} \mid x_{2,i}^{(1)}, u_{0,i}^{(1)})$ and $\prod_{i=1}^{t_2 n} p(x_{0,i}^{(2)} \mid x_{1,i}^{(2)}, u_{0,i}^{(2)})$, respectively.

<u>*Encoding:*</u>

Encoding at the source:

At the odd block b, the source intends to send the message $w^{(b)}$ to the first relay. In order to do that, since the source knows what it has transmitted during the previous block to the second relay, it chooses a codeword $\mathbf{u}_0^{(1)}(q_1)$ such that $\mathbf{u}_0^{(1)}(q_1) \in \mathcal{B}_1(w^{(b)})$ and $\left(\mathbf{u}_0^{(1)}(q_1), \mathbf{x}_2^{(1)}\left(w^{(b-1)}\right)\right) \in A_\epsilon^{(n)}$. and sends $\mathbf{x}_0^{(1)}(\mathbf{u}_0^{(1)}, \mathbf{x}_2^{(1)})$.

At the even block b, the source sends the message $w^{(b)}$ to the second relay in the similar manner.

Encoding at relay 1:

At the even block b, relay 1 encodes $w^{(b-1)} \in \{1, \cdots, 2^{nR^{(1)}}\}$ to $\mathbf{x}_1^{(2)}\left(w^{(b-1)}\right)$.

Encoding at relay 2:

At the odd block b, relay 2 encodes $w^{(b-1)} \in \{1, \cdots, 2^{nR^{(2)}}\}$ to $\mathbf{x}_2^{(1)}\left(w^{(b-1)}\right)$.

<u>*Decoding:*</u>

Decoding at relay 1 and 2:

Having received the odd block b, relay 1 declares $\hat{w}^{(b)} = w^{(b)}$ iff all the sequences $\mathbf{u}_0^{(1)}(q_1)$ which are jointly typical with $\mathbf{y}_1^{(1)}$ belong to a unique bin $\mathcal{B}_1(\hat{w}^{(b)})$. Similarly having received the even block b, relay 2 declares $\hat{w}^{(b)} = w^{(b)}$ iff all the sequences $\mathbf{u}_0^{(2)}(q_2)$ which are jointly typical with $\mathbf{y}_2^{(2)}$ belong to a unique bin $\mathcal{B}_2(\hat{w}^{(b)})$. Therefore, according to the Gelfand-

Pinsker result the following rates for the channels from the source to relay 1 and relay 2 are achievable:

$$R^{(1)} \leq t_1 \left(I(U_0^{(1)}; Y_1^{(1)}) - I(U_0^{(1)}; X_2^{(1)}) \right), \tag{4.8}$$

$$R^{(2)} \leq t_2 \left(I(U_0^{(2)}; Y_2^{(2)}) - I(U_0^{(2)}; X_1^{(2)}) \right). \tag{4.9}$$

Decoding at the destination:

Having received the odd block b, the destination declares $\hat{w}^{(b-1)} = w^{(b-1)}$ iff $\left(\mathbf{x}_2^{(1)} \left(\hat{w}^{(b-1)} \right), \mathbf{y}_3^{(1)} \right) \in A_\epsilon^{(n)}$. Hence, in order to make the probability of error zero, from [14], we have

$$R^{(1)} \leq t_1 I(X_2^{(1)}; Y_3^{(1)}). \tag{4.10}$$

Similarly, for the even block b, we have

$$R^{(2)} \leq t_2 I(X_1^{(2)}; Y_3^{(2)}). \tag{4.11}$$

From (4.8)-(4.11), we obtain (2.9)-(2.11).

Appendix D

Proof of corollary 2.2.1

From Costa's Dirty Paper Coding [33], by having

$$U_0^{(1)} = X_0^{(1)} + \frac{h_{01}h_{12}P_0^{(1)}}{h_{01}^2 P_0^{(1)} + t_1} X_2^{(1)}, \tag{4.12}$$

$$U_0^{(2)} = X_0^{(2)} + \frac{h_{02}h_{12}P_0^{(2)}}{h_{02}^2 P_0^{(2)} + t_2} X_1^{(2)}. \tag{4.13}$$

where $X_0^{(1)} \sim \mathcal{N}(0, P_0^{(1)}), X_0^{(2)} \sim \mathcal{N}(0, P_0^{(2)}), X_2^{(1)} \sim \mathcal{N}(0, P_2),$ and $X_1^{(2)} \sim \mathcal{N}(0, P_1),$ and applying them to Theorem 2.2.1, we obtain corollary 2.2.1.

Appendix E

Proof of Theorem 2.2.2

Codebook Construction:

Let us divide the blocks $1, 2, \cdots, B+2$ into odd and even numbers. The source generates two codebooks $\mathbf{x}_0^{(1)}\left(w^{(b)}|w^{(b-1)}, s_1^{(b-2)}\right)$ and $\mathbf{x}_0^{(2)}\left(w^{(b)}|w^{(b-1)}, s_2^{(b-2)}\right)$ of size $2^{nR^{(1)}}$ and $2^{nR^{(2)}}$ corresponding to even and odd blocks, respectively. The first codebook is generated according to the probability $p(\mathbf{x}_0^{(1)}, \mathbf{x}_2^{(1)}, \mathbf{u}_2^{(1)}) =$
$\prod_{i=1}^{t_1 n} p(u_{2,i}^{(1)})p(x_{2,i}^{(1)}|u_{2,i}^{(1)})p(x_{0,i}^{(1)}|x_{2,i}^{(1)}, u_{2,i}^{(1)})$, and the second codebook is generated according to the probability $p(\mathbf{x}_0^{(2)}, \mathbf{x}_1^{(2)}, \mathbf{u}_1^{(2)}) = \prod_{i=1}^{t_2 n} p(u_{1,i}^{(2)})p(x_{1,i}^{(2)}|u_{1,i}^{(2)})p(x_{0,i}^{(2)}|x_{1,i}^{(2)}, u_{1,i}^{(2)})$.

On the other hand, relay 2 generates $2^{nr_{Bin}^{(1)}}$ i.i.d codewords $\mathbf{u}_2^{(1)}$ and $2^{nR^{(2)}}$ i.i.d codewords $\mathbf{x}_2^{(1)}$ according to the probabilities $p(\mathbf{u}_2^{(1)}) = \prod_{i=1}^{t_1 n} p(u_{2,i}^{(1)})$ and $p(\mathbf{x}_2^{(1)} \mid \mathbf{u}_2^{(1)}) = \prod_{i=1}^{t_1 n} p(x_{2,i}^{(1)} \mid u_{2,i}^{(1)})$ at each odd block and relay 1 generates $2^{nr_{Bin}^{(2)}}$ i.i.d codewords $\mathbf{u}_1^{(2)}$ and $2^{nR^{(1)}}$ i.i.d codewords $\mathbf{x}_1^{(2)}$ according to the probabilities $p(\mathbf{u}_1^{(2)}) = \prod_{i=1}^{t_2 n} p(u_{1,i}^{(2)})$ and $p(\mathbf{x}_1^{(2)} \mid \mathbf{u}_1^{(2)}) = \prod_{i=1}^{t_2 n} p(x_{1,i}^{(2)} \mid u_{1,i}^{(2)})$ at each even block, respectively.

Encoding:

Encoding at the source:

At the odd block b, the source encodes $w^{(b)} \in \{1, \cdots, 2^{nR^{(1)}}\}$ to $\mathbf{x}_0^{(1)}\left(w^{(b)}|w^{(b-1)}, s_1^{(b-2)}\right)$ and at the even block b, it encodes $w^{(b)} \in \{1, \cdots, 2^{nR^{(2)}}\}$ to $\mathbf{x}_0^{(2)}\left(w^{(b)}|w^{(b-1)}, s_2^{(b-2)}\right)$ and sends them in odd and even blocks, respectively.

Encoding at relay 1:

At the even block b, relay 1 encodes the bin index $s_2^{(b-2)}$ of the message $w^{(b-2)}$ it has received from relay 2 in the previous block to $\mathbf{u}_1^{(2)}\left(s_2^{(b-2)}\right)$. Following that, it encodes $w^{(b-1)}$ which was received from the source in block $b-1$ to $\mathbf{x}_1^{(2)}\left(w^{(b-1)}|s_2^{(b-2)}\right)$ and sends it.

Encoding at relay 2:

At the odd block b, relay 2 encodes the bin index $s_1^{(b-2)}$ of the message $w^{(b-2)}$ it has received from relay 1 in the previous block to $\mathbf{u}_2^{(1)}\left(s_1^{(b-2)}\right)$. Following that, it encodes $w^{(b-1)}$ which was received from the source in block $b-1$ to $\mathbf{x}_2^{(1)}\left(w^{(b-1)}|s_1^{(b-2)}\right)$ and sends it.

Decoding:

Decoding at relay 1:

Knowing $w^{(b-2)}$ and consequently $s_1^{(b-2)}$, having received block b, relay 1 declares $(\hat{w}^{(b-1)}, \hat{w}^{(b)}) =$

$(w^{(b-1)}, w^{(b)})$ iff there exits a unique $(\hat{w}^{(b-1)}, \hat{w}^{(b)})$ such that

$$\left(\mathbf{x}_0^{(1)}\left(\hat{w}^{(b)}|\hat{w}^{(b-1)}, s_1^{(b-2)}\right), \mathbf{x}_2^{(1)}\left(\hat{w}^{(b-1)}|s_1^{(b-2)}\right), \mathbf{u}_2^{(1)}(s_1^{(b-2)}), \mathbf{y}_1^{(1)}\right) \in A_\epsilon^{(n)}.$$

Hence, in order to make probability of error zero, from the Extended MAC capacity region (See [14], [29], [30], and [31]), we have

$$R^{(1)} \leq t_1 I(X_0^{(1)}; Y_1^{(1)} \mid X_2^{(1)}, U_2^{(1)}), \tag{4.14}$$

$$R^{(1)} + R^{(2)} \leq t_1 I(X_0^{(1)}, X_2^{(1)}; Y_1^{(1)} \mid U_2^{(1)}). \tag{4.15}$$

Decoding at relay 2:

Knowing $w^{(b-2)}$ and consequently $s_2^{(b-2)}$, having received block b, relay 2 declares $(\hat{w}^{(b-1)}, \hat{w}^{(b)}) = (w^{(b-1)}, w^{(b)})$ iff there exits a unique $(\hat{w}^{(b-1)}, \hat{w}^{(b)})$ such that

$$\left(\mathbf{x}_0^{(2)}\left(\hat{w}^{(b)}|\hat{w}^{(b-1)}, s_2^{(b-2)}\right), \mathbf{x}_1^{(2)}\left(\hat{w}^{(b-1)}|s_2^{(b-2)}\right), \mathbf{u}_1^{(2)}(s_2^{(b-2)}), \mathbf{y}_2^{(2)}\right) \in A_\epsilon^{(n)}.$$

Hence, in order to make the probability of error zero, from the Extended MAC capacity region (See [14], [29], [30], and [31]), we have

$$R^{(2)} \leq t_2 I(X_0^{(2)}; Y_2^{(2)} \mid X_1^{(2)}, U_1^{(2)}), \tag{4.16}$$

$$R^{(1)} + R^{(2)} \leq t_2 I(X_0^{(2)}, X_1^{(2)}; Y_2^{(2)} \mid U_1^{(2)}). \tag{4.17}$$

Decoding at the destination:

Decoding at the destination can be done either *Successively* or *Backwardly* as follows.

1) Successive Decoding:

Having received the odd block b, the destination first declares the bin index $\hat{s}_1^{(b-2)} = s_1^{(b-2)}$ of the message $w^{(b-2)}$ iff there exists a unique $\hat{s}_1^{(b-2)}$ such that $\left(\mathbf{u}_2^{(1)}(\hat{s}_1^{(b-2)}), \mathbf{y}_3^{(1)}\right) \in A_\epsilon^{(n)}$. Hence, in order to make the probability of error zero, from [14] we have

$$r_{Bin}^{(1)} \leq t_1 I(U_2^{(1)}; Y_3^{(1)}). \tag{4.18}$$

Having decoded the bin index $s_1^{(b-2)}$ of the message $w^{(b-2)}$, the destination can resolve its uncertainty about the message $w^{(b-2)}$ and declares $\hat{w}^{(b-2)} = w^{(b-2)}$ iff there exists a unique $\hat{w}^{(b-2)}$ such that
$\left(\mathbf{x}_1^{(2)}(\hat{w}^{(b-2)}|s_2^{(b-3)}), \mathbf{u}_1^{(2)}(s_2^{(b-3)}), \mathbf{y}_3^{(2)}\right) \in A_\epsilon^{(n)}$. Hence, in order to make the probability of error zero, from [14] we have

$$R^{(1)} - r_{Bin}^{(1)} \leq t_2 I(X_1^{(2)}; Y_3^{(2)} \mid U_1^{(2)}). \tag{4.19}$$

Using the same argument for the even block b, we have

$$r_{Bin}^{(2)} \le t_2 I(U_1^{(2)}; Y_3^{(2)}), \tag{4.20}$$
$$R^{(2)} - r_{Bin}^{(2)} \le t_1 I(X_2^{(1)}; Y_3^{(1)} \mid U_2^{(1)}). \tag{4.21}$$

From (4.18)-(4.21), $R^{(1)}$ and $R^{(2)}$ are bounded as follows

$$R^{(1)} \le t_2 I\left(X_1^{(2)}; Y_3^{(2)} \mid U_1^{(2)}\right) + t_1 I\left(U_2^{(1)}; Y_3^{(1)}\right), \tag{4.22}$$
$$R^{(2)} \le t_1 I(X_2^{(1)}; Y_3^{(1)} \mid U_2^{(1)}) + t_2 I(U_1^{(2)}; Y_3^{(2)}). \tag{4.23}$$

From (4.14)-(4.17), (4.22), and (4.23), the achievable rate of BME scheme based on successive decoding is equal to

$$\begin{aligned}
&R_{BME_{succ}} = \max_{0\le t_1,t_2,t_1+t_2=1} R^{(1)} + R^{(2)} \le \max_{0\le t_1,t_2,t_1+t_2=1} \min (\qquad (4.24)\\
&\min\left(t_1 I\left(X_0^{(1)}; Y_1^{(1)} \mid X_2^{(1)}, U_2^{(1)}\right), t_2 I\left(X_1^{(2)}; Y_3^{(2)} \mid U_1^{(2)}\right) + t_1 I\left(U_2^{(1)}; Y_3^{(1)}\right)\right) + \\
&\min\left(t_1 I\left(X_2^{(1)}; Y_3^{(1)} \mid U_2^{(1)}\right) + t_2 I\left(U_1^{(2)}; Y_3^{(2)}\right), t_2 I\left(X_0^{(2)}; Y_2^{(2)} \mid X_1^{(2)}, U_1^{(2)}\right)\right), \\
&t_1 I\left(X_0^{(1)}, X_2^{(1)}; Y_1^{(1)} \mid U_2^{(1)}\right), t_2 I\left(X_0^{(2)}, X_1^{(2)}; Y_2^{(2)} \mid U_1^{(2)}\right)\Big).
\end{aligned}$$

2) Backward Decoding:

Following receiving the sequence corresponding to the $B+2$th block, the destination starts decoding the messages in a backward manner, i.e. from $w^{(B)}$ back to $w^{(1)}$. At the end of odd block b, knowing the value $s_2^{(b-1)}$ from the received signal in block $b+1$, the destination declares $\left(\hat{w}^{(b-1)}, \hat{s}_1^{(b-2)}\right) = \left(w^{(b-1)}, s_1^{(b-2)}\right)$ iff there exists a unique pair $\left(\hat{w}^{(b-1)}, \hat{s}_1^{(b-2)}\right)$ such that $f_{Bin}^{(2)}\left(\hat{w}^{(b-1)}\right) = s_2^{(b-1)}$ and $\left(\mathbf{x}_2^{(1)}\left(\hat{w}^{(b-1)}, \hat{s}_1^{(b-2)}\right), \mathbf{u}_2^{(1)}\left(\hat{s}_1^{(b-2)}\right), \mathbf{y}_3^{(1)}\right) \in A_\epsilon^{(n)}$. Similarly, at the end of even block b, knowing the value $s_1^{(b-1)}$ from the received signal in block $b+1$, the destination declares $\left(\hat{w}^{(b-1)}, \hat{s}_2^{(b-2)}\right) = \left(w^{(b-1)}, s_2^{(b-2)}\right)$ iff there exists a unique pair $\left(\hat{w}^{(b-1)}, \hat{s}_2^{(b-2)}\right)$ such that $f_{Bin}^{(1)}\left(\hat{w}^{(b-1)}\right) = s_1^{(b-1)}$ and $\left(\mathbf{x}_1^{(2)}\left(\hat{w}^{(b-1)}, \hat{s}_1^{(b-2)}\right), \mathbf{u}_1^{(2)}\left(\hat{s}_2^{(b-2)}\right), \mathbf{y}_3^{(2)}\right) \in A_\epsilon^{(n)}$. Hence, in order to make the probability of error zero, from [14] we have

$$r_{Bin}^{(1)} \le R^{(1)}, \tag{4.25}$$
$$r_{Bin}^{(2)} \le R^{(2)}, \tag{4.26}$$
$$R^{(2)} - r_{Bin}^{(2)} \le t_1 I\left(X_2^{(1)}; Y_3^{(1)} \mid U_2^{(1)}\right), \tag{4.27}$$
$$R^{(2)} - r_{Bin}^{(2)} + r_{Bin}^{(1)} \le t_1 I\left(X_2^{(1)}, U_2^{(1)}; Y_3^{(1)}\right), \tag{4.28}$$
$$R^{(1)} - r_{Bin}^{(1)} \le t_2 I\left(X_1^{(2)}; Y_3^{(2)} \mid U_1^{(2)}\right), \tag{4.29}$$
$$R^{(1)} - r_{Bin}^{(1)} + r_{Bin}^{(2)} \le t_2 I\left(X_1^{(2)}, U_1^{(2)}; Y_3^{(2)}\right). \tag{4.30}$$

Hence, by employing BME and Backward decoding, the following rate is achievable subject to (4.14)-(4.17) and (4.25)-(4.30) constraints.

$$R_{BME_{back}} = \max_{0 \leq t_1, t_2, t_1+t_2=1} R^{(1)} + R^{(2)}. \tag{4.31}$$

Optimum input distributions

Now, we prove there exists input probability distributions ($p(x_0^{(1)}, x_2^{(1)}, u_2^{(1)})$ and $p(x_0^{(2)}, x_1^{(2)}, u_1^{(2)})$) which maximize (4.31) and have the following property: $u_2^{(1)}$ is independent from $(x_0^{(1)}, x_2^{(1)})$ and $u_1^{(2)}$ is independent from $(x_0^{(2)}, x_1^{(2)})$. To prove this, consider $p(x_0^{(1)}, x_2^{(1)}, u_2^{(1)})$ and $p(x_0^{(2)}, x_1^{(2)}, u_1^{(2)})$ along with t_1, t_2 which maximize (4.31) subject to the required constraints. Let us define $\hat{p}(x_0^{(1)}, x_2^{(1)}, u_2^{(1)})$ and $\hat{p}(x_0^{(2)}, x_1^{(2)}, u_1^{(2)})$ as

$$\hat{p}(x_0^{(1)}, x_2^{(1)}, u_2^{(1)}) = p(u_2^{(1)})p(x_0^{(1)}, x_2^{(1)}), \tag{4.32}$$
$$\hat{p}(x_0^{(2)}, x_1^{(2)}, u_1^{(2)}) = p(u_1^{(2)})p(x_0^{(2)}, x_1^{(2)}), \tag{4.33}$$

Now, we show that $\hat{p}(x_0^{(1)}, x_2^{(1)}, u_2^{(1)})$ and $\hat{p}(x_0^{(2)}, x_1^{(2)}, u_1^{(2)})$ along with t_1, t_2 achieve at least the same rate as the optimum one. Let us denote the values of mutual information and entropy with respect to the input distributions $p, \hat{p}$ by I_p, H_p and $I_{\hat{p}}, H_{\hat{p}}$, respectively. The right-hand sides of (4.27)-(4.30) with respect to p can be upper-bounded by the ones corresponding to $\hat{p}$ as follows

$$t_1 I_p\left(X_2^{(1)}; Y_3^{(1)} \mid U_2^{(1)}\right) \overset{(a)}{\leq} t_1 I_p\left(X_2^{(1)}; Y_3^{(1)}\right) = t_1 I_{\hat{p}}\left(X_2^{(1)}; Y_3^{(1)}\right), \tag{4.34}$$
$$t_1 I_p\left(X_2^{(1)}, U_2^{(1)}; Y_3^{(1)}\right) \overset{(a)}{=} t_1 I_p\left(X_2^{(1)}; Y_3^{(1)}\right) = t_1 I_{\hat{p}}\left(X_2^{(1)}; Y_3^{(1)}\right), \tag{4.35}$$
$$t_2 I_p\left(X_1^{(2)}; Y_3^{(2)} \mid U_1^{(2)}\right) \overset{(b)}{\leq} t_2 I_p\left(X_1^{(2)}; Y_3^{(2)}\right) = t_2 I_{\hat{p}}\left(X_1^{(2)}; Y_3^{(2)}\right), \tag{4.36}$$
$$t_2 I_p\left(X_1^{(2)}, U_1^{(2)}; Y_3^{(2)}\right) \overset{(b)}{=} t_2 I_p\left(X_1^{(2)}; Y_3^{(2)}\right) = t_2 I_{\hat{p}}\left(X_1^{(2)}; Y_3^{(2)}\right). \tag{4.37}$$

where (a) follows from the fact that $U_2^{(1)} \longrightarrow X_2^{(1)} \longrightarrow Y_3^{(1)}$ form a Markov chain and (b) follows from the fact that $U_1^{(2)} \longrightarrow X_1^{(2)} \longrightarrow Y_3^{(2)}$ form a Markov chain. Moreover as in distribution $\hat{p}$, $u_2^{(1)}$ and $u_1^{(2)}$ are independent from $(x_0^{(1)}, x_2^{(1)})$ and $(x_0^{(2)}, x_1^{(2)})$, it can be easily verified that the right-hand sides of (4.34)-(4.37) are equal to the right-hand sides of (4.27)-(4.30) with the input distribution $\hat{p}$, respectively. Hence, by utilizing $\hat{p}$ instead of p, the region that satisfies (4.27)-(4.30) is enlarged. Now, let us consider the right-hand sides of

(4.14)-(4.17).

$$t_1 I_p\left(X_0^{(1)};Y_1^{(1)}\mid X_2^{(1)},U_2^{(1)}\right)\overset{(a)}{\le} t_1 I_p\left(X_0^{(1)};Y_1^{(1)}\mid X_2^{(1)}\right)= t_1 I_{\hat{p}}\left(X_0^{(1)};Y_1^{(1)}\mid X_2^{(1)}\right) \quad (4.38)$$

$$t_1 I_p\left(X_0^{(1)},X_2^{(1)};Y_1^{(1)}\mid U_2^{(1)}\right)\overset{(a)}{\le} t_1 I_p\left(X_0^{(1)},X_2^{(1)};Y_1^{(1)}\right)= t_1 I_{\hat{p}}\left(X_0^{(1)},X_2^{(1)};Y_1^{(1)}\right) \quad (4.39)$$

$$t_2 I_p\left(X_0^{(2)};Y_2^{(2)}\mid X_1^{(2)},U_1^{(2)}\right)\overset{(b)}{\le} t_2 I_p\left(X_0^{(2)};Y_2^{(2)}\mid X_1^{(2)}\right)= t_2 I_{\hat{p}}\left(X_0^{(2)};Y_2^{(2)}\mid X_1^{(2)}\right) \quad (4.40)$$

$$t_2 I_p\left(X_0^{(2)},X_1^{(2)};Y_2^{(2)}\mid U_1^{(2)}\right)\overset{(b)}{\le} t_2 I_p\left(X_0^{(2)},X_1^{(2)};Y_2^{(2)}\right)= t_2 I_{\hat{p}}\left(X_0^{(2)},X_1^{(2)};Y_2^{(2)}\right) \quad (4.41)$$

where (a) follows from the fact that $U_2^{(1)} \longrightarrow (X_2^{(1)},X_0^{(1)}) \longrightarrow Y_1^{(1)}$ form a Markov chain and (b) follows from the fact that $U_1^{(2)} \longrightarrow (X_1^{(2)},X_0^{(2)}) \longrightarrow Y_2^{(2)}$ form a Markov chain. Similarly, we observe that the right-hand sides of (4.38)-(4.41) represent the right-hand sides of inequalities (4.14)-(4.17) with the input distribution $\hat{p}$. Hence, the region of $(R^{(1)},R^{(2)})$ that satisfies (4.14)-(4.17) and (4.25)-(4.30) is enlarged by utilizing the input distribution $\hat{p}$ instead of p. This proves the independency of input distributions with $u^{(1)}$ and $u^{(2)}$ in the optimum distribution.

<u>*Simplifying the achievable rate*</u>

As we can assume that the input distributions are of the form (4.32) and (4.33), the achievable rate can be simplified as follows.

$$\begin{aligned}
R_{BME_{back}} = &\max_{0\le t_1,t_2,t_1+t_2=1} R^{(1)}+R^{(2)} \le \\
&\max_{0\le t_1,t_2,t_1+t_2=1} \min\left(t_1 I\left(X_0^{(1)},X_2^{(1)};Y_1^{(1)}\right), t_2 I\left(X_0^{(2)},X_1^{(2)};Y_2^{(2)}\right)\right), && (4.42)\\
&\text{subject to}\\
&r_{Bin}^{(1)} \le R^{(1)}, && (4.43)\\
&r_{Bin}^{(2)} \le R^{(2)}, && (4.44)\\
&R^{(1)} \le t_1 I\left(X_0^{(1)};Y_1^{(1)}\mid X_2^{(1)}\right), && (4.45)\\
&R^{(2)} \le t_2 I\left(X_0^{(2)};Y_2^{(2)}\mid X_1^{(2)}\right), && (4.46)\\
&R^{(2)} - r_{Bin}^{(2)} + r_{Bin}^{(1)} \le t_1 I\left(X_2^{(1)};Y_3^{(1)}\right), && (4.47)\\
&R^{(1)} - r_{Bin}^{(1)} + r_{Bin}^{(2)} \le t_2 I\left(X_1^{(2)};Y_3^{(2)}\right). && (4.48)
\end{aligned}$$

with input distributions

$$p(x_0^{(1)},x_2^{(1)}) = p(x_2^{(1)})p(x_0^{(1)}|x_2^{(1)}),$$
$$p(x_0^{(2)},x_1^{(2)}) = p(x_1^{(2)})p(x_0^{(2)}|x_1^{(2)}).$$

Now, we show that (4.42)-(4.48) is equivalent to

$$\begin{aligned}R_{BME_{back}} \leq \max_{0\leq t_1,t_2,t_1+t_2=1} \min\Big(& t_1 I\left(X_0^{(1)}, X_2^{(1)}; Y_1^{(1)}\right), t_2 I\left(X_0^{(2)}, X_1^{(2)}; Y_2^{(2)}\right), \\ & t_1 I\left(X_0^{(1)}; Y_1^{(1)} \mid X_2^{(1)}\right) + t_2 I\left(X_0^{(2)}; Y_2^{(2)} \mid X_1^{(2)}\right), \\ & t_1 I\left(X_2^{(1)}; Y_3^{(1)}\right) + t_2 I\left(X_1^{(2)}; Y_3^{(2)}\right)\Big). \end{aligned} \tag{4.49}$$

First, it is easy to verify that (4.42)-(4.48) imply (4.49). Now, in order to prove that the converse is also true, we show that for every possible rate r satisfying (4.49), there exists a quad-tupple $\left(R^{(1)}, R^{(2)}, r_{Bin}^{(1)}, r_{Bin}^{(2)}\right)$ such that $R^{(1)} + R^{(2)} = r$, $\left(R^{(1)}, R^{(2)}, r_{Bin}^{(1)}, r_{Bin}^{(2)}\right)$ satisfies (4.42)-(4.48), and moreover at least one of bin rates is equal to zero, i.e., $r_{Bin}^{(1)} = 0$ or $r_{Bin}^{(2)} = 0$.

Let us define $R^{(1)} \triangleq \min\left(r, t_1 I\left(X_0^{(1)}; Y_1^{(1)} \mid X_2^{(1)}\right)\right)$, $R^{(2)} \triangleq r - R^{(1)}$. As r satisfies (4.49), we conclude that $(R^{(1)}, R^{(2)})$ satisfies (4.42), (4.45), and (4.46). Furthermore, as $R^{(1)} + R^{(2)} = r \leq t_1 I\left(X_2^{(1)}; Y_3^{(1)}\right) + t_2 I\left(X_1^{(2)}; Y_3^{(2)}\right)$, we conclude that either $R^{(1)} \leq t_2 I\left(X_1^{(2)}; Y_3^{(2)}\right)$ or $R^{(2)} \leq t_1 I\left(X_2^{(1)}; Y_3^{(1)}\right)$. For the sake of symmetry, let us assume that the first case has occurred, i.e., $R^{(1)} \leq t_2 I\left(X_1^{(2)}; Y_3^{(2)}\right)$. Now, we define $r_{Bin}^{(1)} \triangleq 0$ and $r_{Bin}^{(2)} \triangleq \max\left(0, R^{(2)} - t_1 I\left(X_2^{(1)}; Y_3^{(1)}\right)\right)$. Obviously, (4.43), (4.44), and (4.47) are valid. Considering (4.48), we have

$$R^{(1)} - r_{Bin}^{(1)} + r_{Bin}^{(2)} = R^{(1)} + \max\left(0, r - R^{(1)} - t_1 I\left(X_2^{(1)}; Y_3^{(1)}\right)\right) \overset{(a)}{\leq} t_2 I\left(X_1^{(2)}; Y_3^{(2)}\right) \tag{4.50}$$

where (a) follows from the facts that $r \leq t_1 I\left(X_2^{(1)}; Y_3^{(1)}\right) + t_2 I\left(X_1^{(2)}; Y_3^{(2)}\right)$ and $R^{(1)} \leq t_2 I\left(X_1^{(2)}; Y_3^{(2)}\right)$. Hence, (4.48) is also valid. The second case in which $R^{(2)} \leq t_1 I\left(X_2^{(1)}; Y_3^{(1)}\right)$ can be dealt with in a similar manner.

Hence, from the above argument, the achievable rate of the BME scheme with backward decoding can be simplified as follows

$$\begin{aligned}R_{BME_{back}} \leq \max_{0\leq t_1,t_2,t_1+t_2=1} \min\Big(& t_1 I\left(X_0^{(1)}, X_2^{(1)}; Y_1^{(1)}\right), t_2 I\left(X_0^{(2)}, X_1^{(2)}; Y_2^{(2)}\right), \\ & t_1 I\left(X_0^{(1)}; Y_1^{(1)} \mid X_2^{(1)}\right) + t_2 I\left(X_0^{(2)}; Y_2^{(2)} \mid X_1^{(2)}\right), \\ & t_1 I\left(X_2^{(1)}; Y_3^{(1)}\right) + t_2 I\left(X_1^{(2)}; Y_3^{(2)}\right)\Big), \end{aligned} \tag{4.51}$$

with probabilities

$$\begin{aligned} p(x_0^{(1)}, x_2^{(1)}) &= p(x_2^{(1)}) p(x_0^{(1)} | x_2^{(1)}), \\ p(x_0^{(2)}, x_1^{(2)}) &= p(x_1^{(2)}) p(x_0^{(2)} | x_1^{(2)}). \end{aligned}$$

Appendix F

Proof of corollary 2.2.2

Let $V_0^{(1)} \sim \mathcal{N}(0, \alpha_1 P_0^{(1)}), V_0^{(2)} \sim \mathcal{N}(0, \alpha_2 P_0^{(2)})$, $V_2^{(1)} \sim \mathcal{N}(0, \theta_2 P_2), V_1^{(2)} \sim \mathcal{N}(0, \theta_1 P_1)$, $U_2^{(1)} \sim \mathcal{N}(0, \bar{\theta}_2 P_2)$ and $U_1^{(2)} \sim \mathcal{N}(0, \bar{\theta}_1 P_1)$, which are independent of each other.

Letting $X_0^{(1)} = V_0^{(1)} + \sqrt{\frac{\bar{\alpha}_1 P_0^{(1)}}{\theta_2 P_2}} V_2^{(1)}, X_0^{(2)} = V_0^{(2)} + \sqrt{\frac{\bar{\alpha}_2 P_0^{(2)}}{\theta_1 P_1}} V_1^{(2)}, X_2^{(1)} = V_2^{(1)} + U_2^{(1)}$, $X_1^{(2)} = V_1^{(2)} + U_1^{(2)}$ and using the result in the expression for the achievable rate obtained in Theorem 2.2.2, we obtain $R_{BME_{succ}}$ for the Gaussian case, as given in [37] and (2.20), (2.22), and (2.23), respectively.

For backward decoding, let $V_0^{(1)} \sim \mathcal{N}(0, \beta_1 P_0^{(1)}), V_0^{(2)} \sim \mathcal{N}(0, \beta_2 P_0^{(2)})$, $X_2^{(1)} \sim \mathcal{N}(0, P_2)$, and $X_1^{(2)} \sim \mathcal{N}(0, P_1)$, which are independent of each other. By setting $X_0^{(1)} = V_0^{(1)} + \sqrt{\frac{\bar{\beta}_1 P_0^{(1)}}{P_2}} X_2^{(1)}$, $X_0^{(2)} = V_0^{(2)} + \sqrt{\frac{\bar{\beta}_2 P_0^{(2)}}{P_1}} X_1^{(2)}$ and using the result in the expression for the achievable rate obtained in Theorem 2.2.1, we obtain $R_{BME_{back}}$ for the Gaussian case, as given in (2.21).

Appendix G

Proof of Proposition 2.2.2

Consider the sum rate for both the common message and the private message for the extended multiple access channel from relays to the destination,

$$R_p + R_c \leq t_4 C\left(\frac{h_{13}^2 P_{1,p}^{(4)} + (h_{13}\sqrt{P_{1,c}^{(4)}} + h_{23}\sqrt{P_2})^2}{t_4}\right). \tag{4.52}$$

It can be readily verified that subject to the constraint $P_{1,p}^{(4)} + P_{1,c}^{(4)} = P_1$, the right-hand side of (4.52) is a decreasing function of $P_{1,p}^{(4)}$ or equivalently an increasing function of $P_{1,c}^{(4)}$. Now, let us equate R_p in (4.52) with the private rate $\acute{R}_p$ of another MAC which is achieved by successive decoding of common and private messages. Therefore, we have

$$R_p = \acute{R}_p = t_4 C\left(\frac{h_{13}^2 \acute{P}_{1,p}^{(4)}}{t_4}\right) \leq t_4 C\left(\frac{h_{13}^2 P_{1,p}^{(4)}}{t_4}\right). \tag{4.53}$$

According to (4.53), we have (See Fig. 4.7)

$$\begin{aligned} \acute{P}_{1,p}^{(4)} &\leq P_{1,p}^{(4)} \Longrightarrow \\ R_p + R_c &\leq \acute{R}_p + \acute{R}_c, \\ R_c &\leq \acute{R}_c. \end{aligned}$$

Hence, (R_p, R_c) lies in the corner point of the extended MAC with parameters $(\acute{P}_{1,p}^{(4)}, \acute{P}_{1,c}^{(4)})$, i.e., successive decoding of common and private messages achieves the DF rate.

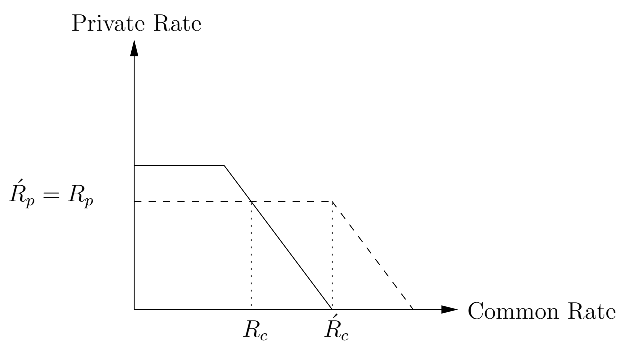


Figure 4.7: The order of decoding *"Common"* and *"Private"* messages.

Appendix H

Proof of Theorem 3.2.1

<u>*Codebook Construction:*</u>

At band α_l, $(l = 1, \cdots, L)$ and β_1, the source generates $2^{nR_{AF_l}}$, $2^{nR_{DF_l}}$, and $2^{nR_{DF}}$ sequences $\mathbf{v}_{BC_l}(w_{AF_l})$, $\mathbf{u}_{BC_l}(w_{DF_l})$, and $\mathbf{x}_{BC}(w_{DF})$ according to $\prod_{i=1}^{\alpha_l n} p(v_{BC_l,i})$, $\prod_{i=1}^{\alpha_l n} p(u_{BC_l,i})$, and $\prod_{i=1}^{\beta_1 n} p(x_{BC,i})$, respectively. V_{BC_l}, U_{BC_l}, and X_{BC} are Gaussian random variables with zero mean and variances P_{s,AF_l}, P_{s,DF_l}, and P_s per dimension, where $P_{s,AF_l} + P_{s,DF_l} = P_s$. Furthermore, at band α_l, the source generates i.i.d sequences $\mathbf{x}_{BC_l}$, where we have $X_{BC_l} = V_{BC_l} + U_{BC_l}$. Hence, $X_{BC_l} \sim \mathcal{N}(0, P_s)$.

All the relays, at band α_l, $(l = 1, \cdots, L)$, and β_2 generate $2^{nR_{DF_l}}$ and $2^{nR_{DF}}$ i.i.d $\mathbf{u}_{r_l}(w_{DF_l})$, and $\mathbf{x}_r(w_{DF})$ sequences according to probabilities $\prod_{i=1}^{\alpha_l n} p(u_{r_l,i})$, and $\prod_{i=1}^{\beta_2 n} p(x_{r,i})$. U_{r_l} and X_r are Gaussian random variables with zero mean and variances P_{r,DF_l} and P_r per dimension. Furthermore, relay m generates i.i.d sequences $\mathbf{x}_{m_l}$, due to

$$X_{m_l} = \sqrt{\frac{P_{r,AF_l}}{P_{s,AF_l} + 1}} (V_{BC_l} + Z_m) + U_{r_l}. \tag{4.54}$$

<u>*Encoding:*</u>

Encoding at the source:

At band α_l, the source encodes $w_{AF_l} \in \{1, \cdots, 2^{nR_{AF_l}}\}$, and $w_{DF_l} \in \{1, \cdots, 2^{nR_{DF_l}}\}$ to $\mathbf{v}_{BC_l}(w_{AF_l})$ and $\mathbf{u}_{BC_l}(w_{DF_l})$ and sends $\mathbf{x}_{BC_l}(w_{AF_l}, w_{DF_l})$ to the relays. Furthermore, at band β_1, the source encodes $w_{DF} \in \{1, \cdots, 2^{nR_{DF}}\}$ to $\mathbf{x}_{BC}(w_{DF})$ and sends it to the relays.

Encoding at relay m:

At band α_l, relay m encodes $w_{DF_l} \in \{1, \cdots, 2^{nR_{DF_l}}\}$ to $\mathbf{u}_{r_l}(w_{DF_l})$ and sends $\mathbf{x}_{m_l}$ as obtained in (4.54), to the destination. Furthermore, at band β_2, relay m encodes $w_{DF} \in \{1, \cdots, 2^{nR_{DF}}\}$ to $\mathbf{x}_r(w_{DF})$ and sends it to the destination.

<u>*Decoding:*</u>

Decoding at relay m:

At band α_l, relay m declares $\hat{w}_{DF_l} = w_{DF_l}$ iff there exits a unique $\mathbf{u}_{BC_l}(w_{DF_l})$, such that $(\mathbf{u}_{BC_l}(w_{DF_l}), \mathbf{y}_{m_l}) \in A_\epsilon^{(n)}$ (See [14]). Hence, in order to make the probability of error zero, we have

$$R_{DF_l} \le \alpha_l C\left(\frac{P_{s,DF_l}}{P_{s,AF_l} + 1}\right). \tag{4.55}$$

Similarly, at band β_1, relay m declares $\hat{w}_{DF} = w_{DF}$ iff there exits a unique $\mathbf{x}_{BC}(w_{DF})$, such that $(\mathbf{x}_{BC}(w_{DF}), \mathbf{y}_m) \in A_\epsilon^{(n)}$. Hence, in order to make the probability of error zero, we have

$$R_{DF} \le \beta_1 C(P_s). \tag{4.56}$$

Decoding at the final destination:
At band α_l, the destination declares $\hat{w}_{AF_l} = w_{AF_l}$ and $\hat{w}_{DF_l} = w_{DF_l}$ iff there exits unique $\mathbf{v}_{BC_l}(w_{AF_l})$ and $\mathbf{u}_{r_l}(w_{DF_l})$, such that $\left(\mathbf{v}_{BC_l}(w_{AF_l}), \mathbf{u}_{r_l}(w_{DF_l}), \mathbf{y}_{MAC_l}\right) \in A_\epsilon^{(n)}$. Hence, in order to make the probability of error zero, we have

$$R_{AF_l} \leq \alpha_l C\left(\frac{M^2 P_{r,AF_l} P_{s,AF_l}}{M P_{r,AF_l} + P_{s,AF_l} + 1}\right), \tag{4.57}$$

$$R_{DF_l} \leq \alpha_l C\left(\frac{M^2 P_{r,DF_l}(P_{s,AF_l} + 1)}{M P_{r,AF_l} + P_{s,AF_l} + 1}\right), \tag{4.58}$$

$$R_{AF_l} + R_{DF_l} \leq \alpha_l C\left(\frac{M^2 P_r P_{s,AF_l} + M^2 P_{r,DF_l}}{M P_{r,AF_l} + P_{s,AF_l} + 1}\right). \tag{4.59}$$

Similarly at band β_2, destination declares $\hat{w}_{DF} = w_{DF}$ iff there exits a unique $\mathbf{x}_r(w_{DF})$, such that $(\mathbf{x}_r(w_{DF}), \mathbf{y}_{MAC}) \in A_\epsilon^{(n)}$. Hence, in order to make the probability of error zero, we have

$$R_{DF} \leq \beta_2 C\left(M^2 P_r\right). \tag{4.60}$$

Noting the fact that $R_{CADF} = \sum_{l=1}^{L}(R_{AF_l} + R_{DF_l}) + R_{DF}$, and from (4.55), (4.56), (4.57), (4.58), (4.59), and (4.60), Theorem 3.2.1 is proved.

Appendix I

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2. Seyed Saeed Changiz Rezaei, Shahab Oveis Gharan, and Amir K. Khandani, "A New Achievable Rate for the Gaussian Parallel Relay Channel", Proc. Of IEEE International Symposium on Information Theory 2009 (ISIT 2009), pp. 194-198.
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