Alternative Models to Analyze Market Power and Financial Transmission Rights in Electricity Markets

by

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I hereby declare that I am the sole author of this thesis. This is a true copy of the thesis, including any required final revisions, as accepted by my examiners.

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Abstract

One of the main concerns with the introduction of competition in the power sector is the strategic behaviour of market participants. Computable models of strategic behaviour are becoming increasingly important to understand the complexities of competition. Such models can help analyze market designs and regulatory policies. In this thesis, further developments on the modelling and analysis of strategic behaviour in electricity markets are presented. This thesis work has been conducted along three research lines.

In the first research line, an oligopolistic model of a joint energy and spinning reserve market is formulated to analyze imperfect competition. Strategic behaviour is introduced by means of conjectured functions. With this integrated formulation for imperfect competition, the opportunity cost between generation and spinning reserve has been analytically derived. Besides, inter-temporal and energy constraints, and financial transmission rights are taken into account. Under such considerations, competition in electricity markets is modelled with more realism. The oligopolistic model is formulated as an equilibrium problem in terms of complementarity conditions.

In the second research line, a methodology to screen and mitigate the potential exacerbation of market power due to the ownership of financial transmission rights is presented. Hedging position ratios are computed to quantify the hedging level of financial transmission rights. They are based on the actual impact that each participant has in the energy market, and on the potential impact that it would have with the ownership of financial transmission rights. Thus, hedging position ratios are used to identify the potential gambling positions from the transmission rights bidders, and, therefore, used to prioritize critical positions in the auction for transmission rights.

In the last research line, alternative equilibrium models of markets for financial transmission rights are formulated. The proposed equilibrium framework is more natural and flexible for modelling markets than the classic cost-minimization markets. Different markets for financial transmission rights are modelled, namely: i) forwards, ii) options, and iii) joint forwards and options. Moreover, one-period, multi-period and multi-round markets for forwards are derived. These equilibrium models are proposed to analyze the bidding strategies of market participants. The potential impact of bidders on congestion prices is modelled by means of conjectured transmission price functions.

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Nomenclature

Acronyms:

AC	Altern Current
ARR	Auction Revenue Right
CM	Congestion Management
DC	Direct Current
EPEC	Equilibrium Problem with Equilibrium Constraints
GenCo	Generation Company
HPR	Hedging Position Ratio
IEEE	Institute of Electrical and Electronic Engineers
ISO	Independent System Operator.
FGR	Flow Gate Right
FTR	Financial Transmission Right
GAMS	General Algebraic Modelling System
LMP	Locational Marginal Pricing/ Locational Marginal Price
LSE	Load Serving Entity
MLCP	Mixed Linear Complementarity Problem
NYISO	New York Independent System Operator
OPF	Optimal Power Flow
PTDF	Power Transfer Distribution Factor
PJM	Pennsylvania-New Jersey-Maryland

Acronyms:

KKT	Karush-Kuhn-Tucker
SR	Spinning Reserve

Indexes:

$h, \hbar$	generation units, or auxiliary indices
i, j, m	nodes in the system
k	transmission lines in the system
r	market rounds
t, Γ	time periods
ℓ	FTRs
ν	market participants; within an energy market ν denotes a GenCo; within an FTR market, ν stands for a portfolio bidder

Sets:

$\mathcal{D}_\nu$	set of operational constraints for consumer ν
$\mathcal{F}_\nu$	set of FTRs obligations (with $\lvert\mathcal{F}_\nu\rvert$ elements) in portfolio ν
$\mathcal{G}_\nu$	set of operational constraints for supplier ν
$\mathcal{H}_i$	set of indices (with $\lvert\mathcal{H}_i\rvert$ elements) for generation units placed at node i
$\mathcal{I}$	set of indices (with $\lvert\mathcal{I}\rvert$ elements) for nodes in the system
$\mathcal{K}$	set of indices (with $\lvert\mathcal{K}\rvert$ elements) for transmission lines in the system
$\mathcal{O}_\nu$	set of FTRs options (with $\lvert\mathcal{O}_\nu\rvert$ elements) in portfolio ν
$\mathcal{R}$, $\hat{\mathcal{R}}$	sets of FTR market rounds
$\mathcal{T}$	set of indices (with $\lvert\mathcal{K}\rvert$ elements) for trading periods
$\mathcal{V}$	set of indices (with $\lvert\mathcal{V}\rvert$ elements) for GenCos
Θ	set of FTRs portfolios (with $\lvert\Theta\rvert$ elements)

Functions, vectors and matrices:

$\mathbb{R}^n$	n-dimensional Euclidian space.
$b : \mathbb{R} \rightarrow \mathbb{R}$	benefit function.
$c_\nu : \mathbb{R} \rightarrow \mathbb{R}$	generation cost function for supplier ν.
$f : \mathbb{R}^p \rightarrow \mathbb{R}$	scalar objective function.
$\rho_\jmath : \mathbb{R} \rightarrow \mathbb{R}$	inverse demand function for consumer $\jmath$.
$\pounds : \mathbb{R}^p \rightarrow \mathbb{R}$	Lagrangian function.
$\nabla_{\boldsymbol{g}} f(\cdot) : \mathbb{R}^n \rightarrow \mathbb{R}$	gradient vector of $f(\cdot)$ with respect to $\boldsymbol{g}$.
$\boldsymbol{d} \in \mathbb{R}^n$	vector of demand variables.
$\boldsymbol{e} \in \mathbb{R}^n$	vector of ones of appropriate dimension, $\boldsymbol{e} = [1, 1, ..., 1]^T$.
$\boldsymbol{g} \in \mathbb{R}^n$	vector of generation variables.
$\boldsymbol{o} \in \mathbb{R}^n$	vector of market prices
$\boldsymbol{x}, \in \mathbb{R}^n$	vector of generic primal variables
$\boldsymbol{z} \in \mathbb{R}^n$	vector of power flows
$\boldsymbol{\beta}_\nu \in \mathbb{R}^{\vert\mathcal{F}_\nu\vert}$	vector of constants associated to the linear term for the benefit of bidder ν
$\boldsymbol{\delta} \in \mathbb{R}^n$	vector of nodal angles
$\boldsymbol{\tau}_\nu \in \mathbb{R}^{\vert\mathcal{F}_\nu\vert}$	vector for FTRs of bidder ν
$\boldsymbol{\omega}_\nu$, $\boldsymbol{\omega}^* \in \mathbb{R}^{\vert\mathcal{I}\vert}$	vector of congestion prices seen by ν and in equilibrium, respectively
$\boldsymbol{A}_\nu \in \mathbb{R}^{\vert\mathcal{I}\vert x \vert\mathcal{I}\vert}$	diagonal matrix of conjecture transmission-price response parameters of bidder ν
$\boldsymbol{B}_\nu \in \mathbb{R}^{\vert\mathcal{F}_\nu\vert x \vert\mathcal{F}_\nu\vert}$	auxiliary matrix of FTRs for bidder ν
$\boldsymbol{G} \in \mathbb{R}^{\vert\mathcal{I}\vert x \vert\mathcal{I}\vert}$	Bus susceptance matrix
$\boldsymbol{H} \in \mathbb{R}^{\vert\mathcal{K}\vert x \vert\mathcal{I}\vert}$	Reactance matrix

Functions, vectors and matrices:

$\boldsymbol{S} \in \mathbb{R}^{\|K\|x(\|\mathcal{I}\|-1)}$	matrix of power distribution factors
$\boldsymbol{\gamma}_\nu \in \mathbb{R}^{\|\mathcal{F}_\nu\|x\|\mathcal{F}_\nu\|}$	diagonal matrix of constants associated to the quadratic term of the benefit function
$\boldsymbol{\xi}_\nu \in \mathbb{R}^{\|\mathcal{I}\|x\|\mathcal{F}_\nu\|}$	incidence matrix for FTRs of bidder ν

Constants:

d_{oi}	quantity intercept for demand at node i (MW)
$e_{\nu,h,i}$	trading energy of unit h, placed at node i and owned by GenCo ν (MWh)
$s_{i,k}$	distribution factor for transmission line k with respect to node i ($p.u.$)
$\overline{r}^t$	spinning reserve requirement for period t (MW)
$\overline{z}_k$	maximum power flow limit for transmission line k (MW)
x_k	reactance for the transmission line k (p.u.)
$A_{\nu,i}$	transmission-price response parameter conjectured by either GenCo or bidder ν $[(\$/MWh)/MW]$ or $[(\$/MW)/MW]$
$A_{\nu,\ell}$	Transmission-price response parameter conjectured by bidder ν for ℓ-th FTR $[(\$/MW)/MW]$
$B_{-\nu,i}$	conjectured rivals response parameter by GenCo ν at node i $[MW/(\$/MWh)]$
C_ν	conjectured spinning reserve price parameter by GenCo ν at node i $[(\$/MWh)/MW]$
$T_{\nu,i,j}$	power to be traded by ν between nodes i and j (MW)
$\alpha_{\nu,h,i}$	marginal cost for the spinning reserve of unit h, placed at node i and owned by GenCo ν ($\$/MWh$)

Constants:

$\beta_{\nu,\ell}$, $\gamma_{\nu,\ell}$ linear and quadratic parameters of the benefit function for FTR ℓ ($\$/MW$ and $\$/MW^2$), respectively

δ_s angle at the slack node (rad)

ε percent of the demand to be set as spinning reserve (p.u.)

ρ_{oi} price intercept for demand at node i ($\$/MWh$)

$\varrho_{k,\ell}$ PTDF in transmission line k for transaction ℓ (p.u.)

$\xi_{\nu,\ell,i}$ i-th element of the incidence vector for the ℓ-th FTR, $\xi_{\nu,\ell,i}$={-1,0,1}

$\phi_{k,\ell}$ hedging position ration in line k for a participant placed at node i (p.u.)

$\hat{\phi}$ threshold for hedging position ratios (p.u.)

$\varphi_{k,i}$ utilization factor of line k for a market participant placed at node i (p.u.)

Υ congestion rents collected by the ISO ($\$/h$)

Ψ_ν congestion charges to be paid by participant ν to the ISO ($\$/h$)

Variables:

a_i, $a_{\nu,i}$ power arbitraged by the ISO, or seen by GenCo ν, at node i (MW)

$d_{\nu,i}$ power sold by GenCo ν at node i (MW)

d_s demand at the slack node (MW)

$g_{\nu,i,h}$ power produced by unit h, placed at node i and owned by GenCo ν (MW)

g_s generation at the slack node (MW)

p_i power injection at node i (MW).

p_i^c fixed power injection at node i (MW)

$r_{\nu,i,h}$ spinning reserve from unit h, placed at i and owned by GenCo ν (MW)

z_k real power flow through transmission line k (MW)

$\rho_{\nu,i}$ price seen by GenCo ν at node i ($\$/MWh$)

Variables:

ρ_s, $\rho_{\nu,s}$	price at the slack node, or seen by GenCo ν ($\$/MWh$)
$\tau_{\nu,m,\ell}$	financial transmission right from node m to node ℓ held by GenCo ν (MW)
$\tau_{\nu,\ell}$	ℓ-th FTR in portfolio ν (MW)
η	market price for spinning reserve ($\$/MWh$)
ρ_i	market price at node i ($\$/MWh$)
$\omega_{\nu,i}$	congestion price seen by GenCo ν at node i ($\$/MWh$)
ω_i^*, ω_ℓ^*	congestion price at equilibrium for node i, and for FTR ℓ ($\$/MW$)
$\overline{\Delta}\, g_{\nu,i,h}$	up-ramp rate for unit h, placed at node i and owned by GenCo ν (MW/h)
$\underline{\Delta}\, g_{\nu,i,h}$	down-ramp rate for unit h, placed at i and owned by GenCo ν (MW/h)
$\Pi_\nu^{(\cdot)}$	profit for GenCo ν given by activity $(\cdot)$ ($\$/h$)

Symbology:

$(\bar{\cdot})$, $(\underline{\cdot})$	maximum and minimum value for variable $(\cdot)$, respectively
$(\cdot)^+$, $(\cdot)^-$	upper and lower bound for $(\cdot)$
$(\cdot)^*$	value for variable $(\cdot)$ in equilibrium
$\lvert(\cdot)\rvert$	cardinality for the finite set $(\cdot)$
$(\cdot)^f$	obligation version for $(\cdot)$
$(\cdot)^o$	option version for $(\cdot)$
$d_{-\nu,i}=$	$\sum_{n\neq\nu} d_{n,i}$, power to be supplied, at node i, by the rivals of ν
$u^t=$	$\{u = 1 \lvert t < \lvert \mathcal{T}\rvert, u = 0 \,\lvert\, t = \lvert\mathcal{T}\rvert\}$
$\delta_i =$	ρ_{oi}/d_{oi}, slope of the demand function
$a \perp b$	complementarity condition between a and b
$\langle a,\, b\rangle$	inner product between a and b

Chapter 1

Introduction

1.1 Electricity Markets

Electric power systems had been traditionally operated as natural monopolies, by vertically-integrated organizations subject to government regulation. Usually, there was a sole provider of the service in certain regions, charging to customers a flat tariff. Since 1982 when Chile shifted the structure of its power sector, deregulation of the power industry has taken place throughout the world. The economic argument to restructuring the power industry has been to increase efficiency by introducing competition and, as a by-product, to reduce energy prices for end-users.

In order to enable competition, it is necessary to unbundle the power sector into administratively independent activities such as generation, transmission and distribution, and to guarantee an open access to these activities. The implementation of electricity markets requires to put in place not only markets for energy, but also transmission –congestion and losses– pricing schemes, together with a secure and reliable operation of the system under the competition environment. Regardless of the market design, a power system under the new business environment is basically conformed by three components: i) a primary/wholesale market where the bulk power is traded; ii) a transmission management system which refers to the mechanism to have an open transmission system, and to implement the energy transactions; and iii) a set of ancillary services to support the reliable and secure operation of the system.

Throughout this worldwide re-structuring trend, different market designs have been put in place; these models have been a product of technical, economical and even political aspects. The performance of different electricity markets has shown that the process of re-structuring has worked in some markets, such as in the NordPool and Pennsylvania-New Jersey-Maryland (PJM) pool, and has deficiencies in others, such as in the California and Ontario markets.

The particular nature of power systems makes the introduction of competition a challenging task. The fact that power systems must be in instantaneous power balance, together with the interactions of its elements, adds a high complexity to their operation. In restructured power systems, although the generation sector has been unbundled, the transmission system usually remains as a monopoly. From a market point of view, the transmission system should be reduced to a system to inject and withdraw the traded power. Unfortunately, power flows in the transmission system are determined by physical laws, and they are limited by the network's transmission constraints. If transmission constraints limit the transactions, congestion arises; this impedes the free movement of power envisioned by economics theory, and may degrade the competitiveness. In a congested network, the cheapest generation cannot be used; instead more expensive generation has to serve the demand. As a consequence of congestion, there are locational marginal prices; the price at each location is always greater than or equal to the marginal cost of generation at such a node. The difference in prices between locations, disregarding losses, is the so-called *congestion cost* for using the transmission system. How congestion is managed has direct implications in the efficiency of the generation market. Therefore, the aim is to have a mechanism to allocate scarce transmission meanwhile it incentives as much as possible competition in the generation sector.

In a centralized market, congestion management is implicitly accomplished in the market clearing process, and power is traded at locational marginal prices. Such prices are the efficient ones for trading power as they consider all the system characteristics. In contrast, in a decentralized market, congestion management is independently carried out from the market clearing process. In hybrid markets, besides the centralized process, bilateral transactions can be implemented. Regardless the market design, all market participants have to face the congestion costs. Since the costs are not known in advance, market participants are exposed to volatile prices, making the transactions risky. An alternative to hedge against congestion costs is by means of Financial Transmission Rights, well known as FTRs. They are purely financial point-to-point rights, and are based upon the price difference between selected locations for a contracted amount of power. Currently, this hedging scheme has been implemented in several markets, *e.g.,* New Zealand, PJM and New York.

Since the introduction of competition in power systems, one of the main concerns has been the ability of some market participants to behave strategically in order to manipulate the market prices, *i.e.*, to exercise *market power*. Due to transmission constraints, generation cannot be transmitted to all nodes of the network, and isolated areas may be exposed to local market power. Unlike any other kind of market, the unique characteristics of power systems[1] may create opportunities for the exercise of market power. For instance, while traditional market power is exercised by restricting production, in electricity markets there can be conditions under which market power can be exercised by increasing generation. This fact is because of the substitutable and complementary relationships among generations through the transmission system. It is recognized that traditional metrics for measuring market power, such as concentration indices, do not work well for the case of electricity markets [1]. This is due to the fact that traditional indices do not consider suppliers' location, network interactions and changing conditions of the system.

1.2 Preliminaries

1.2.1 Modelling competition

In competitive markets, suppliers ignore the actions of the other participants as they do not have any influence (competitive fringe) on the market outcome. On the other side, in the monopolistic market, there is a sole supplier. In oligopolistic models, in contrast, since more than one supplier can alter the market outcome, they have to consider their rivals' behaviour to determine their best strategy. Due to the market structure of power systems, oligopoly models have become a natural framework to study imperfect competition. There is a variety of models which can be classified depending on: i) the system elements taken

[1] Instantaneous match between supply and demand, low elasticity in demand, non-storability of electricity and transmission constraints.

into account; ii) the mechanism for pricing; iii) the way the transactions are made; and iv) the kind of suppliers' conjectures.

Regarding the transmission system modelling, Green and Newbery [2] provide one of the first models for imperfect competition in electricity markets without considering the transmission system. Other models that neglect the transmission system, or consider basic representations of the network, can be found in [3–6]. More sophisticated models have incorporated the network configuration by means of power flow equations [7–17].

Depending on how the transactions are made, two models can be implemented: the bilateral and the pool-like markets; however, in the presence of perfect arbitrage both mechanisms can achieve the same market outcome [18]. In this thesis, a bilateral-like model with perfect arbitrage is considered; this kind of market is characterized by a central pricing mechanism for transmission, which is carried out by an Independent System Operator (ISO), while market participants buy and sell energy among them.

Competition in electricity markets can be also analyzed regarding suppliers strategies. A variety of models have been proposed to predict suppliers behaviour such as Bertrand, Cournot or Stackelberg models [8, 17, 19, 20]. The key difference among these models is the strategic variable that a firm chooses when competing against its rivals. The choice of strategy, e.g. price or quantity, impacts the intensity of competition among suppliers, and consequently, the resulting outcome. In the Bertrand case, a supplier conjectures that rivals will not react to its actions, *i.e.*, rivals will not alter their prices; in the Cournot case, each supplier maximizes its profits considering an expectation of the output power of the other suppliers, *i.e.*, each supplier conjectures that whatever its output power is, rivals will hold their output fixed. A point to be highlighted is that the Cournot equilibrium is also a *Nash equilibrium*; this equilibrium establishes that every supplier achieves its best status given the output of the other suppliers, and consequently, it cannot unilaterally increase its profits. In the Stackelberg case, there is a sophisticated supplier who acts as a leader and recognizes how the others act, meanwhile the remaining suppliers behave naively à la Cournot.

With the unbundled set of goods and services in the de-regulated power sector, and the introduction of new entities in the industry, different agents are simultaneously trying to optimize their goals. Under this kind of multi-agent perspective, economic equilibrium becomes a natural framework for modelling competition. In addition, advances in complementarity theory help better understand some aspects of equilibrium [21]. An equilibrium model can be defined by a set of optimization problems, one per market participant – suppliers, consumers and market operator– which relate prices, generations, demands and power flows to satisfy every market participant's first-order optimality conditions, plus a coordination condition to clear the market, *i.e.*, to match supply and demand of goods and services. If a solution exists for such a problem, then no market participant will unilaterally alter its current position, a Nash equilibrium [13].

Various alternatives to analyze imperfect competition in electricity markets have been proposed by means of equilibrium models [13,15,16,18, 22–27]. Rivier *et al.* [23] propose a complementarity-problem framework to model the long-term operation planning problem of a system with thermal and hydro generation units. Similarly, Bushnell [26] studies the strategic operation planning of hydroelectrical resources within a market environment. The author finds that it becomes more profitable to allocate hydro production to off-peaks hours, opposite to what could be in competitive conditions. On the other hand, for the short-term planning problem, Boucher *et al.* [13, 28] review different market models for power networks; for this analysis, they use a unifying equilibrium framework. Hobbs *et al.* [15,18] introduce a linear complementarity approach for modelling imperfect competition in both the pool and bilateral markets. The authors use a marginal pricing scheme to allocate the transmission to participants. In a step further, Hobbs *et al.* [16, 29] analyze the efficiency of different inter-regional transmission-pricing policies in the presence of oligopolists.

However, most models on imperfect competition have been based upon static models, *i.e.,* they ignore temporal constraints of generation units, such as up- and down-ramp rate limits. Hence, this kind of model may give unrealistic market outcomes, and consequently, misleading conclusions on market power may be achieved [30–32]. Ramos *et al.* [33] propose a competition model based upon a unit commitment problem where first-order optimality conditions of firms are introduced like constraints. In this model, transmission constraints are not taken into account. In [34], a multi-period model for oligopolistic markets is presented; however, this model does not capture the generators' ability to influence the transmission prices, and studies the participants' incentives from the energy-only market. In [35], Mansur proposes an econometric approach for an *ex-post* analysis of market power and its impact on welfare. The author carries out a comparison between static and temporal cases for the PJM market. The author finds that there is a substantial variation from using a static approach.

1.2.2 Energy and spinning reserve markets

The implementation of electricity markets requires not only a market for energy, but also markets for other goods and services such as transmission and ancillary services. It is now well recognized the advantages of using an integrated market [36–41]. In an integrated market, a multi-objective optimization is carried out so that different products can be simultaneously priced and procured. With this approach, the coupling nature of resources, in addition to the system constraints, can be explicitly considered. Hence, a better consistency between prices and the physical dispatch can be achieved. This fact makes the market prices represent more accurately the actual value of the different products, *e.g.*, energy and Spinning Reserve (SR). Nowadays, various real markets use this integrated approach, *e.g.*, Singapore, New York, New Zealand, and the new-brand market of California.

The concern of market power has been traditionally focused on the strategic behaviour of generators within an energy-only market; however, within electricity markets, Generation Companies (GenCos) sell not only energy; indeed, GenCos have incentives from other market activities. For instance, a GenCo can have profits from participating in both the energy and SR markets. Since the levels of energy and SR that a generation unit can provide are limited by the maximum capacity of the unit, a GenCo has to define the optimal levels of both of them simultaneously. On the side of the market, if the energy and SR markets are simultaneously cleared, then there can be an interaction between energy and SR [42–45]; and prices in one market will affect prices in the other. Spinning reserve (which can be considered the most expensive and critical reserve [38)] may motivate generators to behave differently within the energy market due to the opportunity cost between producing and spinning. Sometimes a GenCo can shift power from one market to the other as it is more profitable. As this shift of power can affect the generation schedule, it may impact the energy market efficiency. For instance, if a cheap generation unit decreases further its generation due to a SR incentive, more expensive generation will have to be used.

Although the interaction energy-SR is well-known in competitive markets, few works have addressed such an interaction within an oligopolistic market. In [46], oligopolistic GenCos are considered in separate energy and SR markets; in these markets, the transmission system is not included. In [44], a one-period Cournot model is used, with a transportation-network-like transmission system, to show the interaction of energy and ancillary services. In [47], oligopolistic competition is modelled in the energy market, meanwhile competitive competition is considered in the SR market; the transmission system is modelled by means of a DC approximation. This model is then further extended to include strategic behaviour in the spinning reserve market [48].

Another interaction may occur between energy and pollution permits markets. The usage of such permits to exacerbate suppliers' market power has been studied for the California [49] and PJM [50] markets.

1.2.3 Financial transmission rights and market power

Different studies have been developed to analyze the effects of network conditions in the severity of market power [51–54]. In [55], a survey of market metrics is presented to analyze the fairness, efficiency and competitiveness of electricity markets. Strategic behaviour of FTR holders has been analyzed in a two-node network [11, 56–59]. From this analysis, and with a specific configuration, the conclusion is that if a generator in the importing node holds an FTR, it increases its market power; on the other hand, if the FTR is held by a generator in the exporting node, it has no effect in the market power of the generator. In [11], Stoft shows that FTRs can, in contrast, curb the market power. Unlike the two-node network, meshed networks produce inter-dependencies among nodes and so do participants, making it difficult to analyze the strategic behaviour of FTR holders. Loop flows effect has been addressed with a three-node network [22, 57–60]. Cardell *et al.* [22] analyze the exacerbation of market power with an increase in production by a two-generator supplier. Joskow *et al.* [58,59] have developed an exhaustive analysis of market power in transmission for both two- and three-node systems. They analyze different network configurations for both physical and financial transmission rights. When a generator, holding an FTR, is at a location where it can constrain generation from other generator, the FTR increases its incentives to withhold generation and, therefore, its market power. In a step further, Philpott *et al.* [57] analyze both networks by including uncertainty, achieving similar conclusions. Gilbert *et al.* [60] study two- and three-node networks with oligopolists. They conclude that a perfectly arbitraged single-price auction for FTRs does not let participants enhance their market power. Moreover, they suggest that basing all the FTRs on the least-influenced node does not enhance market power; however, the problem is about the existence of such a node. So far, there is no general methodology to screen and potentially mitigate the exacerbation of market power by the ownership of FTRs.

1.2.4 Markets for transmission rights

FTRs can be acquired through markets or secondary trades. Within markets, FTRs have to be centrally allocated by the ISO in order to ensure the simultaneous feasibility of the FTR awards. These awards are valid for a predefined time period, such as years, seasons or months; even weekly or daily periods of ownership have been proposed. In contrast, secondary trades are implemented in a bilateral basis. In such trades, already-allocated FTRs can be traded among participants. However, only the size of the FTR can be modified. Since there is no FTR re-configuration in regard with the source/sink nodes, any trade does not affect the whole FTR feasibility, and the ISO does not have to intervene in the trade. The ISO only updates the ownership transfers.

On the other hand, the revenue from the auction, collected by the ISO, can be allocated to either transmission owners, entities that pay for upgrading the transmission, Load Serving Entities (LSE) or Auction Revenues Rights (ARR) holders. ARRs are instruments quite similar to FTRs; they are financial instruments to collect a share of the revenue from the FTR market. Their economic value is determined by the clearing prices of the FTR market.

Markets for financial transmission rights have recently arisen and they are still under development [39, 61–63]. Diverse kinds of FTRs, *e.g.*, obligations, options, hybrid, losses and contingent have been proposed [39, 64–67]. Moreover, models for different kind of markets such as static, multi-round and multi-period are still evolving. Due to the recent introduction of FTR markets, the understanding and analysis of bidding strategies from participants play an import role in the enhancement of such markets.

Recently, market efficiency and potential market power in transmission rights markets have become a concern [43, 63, 68–71]. In [69, 70], the authors claim to show that markets for transmission rights are inefficient. They base their analysis upon the performance of the New York market (NYISO) for transmission rights. Because of fears of market power, some market rules have been put in place, such as prohibiting bidders to know each others' bids and/or immediate disclosure of FTR auction data [63]. Nevertheless, bidders need to have available timely data in order to make proper strategies. In [68], a closed-form solution of a market for financial transmission rights is derived. This analytical solution is based on a market with two dominant bidders for the FTR of a major interface. Market power is studied with different strategies for congestion price manipulation. Recently, equilibrium models of competitive markets for financial transmission rights have been proposed in [72]; models of obligations, options and joint obligations and options are presented. Such models are extended in [71] to study the potential impact of FTR bidders on congestion prices. This is done by the inclusion of a conjectured congestion-price function. In a step further, equilibrium models are extended to multi-period and multi-round markets in [73].

1.3 Research Motivation

Although many models of imperfect competition have been developed to analyze market power in the energy market, comprehensive models are still needed to capture more realistic conditions that suppliers face within electricity markets, and more important is that such models can be applied to large systems. On one hand, operational constraints have been neglected in the modelling. On the other hand, the study of the supplier incentives for exercising market power has been focused merely on the energy market, disregarding the incentives from other activities.

The concern about market power exercised by FTRs holders deserves further investigation. Various studies have been carried out to analyze the incidence of the transmission-right ownership in the incentives to exercise market power. So far, all these studies have been limited to analyze cases of two- and three-node networks. Although these simple models are useful to get insights of the problem, this problem has not been addressed in the context of a general power network, and there is no methodology to screen potential market power due to FTRs allocation. The complexity of such a task resides on the way to detect the strategic position that potential FTRs holders can have in the generation market.

Markets for financial transmission rights are becoming increasingly important. New products and markets are envisioned to emerge. This will inevitably create more complex interactions and extra incentives for participants. Due to the incipient experience with this kind of markets, the modelling and development of such markets are still under analysis. Moreover, within markets for financial transmission rights, the study of the impact of bidding strategies on the congestion prices is still needed.

1.4 Structure of the Thesis

This thesis is organized in six chapters and two appendices.

In Chapter 2, economics of restructured power systems is introduced. Suppliers and demands are described; in particular, the profit maximization problem of suppliers is introduced. This problem is then used to characterize competition and the effect of market power in electricity markets. Afterwards, the congestion management problem is reviewed, and the locational marginal pricing scheme is introduced. Basics facts of nodal prices are then derived. The description of financial transmission rights closes the chapter.

In Chapter 3, an equilibrium model for imperfect competition in a joint energy and spinning reserve market is presented. Temporal constraints and energy limits are included in the formulation. Strategic behaviour of suppliers in both markets is considered by means of conjectured functions. Within the market formulation, two kind of agents are considered; on one hand, there is a set of suppliers who act to maximize profits; on the other hand, there is a central entity in charge of allocating the transmission capacity among suppliers, and also of implementing arbitrage. Whereas suppliers are modelled as price-makers, the central entity is considered to act competitively. A profit maximization problem is associated with each agent of the market. Afterwards, optimality conditions are derived for each of these problems. The optimality conditions of all agents are gathered, together with market clearing constraints, to compose an equilibrium problem in terms of complementarities. In a second formulation, FTRs are introduced in the profit-maximization problem of suppliers.

In Chapter 4, a scheme to take into account the potential exacerbation of market power due to the ownership of financial transmission rights is presented. This scheme is based upon the computation of hedging position ratios. For each financial transmission bid, an index is computed to identify the level of hedging that such an FTR would provide to its holder. Afterwards, these hedging ratios are used to weight their corresponding FTRs bids that are to be used in the allocation process. How changes in the networks configuration impact the hedging ratios is also illustrated.

In Chapter 5, equilibrium models of markets for financial transmission rights are derived. An introduction to markets of FTRs, and to obligations and options is first presented. Afterwards, models for obligations, options, and joint obligations and options are formulated. Then multi-round and multi-period markets for obligations are derived. Two kind of agents are considered to participate in the markets: i) agents who bid to acquire FTRs, and ii) a central entity in charge of allocating the transmission rights to those who value them most. The pricing of FTRs is done by means of locational marginal pricing theory. How FTR bidders influence the congestion prices is characterized with conjectures transmission-price functions. Various properties of these markets are mathematically derived throughout the chapter.

A summary of contributions and directions for future research is presented in Chapter 6.

The derivation of the generalized generation and load distribution factors is presented in Appendix A, while the systems' data for the numerical examples are given in Appendix B.

Chapter 2

Economics of Power Systems

In power systems where competition has been introduced, the activities are primarily driven by business-based incentives. Under this new paradigm, the power system economics is being redefined in order to link the operation of power systems with the commercial activities.

In this chapter, basic economic principles of power markets are presented to help introduce the topics covered in this thesis. In §2.1, the power market participants are described. An introduction to market power is presented in §2.2. The congestion management problem is reviewed in §2.3, followed by the introduction of locational marginal pricing in §2.4. The description of transmission rights in §2.5 closes this chapter.

2.1 Market Participants

A market is the set of suppliers (GenCos) and consumers (demands) that interact through potential and real exchanges to determine the price for a commodity [74]. A power market can be based upon the minimization of the social cost, maximizing consumer surplus minus supplier cost. Suppliers and consumers submit respectively their bids, and a market operator is in charge of computing a market equilibrium. The main behavioral assumption to analyze power markets is that both suppliers and consumers act to maximize their profits. In this thesis work, due to the incipient sophistication of consumers so far, the main concern is on the suppliers side.

Definition 2.1: (The supplier problem) Let the variable generation cost of supplier ν be defined by the non-decreasing convex function $c_\nu(g_\nu)$ where g_ν stands for the generation level. Let $\mathcal{G}_\nu$ be the set of operational constraints, such as generation limits, and let ρ be the market price. Thus, the profit-maximization problem of supplier ν can be stated as,

$$\max_{g_\nu} \quad \Pi_\nu(g_\nu) = \big\{\rho g_\nu - c_\nu(g_\nu)\big\}, \tag{2.1}$$

$$s.t. \quad g_\nu \in \mathcal{G}_\nu. \tag{2.2}$$

where $\Pi_\nu(g_\nu)$ is the generation profit function defined as the supplier revenue, ρg_ν, minus its generation cost. The first-order optimality conditions characterize the optimal generation level at which ν maximizes its profit [75], *i.e.*,

$$\rho + g_\nu \frac{d\rho}{dg_\nu} = \frac{dc_\nu(g_\nu)}{dg_\nu}, \tag{2.3}$$

$$g_\nu \in \mathcal{G}_\nu. \tag{2.4}$$

These conditions establish that ν must produce at a level where its marginal revenue equals its marginal cost[1]. In this setting, the market price is actually influenced by ν's decisions, *i.e.*, the market price is a function of the ν's generation level, $\rho(g_v)$. Thus, the supplier sets not only the power it produces, but also the market price at which it sells the power. This fact denotes a *price-setting* behaviour. Depending on the market structure, different theoretical models for analyzing price-setting behaviors (and degrees of

[1]This assertion is valid only for an interior solution; otherwise, operational limits can be appended into the optimality conditions by means of Lagrange multipliers.

competition) have arisen, such as monopoly (a sole supplier) and oligopoly (few dominant suppliers).

A special case occurs when it is assumed that supplier ν takes the market price as given (known as *price-taking* behaviour), such that ρ is an exogenous variable in $\Pi_\nu(g_\nu)$. Consequently, the profit-maximization conditions (2.3) and (2.4) become,

$$\rho = \frac{dc_\nu(g_\nu)}{dg_\nu}, \tag{2.5}$$

$$g_\nu \in \mathcal{G}_\nu. \tag{2.6}$$

These conditions establish that the marginal cost of ν must equal the market price. The fact that ν takes the market price as given leads to a competitive behaviour, as the market price is not distorted by the decisions of ν.

Definition 2.2: (The demand) Let the benefit of consumer $\jmath$ be defined by the non-decreasing concave function $b_\jmath(d_\jmath)$, where $d_\jmath$ stands for the demand level. Let $\mathcal{D}_\jmath$ be the set of operational constraints, such as demand limits. The marginal benefit function $\frac{db_\jmath(d_\jmath)}{dd_\jmath}$, known as the *inverse demand function*, defines the price in terms of demand levels. This demand function is given by $\rho_\jmath(d_\jmath) = \rho_{o\jmath} - (\rho_{o\jmath}/d_{o\jmath})d_\jmath$, where $\rho_{o\jmath}$ and $d_{o\jmath}$ are the price and quantity intersections, respectively. This affine function implies a price-responsive demand. Demands with no price response are represented by fixed demand levels regardless of the market price.

2.2 Competition in Power Markets

For a competitive power market, the main assumption is that suppliers cannot affect the market price, and, consequently, they will bid their true marginal costs. On the opposite side, in a monopoly there is one dominant supplier that can profitably manipulate the market outcome; that is, the supplier has *market power*. By definition, market power is the ability to profitably set prices above competitive levels [76]. However, not all high market prices are a result of exercising market power; some high prices are only a product of scarce generation [77]; the difficulty for making the distinction between causal factors and actual market power leads to more cautious actions in order to avoid negative effects due to a market intervention [78].

In order to highlight the consequences of market power, consider the curves for marginal cost (MC), demand ($\rho(d)$) and marginal revenue (MR) as shown in Figure 2.1. The competitive market outcome is denoted by (*), while the monopolistic market outcome is denoted by (^{m}). Considering the competitive outcome as a benchmark, it follows that:

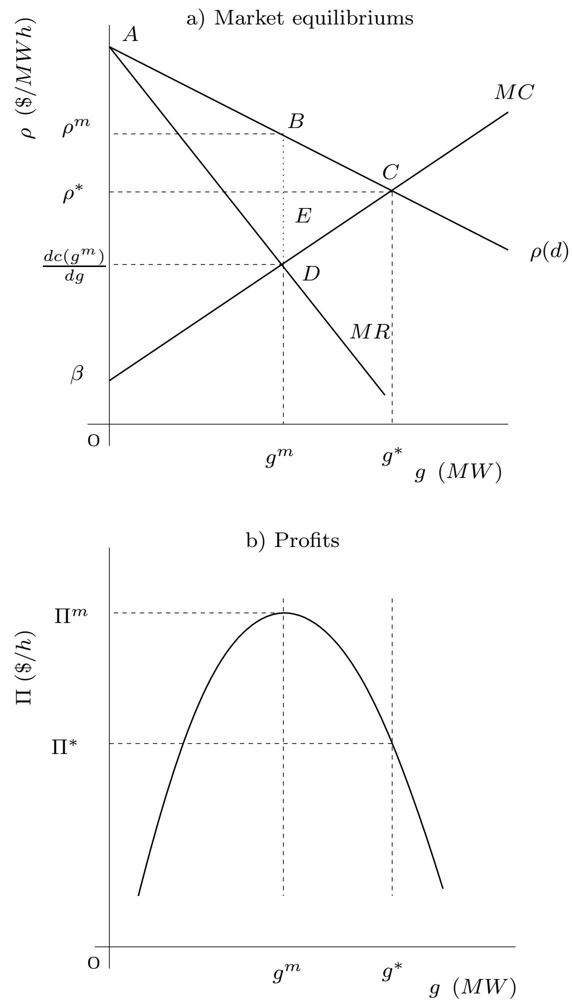

Figure 2.1: Perfect competition *vs.* market power.

(i) The market price goes up from ρ^* to ρ^m, while the output power decreases from g^* to g^m. This reduction on the power outcome is well-known as *withholding of output*. The supplier will lose some money by selling less power; however, it is compensated by selling at a higher market price. As a result, the monopolist's profit increases from Π^* to Π^m.

(ii) There is a net loss of consumer's surplus[2] (area $\rho^*\rho^m BC$ in Figure 2.1) which is composed of two parts:

- a transfer of welfare to the monopolist supplier (area $\rho^*\rho^m BE$ in Figure 2.1) given by $g^m(\rho^m - \rho^*)$; and
- a deadweight loss (area BCE in Figure 2.1), given by $\frac{1}{2}(\rho^m - \rho^*)(g^* - g^m)$.

(iii) There is a net lost supplier's surplus (area CDE in Figure 2.1), given by $\frac{1}{2}(\rho^* - dc(g^m)/dg)(g^* - g^m)$.

(iv) Therefore, there is a net deadweight loss (area BCD in Figure 2.1) given by $\frac{1}{2}(g^* - g^m)(\rho^m - dc(g^m)/dg)$.

The market power causes an inefficiency (known as net deadweight loss) because less power is traded $(g^* - g^m)$ than that under competitive conditions, and represents the social cost of the strategic behaviour. Market power can arise as one of the most critical imperfections of electricity markets, and can become one of the main concerns in the design and monitoring of markets [79–81].

2.3 The Congestion Management Problem

Although the generation sector in deregulated power systems is usually unbundled to allow competition, it is expected that transmission will remain as a regulated monopoly that gives participants an open and nondiscriminatory access to the power market [82]. In order to guarantee such an open access to the transmission, a central independent entity – non-profit or for-profit, *e.g.*, an ISO, is usually in charge of the transmission management.

The inherent features of electric power systems –*e.g.*, permanent changes in demand, inability to store energy, transmission constraints, losses and security requirements– impede that perfect competition can be fairly implemented in power markets. Moreover, the particular nature of power flows in a transmission system, defined by the Kirchhoff's

[2] The consumer surplus is the difference between what a consumer is willing to pay (area $OACg^*$) and what the consumer actually pays (area $O\rho^*Cg^*$). Similarly, the supplier surplus is the difference between what a supplier actually charges (area $O\rho^*Cg^*$) and what a supplier is willing to charge (area $O\beta Cg^*$). Both consumers and suppliers surplus compose what is known as social welfare (area βAC).

laws, entangles their physical management. Ideally, within a power market the participants would trade power at a unique price wherever they were located in the system; the requirements of power would be met by the cheapest generation units –called the *merit-order generation*– so that the social cost incurred were the minimum. However, when the transmission system constraints limit the transactions – commonly known as *congestion* – the cheapest generation may not be used and generation out of merit-order has to be used to meet the demand or, even worse, the demand may not be entirely served. This inefficiency (relative to the non-congested case and inherent in transmission-constrained power systems [83]) may worsen the problems of market power [84]. Different sources of congestion can be identified, such as thermal limits of transmission lines and security constraints. One of the most common security constraint is the *n-1* contingency, where the dispatch has to be feasible after the loss of any transmission line [85].

Congestion Management (CM) refers to the process that is implemented so that power transactions can take place while the transmission system operates within its limits. Although congestion costs represent a small portion of the total volume of energy cost traded in the market[3], the concern must be addressed to their incidence on the market efficiency.

The goal of a CM is threefold:

- To be economically efficient so that market participants can accomplish their transactions, while the system security and the market efficiency are preserved. Ideally, this efficiency must also [87]: (i) limit the cost of congestion to exactly the cost of redispatching the congestion; (ii) give the incentives to alleviate the congestion to the lowest cost; (iii) be able to accommodate bilateral transactions; (iv) assign the cost to those who generate the congestion; and (v) give stable, predictable and known-in-advance prices.

- In the long run, to send efficient signals to encourage transmission and generation investment.

- To facilitate instruments to hedge against congestion.

[3]For instance, during the period 2000-2002 the congestion rents reported by the Pennsylvania-New Jersey-Maryland (PJM) power pool were USD M$833, meanwhile the New York ISO reported USD M$1352 [86].

Along the trend of deregulation in electric power systems, different alternatives have been used to tackle the congestion management problem, varying from physical interruptions to redispatching schemes. They have been based on nodal, zonal and, more recently, on flowgates approaches. Nonetheless, these alternatives can broadly be classified into two classes [38, 82, 88, 89]: (i)a centralized-like system, based on a Locational Marginal Pricing (LMP), which allocates the transmission within the power-market clearing process; and (ii) a decentralized-like system, based on a transaction-based trading, which manages the transmission system independently from the power market. The advocates of a centralized model assert that the power market, together with the congestion management, should be carried out by the ISO so that all the operational requirements of the system can be met. On the other hand, the advocates of a decentralized model assert that the power market must be carried out with the minimum intrusion of a central entity as possible. The ISO should have the sole function of facilitating the transmission.

The Locational Marginal Pricing (or nodal pricing) scheme is based on the theory of spot pricing [62, 90–94]. Spot pricing is the natural extension of the classical market equilibrium theory [95]. In this pricing scheme, a central entity receives voluntary bids from market participants, and, under the assumption that the submitted bids reflect true marginal costs of production and marginal benefit of consumption, a central operator selects the optimal solution via a cost minimization process, typically an Optimal Power Flow (OPF). The outcome satisfies all the transmission constraints, balancing the system at minimum cost, and obtaining Locational Marginal Prices (LMPs). If the transmission system is congested, even with a single binding transmission constraint, the LMPs can be different among each other. The LMPs reflect the locational value of power which depends not only on the generation cost, but also on the transmission system characteristics and the demand willingness to pay. In this process, the power is traded at the corresponding LMPs and all the transactions are done through the central operator.

An AC Optimal Power Flow (OPF) is one of the most useful tools in power systems. Nevertheless, due to linearity, a DC optimal power flow model eases the understanding of economical issues in congestion management; the extension of concepts for an AC model is then straightforward.

2.4 Locational Marginal Pricing

Let us consider a lossless power network where the finite set of nodes is denoted by $\mathcal{I}$ and $|\mathcal{I}|$ stands for its cardinality, and $\mathcal{K}$ stands for the set of transmission lines. The net generation output power at node i is denoted by g_i, while the net demand at node i is denoted by d_i. Let us denote the vectors of generation and demand by $\boldsymbol{g} \in \mathbb{R}^{|\mathcal{I}|}$ and $\boldsymbol{d} \in \mathbb{R}^{|\mathcal{I}|}$, respectively. We also define the vector of nodal angles by $\boldsymbol{\delta} = [\delta_1,\, \delta_2, \ldots \delta_s\,, \ldots,\, \delta_{|\mathcal{I}|}]^T$, where $\delta_s = 0$ is taken as the reference.

The social cost function, $f : \mathbb{R}^{2|\mathcal{I}|} \rightarrow \mathbb{R}$, is given by,

$$f(\boldsymbol{g}, \boldsymbol{d}) = \sum_i \big\{ c_i(g_i) - b_i(d_i) \big\}. \tag{2.7}$$

The real power balance at every system node is a function of nodal voltages and angles. Under linearity assumptions, such as constant nodal voltages and small differences among nodal angles, the nodal power balances can be defined as,

$$\boldsymbol{G\delta} = \boldsymbol{g} - \boldsymbol{d}, \tag{2.8}$$

where $\boldsymbol{G} \in \mathbb{R}^{|\mathcal{I}|x|\mathcal{I}|}$ is the *bus susceptance* matrix.

Likewise, the line power flows limits can be written as

$$\boldsymbol{H\delta} \leq \overline{\boldsymbol{z}}, \tag{2.9}$$

where $\boldsymbol{H} \in \mathbb{R}^{|\mathcal{K}|x|\mathcal{I}|}$ is the *reactance matrix* of the transmission lines; and $\overline{\boldsymbol{z}} \in \mathbb{R}^{|\mathcal{K}|}$ stands for the vector of maximum real power flow limits.

In order to focus on the transmission network issues, a feasible solution $\boldsymbol{p}^*, \boldsymbol{d}^*$ is assumed to be interior; consequently, without any loss of generality, operational limits for generation and demand levels are neglected from the analysis. Therefore, a power auction can be formulated as the following optimal linear power flow problem:

$$f^* = min \quad f(\boldsymbol{g}, \boldsymbol{d}) \tag{2.10}$$

$$s.t. \quad \boldsymbol{G\delta} = \boldsymbol{g} - \boldsymbol{d}, \tag{2.11}$$

$$\boldsymbol{H\delta} \leq \overline{\boldsymbol{z}}. \tag{2.12}$$

The problem (2.10)–(2.12), with a convex objective function and linear constraints, is a convex programming problem [96]. Its associated Lagrange function, $\pounds :\in \mathbb{R}^{4|\mathcal{I}|+|\mathcal{K}|} \rightarrow \mathbb{R}$, can be defined as

$$\mathcal{L}(\boldsymbol{g}, \boldsymbol{d}, \boldsymbol{\delta}, \boldsymbol{\rho}, \boldsymbol{\mu}) = f(\boldsymbol{g}, \boldsymbol{d}) + \boldsymbol{\rho}^T(\boldsymbol{G\delta} - \boldsymbol{g} + \boldsymbol{d}) + \boldsymbol{\mu}^T(\boldsymbol{H\delta} - \overline{\boldsymbol{z}}), \tag{2.13}$$

$$\mu_k \geq 0\,, \quad \forall\, k \in \mathcal{K}, \tag{2.14}$$

where $\boldsymbol{\rho} \in \mathbb{R}^{|\mathcal{I}|}$ and $\boldsymbol{\mu} \in \mathbb{R}^{|\mathcal{K}|}$ are the vectors of *Lagrangian multipliers* – dual variables – associated with Constraints (2.11) and (2.12), respectively.

In order for a point $\boldsymbol{p}^*$, $\boldsymbol{d}^*$ to be optimal, the following necessary conditions need to be satisfied [97]:

$$\nabla_{\boldsymbol{g}} f(\boldsymbol{g}^*, \boldsymbol{d}^*) = \boldsymbol{\rho}^*, \tag{2.15}$$

$$-\nabla_{\boldsymbol{d}} f(\boldsymbol{g}^*, \boldsymbol{d}^*) = \boldsymbol{\rho}^*, \tag{2.16}$$

$$\boldsymbol{G}^T \rho^* + \boldsymbol{H}^T \mu^* = 0, \tag{2.17}$$

$$\boldsymbol{G\delta} - \boldsymbol{g}^* + \boldsymbol{d}^* = 0, \tag{2.18}$$

$$\boldsymbol{H\delta} - \overline{\boldsymbol{z}} \leq 0, \tag{2.19}$$

$$\mu_k^* \geq 0\,, \quad \forall\, k \in \mathcal{K}, \tag{2.20}$$

$$\mu_k^*(H_k \boldsymbol{\delta} - \overline{z}_k) = 0\,, \quad \forall\, k \in \mathcal{K}. \tag{2.21}$$

Conditions (2.15) and (2.16) make suppliers and consumers fix their operation point such that their marginal cost and marginal benefit match the corresponding market price. Condition (2.18) matches generation and demand at each system node. Condition (2.19) maintains the transmission system feasibility. The remaining conditions force the market prices to agree with the optimal operation of the system, *i.e.*, to have the Lagrange multipliers as prices. With such a constraint, there will be no arbitrage opportunities [98].

The second-order optimality conditions are satisfied because of the convexity of (2.10)–(2.12); hence, the solution $\boldsymbol{g^*}, \boldsymbol{d^*}$ is a global minimizer. In the economical sense, the Lagrange multiplier ρ_i (also known as *shadow price*) associated with the power balance at the i-th node can be interpreted as the optimal price because it quantifies the cost (or value, from demand side) for supplying (or consuming) an additional MW at the i-th node of the network. Therefore, the vector of nodal prices in the system is the vector of Lagrange multipliers $\boldsymbol{\rho}^*$. On the other hand, the Lagrange multiplier μ_k associated with the power flow limit of the k-th transmission line is interpreted as the variation in social cost ($\nabla_{\overline{\boldsymbol{z}}} \mathcal{L} = -\boldsymbol{\mu}$) if the transmission capacity is relaxed; it is called *congestion multiplier*.

Nonetheless, the variable μ_k does not represent the congestion price between the nodes of the k-th transmission line. In fact, such a congestion multiplier is greater than the congestion price due to loop flows in the network; only in the case of a radial network both quantities will be equal.

Proposition 2.1: (Congestion rents) *Given the vectors $\boldsymbol{g}^*, \boldsymbol{d}^*$ that solve (2.10)–(2.12), and $\boldsymbol{\rho}^*$, $\boldsymbol{\mu}^*$ exist such that (2.15)–(2.21) hold. If congestion occurs, then the market operator collects money in excess –the net money collected from demands is higher than the money paid to suppliers.*

Proof. Let us consider optimality conditions (2.15)–(2.21); if (2.17) is multiplied by $\boldsymbol{\delta}$, and substituting (2.18) in the first term and (2.19) in the second term, one obtains

$$\langle\, \boldsymbol{\rho}^* \,,\, \boldsymbol{g}^* - \boldsymbol{d}^* \,\rangle \,+\, \langle\, \boldsymbol{\mu}^* \,,\, \boldsymbol{z}^* \,\rangle = 0. \tag{2.22}$$

where $\boldsymbol{z}^* = \boldsymbol{H}\boldsymbol{\delta}^*$. By rearranging (2.22) in terms of scalars,

$$\Upsilon = \sum_{i\in\mathcal{I}} \rho_i^* \big\{ d_i^* - g_i^* \big\} = \sum_{k\in\mathcal{K}} \mu_k^* z_k^*, \tag{2.23}$$

where Υ is the money collected by the market operator, known as congestion rents. Since $\boldsymbol{g}^*$,$\boldsymbol{d}^*$ satisfies (2.10)–(2.12), and given the vector $\boldsymbol{\mu}^*$ that satisfies the KKT conditions, by the complementary conditions (2.19)–(2.21), if $\mu_k^* > 0$, the k-th transmission line is binding ($z_k^* = \overline{z}_k$), and the system is congested; thus, the congestion rents are

$$\Upsilon = \sum_{k\in\tilde{\mathcal{K}}} \mu_k^* \, \overline{z}_k^* > 0, \tag{2.24}$$

where $\tilde{\mathcal{K}} \subset \mathcal{K}$ stands for the set of binding transmission lines; in this context, μ_k represents a congestion charge per each megawatt that flows through the k-th transmission line in congested conditions; therefore, it follows from substituting (2.23) into (2.24) that in a congested system, the net payment from consumers is greater than the net payment to suppliers, *i.e.*,

$$\sum_{i\in\mathcal{I}} \rho_i^* d_i^* > \sum_{i\in\mathcal{I}} \rho_i^* p_i^*. \tag{2.25}$$

as claimed. ■

At the solution, if no transmission constraint is binding, $\boldsymbol{\mu} = 0$ and $\Upsilon = 0$. Furthermore, (2.25) also implies a spatial price discrimination of power; otherwise, the congestion rents would always be zero.

Proposition 2.2: *The congestion rents are a welfare transfer from market participants to the ISO.*

Proof. Let the surplus of a power market be composed by the demands surplus

$$\sum_i \{b_i(d_i) - \rho_i d_i\}, \tag{2.26}$$

and the suppliers profits

$$\sum_i \{\rho_i g_i - c_i(g_i)\}. \tag{2.27}$$

Combining (2.26) and (2.27),

$$f = \sum_i \big\{b_i(d_i) - c_i(g_i)\big\} - \sum_j \big\{\rho_j d_j - \rho_j g_j\big\}. \tag{2.28}$$

By Proposition 2.1, in the absence of congestion $\sum_i \{\rho_i d_i - \rho_i g_i\} = 0$; then the social welfare is

$$f = \sum_i \big\{b_i(d_i) - c_i(g_i)\big\}. \tag{2.29}$$

However, if congestion occurs, then $\sum_i \{\rho_i d_i - \rho_i g_i\} > 0$. Hence, the welfare from all market participants (net market surplus) is lower than the net social welfare of the market; the difference is the welfare captured by the ISO. ■

From an economic point of view, transmission constraints have an analogous effect of taxes on the market efficiency.

Proposition 2.3: (Locational marginal prices) *Within a power market, if congestion occurs, then there will be locational price discrimination of power, resulting in locational marginal prices.*

Proof. Let us consider $\boldsymbol{g} \in \mathbb{R}^{|\mathcal{I}|-1}$ and $\boldsymbol{d} \in \mathbb{R}^{|\mathcal{I}|-1}$ be the reduced vectors of generation and demand, respectively; and g_s and d_s be the excluded elements which denote the generation and demand at the slack node, respectively.

The vector of maximum real power flow limits, $\overline{\boldsymbol{z}}$, for the transmission lines can be alternatively defined in terms of nodal power injections by using the power distribution factors, –see Definition A.1,

$$\boldsymbol{S}(\boldsymbol{g}-\boldsymbol{d}) \leq \overline{\boldsymbol{z}}, \tag{2.30}$$

where $\boldsymbol{S} \in \mathbb{R}^{|K|x(|\mathcal{I}|-1)}$ denotes the *sensitivity matrix*; this matrix maps nodal power changes into lines power flows.

Thus, for an interior solution, the congestion management problem can now be modelled as follows:

$$f^* = min \qquad f_s + f(\boldsymbol{g},\boldsymbol{d}) \tag{2.31}$$

$$s.t. \qquad g_s - d_s + \langle\, \boldsymbol{e}\,,\, \boldsymbol{g}-\boldsymbol{d}\,\rangle = 0, \tag{2.32}$$

$$\boldsymbol{S}(\boldsymbol{g}-\boldsymbol{d}) \leq \overline{\boldsymbol{z}}. \tag{2.33}$$

where $f_s = c_s(g_s) - b_s(d_s)$; and $\boldsymbol{e} \in \mathbb{R}^{\mathcal{I}-1}$ is an auxiliary vector of ones. Expression (2.32) stands for the power balance of the system.

The problem (2.31)–(2.33) is a convex programming problem; its Lagrangian, $\pounds : \mathbb{R}^{2|\mathcal{I}|+|\mathcal{K}|+1} \rightarrow \mathbb{R}$, can be defined as,

$$\pounds(g_s, d_s, \boldsymbol{g}, \boldsymbol{d}, \rho_s, \boldsymbol{\mu}) = f_s + f(\boldsymbol{g},\boldsymbol{d}) - \rho_s\big\{g_s - d_s + \langle\, \boldsymbol{e}\,,\, \boldsymbol{g}-\boldsymbol{d}\,\rangle\big\} - \boldsymbol{\mu}^T\big\{\boldsymbol{S}(\boldsymbol{g}-\boldsymbol{d}) - \overline{\boldsymbol{z}}\big\}, \tag{2.34}$$

$$\mu_k \leq 0\,, \quad \forall\, k \in \mathcal{K}. \tag{2.35}$$

where ρ_s is the Lagrange multiplier for the power balance of the system. The first-order optimality conditions with respect to generation are,

$$\frac{df_s}{dg_s} = \rho_s^*, \tag{2.36}$$

$$\nabla_{\boldsymbol{g}} f(\boldsymbol{g},\boldsymbol{d}) = \rho_s^* \boldsymbol{e} + \boldsymbol{S}^T \boldsymbol{\mu}^*. \tag{2.37}$$

With ρ_s being the market price at the slack node, it follows from (2.37) that the market prices at the remaining nodes are

$$\boldsymbol{\rho}^* = \rho_s^* \boldsymbol{e} + \boldsymbol{\omega}^*, \tag{2.38}$$

where $\boldsymbol{\omega}^* = \boldsymbol{S}^T \boldsymbol{\mu}^*$ is the vector of congestion prices. The congestion prices are given by the linear combination of the congestion charges from the binding transmission constraints.

The congestion prices are added to or subtracted from the energy component depending on whether power injections increase or decrease congestion. Under no congestion, $\boldsymbol{\omega} = \mathbf{0}$, and all the LMPs are equal to ρ_s^*. On the other hand, if congestion occurs, the LMPs are composed by the slack nodal price and the congestion prices. ■

2.5 Transmission Rights

Since market participants have to use the transmission in order to carry out their transactions, if congestion occurs, market participants have to pay congestion charges despite their location. These congestion charges arise from the use of scarce transmission capacity. Unfortunately, because the market outcome is not known in advance, market participants are exposed to volatile congestion charges. Therefore, the congestion charges introduce risk and discourage the transactions.

If market participants acquired *ex ante* the required transmission capacity to implement their transactions, and then entered in the power market, they would avoid the congestion charges. The transmission capacity allocated *ex ante* is known as a transmission right. A *transmission right* is the –physical or financial– right to access a transmission capacity. Such rights can be classified into three different types [61]: (i) rights for an exclusive use of the transmission capacity; (ii) rights to use the transmission capacity; and (iii) rights to collect the congestion rents.

In the short term, the goal of transmission rights is to facilitate –hedge– the transactions in the forward power market by locking a transmission usage price in order to avoid the risk of the volatile congestion charges. In the long term, the objective is to give the right signals to the market participants to motivate transmission investments [99].

In order to hedge against congestion costs, two kind of transmission rights have been proposed. The first one is based upon Financial Transmission Rights. They are purely financial as there is no physical (actual) reservation for the use of the transmission capacity. They are point-to-point rights, and are exclusively based upon the prices at selected locations for an awarded quantity of power; the holder of this kind of instruments receives a share of the congestion rents. Currently, the FTR scheme has been or is in the process to be implemented in several markets, *e.g.,* PJM, New York, Australia and New Zealand.

The second scheme for hedging is built up Flowgate Rights (FGRs); FGRs are based on the physical distribution of power flows in a transmission system. Each (bilateral) transaction is decomposed into the power flow share in every – economically significant – congested line (called flow-gate). Although FGRs are related to the physical distribution of power flows, more recently FGRs are proposed to be used as pure financial rights [39]. Thus, a market participant holding FGRs will have an associated payoff (rather than a physical priority) only when the corresponding flow-gates are binding. In this thesis work, emphasis is given to the analysis of FTRs.

An FTRs' scheme can be seen as the natural complement of the locational marginal pricing, and it is based on three requirements [62]:

(i) transmission rights must be subject to preserve the system reliability;

(ii) they must be associated with a usage pricing mechanism; and

(iii) they must be available to all market participants.

FTRs provide financial protection to their holders by reimbursing the congestion costs for a point-to-point transaction. Hence, FTRs can be seen as hedges against price risk of transmission congestion, even if there are large changes in net injections which can produce changes in flow patterns, distribution factors and LMPs. FTRs naturally comprise the power flows, dispatch constraints, contingencies, power distribution factors and non-linear relationships that define the effect of congestion. FTRs help reduce the risk in energy markets and can substantially impact the profitability of a market participant [100].

Proposition 2.4 *A participant ν of a power market, with a transaction $T_{\nu,i,j}$ between the injection node i and the withdrawal node j, can be fully hedged against congestion charges by holding an FTR, $\tau_{\nu,i,j}$.*

Proof. Given a market outcome with prices $\boldsymbol{\rho}$, ν sells $T_{\nu,i,j}$ at price ρ_i and buys $T_{\nu,i,j}$ at price ρ_j. Disregarding losses, the congestion charge that ν pays for its transaction is defined as the price difference between trading locations,

$$\Psi_\nu = T_{\nu,i,j}(\rho_j - \rho_i). \tag{2.39}$$

Recalling the generation and congestion components of LMPs, (2.39) becomes,

$$\Psi_\nu = T_{\nu,i,j}(\omega_j - \omega_i). \tag{2.40}$$

Thus, the transmission cost for a point-to-point transaction, in the presence of congestion, is the difference in congestion components between the trading locations, regardless of the choice of the slack node. If a transaction contributes to congestion, $\Psi_\nu > 0$ is a charge for ν; in contrast, if a transaction relieves congestion, $\Psi_\nu < 0$ is a payment to ν.

If ν holds an FTR, $\tau_{\nu,i,j}$, between nodes i and j, it will receive the following payoff:

$$\Pi_\nu = \tau_{\nu,i,j}(\rho_j - \rho_i) = \tau_{\nu,i,j}(\omega_j - \omega_i). \tag{2.41}$$

A full hedge occurs when $\tau_{\nu,i,j} \geq T_{\nu,i,j}$, such that the reimbursement Π_ν cancels out the congestion charges Ψ_ν, regardless of the power market outcomes. ■

The transaction between nodes i and j can be represented as the superposition of two transactions: i) one transaction between the i and slack nodes, and ii) the other between the slack and j nodes,

$$\Psi_\nu = T_{\nu,i,j}\big[(\rho_s - \rho_i) + (\rho_j - \rho_s)\big]. \tag{2.42}$$

This implies that ν will be paid ω_i $\$/MWh$ to inject 1 MW to the slack node from its generator placed at i. On the other side, ν will be charged ω_j $\$/MWh$ to withdraw that 1 MW from the slack node to node j. Moreover, the congestion rents collected by the system operator from congestion are the money used to honor the FTRs.

Although FTR formulations can be extended to include losses [67, 101], so far the implementation of FTR markets has been focused on congestion. Three underlaying features of FTRs that make them an attractive alternative for hedging are:

- They are only related to LMPs – injection and withdrawal points– and not to particular lines and flows; thus, the physical reality of the network (say, transmission configuration and operational limits) is implicitly included. This also implies that the FTRs' holders do not have an exclusive use of the transmission system.

- Since FTRs are financial instruments, the holder collects the payments from the contract. This allows to achieve an efficient dispatch of the system because the FTRs are independently traded from the energy market. Therefore, FTRs do not distort the locational marginal prices [65].

- As FTRs hedge against congestion cost, the system operator does not have to socialize costs when the system conditions change.

However, FTRs have drawbacks such as:

- Although FTRs are theoretically perfect to hedge congestion, they do not completely eliminate the trading risk [102]. The holders acquire FTRs based on expectations of future system conditions and transmission congestions; but there is not a complete guarantee that such conditions take place.

- The allocation of FTRs can be unequitable. There may be participants that are completely exposed to congestion. For equity, the FTRs should be allocated to all network participants in proportion to their market shares. In [103, 104], a scheme for the equitably allocation of congestion rents is proposed. This method is based on the actual contribution that each market participant has on the market outcome.

- The main problem of having a price per location is to evaluate the set of FTRs combinations in order to compare the costs of different supply options and to find the most convenient one. Given a transmission system with n nodes, there are $n(n-1)$ potential FTRs. This drawback worsens off the liquidity and price certainty of FTRs [105]. Such a liquidity may be improved as FTRs auctions are enhanced [106].

- FTRs can be used for gaming strategies. Since FTRs represent another money stream, some market participants holding FTRs may have an extra incentive for strategic behavior [22, 59, 107, 108].

- The issuance and reconfiguration of FTRs have to be subject to a simultaneous feasibility test, which has to be centrally done. This makes difficult the trading of FTRs in order to hedge the changing needs of market participants.

Chapter 3

An Oligopolistic Model for Power Networks

A model for oligopolistic competition in electricity markets is presented in this chapter. Most previous proposed models have been static, and they focus only on the energy market incentives for strategic behaviour. In a step further, in this proposed model, inter-temporal constraints, the spinning reserve market and the ownership of FTRs are taken into account. Under such considerations, the competition among participants is modelled with more realism. Thus, potential of market power can be analyzed by considering the incentives that generators may have from different activities to behave strategically. Competition in the energy, transmission and SR markets is modelled by means of conjectured functions. The resulting equilibrium problem is modelled in terms of complementarity conditions. The proposed model is illustrated by two- and six-node power systems, and then extended to 54- and 118-node systems using a DC-network approximation.

Preliminary definitions are given in §3.1. In §3.2, an integrated market for energy and spinning reserve is described, and its equilibrium model is derived. Numerical examples are given in §3.3 and 3.4. The inclusion of FTRs in the profit-maximization problem of suppliers is presented in §3.5. Final remarks are given in §3.6.

3.1 Definitions

The following definitions are introduced as preliminary to the description of the model presented in this chapter.

Definition 3.1 (Complementarity conditions) Given the following primal problem:

$$\begin{aligned} \max_{\boldsymbol{x},\boldsymbol{y}} \;\; & f(\boldsymbol{x},\boldsymbol{y}) \\ s.t. \;\; & \boldsymbol{h}(\boldsymbol{x},\boldsymbol{y}) = \boldsymbol{0}, \\ & \boldsymbol{g}(\boldsymbol{x},\boldsymbol{y}) \leq \boldsymbol{0}. \end{aligned} \tag{3.1}$$

where $[\boldsymbol{x^T} \; \boldsymbol{y^T}]^T \in \mathbb{R}^n$, with $\boldsymbol{x} \geq \boldsymbol{0}$ and $\boldsymbol{y}$ unconstrained; $f : \mathbb{R}^n \rightarrow \mathbb{R}$ is the objective function; $\boldsymbol{h} : \mathbb{R}^n \rightarrow \mathbb{R}^m$ and $\boldsymbol{g} : \mathbb{R}^n \rightarrow \mathbb{R}^p$ are the vectors of constraints.

Let the Lagrangian function $\pounds : \mathbb{R}^{n+m+p} \rightarrow \mathbb{R}$ be defined by

$$\begin{aligned} \pounds(\boldsymbol{x},\boldsymbol{y},\boldsymbol{\lambda},\boldsymbol{\mu}) =& f(\boldsymbol{x},\boldsymbol{y}) - \boldsymbol{\lambda}^T\boldsymbol{h}(\boldsymbol{x},\boldsymbol{y}) - \boldsymbol{\mu}^T\boldsymbol{g}(\boldsymbol{x},\boldsymbol{y}), \\ & \mu_j \geq 0, \quad \forall j, \\ & \lambda_i \quad free, \quad \forall i, \end{aligned} \tag{3.2}$$

where $\boldsymbol{\lambda} \in \mathbb{R}^m$ and $\boldsymbol{\mu} \in \mathbb{R}^p$ are the vectors of Lagrange multipliers –dual variables– associated with $\boldsymbol{h}(\boldsymbol{x},\boldsymbol{y})$ and $\boldsymbol{g}(\boldsymbol{x},\boldsymbol{y})$, respectively.

Thus, the first-order optimality necessary –KKT– conditions are [96],

$$\nabla_{\boldsymbol{x}} f(\boldsymbol{x},\boldsymbol{y}) - \nabla_{\boldsymbol{x}}^T\boldsymbol{h}(\boldsymbol{x},\boldsymbol{y})\boldsymbol{\lambda} - \nabla_{\boldsymbol{x}}^T\boldsymbol{g}(\boldsymbol{x},\boldsymbol{y})\boldsymbol{\mu} \leq \boldsymbol{0}; \quad \boldsymbol{x} \geq \boldsymbol{0}, \tag{3.3}$$

$$x_k\Big(\frac{\partial f(\boldsymbol{x},\boldsymbol{y})}{\partial x_k} - \sum_i \lambda_i \frac{\partial h_i(\boldsymbol{x},\boldsymbol{y})}{\partial x_k} - \sum_j \mu_j \frac{\partial g_j(\boldsymbol{x},\boldsymbol{y})}{\partial x_k}\Big) = 0, \quad \forall k, \tag{3.4}$$

$$\nabla_{\boldsymbol{y}} f(\boldsymbol{x},\boldsymbol{y}) - \nabla_{\boldsymbol{y}}^T\boldsymbol{h}(\boldsymbol{x},\boldsymbol{y})\boldsymbol{\lambda} - \nabla_{\boldsymbol{y}}^T\boldsymbol{g}(\boldsymbol{x},\boldsymbol{y})\boldsymbol{\mu} = \boldsymbol{0}; \quad \boldsymbol{y} \quad free, \tag{3.5}$$

$$\boldsymbol{h}(\boldsymbol{x},\boldsymbol{y}) = \boldsymbol{0}; \quad \boldsymbol{\lambda} \quad free, \tag{3.6}$$

$$\boldsymbol{g}(\boldsymbol{x},\boldsymbol{y}) \leq \boldsymbol{0}; \quad \boldsymbol{\mu} \geq \boldsymbol{0}, \tag{3.7}$$

$$\mu_j g_j(\boldsymbol{x},\boldsymbol{y}) = 0, \quad \forall j. \tag{3.8}$$

Because of the nonnegativity condition of $\boldsymbol{x}$ and $\boldsymbol{\mu}$, their corresponding KKT equations are defined by complementarity conditions. In order to compactly denote such conditions, the symbol $\perp$ is henceforth used in this thesis. Thus, the first-order optimality conditions of (3.1) can be characterized as a complementarity problem, *i.e.*,

$$\mathbf{0} \geq \nabla_{\boldsymbol{x}} f(\boldsymbol{x},\boldsymbol{y}) - \nabla_{\boldsymbol{x}}^T \boldsymbol{h}(\boldsymbol{x},\boldsymbol{y})\boldsymbol{\lambda} - \nabla_{\boldsymbol{x}}^T \boldsymbol{g}(\boldsymbol{x},\boldsymbol{y})\boldsymbol{\mu} \quad \perp \quad \boldsymbol{x} \geq \mathbf{0}, \tag{3.9}$$

$$\mathbf{0} \geq \boldsymbol{g}(\boldsymbol{x},\boldsymbol{y}) \quad \perp \quad \boldsymbol{\mu} \geq \mathbf{0}, \tag{3.10}$$

$$\mathbf{0} = \nabla_{\boldsymbol{y}} f(\boldsymbol{x},\boldsymbol{y}) - \nabla_{\boldsymbol{y}}^T \boldsymbol{h}(\boldsymbol{x},\boldsymbol{y})\boldsymbol{\lambda} - \nabla_{\boldsymbol{y}}^T \boldsymbol{g}(\boldsymbol{x},\boldsymbol{y})\boldsymbol{\mu}; \qquad \boldsymbol{y} \quad free, \tag{3.11}$$

$$\mathbf{0} = \boldsymbol{h}(\boldsymbol{x},\boldsymbol{y}); \qquad \boldsymbol{\lambda} \quad free. \tag{3.12}$$

In standard notation, (3.9)–(3.12) can be written as,

$$\mathbf{0} \leq \boldsymbol{m}(\boldsymbol{x},\boldsymbol{y}) \quad \perp \quad \boldsymbol{x} \geq \mathbf{0}, \tag{3.13}$$

$$\mathbf{0} = \boldsymbol{q}(\boldsymbol{x},\boldsymbol{y}); \qquad \boldsymbol{y} \quad free. \tag{3.14}$$

As long as the functions $\boldsymbol{m}(\boldsymbol{x},\boldsymbol{y})$ and $\boldsymbol{q}(\boldsymbol{x},\boldsymbol{y})$ are affine, (3.13)–(3.14) is a Mixed Linear Complementarity Problem (MLCP) [109].

Definition 3.2 (Conjectured supply function) Given the ν-th GenCo, with a set $\mathcal{H}_i$ of generation units placed at node i, the power output of its generation unit $h \in \mathcal{H}_i$, is represented by $g_{\nu,h,i}$. Thus, the net power generation by GenCo ν at node i is $\sum_h g_{\nu,h,i}$. On the other hand, the net power supplied (sales) by ν at node i is denoted by $d_{\nu,i}$.

Let the demand at node i, d_i, be satisfied by both GenCos and ISO (by means of the arbitraged power a_i), *i.e.*, $d_i = \sum_\nu d_{\nu,i} + a_i$. As the power supplied by GenCos is composed by the own power of ν ($d_{\nu,i}$) and its rivals power ($\sum_{f\neq\nu} d_{f,i} = d_{-\nu,i}$), the demand at node i is $d_i = d_{\nu,i} + d_{-\nu,i} + a_i$.

Under a Cournot strategy, GenCo ν conjectures that rivals will not react to its actions, and, consequently, the level of power supplied by its rivals is taken as an exogenous variable[1] into its profit maximization problem, $d_{-\nu,i} = d^*_{-\nu,i}$.

In a step further, a general conjecture function can be used to model the rivals' response to the strategy of ν, *e.g.*, to quantify how ν expects that changes in nodal prices alter the

[1]Exogenous variables will be denoted by an asterisk, such as $d^*_{-\nu,i}$. Even though they are fixed into participants subproblems, within the equilibrium problem as a whole, they are variables that will actually match the equilibrium values.

level of power supplied by its rivals [18], *i.e.*,

$$d_{-\nu,i} = d^*_{-\nu,i} + (\rho_{\nu,i} - \rho_i^{*t})B_{-\nu,i}, \quad \forall i \in \mathcal{I}, \tag{3.15}$$

where $\rho_{\nu,i}$ is the price, at node i, seen by ν; however, at equilibrium all participants will see the same price ρ_i^*. The rivals supply response, conjectured by ν, is denoted by the parameter $B_{-\nu,i}$. The higher the parameter $B_{-\nu,i}$, the stronger the rivals response when ν attempts to jack up the price, and, consequently, the more competitive the market. For the standard Cournot outcome, $B_{-\nu,i} \equiv 0$.

Definition 3.3 (Conjectured reserve-price function) Let $r_{\nu,h,i}$ be the spinning reserve from unit h, at node i, owned by GenCo ν, and, thus, $\sum_{h,i} r_{\nu,h,i}$ be the net spinning reserve to be provided by GenCo ν. A conjectured reserve-price function can be defined as [48],

$$\eta_\nu = \eta^* + \Big(\sum_{h,i} r_{\nu,h,i} - \sum_{\hbar,j} r^*_{\nu,\hbar,j}\Big)C_\nu, \tag{3.16}$$

where η_ν and η^* stand for the spinning reserve price seen by ν, and the spinning reserve price at equilibrium, respectively. The SR price function (3.16) allows one to model the ability of ν to influence the spinning reserve prices; *i.e.*, how ν expects the SR price varies if ν modifies its provision of reserve. For the standard competitive market outcome $C_\nu \equiv 0$.

All the conjectured functions described *supra* can be seen as a first-order approximation of ν's expectations at the equilibrium point. The values of $B_{-\nu,i}$ and C_ν can be parametrically varied to analyze the potential impact of different degrees of price manipulation by GenCos.

3.2 An Integrated Market for Energy and Spinning Reserve

A spatial and dynamic oligopoly model for an integrated electricity market is formulated in this section. In this model, GenCos can behave strategically to influence both the energy and spinning reserve prices. A lossless DC model is used to consider the transmission system. The market is modelled with one maximization problem per GenCo, one maximization problem for the ISO, and a set of market clearing conditions to meet the demand requirements of energy, transmission and spinning reserve. The model also includes temporal constraints; however, commitment decisions, such as those arising from modelling startup cost, are not considered into the model since it would require the use of binary variables [110]. Such kind of variables cannot be used in the standard complementarity model. The proposed model is an extension of the equilibrium framework suggested in [15].

3.2.1 GenCo problem

For a GenCo participating in an integrated market for energy and spinning reserve, its objective is to maximize profits from both activities. In an oligopolistic market, a GenCo has also to consider its rivals' decisions within its optimization problem. Its decision variables are the generation, spinning reserve and sale levels. Hence , the profit-maximization problem for GenCo ν can be mathematically stated as,

$$\begin{aligned} \max \;\; & \Pi_\nu = \Pi_\nu^G + \Pi_\nu^R \\ s.t. \;\; & \boldsymbol{h} = \boldsymbol{0}, \\ & \boldsymbol{g} \leq \boldsymbol{0}. \end{aligned} \tag{3.17}$$

The profit maximization is subject to supply constraints specifications $\boldsymbol{h}$ and $\boldsymbol{g}$. Each component of problem (3.17) is next described.

Profit components

Each GenCo can have money streams from the energy and spinning reserve markets.

- The profit from participating in the energy market is given by,

$$\Pi_\nu^G = \sum_t \Big\{ \sum_j \big(\rho_{\nu,j}^t - \omega_j^{*t}\big) d_{\nu,j}^t - \sum_{h,i} (\beta_{\nu,h,i} + \gamma_{\nu,h,i} g_{\nu,h,i}^t - \omega_i^{*t}) g_{\nu,h,i}^t \Big\}. \tag{3.18}$$

The first term stands for the revenue from selling power throughout the system's nodes at the corresponding locational price $\rho_{\nu,j}^t$, across all time periods t. The second term is the cost for supplying power from all the generation units. The generation cost is denoted by $(\beta_{\nu,h,i} + \gamma_{\nu,h,i} g_{\nu,h,i}^t) g_{\nu,h,i}^t$, where $\beta_{\nu,h,i}$ and $\gamma_{\nu,h,i}$ are constants; hence, the marginal cost is $\beta_{\nu,h,i} + 2\gamma_{\nu,h,i} g_{\nu,h,i}^t$. In order to implement their transactions, GenCos have to pay the associated congestion costs. Both revenues and costs are affected by the congestion costs.

On one side, ν minimizes the congestion charges $(\omega_j^{*t} d_{\nu,j}^t)$ that it has to pay for withdrawing power at the sale point; on the other side, ν maximizes the congestion charges $(\sum_h \omega_i^{*t} g_{\nu,h,i}^t)$ that it receives by injecting power at nodes where it has generation units –see Proof 2.4.

Under oligopolistic competition, the market prices $(\rho_{\nu,j}^t, \forall j, t)$ are considered endogenous variables. These prices are given by the inverse demand function,

$$\rho_{\nu,i}^t = \rho_{oi}^t - \delta_i^t (d_{\nu,i}^t + d_{-\nu,i}^t + a_i^{*t}). \tag{3.19}$$

By introducing the conjectured supply function (3.15) into (3.19), the price function becomes

$$\rho_{\nu,i}^t = \frac{\rho_{oi}^t - \delta_i^t (d_{\nu,i}^t + d_{-\nu,i}^{*t} - B_{-\nu,i} \rho_i^{*t} + a_i^{*t})}{1 + B_{-\nu,i} \delta_i^t}. \tag{3.20}$$

Thus, considering the market prices, ν adjusts its generation and sale levels according to its conjectures of rivals responses. Nonetheless, the congestion prices $(\omega_i^{*t}, \forall i, t)$ are taken as fixed within each GenCo problem. This assumption implies a competitive behaviour of GenCos in the transmission market. In addition, each GenCo takes the arbitrage power $(a_i^{*t}, \forall i, t)$ as exogenous too; this means that GenCos believe that they cannot influence the arbitrage of power.

• A second money stream comes from participating in the market for spinning reserve; the profit is given by the difference between the revenue from selling reserve at the SR market price and the cost of providing such a reserve,

$$\Pi_\nu^R = \sum_{t,h,i} (\eta_\nu^t - \alpha_{\nu,h,i}^t) r_{\nu,h,i}^t, \tag{3.21}$$

where $\alpha_{\nu,h,i}^t$ stands for the marginal SR cost. Since the SR price, η_ν^t, is taken as an endogenous variable into the maximization problem, ν will adjust its SR levels based upon its belief of how it can alter the SR price –see *supra* Definition 3.3.

Constraints

The maximization of the GenCo profit is subject to different constraints, such as:

• Power balance. This constraint makes a GenCo produce, by its own, all the power it sells; *i.e.*, this constraint avoids arbitrage from ν in the energy market,

$$\sum_i d_{\nu,i}^t - \sum_{h,j} g_{\nu,h,j}^t = 0, \quad \forall t \in \mathcal{T}. \tag{3.22}$$

• Conjectured functions for generation and reserve-price responses,

$$-(\rho^{t}_{\nu,i} - \rho^{*t}_{i})B_{-\nu,i} + d^{t}_{-\nu,i} - d^{t^*}_{-\nu,i} = 0, \quad \forall i \in \mathcal{I}, \quad t \in \mathcal{T}, \tag{3.23}$$

$$-\Big(\sum_{h,i} r^{t}_{\nu,h,i} - \sum_{h,j} r^{*t}_{\nu,h,j}\Big)C_{\nu} + \eta^{t}_{\nu} - \eta^{*t} = 0, \quad t \in \mathcal{T}. \tag{3.24}$$

Both conjectured supply and reserve-price response functions are used to substitute $d^{t}_{-\nu,i}$ and η^{t}_{ν} into the profit functions Π^{G}_{ν} and Π^{R}_{ν}, respectively; hence, (3.23) and (3.24) are not explicitly used in the model as constraints, and no dual variables are associated to them.

• Generation, spinning reserve and sale limits. These constraints follow the classical limits of capacity, and also a maximum limit to provide spinning reserve,

$$g^{t}_{\nu,h,i} + r^{t}_{\nu,h,i} - \overline{g}_{\nu,h,i} \leq 0, \quad \forall i \in \mathcal{I},\, h \in \mathcal{H}_i,\, t \in \mathcal{T}, \tag{3.25}$$

$$\underline{g}_{\nu,h,i} - g^{t}_{\nu,h,i} \leq 0, \quad \forall i \in \mathcal{I},\, h \in \mathcal{H}_i,\, t \in \mathcal{T}, \tag{3.26}$$

$$r^{t}_{\nu,h,i} - \overline{r}^{t}_{\nu,h,i} \leq 0, \quad \forall i \in \mathcal{I},\, h \in \mathcal{H}_i,\, t \in \mathcal{T}, \tag{3.27}$$

$$r^{t}_{\nu,h,i},\, d^{t}_{\upsilon,i} \geq 0, \quad \forall i \in \mathcal{I},\, h \in \mathcal{H}_i,\, t \in \mathcal{T}, \tag{3.28}$$

where $(\bar{\cdot})$ and $(\underline{\cdot})$ stand for the maximum and minimum limits.

• Up- and down-ramp rates. These constraints represent the physical limitations of the thermal units to increase/decrease their levels of output. These constraints couple together every period with the previous and following ones,

$$g^{t}_{\nu,h,i} - g^{t-1}_{\nu,h,i} \leq \overline{\Delta} g_{\nu,h,i}, \quad \forall i \in \mathcal{I},\, h \in \mathcal{H}_i,\, t \in \mathcal{T}, \tag{3.29}$$

$$g^{t-1}_{\nu,h,i} - g^{t}_{\nu,h,i} \leq \underline{\Delta} g_{\nu,h,i}, \quad \forall i \in \mathcal{I},\, h \in \mathcal{H}_i,\, t \in \mathcal{T}, \tag{3.30}$$

where $\overline{\Delta} g_{\nu,h,i}$ and $\underline{\Delta} g_{\nu,h,i}$ are the maximum up and down limits, respectively.

• Maximum energy supplied in the trading period. There can be energy-limited generators due to operating conditions, *e.g.*, fuel or environmental constraints. Thus, there is a limit on the net amount of energy that a generation unit can produce in all the trading horizon,

$$\sum_{t} g^{t}_{\nu,h,i} \leq \overline{e}_{\nu,h,i}, \ \forall i \in \mathcal{I},\, h \in \mathcal{H}_i, \tag{3.31}$$

where $\overline{e}_{\nu,h,i}$ is the maximum energy limit of unit h. In this work, hydro-electric generators are not considered.

The Lagrangian for the problem (3.18), (3.21)–(3.31) can be written as,

$$\pounds = \sum_{t} \Bigg\{ \sum_{j} \Bigg[\frac{\rho^t_{oj} - \delta^t_j (d^t_{\nu,j} + d^{*t}_{-\nu,j} - B_{-\nu,j}\rho^{*t}_j + a^{*t}_j)}{1 + B_{-\nu,j}\delta^t_j} - \omega^{t^*}_j \Bigg] d^t_{\nu,j} - \sum_{h,i} \Bigg[(\beta_{\nu,h,i} + \gamma_{\nu,h,i} g^t_{\nu,h,i} - \omega^{t^*}_i) g^t_{\nu,h,i} - \Big[\eta^* + \Big(\sum_{\hbar,\jmath} r_{\nu,\hbar,\jmath} - \sum_{\hbar,j} r^*_{\nu,\hbar,j}\Big) C_\nu - \alpha^t_{\nu,h,i}\Big] r^t_{\nu,h,i} + \overline{\mu}^t_{\nu,h,i}\big(g^t_{\nu,h,i} + r^t_{\nu,h,i} - \overline{g}^t_{\nu,h,i}\big) + \underline{\mu}^t_{\nu,h,i}\big(\underline{g}_{\nu,h,i} - g^t_{\nu,h,i}\big) + \phi^t_{\nu,h,i}(r^t_{\nu,h,i} - \overline{r}_{\nu,h,i}) + \overline{\psi}^t_{\nu,h,i}\big(g^t_{\nu,h,i} - g^{t-1}_{\nu,h,i} - \overline{\Delta} g_{\nu,h,i}\big) + \underline{\psi}^t_{\nu,h,i}\big(g^{t-1}_{\nu,h,i} - g^t_{\nu,h,i} - \underline{\Delta} g_{\nu,h,i}\big) \Bigg] - \lambda^t_\nu \big(\sum_i d^t_{\nu,i} - \sum_{j,h} g^t_{\nu,h,i}\big) \Bigg\} - \sum_{h,j} \sigma^t_{\nu,h,j}\big(\sum_t g^t_{\nu,h,j} - \overline{e}_{\nu,h,j}\big), \tag{3.32}$$

$$d^t_{\nu,i},\ r^t_{\nu,h,i},\ \overline{\mu}^t_{\nu,h,i},\ \underline{\mu}^t_{\nu,h,i},\ \phi^t_{\nu,h,i},\ \overline{\psi}^t_{\nu,h,i},\ \underline{\psi}^t_{\nu,h,i},\ \sigma^t_{\nu,h,i} \geq 0,\ \forall i \in \mathcal{I},\ h \in \mathcal{H}_i,\ t \in \mathcal{T},$$

$$g_{\nu,h,i},\ \lambda^t_\nu,\quad free,\quad \forall i \in \mathcal{I},\ h \in \mathcal{H}_i,\ t \in \mathcal{T},$$

where λ^t_ν, $\overline{\mu}^t_{\nu,h,i}$, $\underline{\mu}^t_{\nu,h,i}$, $\phi^t_{\nu,h,i}$, $\sigma^t_{\nu,h,i}$, $\overline{\psi}^t_{\nu,h,i}$, $\underline{\psi}^t_{\nu,h,i}$ are the corresponding dual variables associated with the GenCos' constraints described *supra*.

3.2.2 ISO problem

The ISO objective is to efficiently allocate the scarce transmission among GenCos. This problem can be seen as a transmission rights auction where the ISO sells transmission rights so that GenCos can implement their bilateral transactions. These transmission rights can be seen as reservations to inject/withdraw power among nodes. It is required that balanced transmission rights be sold to each GenCo for their bilateral transactions. Furthermore, the ISO carries out locational arbitrage of power in order to eliminate any price difference that is beyond transmission costs. Consequently, the arbitrage leads the transmission price between two nodes to be the price difference between such nodes [18]; *i.e.*, to obtain LMPs. The ISO's objective can be casted as,

$$max \sum_{t,i} \Big[\omega^{*t}_i p^t_i + (\rho^{*t}_i - \omega^{*t}_i) a^t_i\Big] \tag{3.33}$$

where p^t_i, $\forall i, t$ stands for the nodal power injections to be determined by the ISO. Likewise the GenCos problem, the congestion prices are considered exogenous variables within the ISO problem in order to have the ISO behave competitively. On the other hand, the ISO's decisions (nodal injections) have to be feasible for the transmission system constraints.

These constraints are defined by the power flow equations[2],

$$\Big|\sum_i s_{i,k} p_i^t\Big| - \overline{z}_k \leq 0, \quad \forall k \in \mathcal{K},\, t \in \mathcal{T}, \tag{3.34}$$

where $s_{i,k}$ denotes the sensitivity factor for transmission line k with respect to an injection at node i (see Definition A.1, Appendix A); while $\overline{z}_k$ stands for the maximum power flow limit of line k.

All the power that the ISO buys has to be sold, *i.e.*, the ISO has to be only an arbitrager,

$$\sum_i a_i^t = 0, \quad \forall t \in \mathcal{T}. \tag{3.35}$$

For the ISO problem, the variables p_i^t and a_i^t, $\forall i, t$ are unconstrained.

Thus, the Lagrangian for the problem (3.33)–(3.35) is

$$\begin{aligned} \pounds = & \sum_t \Big\{ \sum_i \Big[\omega_i^{*t} p_i^t + (\rho_i^{*t} - \omega_i^{*t}) a_i^t\Big] - \rho_s^t \sum_i a_i^t - \\ & \sum_k \Big[\lambda_k^{t+}\big(\sum_i s_{i,k} p_i^t - \overline{z}_k\big) + \lambda_k^{t-}\big(-\sum_i s_{i,k} p_i^t - \overline{z}_k\big)\Big]\Big\}, \\ & \lambda_k^{t+},\ \lambda_k^{t-} \quad \geq 0, \quad \forall k \in \mathcal{K},\, t \in \mathcal{T}, \\ & p_i^t,\ a_i^t,\ \rho_s^t, \quad free, \forall i \in \mathcal{I},\, t \in \mathcal{T}, \end{aligned} \tag{3.36}$$

where λ_k^{t+} and λ_k^{t-}, and ρ_s^t stand for the dual variables associated with constraints (3.34) and (3.35), respectively.

3.2.3 Markets clearing conditions

In addition to the GenCos and ISO problems, there need be a set of conditions for clearing the markets. These conditions will force to balance demand and supply of energy, transmission and spinning reserves; they are next described.

Energy services

This condition establishes that, under equilibrium, the price ($\rho_{\nu,i}^t$) assumed by every GenCo is the actual market price, $\rho_i^{t^*}$; this constraint matches supply and demand of energy,

$$\rho_i^{*t} = \rho_{oi}^t - \delta_i^t\big(d_{\nu,i}^t + d_{-\nu,i}^t + a_i^t\big), \quad \forall i \in \mathcal{I},\, t \in \mathcal{T}. \tag{3.37}$$

[2]Each transmission line flow is modelled by means of two constraints; one constraint per power-flow direction.

Transmission services

This market condition is related to the injections/withdrawals of power in the transmission system, such that the nodal power balance is preserved at each system node; *i.e.*, it matches supply of transmission by the ISO (p_i^t), and demand of transmission from GenCos and arbitrage,

$$p_i^t = a_i^t + \sum_{\nu} \big(d_{\nu,i}^t - \sum_{h} g_{\nu,h,i}^t \big), \quad \forall i \in \mathcal{I},\, t \in \mathcal{T}. \tag{3.38}$$

Reserve services

This condition establishes that the SR requirement, $\overline{r}^t$, of the system must be satisfied, *i.e.*,

$$\overline{r}^t \leq \sum_{\nu,h,i} r_{\nu,h,i}^t, \quad \forall t \in \mathcal{T}. \tag{3.39}$$

Such a level is based upon the ISO demand forecast and other operating conditions [40]. The spinning reserve can be set equal to some percent of the demand, as in the Spanish system [111]; or equal to the largest loss of power due to a single –generator or line– contingency, as in the Ontario system [112]. Spinning reserve can also be provided by loads that can decrease their consumption [113–115]. More sophisticated approaches can be used to set the reserve requirements with either a probabilistic criterion [116], or a demand function for reserve [117]. In this proposed model, the SR requirement is defined as a percent (ε) of the demand; it can be modelled as a given and fixed value, or can be implicity computed within the equilibrium model. For the latter, one has

$$\overline{r}^t = \varepsilon \sum_{\nu,i} \big(d_{\nu,i}^t + a_i^t \big), \quad \forall t \in \mathcal{T}. \tag{3.40}$$

3.2.4 Equilibrium model

Let $\boldsymbol{x}_\nu \in \mathcal{X}_\nu$ be a strategy of GenCo ν; this strategy is composed by the decision variables of ν,

$$\boldsymbol{x}_\nu = \begin{pmatrix} g_{\nu,h,i}^t, & \forall h \in \mathcal{H}_i,\, i \in \mathcal{I},\, t \in \mathcal{T} \\ d_{\nu,i}^t, & \forall i \in \mathcal{I},\, t \in \mathcal{T} \\ r_{\nu,h,i}^t, & \forall h \in \mathcal{H}_i,\, i \in \mathcal{I},\, t \in \mathcal{T} \end{pmatrix}.$$

Let $\boldsymbol{x}_{-\nu} = \{\boldsymbol{x}_\imath | \ \imath \neq \nu\}$ be a vector of decision variables but ν. GenCo ν, by means of the conjectured functions described *supra*, is conjecturing that the other GenCos will react in certain way, *i.e.*, $\boldsymbol{x}_{-\nu}(\boldsymbol{x}_\nu)$.

Also let $\boldsymbol{y} \in \mathcal{Y}$ be a strategy of the ISO, which is composed by its decision variables,

$$\boldsymbol{y} = \begin{pmatrix} p_i^t, & \forall i \in \mathcal{I},\ t \in \mathcal{T} \\ a_i^t, & \forall i \in \mathcal{I},\ t \in \mathcal{T} \end{pmatrix},$$

and $\boldsymbol{o} \in \mathcal{O}$ be a vector of markets prices.

$$\boldsymbol{o} = \begin{pmatrix} \rho_i^t, & \forall i \in \mathcal{I},\ t \in \mathcal{T} \\ \omega_i^t, & \forall i \in \mathcal{I},\ t \in \mathcal{T} \\ \eta^t, & \forall i \in \mathcal{I},\ t \in \mathcal{T} \end{pmatrix}.$$

Definition 3.4 (Market equilibrium) The point $(\boldsymbol{x}_\nu^*,\ \forall \nu \in \mathcal{V},\ \boldsymbol{y}^*,\ \boldsymbol{o}^*)$ constitutes an equilibrium of the electricity market if and only if

(i) $\Pi_\nu(\boldsymbol{x}_\nu^*, \boldsymbol{x}_{-\nu}^*(\boldsymbol{x}_\nu^*)) \geq \Pi_\nu(\boldsymbol{x}_\nu, \boldsymbol{x}_{-\nu}(\boldsymbol{x}_\nu))$, $\forall \boldsymbol{x}_\nu \in \mathcal{X}_\nu$, $\nu \in \mathcal{V}$ with market prices $\boldsymbol{o}^*$; *i.e.*, $\boldsymbol{x}_\nu^*$ solves (3.17) for all ν;

(ii) $\Pi_{ISO}(\boldsymbol{y}^*) \geq \Pi_{ISO}(\boldsymbol{y})$, $\forall \boldsymbol{y} \in \mathcal{Y}$ for prices $\boldsymbol{o}^*$; *i.e.*, $\boldsymbol{y}^*$ solves (3.33)–(3.35);

(iii) the vectors $\boldsymbol{x}_\nu^*$ and $\boldsymbol{y}^*$ balance the supply and demand for energy, transmission and spinning reserve at markets prices $\boldsymbol{o}^*$; *i.e.*, $\boldsymbol{x}_\nu^*$ and $\boldsymbol{y}^*$ satisfy (3.37)–(3.39).

As each GenCo and ISO problem can be defined via KKT optimality conditions, an equilibrium will be a point that simultaneously satisfies the first-order optimality conditions of all market participants while balancing the markets.

By Definition 3.1, it follows that the KKT conditions of each GenCo and ISO problem is a MLCP; this set of MLCPs together with the market clearing conditions can be written as the following and single MLCP:

- For $d_{\nu,i}^t, \quad \forall i \in \mathcal{I},\ \nu \in \mathcal{V},\ t \in \mathcal{T}$:

$$0 \geq \frac{\rho_{oi}^t - \delta_i^t(2d_{\nu,i}^t + d_{-\nu,i}^{*t} - B_{-\nu,i}\rho_i^* + a_i^{*t})}{1 + B_{-\nu,i}\delta_i^t} - \omega_i^{*t} - \lambda_\nu^t \quad \perp \quad d_{\nu,i}^t \geq 0 \tag{3.41}$$

- For $r^t_{\nu,h,i}$, $\forall i \in \mathcal{I},\ h \in \mathcal{H}_i,\ \nu \in \mathcal{V},\ t \in \mathcal{T}$:

$$0 \geq \eta^{*t} + C_\nu \sum_{\hbar,\jmath} r^t_{\nu,\hbar,\jmath} - \alpha^t_{\nu,h,i} - \overline{\mu}^t_{\nu,h,i} - \phi^t_{\nu,h,i} \quad \perp \quad r^t_{\nu,h,i} \geq 0 \tag{3.42}$$

- For $\overline{\mu}^t_{\nu,h,i}$, $\forall i \in \mathcal{I},\ h \in \mathcal{H}_i,\ \nu \in \mathcal{V},\ t \in \mathcal{T}$:

$$0 \geq g^t_{\nu,h,i} + r^t_{\nu,h,i} - \overline{g}_{\nu,h,i} \quad \perp \quad \overline{\mu}^t_{\nu,h,i} \geq 0 \tag{3.43}$$

- For $\underline{\mu}^t_{\nu,h,i}$, $\forall i \in \mathcal{I},\ h \in \mathcal{H}_i,\ \nu \in \mathcal{V},\ t \in \mathcal{T}$:

$$0 \geq \underline{g}_{\nu,h,i} - g_{\nu,h,i} \quad \perp \quad \underline{\mu}^t_{\nu,h,i} \geq 0 \tag{3.44}$$

- For $\phi^t_{\nu,h,i}$, $\forall i \in \mathcal{I},\ h \in \mathcal{H}_i,\ \nu \in \mathcal{V},\ t \in \mathcal{T}$:

$$0 \geq r^t_{\nu,h,i} - \overline{r}^t_{\nu,h,i} \quad \perp \quad \phi^t_{\nu,h,i} \geq 0 \tag{3.45}$$

- For $\overline{\psi}^t_{\nu,h,i}$, $\forall i \in \mathcal{I},\ h \in \mathcal{H}_i,\ \nu \in \mathcal{V},\ t \in \mathcal{T}$:

$$0 \geq g^t_{\nu,h,i} - g^{t-1}_{\nu,h,i} - \overline{\Delta} g_{\nu,h,i} \quad \perp \quad \overline{\psi}^t_{\nu,h,i} \geq 0 \tag{3.46}$$

- For $\underline{\psi}^t_{\nu,h,i}$, $\forall i \in \mathcal{I},\ h \in \mathcal{H}_i,\ \nu \in \mathcal{V},\ t \in \mathcal{T}$:

$$0 \geq g^{t-1}_{\nu,h,i} - g^t_{\nu,h,i} - \underline{\Delta} g_{\nu,h,i} \quad \perp \quad \underline{\psi}^t_{\nu,h,i} \geq 0 \tag{3.47}$$

- For $\sigma_{\nu,h,i}$, $\forall i \in \mathcal{I},\ h \in \mathcal{H}_i,\ \nu \in \mathcal{V}$:

$$0 \geq \sum_t g^t_{\nu,h,i} - \overline{e}_{\nu,h,i} \quad \perp \quad \sigma_{\nu,h,i} \geq 0 \tag{3.48}$$

- For λ^{t+}_k, $\forall k \in \mathcal{K},\ t \in \mathcal{T}$:

$$0 \geq \sum_i s_{i,k} p^t_i - \overline{z}_k \quad \perp \quad \lambda^{t+}_k \geq 0 \tag{3.49}$$

- For λ_k^{t-}, $\forall k \in \mathcal{K}$, $t \in \mathcal{T}$:

$$0 \geq -\sum_i s_{i,k} p_i^t - \overline{z}_k \quad \perp \quad \lambda_k^{t-} \geq 0 \tag{3.50}$$

- For η_i^{*t}, $\forall t \in \mathcal{T}$:

$$0 \geq \overline{r}^t - \sum_{\nu,h,i} r_{\nu,h,i}^t \quad \perp \quad \eta_i^{*t} \geq 0 \tag{3.51}$$

- For $g_{\nu,h,i}^t$, $\forall i \in \mathcal{I}$, $h \in \mathcal{H}_i$, $\nu \in \mathcal{V}$, $t \in \mathcal{T}$:

$$\begin{aligned} 0 = &\lambda_\nu^t - \overline{\mu}_{\nu,h,i}^t + \underline{\mu}_{\nu,h,i}^t - \beta_{\nu,h,i} - 2\gamma_{\nu,h,i} g_{\nu,h,i}^t + \omega_i^{*t} + \big(\underline{\psi}_{\nu,h,i}^t - \overline{\psi}_{\nu,h,i}^t\big) - \\ &u^t\big(\underline{\psi}_{\nu,h,i}^{t+1} - \overline{\psi}_{\nu,h,i}^{t+1}\big) - \sigma_{\nu,h,i}; \quad g_{\nu,h,i}^t \ free, \quad \boldsymbol{u}^t = [1, 1, ..., 1, 1, 0]^T \end{aligned} \tag{3.52}$$

- For λ_ν^t, $\forall \nu \in \mathcal{V}$, $t \in \mathcal{T}$:

$$0 = \sum_i d_{\nu,i}^t - \sum_{h,j} g_{\nu,h,j}^t; \quad \lambda_\nu^t \ free \tag{3.53}$$

- For p_i^t, $\forall i \in \mathcal{I}$, $t \in \mathcal{T}$:

$$0 = \omega_i^{*t} + \sum_k s_{i,k}(\lambda_k^{t-} - \lambda_k^{t+}); \quad p_i^t \ free \tag{3.54}$$

- For a_i^t, $\forall i \in \mathcal{I}$, $t \in \mathcal{T}$:

$$0 = \rho_i^{*t} - \omega_i^{*t} - \rho_s^t; \quad a_i^t \ free \tag{3.55}$$

- For ρ_s^t, $\forall t \in \mathcal{T}$:

$$0 = \sum_i a_i^t; \quad \rho_s^t \ free \tag{3.56}$$

- For ρ_i^{*t}, $\forall i \in \mathcal{I},\, t \in \mathcal{T}$:

$$0 = \rho_i^{*t} - \rho_{oi}^t + \delta_i^t\Big(d_{\nu,i}^t + d_{-\nu,i}^t + a_i^t\Big); \quad \rho_i^{*t}\ free \tag{3.57}$$

- For ω_i^{*t}, $\forall i \in \mathcal{I},\, t \in \mathcal{T}$:

$$0 = p_i^t - a_i^t - \sum_{\nu}\big(d_{\nu,i}^t - \sum_h g_{\nu,h,i}^t\big); \quad \omega_i^{*t}\ free. \tag{3.58}$$

Simultaneously solving for the primal ($d_{\nu,i}^t$, $g_{\nu,i,h}^t$, $r_{\nu,i,h}^t$, p_i^t, a_i^t) and dual (λ_ν^t, $\overline{\mu}_{\nu,i,h}^t$, $\underline{\mu}_{\nu,i,h}^t$ λ_k^{t+}, λ_k^{t-}, ρ_s^t, $\phi_{\nu,i,h}^t$, $\overline{\psi}_{\nu,i,h}^t$, $\underline{\psi}_{\nu,i,h}^t$, $\sigma_{\nu,i,h}$) variables, with prices (ρ_i^{*t}, ω_i^{*t}, η^{*t}), gives an equilibrium point of the multi-period market. In this thesis work, the equilibrium model defined by the MLCP has been formulated in GAMS and solved using the PATH solver [118]. PATH is a numerical algorithm based on the Newton method to solve mixed complementarity problems. It uses a path-following technique which is used to construct a piecewise linear path from the current point to the Newton point 119.

3.2.5 Energy and spinning reserve interaction

A generation unit may also provide spinning reserve rather than only energy. Nonetheless, capacity limits impose restrictions on the available resources; *e.g.*, the spinning reserve that it can provide depends on the generation level scheduled for the energy market; hence, a GenCo has to choose between supplying energy or spinning reserve [42]. This interdependency has to be considered into the market model in order to have a realistic analysis of the GenCo incentives.

Proposition 3.1 (Opportunity cost) *Let us consider a generation unit participating in an integrated market for energy and spinning reserve, and its maximum capacity limit be defined by (3.25). If the generation unit is providing both energy and spinning reserve, then there may be an opportunity cost between producing and spinning if and only if its maximum capacity limit is binding.*

Proof. Consider only the terms related to energy and spinning reserve for a static case from §3.2.4. As $d_{\nu,i}$, $r_{\nu,h,i} > 0$, by complementarity conditions, Expressions (3.41) and (3.42) are strictly satisfied; thus, (3.41), (3.42) and (3.52) can be written as

$$\rho_{\nu,i} - \frac{\delta_i d_{\nu,i}}{1+\delta_i B_{-\nu,i}} = \omega_i^* + \lambda_\nu, \quad \forall\, d_{\nu,i} > 0, \tag{3.59}$$

$$\eta^* = \alpha_{\nu,h,i} - C_\nu \sum_{\hbar,\jmath} r_{\nu,\hbar,\jmath} + \overline{\mu}_{\nu,h,i} + \phi_{\nu,h,i}, \quad \forall\, r_{\nu,h,i} > 0, \tag{3.60}$$

$$\overline{\mu}_{\nu,h,i} = \lambda_\nu + \underline{\mu}_{\nu,h,i} - \beta_{\nu,h,i} - 2\gamma_{\nu,h,i} g_{\nu,h,i} + \omega_i^*. \tag{3.61}$$

When the maximum limit of capacity is binding, the minimum capacity limit cannot be active, and, thus, $\underline{\mu}_{\nu,h,i} = 0$. By substituting (3.59) and (3.61) into (3.60), the market price for spinning reserve is given by

$$\eta^* = \Big\{\alpha_{\nu,h,i} - C_\nu \sum_{\hbar,\jmath} r_{\nu,\hbar,\jmath}\Big\} + \left\{\rho_{\nu,i} - \big(\beta_{\nu,h,i} + 2\gamma_{\nu,h,i} g_{\nu,h,i}\big) - \frac{\delta_i d_{\nu,i}}{1+\delta_i B_{-\nu,i}}\right\} + \phi_{\nu,h,i}. \tag{3.62}$$

Notice that in equilibrium: $\rho_{\nu,i} = \rho_i^*$. The first term in brackets is the marginal cost of SR affected by the ability of the generation unit to manipulate the SR price, while the second term in brackets represents the opportunity cost for supplying spinning reserve instead of energy. This term is composed by the price difference between the nodal price where the generation unit h, i is placed, and the marginal cost of such a unit and its impact on the price by means of its sales at i. Only when the generation unit has reached its spinning reserve limit, the term $\phi_{\nu,i,h}$ may count. ■

For the Cournot outcome, the spinning reserve price is

$$\eta^*\Big|_{B_{-\nu,i}=0} = \Big\{\alpha_{\nu,h,i} - C_\nu \sum_{\hbar,\jmath} r_{\nu,\hbar,\jmath}\Big\} + \Big\{\rho_{\nu,i} - \big(\beta_{\nu,h,i} + 2\gamma_{\nu,h,i} g_{\nu,h,i}\big) - \delta_i d_{\nu,i}\Big\} + \phi_{\nu,h,i}. \tag{3.63}$$

On the other side, for competitive energy and spinning reserve markets, one gets

$$\lim_{B_{-\nu,i}\to\infty} \eta^* = \alpha_{\nu,h,i} + \Big\{\rho_i^* - \big(\beta_{\nu,h,i} + 2\gamma_{\nu,h,i} g_{\nu,h,i}\big)\Big\} + \phi_{\nu,h,i}. \tag{3.64}$$

In this case, the opportunity cost is given by the difference between the corresponding locational marginal price and the energy marginal cost. A similar relationship has been derived in [40] for a competitive energy market. The terms related to ramp-rates and energy constraints can also be included to analyze their impact on the opportunity cost.

3.3 Illustrative Six-Node System

A six-node system is used to represent the transmission system, as shown in Figure 3.1. The network is modelled as lossless and with equal transmission line reactances. Transmission limits for lines 1-6 and 2-5 are 200 MW; these lines are labelled as 1 and 2, respectively. The limits of the other transmission lines are large enough to be neglected.

The data related to generation and demand are given in Table 3.1. All generation units have unlimited capacity (otherwise specified); generation units and demands are accordingly labelled as shown in Figure 3.1.

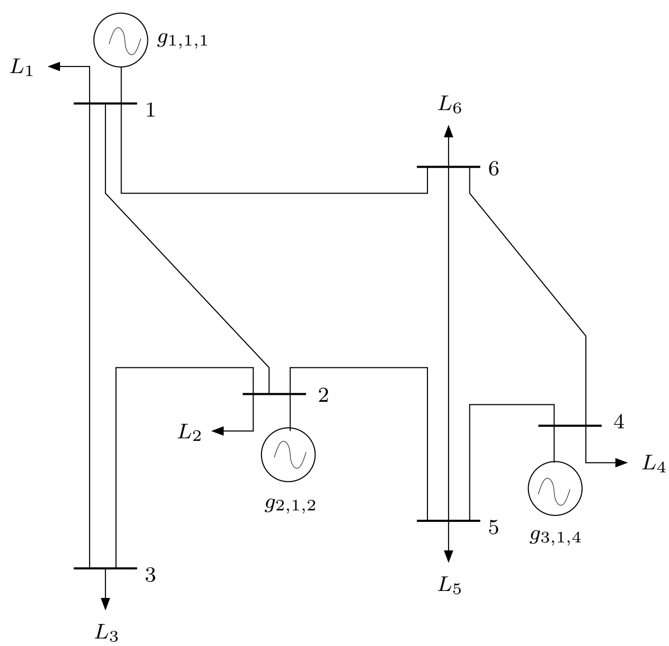

Figure 3.1: A six-node system.

3.3.1 Competitive energy market

Let us consider a static case for an integrated energy and spinning reserve market. The energy market is considered to be competitive through all the cases; meanwhile, the SR market is considered as competitive for all cases, but Case D. The reserve requirement is set to be 10% of the total demand. The market outcomes are summarized in Tables 3.2 and 3.3.

Table 3.1: Generation and demand data. Six-node system.

	Generation					**D**emand	
Node	GenCo	Unit	$\alpha_{\nu,h,i}$	$\beta_{\nu,h,i}$	$\gamma_{\nu,h,i}$	ρ_{oi}	d_{oi}
i	ν	h	$(\$/MWh)$	$(\$/MWh)$	$(\$/MW^2h)$	$(\$/MWh)$	(MW)
1	1	1	4	10	0.003	60	300
2	2	1	6	15	0.005	80	400
3	–	–	–	–	–	50	250
4	3	1	7	20	0.01	60	300
5	–	–	–	–	–	100	500
6	–	–	–	–	–	80	400

Case A. No generation and SR limits

Under competitive conditions in the energy market, all generation units provide energy. Since the power-flow limit of transmission line 1 becomes active, locational price discrimination arises, and the ISO collects congestion rents of $\$4800/h$. In this case, all generation units are marginal; *i.e.,* the nodal prices where they are placed equal their corresponding marginal cost. Due to the fact that no generator is constrained by its maximum capacity limit, the joint optimization of the SR market does not modify the energy-only market outcome. The SR requirement of 147.7 MW is being provided by the cheapest generator ($g_{1,1,1}$) at a price of $\$4/MWh$.

Case B. No SR limits and $\overline{g}_{1,1,1}$=750 MW

As $g_{1,1,1}$ is the cheapest supplier of both energy and SR, its full capacity (750 MW) is used. If there were no opportunity cost between energy and SR, $g_{1,1,1}$ would supply 672 MW for the energy market (as in Case A), and its remaining capacity (77.92 MW) would be used to provide reserve. Nonetheless, as such an opportunity cost exists, $g_{1,1,1}$ provides more spinning reserve (146.68 MW) by reducing the power to be supplied into the energy market up to 603.31 MW. This withholding of capacity in the energy market is actually due to an incentive from the SR market, not an exercise of market power. Moreover, this shift of

Table 3.2: Energy and spinning reserve interaction. Competitive case[†].

Case	A	B	C	D
$g_{1,1,1}$	672	603.3	650	668.6
$g_{2,1,2}$	383.2	463.8	409	387.2
$g_{3,1,4}$	421.6	399.6	414.5	420.5
$r_{1,1,1}$	147.7	146.6	100	81.3
$r_{2,1,2}$	0.0	0.0	47.3	43.1
$r_{3,1,4}$	0.0	0.0	0.0	23.1
ρ_1	14.0	15.4	14.4	14.1
ρ_2	18.8	19.6	19.0	18.8
ρ_3	16.4	17.5	16.7	16.4
ρ_4	28.4	27.9	28.2	28.4
ρ_5	26.0	25.9	25.9	26.0
ρ_6	30.8	30.0	30.5	30.7
d_1	229.8	222.6	227.5	229.4
d_2	305.8	301.8	304.5	305.6
d_3	167.8	162.2	166.0	167.5
d_4	157.8	160.0	158.5	157.9
d_5	369.8	370.4	370.0	369.8
d_6	245.8	249.5	247.0	246.0
z_1	200.0	200.0	200.0	200.0
z_2	151.8	180.4	161	153.2
η^*	4	5.8	6	8.1
λ_1^+	24	20.8	23	23.8
λ_2^+	0.0	0.0	0.0	0.0

† Power in MW, prices in $\$/MWh$.

power causes a different market outcome (see prices and demand levels). Because cheap power is reduced, more power flows through line 2, resulting in less congestion in line 1. GenCos surplus increases mainly by the higher prices at which GenCos 1 and 2 now sell; in contrast, the loads surplus is slightly reduced. Although the net surplus increases, the social welfare decreases because less congestion rents (welfare transfer from demands and GenCos to the ISO) are collected. In addition, $g_{1,1,1}$ sets a higher SR price of $5.84/$MWh$. This price is composed by the SR marginal cost of $g_{1,1,1}$ ($4/$MWh$) plus the opportunity cost

Table 3.3: Profits comparison in $/$h$. Competitive market.

Case	A	B	C	D
Π_1^G	1,355	2,203.3	1,651.9	1,402.4
Π_2^G	734.4	1,076.0	837.0	749.8
Π_3^G	1,777.9	1,597.1	1,718.8	1,768.7
Π_1^R	0.0	270.2	200.0	338.1
Π_2^R	0.0	0.0	0.0	93.1
Π_3^R	0.0	0.0	0.0	26.8
Π_1	1,355	2,473.6	1,851.9	1,740.6
Π_2	734.4	1,076.0	837.0	843.0
Π_3	1,777.9	1,597.1	1,718.8	1,795.6
Congestion rents	4,800.0	4,176.8	4,600.0	4,769.3
GenCos' surplus	3,867.5	4,876.5	4,207.8	3,921.1
Demands' surplus	39,665.3	39,216.4	39,518.6	39,642.5
Social welfare	48,333.8	48,269.7	48,326.5	48,332.9

of $1.84/$MWh$. This opportunity cost is simply the difference between the corresponding price at node 1 ($15.46/$MWh$) and the energy marginal cost ($13.61/$MWh$) of $g_{1,1,1}$ –see (3.64). Since the SR price is above the SR marginal cost of $g_{1,1,1}$, this GenCo has a profit of $270.2/$h$ by providing spinning reserve.

Case C. $\overline{g}_{1,1,1}$**=750 MW and** $\overline{r}_{1,1,1}$**=100 MW**

Let us now consider that $g_{1,1,1}$, besides $\overline{g}_{1,1,1} = 750$ MW, has also a maximum SR limit. If $\overline{r}_{1,1,1} \in (77.92,\, 146.6)$, the inclusion of a SR limit will alter the energy market outcome[3]. Taking this into account, consider a SR limit of, say, 100 MW. In this case, $g_{1,1,1}$ now provides 650 MW for energy and 100 MW for SR. To satisfy the SR requirement, now $g_{2,1,2}$ provides 47.37 MW, and sets the SR market price at \$6/MWh. The introduction of the SR limit curbs the decrease (motivated from the interaction of the energy and reserve markets) in generation of $g_{1,1,1}$, which produces 650 MW rather than 603.3 MW. As more –cheap– generation is used (in comparison to Case B), more congestion occurs in line 1; however the market is better off as the welfare increases. Consequently, the incentive from the SR market will impact less severely the energy market.

Case D. Market power in the SR market

In this case, GenCos can manipulate the SR market price; this is done by means of the conjectured reserve-price function described in Definition 3.3. Assume that all GenCos behave strategically (C_ν=-0.05, $\forall\, \nu$). The market outcome is computed by including both energy and SR limits for $g_{1,1,1}$, as in Case C. The strategy for $g_{1,1,1}$ is to shift power (100-83.1 MW) from the SR market to the energy market. Such an increase of generation makes $g_{2,1,2}$ decrease its generation (leading to lower energy prices at nodes 1-3), and $g_{3,1,4}$ increase its generation. As more –cheap– power from $g_{1,1,1}$ is produced, the transmission line 1 becomes more congested, increasing the congestion rents by \$169.3/h. The reduction of SR from $g_{1,1,1}$ is mainly compensated by $g_{3,1,4}$, which leads a higher SR price (\$8.1/MWh). On the other hand, this manipulation in the SR market has been not profitable for $g_{1,1,1}$. This GenCo is now obtaining a higher profit from the SR market; nonetheless, this profit is not

[3]If a spinning reserve limit $\overline{r}_{1,1,1} \in [0,\, 77.92]$ MW is chosen, the energy market outcome will be as that of Case A. This is because $\overline{r}_{1,1,1}$ would be lower than the SR level that $g_{1,1,1}$ has available (750-672 MW). Then the SR that $g_{1,1,1}$ provides would be such a chosen SR limit. On the other hand, if a spinning reserve limit $\overline{r}_{1,1,1} \in [146.68,\, \infty)$ is chosen, the energy market outcome will be as that of Case B. This is because $\overline{r}_{1,1,1}$ will be higher than the optimal SR level of $g_{1,1,1}$. Then the SR that $g_{1,1,1}$ provides will be 146.68 MW.

enough to compensate the lost profit from the energy market[4]. The other GenCos have seen a net profit increase.

3.3.2 Oligopolistic energy market

In this section, all generators are considered to compete à la Cournot in the energy market ($B_{-\nu,i}$=0.0, $\forall\,\nu, i$), while the SR market remains competitive for all cases but Case D. The interaction becomes more complex for the oligopolistic market, although the logic is similar to the competitive case. The market outcomes comparison is presented in Tables 3.4 and 3.5.

Case A. No generation limits

In comparison to the competitive case, there is a reduction of generation from all GenCos that jacks up the market prices. Due to higher energy prices, less demand is served; this causes not only less congestion in the system[5] but also a lower requirement of spinning reserve (from 147.7 MW to 116.5 MW). Although GenCos produces less power, they earn higher profits since they are charging much higher energy prices. The strategic behaviour increases the GenCos surplus by 365%, while the demands surplus is reduced by 34%; this represents a large welfare transfer from consumers to GenCos. In comparison to the competitive case, the net welfare decreases from $\$48,333.8/h$ to $\$46,205.5/h$.

On the other hand, as no generator is constrained by a maximum capacity, the SR market does not affect the energy market outcome; then, by the complementarity principle, $\overline{\mu}_{\nu,h,i}$=0, and there is no opportunity cost between producing and spinning. The SR price is set again by $g_{1,1,1}$ at η^{*}=$\alpha_{1,1,1}$=$\$4/MWh$.

[4]This is due to $g_{1,1,1}$'s position in the SR market. This generator is the cheapest unit to provide SR, and cannot supply all of it; hence, this generator is not the marginal unit for SR, and its strategies are limited by the other GenCos strategies.

[5]The dual variable for the transmission line 1 decreases from $\$24/MWh$ to $\$9.8/MWh$, and, consequently, the congestion rents shrink by 58.8%.

Table 3.4: Energy and spinning reserve interaction. Oligopolistic case†.

Case	A	B	C	D
$g_{1,1,1}$	494.8	472.4	480	487.1
$g_{3,1,2}$	379.2	394.9	389.6	384.6
$g_{3,1,4}$	288.4	284.6	285.9	287.1
$r_{1,1,1}$	116.2	77.6	70	62.8
$r_{2,1,2}$	0.0	37.6	45.5	36.5
$r_{3,1,4}$	0.0	0.0	0.0	16.5
ρ_1	29.4	30.5	30.2	29.8
ρ_2	31.4	32.1	31.8	31.6
ρ_3	30.4	31.3	31.0	30.7
ρ_4	35.3	35.1	35.2	35.3
ρ_5	34.3	34.4	34.4	34.4
ρ_6	36.3	35.9	36.0	36.2
d_1	152.6	147.0	148.9	150.7
d_2	242.8	239.4	240.5	241.6
d_3	97.7	93.2	94.7	96.2
d_4	123.0	124.0	123.7	123.4
d_5	328.0	327.9	327.9	327.9
d_6	218.1	220.2	219.5	218.8
z_1	200.0	200.0	200.0	200.0
z_2	180.8	187.5	185.2	183.1
η^*	4	6	6	7.8
λ_1^+	9.8	7.6	8.4	9.1
λ_2^+	0.0	0.0	0.0	0.0

† Power in MW, prices in $\$/MWh$.

Case B. No SR limits and $\overline{g}_{1,1,1}$=550 MW

For this case, assume $g_{1,1,1}$ has a maximum generation limit of, say, $550MW$; because of this constraint, $g_{1,1,1}$ will not be able to provide all the SR, as in Case A. Due to the opportunity cost, $g_{1,1,1}$ reduces generation (from 494.81 MW to 472.39 MW) in order to provide a larger amount of SR (77.6 MW instead of 55.19 MW). Now, $g_{2,1,2}$ provides the remaining amount of SR (37.6 MW), and becomes the marginal unit for it, setting the SR market price at $\$6/MWh$. The SR price, η^{*}=$\$6/MWh$, is composed by the SR marginal cost of $g_{1,1,1}$ ($\alpha_{1,1,1}$=$\$4/MWh$) and the opportunity cost between energy and spinning reserve ($\$2/MWh$). This opportunity cost is not only defined by the difference between

Table 3.5: Profits comparison in $\$/h$. Oligopolistic market.

Case	A	B	C	D
Π_1^G	8,895.8	9,052.9	9,005.6	8,955.5
Π_2^G	5,514.6	5,979.7	5,819.9	5,671.5
Π_3^G	3,604.7	3,511.7	3,543.1	3,572.8
Π_1^R	0.0	155.2	140.0	240.4
Π_2^R	0.0	0.0	0.0	66.6
Π_3^R	0.0	0.0	0.0	13.6
Π_1	8,895.8	9,208.1	9,145.6	9,196.0
Π_2	5,514.6	5,979.7	5,819.9	5,738.2
Π_3	3,604.7	3,511.7	3,543.1	3,586.4
Congestion rents	1,973.2	1,533.9	1,682.9	1,823.0
GenCos' surplus	18,015.2	18,544.5	18,368.7	18,199.9
Demands' surplus	26,217.0	25,909.8	26,012.5	26,110.5
Social welfare	46,205.5	45,988.3	46,063.9	46,133.4

the locational price at node 2 and the marginal cost of $g_{1,1,1}$ (like in the competitive case), but also by the price that $g_{1,1,1}$ sets upon its power sales at node 1 ($\delta_1 d_{1,1}$=$\$15.74/MWh$) –see (3.63). Generator $g_{1,1,1}$ is producing less power than it would be in the case of an energy-only market; such a reduction causes an increase of the energy prices at nodes 1-3. This further increase in prices is due to the incentives from the SR market; nonetheless, this incentive now depends also upon the ability of $g_{1,1,1}$ to influence the energy market.

Case C. $\overline{g}_{1,1,1}$=550 MW and $\overline{r}_{1,1,1}$=70 MW

Consider now that $g_{1,1,1}$ has also a maximum SR limit of 70 MW. Due to this limit, $g_{1,1,1}$ is constrained to provide 70 MW for SR, while its remaining capacity (480 MW) is used for the energy market. The SR limit makes $g_{1,1,1}$ increase its generation from 472.4 MW (Case B) to 480 MW; this increase leads lower energy prices at nodes 1–3, but higher prices at nodes 4–6. As more (cheap) power from $g_{1,1,1}$ is used to supply the demands, the transmission line 1 becomes more congested, increasing the congestion rents by 9.7%. On the other hand, the SR price, from the point of view of $g_{1,1,1}$, is now composed by its SR marginal cost ($\alpha_{1,1,1}$=\$4/MWh), the opportunity cost (\$1.32/MWh), and the dual variable of the SR limit ($\phi_{1,1,1}$=\$0.67/MWh).

Case D. Market power in the SR market

Consider Case C of this section and assume now that all GenCos behave strategically in the SR market (C_ν=-0.05, $\forall\,\nu$). The strategies of $g_{1,1,1}$ and $g_{2,1,2}$ is to reduce the provision of spinning reserve. Such reductions make $g_{3,1,4}$ provide 16.5 MW of spinning reserve at price of \$7.82/MWh. For $g_{1,1,1}$, the SR price is composed by its SR marginal cost ($\alpha_{1,1,1}$=\$4/MWh), the SR price manipulation (\$3.14/MWh), and the opportunity cost between the provision of energy and spinning reserve (\$0.68/MWh). Such an opportunity cost is reduced (from \$1.32/MWh to \$0.68/MWh) because $g_{1,1,1}$ is shifting power from the SR market to the energy market. As long as its strategic behaviour in the SR market increases, the opportunity cost reduces to zero. Hence, at some point $g_{1,1,1}$ will get the energy market outcome from Case A (this is equivalent to have no limits in capacity and spinning reserve) as its manipulation of the SR market will not have anymore an effect on the energy market. That is, depending on its degree of strategic behaviour in the SR market, the generation level of $g_{1,1,1}$ can fall between 480 MW (competitive in the spinning reserve market) and 494.81 MW (highly strategic in the SR market). This represents a tradeoff between strategic behaviour in the SR market and the incentive of the opportunity cost between the energy and spinning reserve markets.

3.3.3 Multi-period market

In another simulation, consider an extension for 24 trading periods of the test system above described. The profile for all six demands is shown in Figure 3.2. This profile resembles a classical daily pattern where there is a peak. The demands are defined with a reference price of \$20/MWh and a demand function slope (δ_i) of 0.2. Upper generation limits are $\overline{g}_{1,1,1}$ =550 MW, $\overline{g}_{2,1,2}$=800 MW and $\overline{g}_{3,1,4}$=800 MW; while all generating units are assumed to have up- and down-ramp limits of 50MW/h. Through all the cases, GenCos behave strategically in both the energy and SR markets, with $C_{\nu,i}$=-0.05, $B_{-\nu,i}$=10, $\forall\, \nu, i$. The main results are presented in Table 3.6. In the comparisons, only the means of the nodal prices are shown.

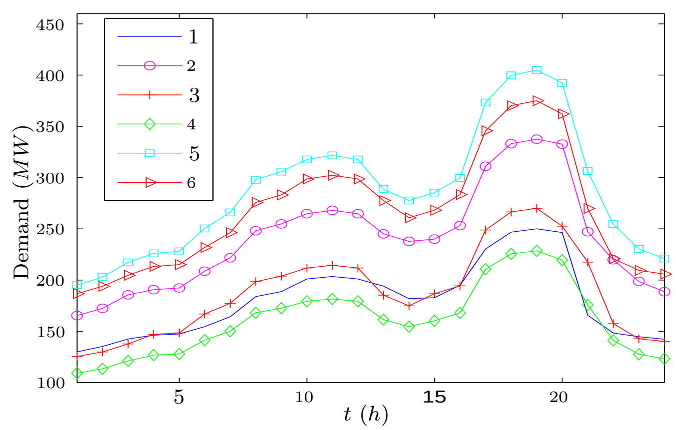

Figure 3.2: Demand profile at each node.

Case A. No ramp-rate limits

For the sake of comparison, a market outcome is obtained by relaxing the ramp-rate limits; this is equivalent to have 24 independent and static market outcomes. The optimal solution for the whole trading horizon is the set of all the static (local) optima. When ramp-rate limits are neglected, generation units can be scheduled to any level from one period to another, and, hence, any unit can avoid low prices before incurring generation losses.

Table 3.6: Ramp-rate limits effect on the market outcome. Static *vs.* dynamic case†.

Case	A	B	C
ρ_1	22.09	22.14	22.89
ρ_2	23.71	23.82	24.23
ρ_3	22.90	22.98	23.56
ρ_4	27.24	27.20	27.05
ρ_5	26.43	26.36	26.38
ρ_6	28.06	28.04	27.72
Π_1^G	133,307.1	133,232.3	137,909.0
Π_2^G	69,255.2	71,668.2	77,311.3
Π_3^G	31,913.1	32,412.0	31,440.4
Π_1^R	1,532.5	1,792.5	2,849.1
Π_2^R	2,015.0	1,879.4	1,339.7
Π_3^R	34.2	27.0	21.7
Π_1	134,839.7	135,024.9	140,758.1
Π_2	71,270.3	73,547.7	78,651.1
Π_3	31,947.3	32,439.1	31,462.1
Congestion rents	41,697.0	40,507.8	33,520.5
Demands' surplus	594,000.7	590,273.1	585,374.1
GenCos' surplus	234,475.5	237,312.6	246,660.7
Social welfare	870,173.2	868,093.5	865,555.3
$e_{1,1,1}$	12,605.3	12,499.0	12,000.0
$e_{2,1,2}$	9,910.5	10,038.0	10,496.9
$e_{3,1,4}$	5,592.4	5,558.4	5,443.6

† Power in MW, prices in $\$/MWh$, energy in MWh and profits in \$.

Case B. Inclusion of ramp-rate limits

As every period is linked with the previous and subsequent ones, the optimal solution for every period may not be the same as the static optimal from Case A –see Figure 3.3. This fact becomes evident in the periods around the peak. For instance, the generation of $g_{2,1,2}$ becomes limited up and down by the ramp-rate limits. On one side, generation levels will be lower (in comparison to those of Case A) for the up and peak periods (16-20), leading to higher energy prices and higher profits. On the other side, generation levels are higher for the down periods (21-24), leading to lower prices and profits. However, the

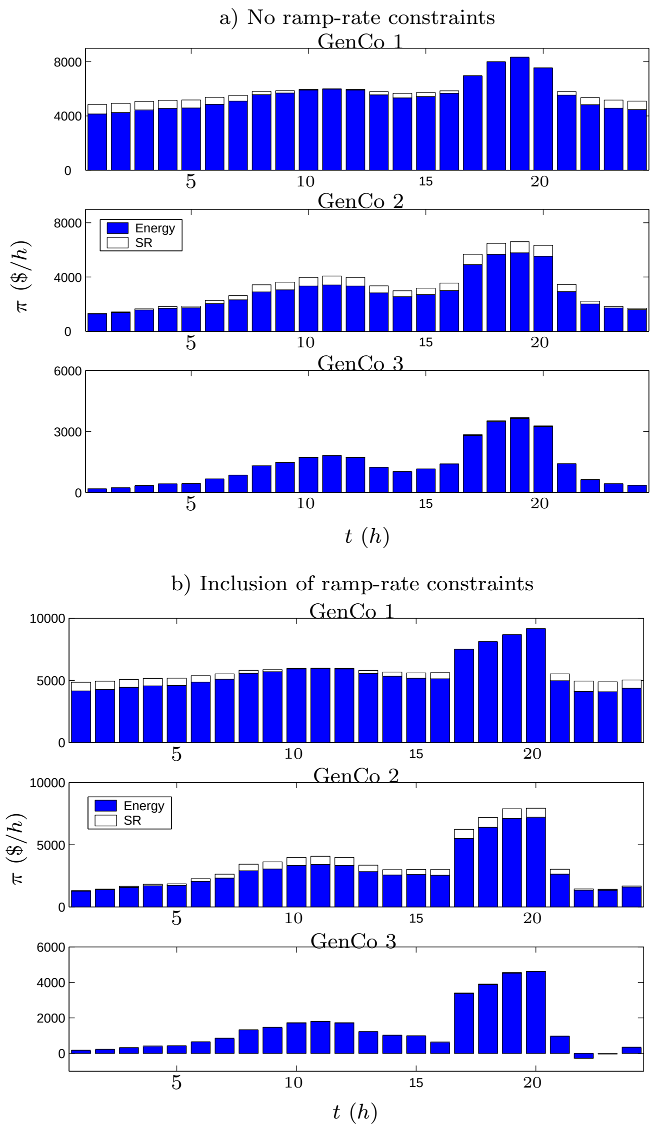

Figure 3.3: GenCos profits. Multi-period cases.

money collected from the up periods is higher than the money lost from the down periods; consequently, $g_{2,1,2}$ has a net increase in profit. The decrease in generation (in comparison to those from Case A) in up and peak hours arises from inter-temporal constraints, not from a market power exercise *per se*. As expected, misleading conclusions on market power could be derived from an analysis built upon a static model.

In the case of $g_{1,1,1}$, the ramp-rate limits make it produce less power throughout the horizon and follow a different strategy. For the up periods, $g_{1,1,1}$ has no changes in generation, and it sells the same amount of power at higher prices, earning higher profits. For the down periods, $g_{1,1,1}$ reduces generation in order to compensate the $g_{2,1,2}$'s effect on prices, selling less power to avoid lower prices. The result is that cheap power is substituted by more expensive generation.

This logic is more vivid for $g_{3,1,4}$; since this generator is the most expensive, changes in its generation have a stronger impact on prices. In peak periods, as $g_{3,1,4}$ finds it very profitable to produce, it is at high generation levels. However, in subsequent periods the demand becomes low and so do prices; then this generator cannot freely ramp down to be scheduled at static optimal generation levels, and it has to produce at prices below its marginal costs, incurring losses. Nonetheless, the high profits earned from peak periods offset the losses from down periods, resulting in a net increase of profits for $g_{3,1,4}$.

Furthermore, the change in generation also impacts on the SR level to be provided by each GenCo. In periods 15-20, less demand, and, hence, less spinning reserve is required (in comparison to Case A); the reduction in SR comes from $g_{2,1,2}$ and $g_{3,1,4}$. In periods 21-24, there is an increase in demand, and, thus, in SR; this increase is provided by $g_{1,1,1}$. There is also an extra shift of power to the spinning reserve; due to ramp-rate constraints, $g_{1,1,1}$ reduces generation, and it has now more capacity available for spinning reserve (this generator has cheaper SR). Thus, $g_{1,1,1}$ will substitute spinning reserves from $g_{2,1,2}$, leading to a lower SR price in these periods. In the remaining periods, there is no change in the profile of the SR provision. This results in a net profit increase from spinning reserve for $g_{1,1,1}$, and a profit decrease for the other GenCos.

Case C. $\overline{e}_{1,1,1}$=12000 MWh

Let us consider Case B but $g_{1,1,1}$ is now limited by the maximum energy that it can provide in the trading horizon. From Case B, the net energy supplied by $g_{1,1,1}$ is 12499 MWh; to see how the energy limit affects its strategies, assume an energy limit of, say, 12000 MWh. Due to this constraint, $g_{1,1,1}$ has to reduce throughout the horizon a net amount of 499 MWh. For periods 17-20, with the highest prices and also the most profitable ones, there is no change in generation. For most of the remaining periods, the trend is to have a larger reduction where both generation levels and prices are higher; a generation decrease where prices are already high will lead to higher prices, and, hence, higher profits. The extra available capacity of $g_{1,1,1}$ is now used to provide spinning reserve; therefore, cheaper prices (except for periods 17-20) for spinning reserve are obtained through the trading periods. On the other hand, more expensive power from $g_{2,1,2}$ has to be used to compensate the reductions of $g_{1,1,1}$, leading to higher prices at nodes 1-3 (except at periods 17-20); this is followed by a decrease in generation by $g_{3,1,4}$ in order to avoid sell power at a lower price.

The variation of nodal prices due to the inclusion of ramp-rate limits is depicted in Figure 3.4; such a variation is computed as the difference between the nodal prices of Cases A and B. When ramp-rate limits are not binding, there are no price changes; see for instance, periods 1-14. Nonetheless, when they are binding, as in most of the on-peak periods, ramp-rate constraints increase the nodal prices, meanwhile the nodal prices are diminished in off-peak hours.

The inclusion of more constraints may lead to complicated interactions within the market. Although more constraints are considered into the system, the GenCos profits become higher; this counter-intuitive result comes from the fact that the cheapest generation is which becomes more limited; hence, power from more expensive units is used, resulting in higher prices and profits. The decrease of cheap power also causes less congestion in the system. On the other hand, the inclusion of more constraints results in a decrease of the social welfare.

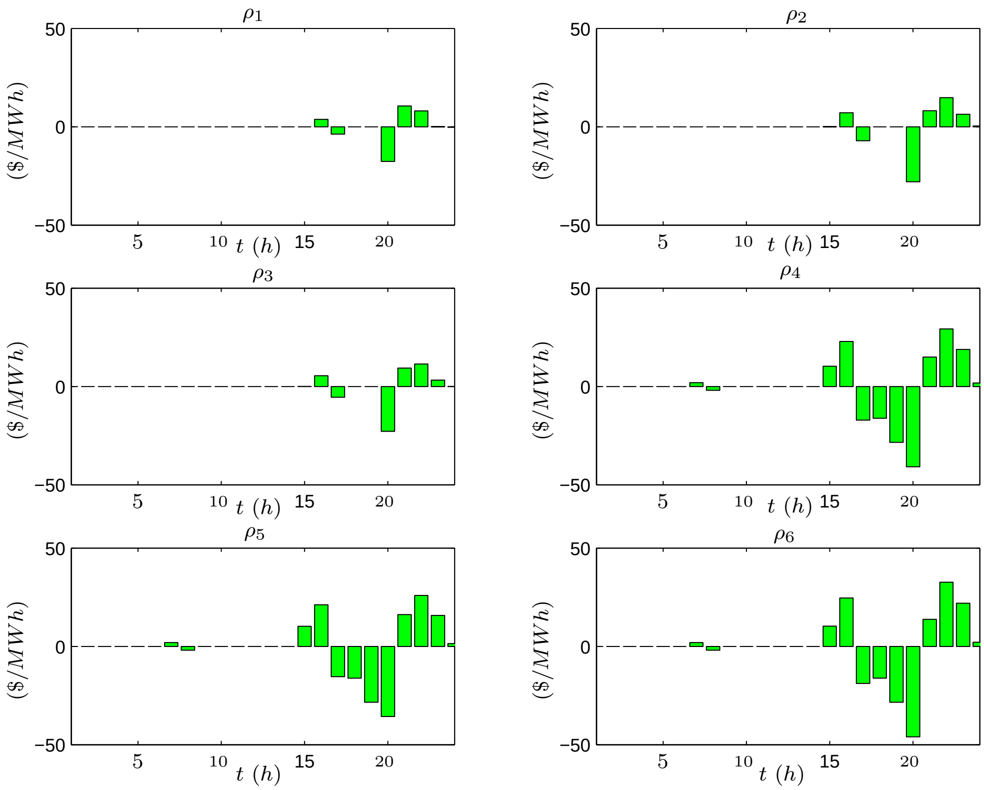

Figure 3.4: Effect of ramping constraints on prices.

3.4 A 57-node System

In another simulation, the standard IEEE-based test power system of 57 nodes and 80 transmission lines [120] is considered for 24 trading periods. There are 7 GenCos participating in the energy and spinning reserve markets; their generation characteristics are listed in Table B.8, Appendix B. All generation units are assumed to have linear cost functions and ramp-rate limits of $40MW/h$. The peak values for the time-varying demands are presented in Table B.9, Appendix B. GenCos 1 and 2 are considered to behave strategically in both the energy and spinning reserve markets ($B_{-\nu,i}$=10, C_ν=-0.05, $\nu = 1, 2,\ \forall i$), while the remaining GenCos are considered as a competitive fringe.

For this case study, a simulation has been also performed by using the solver PATH under GAMS [118]. All the system and generation constraints are considered. The net GenCos' profits are given in Table 3.7.

Table 3.7: Net profits for GenCos. 57-node system (in $).

ν	Π_ν^G	Π_ν^R	Π_ν
1	26,299.7	1,692.7	27,992.4
2	98,128.4	111.3	98,239.7
3	19,984.6	3,635.0	23,619.6
4	5,023.2	0.0	5,023.2
5	3,263.6	0.0	3,263.6
6	49,716.4	40.3	49,756.8
7	815.7	0.0	815.7

Besides GenCos 1 and 2, GenCos 3 and 6 earn high profits; this is because they have cheap units and because the market prices, at which these GenCos sell power, is set above their marginal cost. These high prices arise from the system interactions and the strategic behaviour of GenCos 1 and 2 rather than by the GenCos 3 and 6 behaviour. In the reserve market, GenCos 1, 2, 3 and 6 have profits since they are the cheapest units for spinning reserves. The SR price through the trading periods is depicted in Figure 3.5. The SR price varies accordingly to the demand levels; the more the demand and, thus, the SR requirement, the higher the spinning reserve price.

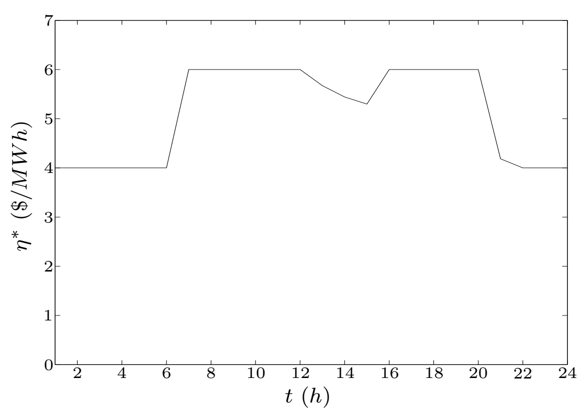

Figure 3.5: Spinning reserve price over time.

3.5 FTR Impact on Market Power

The introduction of hedging instruments, such as transmission rights, may add more room for market power. Although extensive work has been carried out from an analytical point of view, with two- and three-node systems, few works have studied this interaction for more real systems. In [121], a methodology for screening exacerbation of market power due to FTRs is presented. In a pool-like model, [22], the effect of FTRs on the generators strategies is analyzed; in this model, two sets of GenCos are considered: competitive and Cournot GenCos. All the competitive GenCos are modelled as a fringe, and its profit-maximization problem is casted as a social welfare maximization. Then the first-order optimality conditions of the competitive fringe are incorporated into the optimization problems of each Cournot GenCo. In this way, Cournot participants foresee how competitive participants will act, and how their actions can impact transmission prices. Thus, the profit-maximization problem for each Cournot GenCo becomes a *Mathematical Problem with Equilibrium Constraints* (MPEC) [122], which is an inherently non-convex and a hard-to-solve problem. An alternative model to ease the problem is to include smooth functions for modelling the manipulation of the transmission prices. In this way, the problem can be treated as being convex, and modelled as a MLCP [47, 123] which leads to a computationally tractable model.

Let us now consider that GenCos behave in a more sophisticated manner, such that they anticipate how arbitrage is affected by their strategies. This can be done by incorporating the equilibrium conditions –(3.55) and (3.56)– of arbitrage into each GenCo problem. Because the variables related to arbitrage are unrestricted, the corresponding equilibrium conditions are equalities. Thus, the equilibrium conditions of arbitrage are simply extra linear constraints, and the GenCos profit-maximization problem remains as convex. In addition, the arbitrage and the price at the slack node become endogenous in each GenCo problem, consequently, they are now denoted by $a_{\nu,i}$ and $\rho_{\nu,s}$, respectively. This is similar to what has been done for a pool model in [15].

Similar to the conjectures functions described in §3.1, a conjectured transmission-price response function can be introduced in order to model how GenCos can influence the transmission prices. Consider the net power injection at node i given by $a_{\nu,i} + \sum_\nu \big(d_{\nu,i} - \sum_h g_{\nu,h,i}\big)$ –see Expression (3.38). Since the net transmission service required by ν, at node

i, is $d_{\nu,i} - \sum_h g_{\nu,h,i}$, the power injection at i can be decomposed as

$$(a_{\nu,i} + d_{\nu,i} - \sum_{h} g_{\nu,h,i}) + \big(d_{-\nu,i} - \sum_{f \neq \nu} \sum_{\hbar} g_{f,\hbar,i}\big), \tag{3.65}$$

where the first term is the power under control of ν, while the second term is the power of the other GenCos.

Definition 3.5 (Conjectured transmission-price function) A conjectured transmission-price function can be defined as,

$$\omega_{\nu,i} = \omega_i^* + \Big[(a_{\nu,i} + d_{\nu,i} - \sum_{h} g_{\nu,h,i}) - (a^*_{\nu,i} + d^*_{\nu,i} - \sum_{\hbar} g^*_{\nu,\hbar,i})\Big] A_{\nu,i}, \quad \forall i \in \mathcal{I}, \tag{3.66}$$

where $\omega_{\nu,i}$ and ω_i^* are the congestion price seen by ν at i, and the transmission price at equilibrium, respectively. $A_{\nu,i}$ is the conjectured transmission-price response parameter for ν. For the standard competitive outcome, $A_{\nu,i} \equiv 0$. The transmission-price function allows one to model the ability of ν to influence the transmission prices; *i.e.*, to include how ν expects the transmission prices vary if ν modifies the power injection at node i.

3.5.1 GenCo problem

Let us consider a one-period market for energy and transmission where participants hold a portfolio of FTRs –which has been previously allocated. Besides the profit from the energy market, the FTR payoff represents another money stream. As FTRs can be defined by an amount of power $\tau_{\nu,m,\ell}$, and the injection (m) and withdrawal (ℓ) nodes, the payoff associated with such an FTR is then $\tau_{\nu,m,\ell}(\rho_{\nu,\ell} - \rho_{\nu,m})$. The price $\rho_{\nu,i}$ seen by GenCo ν, at node i, is

$$\rho_{\nu,i} = \frac{\rho_{oi} - \delta_i(d_{\nu,i} + d^*_{-\nu,i} - B_{-\nu,i}\rho_i^* + a_{\nu,i})}{1 + B_{-\nu,i}\delta_i}. \tag{3.67}$$

Thus, the profit-maximization problem for GenCo ν (without taking into account the cost incurred from acquiring the FTRs) is,

$$
\begin{aligned}
max \sum_{j} \big(\rho_{\nu,j} - \omega_{\nu,j}\big) d_{\nu,j} - \sum_{h,i} (\beta_{\nu,h,i} + \gamma_{\nu,h,i} g_{\nu,h,i} - \omega_{\nu,i}) g_{\nu,h,i} + \\
\sum_{m} \sum_{\ell \neq m} \tau_{\nu,m,\ell} (\rho_{\nu,\ell} - \rho_{\nu,m}) \qquad (3.68) \\
s.t. \sum_{i} d_{\nu,i} - \sum_{h,j} g_{\nu,h,j} = 0, \qquad (3.69) \\
g_{\nu,h,i} - \overline{g}_{\nu,h,i} \leq 0, \quad \forall i \in \mathcal{I},\, h \in \mathcal{H}_i, \qquad (3.70) \\
\rho_{\nu,i} - \rho_{\nu,s} - \omega_{\nu,i} = 0, \quad \forall i \in \mathcal{I}, \qquad (3.71) \\
\sum_{i} a_{\nu,i} = 0, \qquad (3.72) \\
d_{\nu,i},\, g_{\nu,h,i}, \geq 0, \quad \forall i \in \mathcal{I}, \quad h \in \mathcal{H}_i, \qquad (3.73)
\end{aligned}
$$

The conjectured supply and transmission-response functions, (in Definitions 3.2 and 3.5) are used to substitute $d_{-\nu,i}$ and $\omega_{\nu,i}$, respectively, into the problem (3.68)–(3.73). The Lagrangian for the GenCo problem then can be casted as follows:

$$
\begin{aligned}
\pounds = & \sum_{j} \big(\rho_{\nu,j} - \omega_{\nu,j}\big) d_{\nu,j} - \sum_{h,i} (\beta_{\nu,h,i} + \gamma_{\nu,h,i} g_{\nu,h,i} - \omega_{\nu,i}) g_{\nu,h,i} + \\
& \sum_{m} \sum_{\ell \neq m} \tau_{\nu,m,\ell} (\rho_{\nu,\ell} - \rho_{\nu,m}) - \vartheta_\nu \big(\sum_{i} d_{\nu,i} - \sum_{h,j} g_{\nu,h,j} \big) - \\
& \sum_{h,i} \mu_{\nu,h,i} \big(g_{\nu,h,i} - \overline{g}_{\nu,h,i}\big) - \sum_{i} \lambda_{\nu,i} \big(\rho_{\nu,i} - \rho_{\nu,s} - \omega_{\nu,i}\big) - \varphi_\nu \sum_{i} a_{\nu,i}, \qquad (3.74) \\
& g_{\nu,h,i},\, d_{\nu,i},\, \mu_{\nu,h,i} \geq 0, \quad \forall i \in \mathcal{I}, \quad h \in \mathcal{H}_i, \\
& \rho_{\nu,s},\, a_{\nu,i}, \vartheta_\nu,\, \lambda_{\nu,i}, \varphi_\nu \quad free \quad \forall i \in \mathcal{I}.
\end{aligned}
$$

Based on this Lagrangian, the equilibrium conditions for GenCo ν are derived,

- For $d_{\nu,i}, \quad \forall i \in \mathcal{I}$:

$$
\begin{aligned}
0 \geq & \frac{\rho_{oi} - \delta_i (d_{\nu,i} + d^*_{-\nu,i} - B_{-\nu,i} \rho^*_i + a_{\nu,i})}{1 + B_{-\nu,i} \delta_i} - \Big[\omega^*_i + A_{\nu,i} (d_{\nu,i} - \sum_{h} g_{\nu,h,i}) \Big] - \\
& \frac{\delta_i}{1 + \delta_i B_{-\nu,i}} \Big[d_{\nu,i} - \lambda_{\nu,i} + \sum_{\ell \neq i} \big(\tau_{\nu,\ell,i} - \tau_{\nu,i,\ell} \big) \Big] - \vartheta_\nu + A_{\nu,i} \lambda_{\nu,i} \quad \perp \quad d_{\nu,i} \geq 0 \qquad (3.75)
\end{aligned}
$$

- For $g_{\nu,h,i}, \quad \forall i \in \mathcal{I}, \quad h \in \mathcal{H}_i$:

$$\begin{aligned} 0 \geq & -\beta_{\nu,h,i} - 2\gamma_{\nu,h,i} g_{\nu,h,i} + \Big[\omega_i^* + A_{\nu,i}(d_{\nu,i} - \sum_{\hbar} g_{\nu,\hbar,i})\Big] + \vartheta_\nu \\ & - A_{\nu,i}\lambda_{\nu,i} - \mu_{\nu,h,i} \quad \perp \quad g_{\nu,h,i} \geq 0 \end{aligned} \tag{3.76}$$

- For $\mu_{\nu,h,i}, \quad \forall i \in \mathcal{I}, h \in \mathcal{H}_i$:

$$0 \geq g_{\nu,h,i} - \overline{g}_{\nu,h,i} \quad \perp \quad \mu_{\nu,h,i} \geq 0 \tag{3.77}$$

- For $a_{\nu,i}, \quad \forall i \in \mathcal{I}$:

$$\begin{aligned} 0 = & \frac{-\delta_i}{1 + \delta_i B_{-\nu,i}} \Big[d_{\nu,i} - \lambda_{\nu,i} + \sum_{\ell \neq i} \big(\tau_{\nu,\ell,i} - \tau_{\nu,i,\ell}\big)\Big] + \\ & A_{\nu,i}\Big[\lambda_{\nu,i} - (d_{\nu,i} - \sum_{\hbar} g_{\nu,\hbar,i})\Big] - \varphi_\nu; \quad a_{\nu,i} \ free \end{aligned} \tag{3.78}$$

- For $\rho_{\nu,s}$:

$$0 = \sum_i \lambda_{\nu,i}; \quad \rho_{\nu,s} \ free \tag{3.79}$$

- For ϑ_ν:

$$0 = \sum_i d_{\nu,i} - \sum_{h,j} g_{\nu,h,j}; \quad \vartheta_\nu \ free \tag{3.80}$$

- For $\lambda_{\nu,i}, \quad \forall i \in \mathcal{I}$:

$$0 = \rho_{\nu,i} - \rho_{\nu,s} - \omega_{\nu,i}; \quad \lambda_{\nu,i} \ free \tag{3.81}$$

- For φ_ν:

$$0 = \sum_i a_{\nu,i}; \quad a_{\nu,i} \ free \tag{3.82}$$

3.5.2 ISO problem

Since the arbitrage conditions have been introduced into the GenCos' problem, such conditions do not have to be explicitly modelled into the ISO problem anymore. The ISO problem is now to efficiently allocate the scarce transmission among GenCos. Recalling the ISO problem from §3.2.2, it can be casted as,

$$max \sum_i \omega_i^* p_i \tag{3.83}$$

$$s.t. \left|\sum_i s_{i,k} p_i\right| - \overline{z}_k \leq 0, \quad \forall k \in \mathcal{K}, \tag{3.84}$$

$$p_i \; free, \quad \forall i \in \mathcal{I}. \tag{3.85}$$

The derivation of the corresponding equilibrium conditions are straightforward; they are

- For $\lambda_k^+, \quad \forall k \in \mathcal{K}$:

$$0 \geq \sum_i s_{i,k} p_i - \overline{z}_k \quad \perp \quad \lambda_k^+ \geq 0 \tag{3.86}$$

- For $\lambda_k^-, \quad \forall k \in \mathcal{K}$:

$$0 \geq -\sum_i s_{i,k} p_i - \overline{z}_k \quad \perp \quad \lambda_k^- \geq 0 \tag{3.87}$$

- For $p_i, \quad \forall i \in \mathcal{I}$:

$$0 = \omega_i^* + \sum_k s_{i,k}(\lambda_k^- - \lambda_k^+); \quad p_i \; free \tag{3.88}$$

3.5.3 Market clearing conditions

In addition to match demand and supply of energy and transmission, market clearing conditions characterize the market prices for such services. The first condition is for the energy market. This condition ensures that, under equilibrium, the nodal prices are equal to the prices seen by each GenCo,

$$\rho_i^* = \rho_{oi} - \delta_i\big(d_{\nu,i} + d_{-\nu,i} + a_{\nu,i}\big), \quad i \in \mathcal{I}, \quad \nu \in \mathcal{V}. \tag{3.89}$$

A second condition is that the power balance at each node needs to be preserved,

$$p_i = a_{\nu,i} + \sum_f (d_{f,i} - \sum_h g_{f,h,i}), \quad \forall i \in \mathcal{I}, \quad \nu \in \mathcal{V}. \tag{3.90}$$

Notice that at equilibrium: $\omega_{\nu,i} = \omega_i^*, \forall \nu, i$. Although arbitrage and the slack-node price are considered endogenous within each GenCo problem, at equilibrium, they will be the same for all GenCos, *i.e.*, $a_{f,i} = a_{\nu,i}$ and $\rho_{f,s} = \rho_{\nu,s}, \forall f \neq \nu$.

Likewise the model in §3.2, the complementarity conditions of all market participants together with the market clearing conditions define a market equilibrium problem. The resulting complementarity model has been solved with PATH under GAMS [118].

3.5.4 An illustrative example

Let us consider the two-zone system shown in Figure 3.6. Assume that suppliers at every zone can be represented by a sole GenCo with unlimited capacity (otherwise specified), and denoted by $g_{1,1}$ and $g_{2,2}$, respectively. They have energy marginal costs of \$15/MWh and \$14/MWh, respectively. Demands are given by the affine functions ρ_1=150 − 0.15d_1 and ρ_2=149 − 0.15d_2. The transmission link, z_1, between zones is modelled as lossless and with a transmission limit of 100 MW.

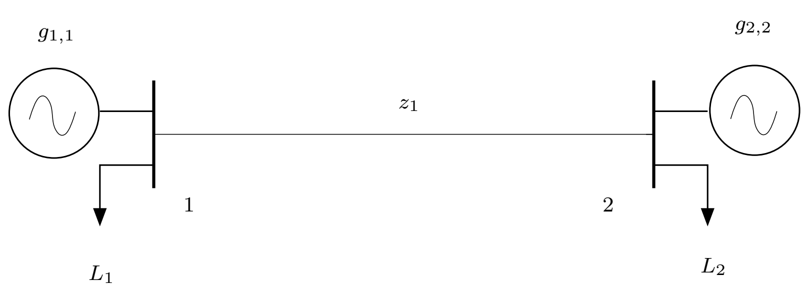

Figure 3.6: A two-zone system.

Consider a static market where $g_{1,1}$ behaves in a Cournot fashion ($B_{-1,i} = 0$) and $g_{2,2}$ behaves competitively ($B_{-2,i} \to \infty$). Furthermore, assume that $g_{1,1}$ can influence the transmission prices ($A_{1,i} = 0.5, \quad i = \{1, 2\}$). With this market configuration, the price at zone 2 tends to be more competitive ($\rho_2 \to \$14$ as $B_{-2,i} \to \infty$), and $g_{2,2}$ becomes an exporter. Hence, the transmission link will become congested ($\lambda_1^- > 0$) as cheap power flows from zone 2 to zone 1. Here, three cases are considered under different assumptions of FTR ownership. In all three cases, the outcomes of $g_{2,2}$ do not vary; different market outcomes will occur only if the strategy $g_{1,1}$ changes. Results of these simulations are presented in Table 3.8.

Case A: No FTRs

Let us consider that $g_{1,1}$ holds an FTR of, say, 75 MW from zone 2 to zone 1 ($\tau_{1,2,1}$), but $g_{1,1}$ does not include such an FTR into its profit maximization problem; *i.e,* $g_{1,1}$ considers the FTR payoff as an exogenous revenue. The payoff for holding $\tau_{1,2,1}$ will be $\Pi_1^F = 75(\rho_1^* - \rho_2^*)$ after the energy market is cleared. Due to strategic behaviour of $g_{1,1}$, the market price at its zone is \$71.32/MWh; while the congestion cost is \$57.3/MWh. The market operator collects \$5,732.65/h from congestion; from this surplus, \$4,299.5/h are used to honor the $\tau_{1,2,1}$ payoff to $g_{1,1}$.

Case B: Inclusion of $\tau_{1,2,1}$

For this case, assume that $g_{1,1}$ takes into account the FTR ownership within its profit-maximization problem. The result is a further decrease in production (393.87 MW) with an increase in its zone price, and, therefore, an increase in congestion (from \$57.32/MWh to \$61.91MWh). Its net profit is now \$23,994.3/h by selling energy, plus \$4,643.8/h from the FTR payoff. By including the FTR into its strategy, $g_{1,1,1}$ has increased the $\tau_{1,2,1}$ payoff from \$4,299.5/h to \$4,643.8/h. Hence, the ownership of $\tau_{1,2,1}$ has exacerbated the market power of $g_{1,1}$, shrinking the social welfare. For a similar system configuration, Joskow *et al.*, [59], have analytically reached this conclusion.

Case C: Inclusion of $\tau_{1,1,2}$

Let us now consider that $g_{1,1}$ holds an FTR in the opposite direction; *i.e.,* $\tau_{1,1,2} = 75$ MW. This FTR, going from zone 1 to zone 2, has a negative payoff, resulting in a liability for the holder. Consequently, $\tau_{1,2,1}$ is now an obligation rather than a right. The best strategy for $g_{1,1}$ is to increase production in order to reduce the price of its zone, and, thus, reduce the congestion cost (from \$57.3/MWh to \$55.3/MWh). The liability for holding $\tau_{1,1,2}$ decreases from \$4,299.5 to \$4153.12. In this case, the ownership of $\tau_{1,1,2}$ mitigates the market power of $g_{1,1}$, increasing the social welfare of the market.

Case D: Pure monopoly

Alternatively, all cases can be solved analytically to get the generation level and price at zone 1 in terms of the allocated amount of $\tau_{1,2,1}$.

Given the system configuration, it is straightforward to state the profit-maximization problem of $g_{1,1}$ for an energy-only market with the ownership $\tau_{1,2,1}$; this is as follows:

$$\max_{g_{1,1}} \Pi_1 = \rho_{1,1} g_{1,1} - \beta_{1,1} g_{1,1} + \tau_{1,2,1}(\rho_{1,1} - \rho_2^*), \tag{3.91}$$

where $\rho_{1,1} = 150 - 0.15d_1$ is the inverse demand function, and $\rho_2^* = \$14/MWh$ is the

Table 3.8: Market outcomes comparison with different FTR ownerships[‡].

Case	A	B	C	D
$A_{1,i}, i = \{1, 2\}$	0.5	0.5	0.5	$\to \infty$
$\tau_{1,2,1}$	0.0	75	-75	75
$g_{1,1}$	424.5	393.8	437.5	362.5
$g_{2,2}$	1,000.0	1,000.0	1,000.0	1,000.0
d_1	524.5	493.8	537.5	462.5
d_2	900.0	900.0	900.0	900.0
z_1	100.0	100.0	100.0	100.0
ρ_1^*	71.3	75.9	69.3	80.6
ρ_2^*	14.0	14.0	14.0	14.0
λ_1	57.3	61.9	55.3	66.6
Π_1^G	23,910.0	23,994.3	23,789.0	23,789.0
Π_1^F	4,299.5	4,643.8	-4,153.1	4,996.8
Π_1	28,209.5	28,638.1	19,635.9	28,785.8
Π_2	0.0	0.0	0.0	0.0
Congestion Rents	5,732.6	6,191.8	5,537.5	6,850.0
GenCos' surplus	23,910.0	23,994.3	23,789.0	23,789.0
Loads' surplus	81,381.5	79,043.4	82,417.7	76,792.7
Social welfare	111,024.1	109,229.5	111,744.3	107,431.7

‡ Power in MW, prices in $\$/MWh$, any other in $\$/h$.

competitive price at zone 2. Because $g_{2,2}$ is cheaper and behaves competitively, the system will be congested from zone 2 to zone 1; hence, $\overline{z}_1$ will be binding. As long as this congestion pattern is acknowledged by $g_{1,1}$, this GenCo will see a residual demand of $d_1 - 100$ MW. Thus, simplifying (3.91) one gets

$$\max_{g_{1,1}} \Pi_1 = 120 - 0.15g_{1,1}^2 + \tau_{1,2,1}(121 - 0.15g_{1,1}). \tag{3.92}$$

By the first optimality conditions, $g_{1,1}^* = 400 - 0.5\tau_{1,2,1}$ MW and $\rho_1^* = 75 + 0.075\tau_{1,2,1}$ $\$/MWh$. As it can be seen, the Case B falls between the monopoly without FTRs ($g_{1,1}^* =$ 400 MW, $\rho_1^* = \$75/MWh$) and the perfect monopoly with $\tau_{1,2,1} = 75$ MW ($g_{1,1}^* = 362.5$ MW, $\rho_1^* = \$80.62/MWh$). With the proposed computable model, the market outcome from Case D can be obtained by using larger values of $A_{1,i}\,\forall\, i$.

3.5.5 A 118-node system

In this simulation, the standard IEEE-based test power system of 118 nodes and 186 transmission lines [120] is used to represent the transmission system. For this case, 54 generating units are considered and organized in 15 GenCos. Furthermore, GenCos hold portfolios of FTRs from an *ex ante* allocation process. Generation, demand and FTR data are listed in §B.4, Appendix B.

For a case study, GenCos 1 and 2 are considered to behave à la Cournot in the energy market, while they are also able to alter the transmission prices ($A_{\nu,i} = 0.01$, $\nu = \{1, 2\}$, $\forall i$). The remaining GenCos are modelled as a competitive fringe. For this system, two cases are analyzed. In Case A, although GenCos hold FTRs, such FTRs are no included in the GenCos strategies. In this way, the FTR payoffs are modelled as exogenous to the GenCos profit-maximization problem. In contrast, for Case B, the FTR payoffs are considered endogenous within the GenCo profit optimization problems, as derived *supra* in §3.5.1.

When GenCos recognize the extra room that FTRs can provide for exercising market power, the peak prices (both lower and upper) are increased, making larger the price differences throughout the network –see Figure 3.7. For instance, the price at node 69, which is the second lowest one, decreases from $\$21.33/MWh$ to $\$20.27/MWh$; on the other hand, the price at node 75, which is the highest one, increases from $\$34.08/MWh$ to $\$36.55/MWh$. Consequently, those FTRs defined between such extreme prices become more valuable. Consider, for instance, $\tau_{1,69,75}$; for Case A, this FTR has a value

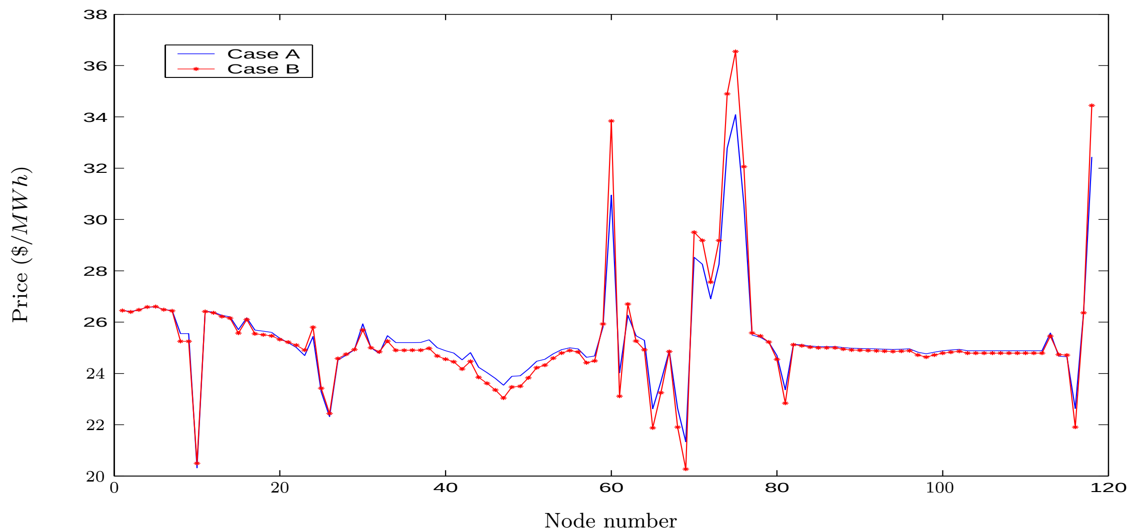

Figure 3.7: Comparison of nodal prices. 118-node system.

of $34.08 - 21.33 = 12.75\$/MWh$; meanwhile, for Case B, its value becomes $36.55 - 20.27 = 16.28\$/MWh$. By a further manipulation of the market, this FTR is $\$3.53/MWh$ more valuable.

In order to manipulate the market prices, GenCos have to modify their generation schedules. GenCo 1, placed at node 69, actually increases its generation from 742.9 MW to 820 MW (full capacity) in order to decrease the corresponding nodal price[6]. Due to this kind of adjustments, the generation schedules may change when FTRs are included. The most vivid case is the generation unit of GenCo 8, which is placed at node 55. For Case A, this unit produces 104.2 MW; nonetheless, when FTRs are considered, this generation unit is not scheduled anymore. The comparison of generation schedules for both cases is given in Table 3.9.

[6]At first glance, this increase in generation might not be seen as an exercise of market power, as the traditional exercise of market power is based upon a withholding of capacity. This counter-intuitive way to exercise market power is due to inherent interdependencies among suppliers through the transmission network [22].

Table 3.9: Comparison of generation schedules in MW.

ν	h	i	Case A	Case B
1	1	49	344.6	400.0
1	1	59	334.7	323.1
1	1	65	471.4	473.4
1	1	69	742.9	820.0
1	1	89	700.0	700.0
2	1	10	300.1	300.1
2	1	25	196.6	192.6
2	1	26	299.9	294.2
2	1	61	270.9	311.7
2	1	66	274.7	331.4
2	1	80	675.0	675.0
2	1	100	350.0	350.0
3	1	4	220.0	220.0
3	1	6	200.0	200.0
4	1	15	200.0	200.0
4	1	18	200.0	200.0
5	1	31	90.6	111.6
6	1	32	200.0	200.0
6	1	34	200.0	200.0
7	1	85	200.0	12.9
8	1	55	104.2	0.0
8	1	62	200.0	200.0
9	1	56	200.0	200.0
11	1	77	152.8	200.0
14	1	105	200.0	200.0
14	1	110	200.0	200.0

As a result of the inclusion of FTRs into the profit-maximization problems, GenCos 1 and 2 have a profit increase not only from the FTR payoffs, but also from participating in the energy market –see Table 3.10. While some of the remaining GenCos (3, 5, 7, 8, 11, 12, 13 and 14) are better off for the strategic behaviour of GenCo 1 and 2, other GenCos (4, 6 and 9) have their profits decreased. Which (competitive) GenCos are better off or worsened depends basically upon the system configuration and the generation units costs.

Table 3.10: Comparison of GenCos profits in $\$/h$.

	Case A			**C**ase B		
ν	Π_ν^G	Π_ν^F	Π_ν	Π_ν^G	Π_ν^F	Π_ν
1	32,123.4	3,463.7	35,587.1	32,165.1	4,457.9	36,623.0
2	24,086.8	2,504.8	26,591.6	24,330.1	3,264.3	27,594.5
3	445.9	0.0	445.9	446.7	0.0	446.7
4	270.4	301.1	571.5	215.3	259.7	475.0
5	0.0	959.4	959.4	0.0	1,192.5	1,192.5
6	396.9	71.8	468.7	348.4	69.8	418.3
7	9.6	502.6	512.3	0.0	643.2	643.2
8	53.9	301.6	355.5	140.6	446.8	587.4
9	191.5	102.2	293.7	167.5	107.0	274.5
11	0.0	515.9	515.9	15.7	660.6	676.3
12	0.0	13.9	13.9	0.0	17.8	17.8
13	0.0	278.3	278.3	0.0	356.1	356.1
14	953.4	392.2	1,345.6	915.4	471.7	1,387.2

3.6 Summary

In this chapter, an equilibrium model to analyze imperfect competition in an integrated market for energy and spinning reserve has been presented. By including temporal constraints, a dynamic model is analyzed. It is also considered that GenCos can influence the prices of energy, transmission and spinning reserve.

The integration of different goods in one market creates complex interactions and interdependencies among suppliers resources due to system constraints. If such interactions and constraints are not taken into account, misleading conclusions of market power can be obtained. With the proposed model, one attempts to add more realism to the modelling and analysis of market power in electricity markets.

The proposed MLCP is obtained by gathering the KKT conditions of the optimization problem for each market participant –GenCos and ISO– and the market clearing conditions. One of the advantages of using a linear-complementarity framework, for modelling imperfect competition, is the convexity nature of the equilibrium problem, and its potential applicability for large power systems.

Based upon optimality conditions, a general expression of the opportunity cost between energy and spinning reserve markets has been derived. This derivation allows one to identify the components of the opportunity cost between generating and spinning within a range of strategies, going from the Cournot case to the competitive one, and also to identify the impact of manipulation on the spinning reserve market. Within this model, the effect of financial transmission rights allocation upon GenCos incentives have been considered; moreover, complex strategies for GenCos (with a set of generators and a portfolio of FTRs) can be easily included.

It has been shown that even a competitive spinning reserve market may have an effect on the energy market efficiency. For instance, when a capacity-limited generation unit procures both energy and spinning reserve, such a unit shifts power from the energy market to the spinning reserve market. Therefore, more expensive energy has to be used, with a decrease of the social welfare. However, if this unit attempts to increase the SR price, it has to reduce its SR capacity. This fact can be an opposite incentive to that from the opportunity cost, since now this unit would have more capacity available for the energy market.

Due to the inherent characteristics of power systems, oligopoly models are currently envisioned as a proper modelling framework. However, it is recognized that, they are not enough to address all the concerns of imperfect competition in the power sector [32, 124]. So far, due to its nice numerical properties, equilibrium models based upon conjectures functions have arisen as a practical alternative to analyze imperfect competition in electricity markets [32].

One drawback could seem to be the computation of the conjectured parameters as they are exogenous factors to the oligopoly models, and are not directly observed in the markets. However, it has been shown that the conjectures parameters can be either obtained from market data or can be calibrated to replicate market outcomes [15,29,50,111]. Moreover, the conjecture-based models have the flexibility to analyze different settings by parametrically varying the conjectures.

Chapter 4

Exacerbation of Market Power due to FTR Ownership

In this chapter, a methodology is proposed for taking into account potential exacerbation of market power due to financial transmission rights allocations. Hedging Position Ratios (HPRs) are computed for FTR bids. These ratios quantify the relationship between the positions of an FTR bidder in the energy market, and in the transmission rights allocation (based on the transmission rights bids). They are used to identify the potential gambling positions from the transmission rights bidders and, therefore, to prioritize critical positions in the auction. The transmission-right auction is modelled as a quadratic programming problem with a DC-network approximation. The methodology is illustrated by a three-node system, and then extended to larger systems.

This chapter is organized as follows. The proposed scheme for the screening and mitigation of exacerbated market power due to an FTR allocation is presented in §4.1. Numerical results are presented in §4.2. Final remarks close the chapter in §4.3.

4.1 Proposed Methodology

4.1.1 A market for FTRs

A transmission-right market can provide a way to determine an allocation of FTRs and their respective prices. The participants bid their willingness to trade FTRs. As an outcome, the auction allocates the set of feasible FTRs. The objective of the market is twofold: (i) to provide a means so that participants can access FTRs; and (ii) to maximize the revenues from the FTR trading. An FTR market can be modelled similar to an energy auction, maximizing surplus while maintaining the network feasibility for the set of awarded FTRs; however, it does not represent an actual energy transaction.

For a power system, where the set of nodes is denoted by $\mathcal{I}$ and $|\mathcal{I}|$ stands for its cardinality, an FTR can be modelled with a variable τ_ℓ for the real power to be contracted, and an incidence vector $\boldsymbol{\xi}_\ell \in \Re^{|\mathcal{I}|}$ for injections/withdrawals of real power. The elements of $\boldsymbol{\xi}_\ell$ are factors (in per unit) that describe how an FTR is distributed among the network nodes. Thus, every FTR can be modelled with either a pair or a cluster of nodes for the source and sink.

In the proposed model, FTRs are defined with two nodes, one for injection (source) and the other for withdrawal (sink). Let us consider a DC-model approach for an FTR auction. The set $\mathcal{F}$ of all FTRs represents the net power injection into the system; thus, the nodal real power balance in the system is given by

$$\boldsymbol{G}\boldsymbol{\delta} = \sum_{\ell \in \mathcal{F}} \tau_\ell \boldsymbol{\xi}_\ell, \tag{4.1}$$

where $\boldsymbol{G}$ stands for the susceptance matrix and $\boldsymbol{\delta}$ is the vector of nodal angles [125]. The set $\mathcal{F}$ can have three kinds of FTRs [126]: (i) the subset $\mathcal{F}_f$ which contains fixed FTRs; these FTRs are not traded in the auction but they must be included to consider their effect; (ii) $\mathcal{F}_p$ which contains FTRs to be purchased; and (iii) $\mathcal{K}_s$ that contains FTRs to be sold, *i.e.*,

$$\mathcal{F} = \mathcal{F}_f \cup \mathcal{F}_p \cup \mathcal{F}_s, \tag{4.2}$$

such that

$$\sum_{\ell \in \mathcal{F}} \tau_\ell \boldsymbol{\xi}_\ell = \sum_{\ell \in \mathcal{F}_f} \tau_\ell \boldsymbol{\xi}_\ell + \sum_{\ell \in \mathcal{F}_p} \tau_\ell \boldsymbol{\xi}_\ell - \sum_{\ell \in \mathcal{F}_s} \tau_\ell \boldsymbol{\xi}_\ell. \tag{4.3}$$

In addition, the feasible values of the FTRs are defined as,

$$0 \leq \tau_\ell \leq \overline{\tau}_\ell, \quad \forall \ell \in \mathcal{F}, \tag{4.4}$$

where $\overline{\tau}_\ell$ denotes the maximum value of the ℓ-th FTR. Such a maximum value is estimated by the bidders based upon their requirements of hedging, and submitted as a part of their bid to the ISO.

For the case of FTRs to be purchased, their allocation can be stated as the following optimization problem:

$$max \sum_{\ell \in \mathcal{F}} f_\ell(\tau_\ell) \tag{4.5}$$

$$s.t. \;\; \boldsymbol{G\delta} = \sum_{\ell \in \mathcal{F}} \tau_\ell \boldsymbol{\xi}_\ell, \tag{4.6}$$

$$\boldsymbol{H\delta} \leq \overline{\boldsymbol{z}}, \tag{4.7}$$

$$0 \leq \tau_\ell \leq \overline{\tau}_\ell, \quad \forall \ell \in \mathcal{F}. \tag{4.8}$$

where $f_\ell(\tau_\ell)$ stands for the concave benefit function for the ℓ-th FTR. This benefit function is estimated by bidders according to their hedging needs, and represents the bidder willingness to acquire such FTRs. Equation (4.7) stands for the real power-flow transmission-line limits; $\boldsymbol{H}$ is the reactance matrix of the transmission lines [125]; and $\overline{\boldsymbol{z}}$ denotes the vector of maximum real power flows on the transmission lines. The solution to the problem (4.5)–(4.8) gives the optimal FTR allocation. The dual variables related to (4.6) yield the nodal prices; since FTRs are defined between two locations, the price for every FTR is given by the price difference between trading locations.

4.1.2 Transmission usage factors for market participants

Since the sensitivity factors are dependent on the slack-node choice, and quantify incremental changes, they may not be used to quantify the net utilization of the network by participants. Different methods to compute the transmission usage by market participants have been proposed, such as Generalized Distribution Factors (GDFs) [127], a proportionality method [128] and the tracing method [129]. Due to the closeness with the sensitivity factors, GDFs are used in this chapter. For details of the derivation of such factors, please refer to Definition A.3, Appendix A. The sound feature of these factors is their independence of the slack-node choice which yields no conflict of interest; and secondly, they are based upon an actual operation point of the power system.

Given a market outcome, the vector of power flows $\boldsymbol{z} \in \mathbb{R}^{|\mathcal{K}|}$, scheduled generations $\boldsymbol{g} \in \mathbb{R}^{|\mathcal{I}|}$ and loads $\boldsymbol{d} \in \mathbb{R}^{|\mathcal{I}|}$ are taken to compute the GDFs. By Definition A.3, the usage factor, $\varphi_{k,i}$, of the k-th transmission line, by a market participant placed at node i, is given by

$$\varphi_{k,i} = s_{k,i} + \varphi_{k,s}, \tag{4.9}$$

where

$$\varphi_{k,s} = \begin{cases} \frac{z_k - \langle \boldsymbol{s}_k, \boldsymbol{g} \rangle}{\langle \boldsymbol{e}, \boldsymbol{g} \rangle}, & \text{for suppliers,} \\ \\ \frac{\langle \boldsymbol{s}_{ij}, \boldsymbol{d} \rangle - z_k}{\langle \boldsymbol{e}, \boldsymbol{d} \rangle}, & \text{for loads.} \end{cases} \tag{4.10}$$

4.1.3 Transmission usage factors for FTRs

As an FTR τ_ℓ is defined by a point-to-point transaction, by Definition A.2 , the transmission usage factors for ℓ-th FTR is given by the power transfer distribution factors $\varrho_{k,\ell}$.

4.1.4 Hedging position ratios

Once the distribution factors have been computed, the power capacity that market participants are using of each transmission line can be easily obtained. For a market participant placed at node i, with a power injection p_i, the power that is being sent through the transmission line k is $p_i \varphi_{k,i}$. Similarly, for an FTR bid τ_ℓ, the power that this FTR would send through the transmission line k is $\overline{\tau}_\ell \varrho_{k,\ell}$. The former value is a metric of the size of the exposure, while the latter is a metric of the size of the hedge. The proportion between the position taken in the hedge and the position in the exposure will be called Hedging Position Ratio (HPR), and denoted by $\phi_{k,\ell}$.

For a market participant placed at node i and submitting a bid for $\overline{\tau}_\ell$, its HPR for line k is given by

$$\phi_{k,\ell} = \frac{p_i}{\overline{\tau}_\ell} \frac{\varphi_{k,i}}{\varrho_{k,\ell}}. \tag{4.11}$$

The HPR contains two factors; the first one is $p_i/\overline{\tau}_\ell$ which is the ratio of the power traded in the market and the power bid for hedging; this ratio is independent of the transmission line. If $p_i < \overline{\tau}_\ell$, then $p_i/\overline{\tau}_\ell < 1$; hence, the FTR bid is implicitly penalized since the bidder is hedging more power than the bidder is actually transacting in the market. On the other hand, if $p_i > \overline{\tau}_\ell$, then $p_i/\overline{\tau}_\ell > 1$; such an FTR bid is implicitly encouraged. This factor, thus, gets the market participants to bid the amount of power they have been actually transacting in the energy market. The second factor, $\varphi_{k,i}/\varrho_{k,\ell}$, is more exogenous since it depends not only upon the participant's bid but also upon the network configuration, the participant's location and even upon the position of the other participants.

Because some power flows of the FTR bids can be opposite to those flows in the energy market, their corresponding HPRs do not represent an actual hedge, and, thus, they are left out, *i.e.*, $\phi_{k,\ell} = 0 \,|\, \phi_{k,\ell} < 0$.

Afterwards, the HPRs are normalized in every transmission line in order to evenly take into account the contribution of every transmission line to the hedging position of market participants, *i.e.*,

$$\overline{\phi}_{k,\ell} = \frac{\phi_{k,\ell}}{\underset{\ell}{max} \;\; \phi_{k,\ell}}, \quad \forall k \in \mathcal{K}. \tag{4.12}$$

The net HPR in the system for a market participant, biding for τ_ℓ, is then computed as the sum of all HPRs from the system transmission lines, *i.e.*,

$$\phi_\ell = \sum_{k \in \mathcal{K}} \overline{\phi}_{k,\ell} \quad \forall \ell \in \mathcal{F}. \tag{4.13}$$

In order to use the HPRs as weights they are normalized as well,

$$\overline{\phi}_\ell = \frac{\phi_\ell}{\underset{\ell}{max} \;\; \phi_\ell}, \quad \forall \ell \in \mathcal{F}. \tag{4.14}$$

Notice that $\overline{\phi}_\ell \in [0,1]$, $\forall \, \ell \in \mathcal{F}$. The HPRs comprise the market outcome, the network conditions and the participants' position in the market; hence, they may be used to screen potential gambling positions by FTR bidders. As HPRs quantify the level of hedging that FTRs provide to their potential holders, the lower the HPR value ($\overline{\phi}_\ell$), the more the gambling position a market participant may have with τ_ℓ.

4.1.5 Modified auction for FTRs

In order to detect any gambling position, it is required to analyze the HPRs for the historic market outcomes; the HPR analysis can be grouped into different periods such as on-peak and off-peak hours. An alternative, for instance, is to weight the FTR bids, $\overline{\tau}$, based on the mean value of the HPRs from historic market outcomes (say, the previous month for a monthly auction). Thus, an FTR auction can now be mathematically stated as,

$$max \sum_{\ell \in \mathcal{F}} f_\ell(\tau_\ell) \tag{4.15}$$

$$s.t. \quad \boldsymbol{G\delta} = \sum_{\ell \in \mathcal{F}} \tau_\ell \boldsymbol{\xi}_\ell, \tag{4.16}$$

$$\boldsymbol{H\delta} \leq \overline{\boldsymbol{z}}, \tag{4.17}$$

$$0 \leq \tau_\ell \leq \overline{\phi}_\ell \overline{\tau}_\ell, \quad \forall \ell \in \mathcal{F}. \tag{4.18}$$

In (4.18), the bids are proportionally weighted to the actual position that bidders have historically had in the market. Thus, market participants with low HPRs will have their maximum quantity for FTRs reduced. Since $\overline{\phi}_\ell \geq 0$, this scheme may still award a portion of strategic FTRs (although in smaller amounts) to participants with gambling positions. A stronger prioritization is to introduce a threshold HPR, $\hat{\phi}$, such that the ℓ-th FTR bid is accepted if $\overline{\phi}_\ell \geq \hat{\phi}$; otherwise, the bid is accepted as given. Then a challenge would be to identify which value is high enough for this threshold.

4.2 Numerical Examples

4.2.1 A three-node system

The methodology is firstly illustrated by means of a three-node system, which is shown in Figure 4.1. It is assumed that both suppliers G_1 and G_2 have unlimited capacity and marginal costs of $\$15/MWh$ and $\$20/MWh$, respectively; there is a price-responsive consumer at node 3 with a demand function $p_3 = 30 - 0.5\rho_3$. Furthermore, assume that G_1 behaves competitively, while G_2 is a price-maker. The network is modelled as lossless and with equal transmission lines' reactance; the transmission limit on line 1-2 is $4MW$, while the limits for the others are large enough to be disregarded. For a market outcome, p_i and ρ_i for $i \in \{1, 2, 3\}$ denote the net power injection (generation/demand) and the locational

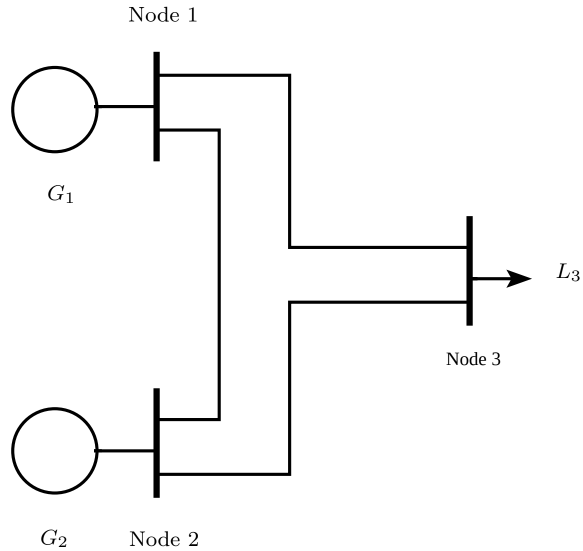

Figure 4.1: Three-node power system.

marginal prices, respectively; μ_{12} stands for the congestion multiplier of transmission line 1-2, while $4\mu_{12}$ is the congestion rent.

In this work, unlike [59], a point-to-point-based rights approach is analyzed. In order to show the incidence of FTR ownership by the price-maker supplier, only the case where G_2 is awarded with an FTR going from node 1 to node 2 (τ_1) is analyzed. The market outcomes under different strategies G_2 are shown in Table 4.1 for $\tau_1 \in [\,0,\ 4.625\,]$; the superscripts (*), (^{m}) and (^{f}) stand for the competitive market, market power with no FTR ownership and market power with τ_1, respectively. Π^G, Π^F and Π^{G+F} denote the profit by selling energy, by holding an FTR, and by both selling energy and holding and FTR, respectively.

Under competitive conditions, and with no transmission constraints, generator G_1 supplies all the demand; however, the transmission constraint in line 1-2 restraints the generation at node 1 and, thus, more expensive generation from G_2 has to be produced to serve the demand. Due to the system configuration, it can be seen that G_1 and G_2 are complements. In this case, G_2 produces a counterflow for G_1 which decreases congestion in line 1-2. An increase in p_2 allows an increase in p_1; hence, both generators have to be scheduled to obtain the optimal outcome. For the constrained case, the demand is 21.25 MW, while the congestion rent is \$30/h. The prices at nodes 1 and 2 equal the marginal cost of G_1 and G_2, respectively, while the price at node 3 is $(\rho_1 + \rho_2)/2$; therefore, G_2 has no profit by selling energy. On the other hand, if G_2 holds an FTR, τ_1, its payoff is the nodal price difference $(\rho_2 - \rho_1)$ times the FTR capacity.

Under monopoly conditions, the market power exercised by G_2 (by withholding capacity) worsens the market outcome; the larger the withheld capacity Δp_2^m, the lower the demand level, the higher the nodal prices ρ_2 and ρ_3, the higher the congestion cost, and the larger the G_2's profit Π^m. In order to maximize its profit, G_2 has to withhold $\Delta p_2^m = p_2^*/2$; hence, under monopolistic behaviour the production of G_2 is $p_2^m = p_2^*/2$, as traditional economics establishes[1]. By their complementary effect, a reduction in p_2 is followed by a

Table 4.1: Market outcomes comparison†

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	$(^*)$	$(^m)$	$(^f)$
ρ_1	15	15	15
ρ_2	20	38.50	38.50 +4 τ_1
ρ_3	17.50	26.75	26.75 +2 τ_1
p_1	16.62	14.31	14.31 -.5 τ_1
p_2	4.62	2.31	2.31 -.5 τ_1
p_3	21.25	16.62	16.62 - τ_1
μ_{12}	7.50	35.25	35.25 + $6\tau_1$
$4\mu_{12}$	30	141	141+24 τ_1
Π_G	0.0	42.78	42.78 - 2 τ_1^2
Π_F	5 τ_1	0.0	$23.5\tau_1$ + $4\tau_1^2$
Π_{G+F}	5 τ_1	42.78	42.78+ 23.5 τ_1 + 2 τ_1^2

† Prices in $\$/MWh$, power in MW and profits in $\$/h$.

[1]This result comes from the fact that with a linear demand, the marginal revenue curve (for monopolistic pricing) is twice as steep as the demand curve (for competitive pricing); hence, the marginal revenue curve hits the marginal cost curve at half the value of the demand curve.

reduction in p_1. On the other hand, as a result of a higher price at node 3, the demand level is reduced to 16.62 MW, and the congestion rent goes up to \$141/h.

Furthermore, if G_2 has also been awarded with an FTR, say, τ_1, its market power can be exacerbated; now G_2 can increase its profit further than in the pure monopolistic case since its profit is composed not only by energy sold, but also by the revenue from holding an FTR. Thus, by increasing its withheld amount by Δp_2^f, beyond the pure monopolistic level, G_2 can increase its net profit by $\Delta\Pi^f$ –see Figure 4.2. As expected, the extra withheld output to maximize the profit of G_2 depends upon the FTR amount, as shown in Figures 4.3 and 4.4. With no FTR ownership (the initial point $\tau_1 = 0$), the optimal strategy of G_2 is the output under pure monopolistic behaviour –see second column of Table 4.1. As long

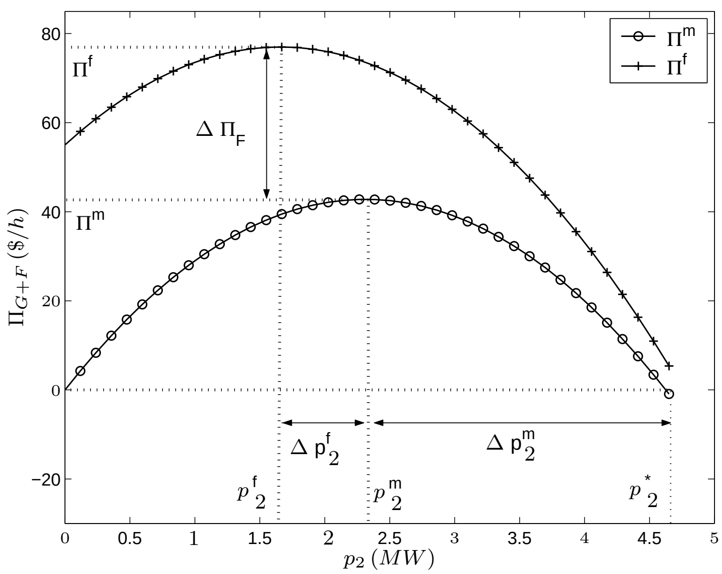

Figure 4.2: G_2's profit with and without $\tau_1 = 1.4$ MW.

as the awarded FTR increases, the generation level p_2 (to maximize G_2's profit) is lower, the price at node 2 is higher, and hence, the demand level is lower. This trade-off occurs up to $\tau_1 = 4.625MW$; beyond this value, G_2's optimal strategy is a generation of zero. The path of this optimal generation output is traced within Figure 4.4. With an awarded FTR of 6 MW, G_2 is able to extract all the congestion rent of \$252/h.

Consider now the pure monopolistic case; with this system configuration there are three profitable FTRs: (i) from node 1 to node 3: $\ell = 1$; (ii) from node 1 to node 2: $\ell = 2$; and (iii) from node 3 to node 2: $\ell = 3$. Any of these FTRs can exacerbate the market

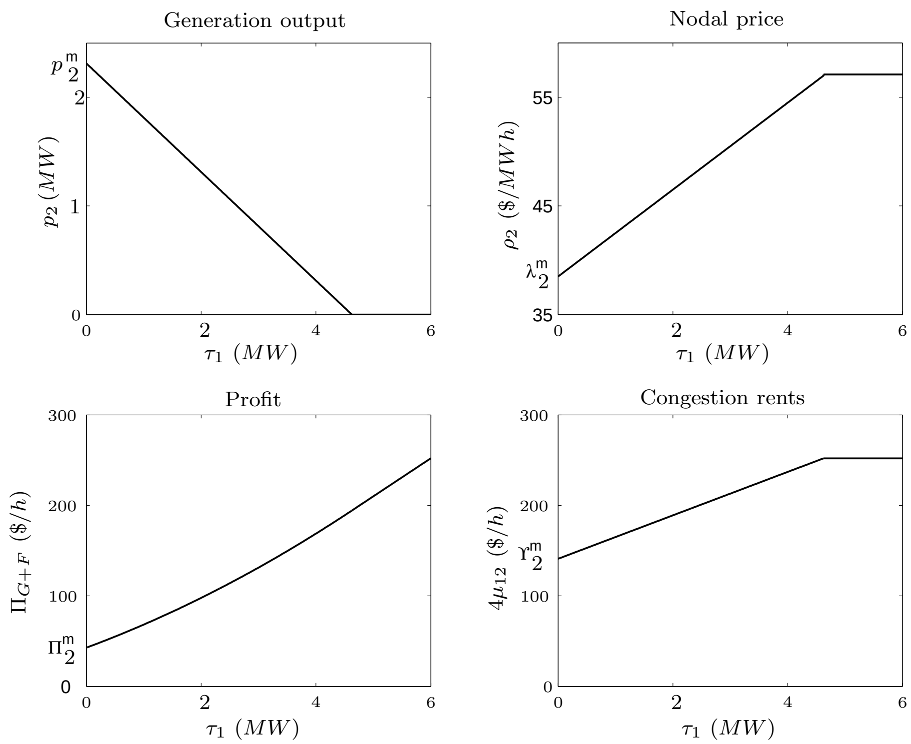

Figure 4.3: G_2's optimal strategy for different values of τ_1.

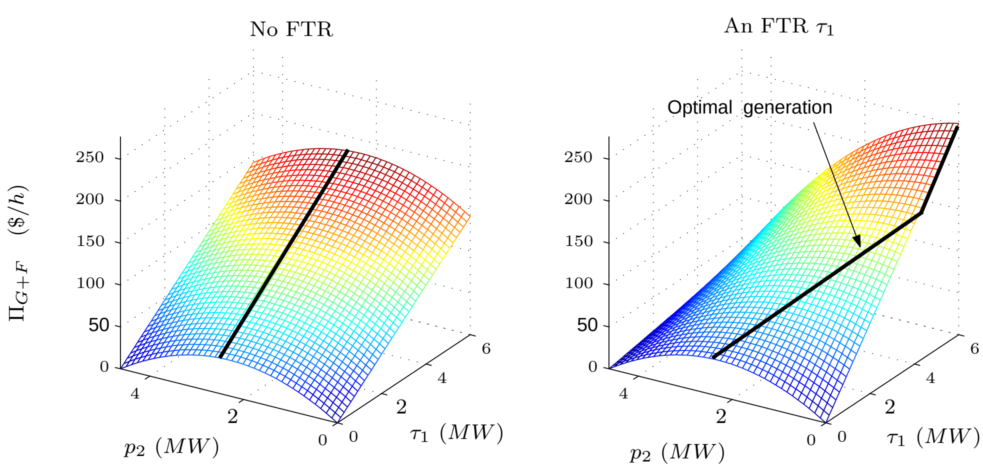

Figure 4.4: Effect of an FTR ownership on G_2's strategies.

power of G_2. Let us study these FTRs as if they were bid by either G_1 or G_2 . It is expected that G_2 bids for an FTR amount which makes G_2 maximize its profit. In the standard FTR market, suppliers can bid for any quantity they are willing to hold; but in the proposed methodology, if they bid for a quantity greater than the value of power they have historically sold in the market, their bids are implicitly penalized in the HPRs –see Equation (4.11). For this example, the transmission usage by participants are shown in Figures 4.5 and 4.6, and the HPRs are summarized in Table 4.2.

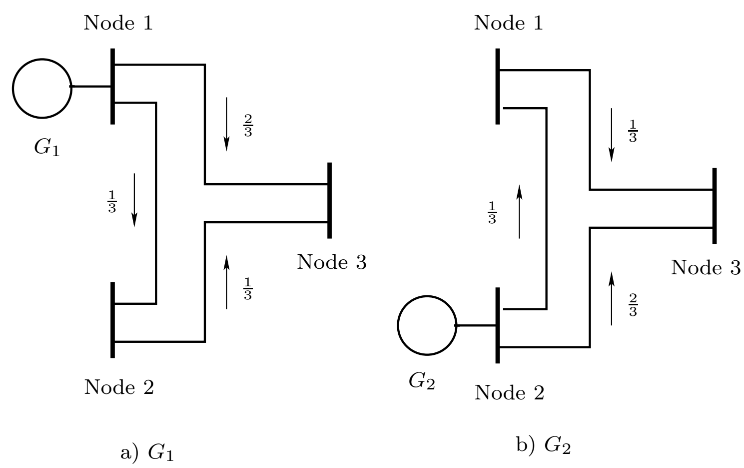

Figure 4.5: Generalized generation distribution factors for suppliers.

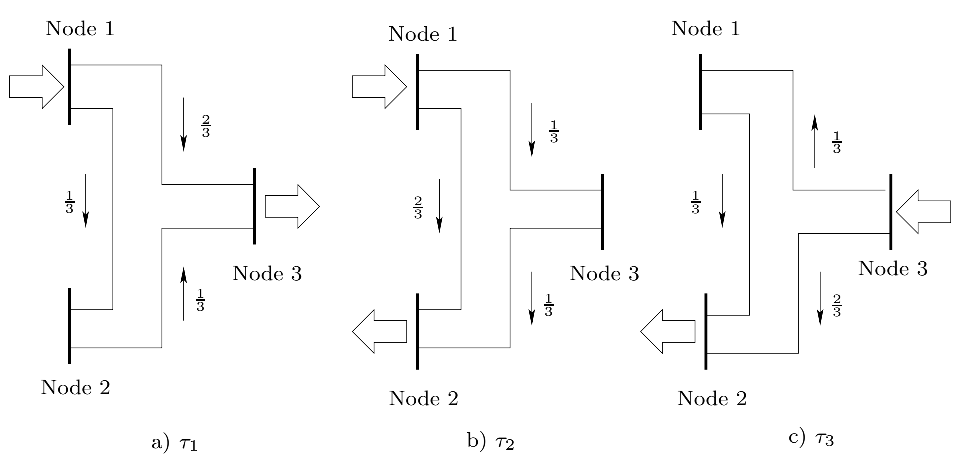

Figure 4.6: Power transfer distribution factors for different FTRs.

The HPRs identify the low-hedging positions that G_2 would have with any of the FTRs; for instance, the HPR corresponding to τ_1 is 7.7%, showing its low level of hedging. Consequently, the maximum value of τ_1 that G_2 could hold is $0.46MW$ (against $6MW$ that

Table 4.2: HPRs for different FTR bids.

bidder	ℓ	Source i	Sink j	$\overline{\phi}_\ell$ $(p.u.)$	$\overline{\tau}_\ell$ (MW)
G_1	1	1	3	1.00	14.31
G_2	1	1	3	0.17	12
G_1	2	1	2	0.60	14.31
G_2	2	1	2	0.07	6
G_1	3	3	2	0.40	14.31
G_2	3	3	2	0.00	12

G_2 needs to extract all the congestion rents). In the case of τ_3, its HPR results in a zero hedging level; as can be seen, this result comes from the fact that G_2 is bidding for an FTR which is not related to its actual position in the market. Moreover, as can be expected, G_1 would have the highest HPR value (100%) if it bid for τ_1, since G_1 is actually looking for hedging its position in the market.

4.2.2 Larger systems

Let us apply the above methodology to a five-node system; the test system is described in Appendix B. It follows the idea of having loads with expensive local generation, and cheaper generation in other areas. This illustrative power system has been used in different FTR literature of PJM, see for instance [130].

Based on nodal prices, it can be seen that different FTRs can be profitable, as shown in Table 4.3. On one hand, by market simulations, it is seen that G_4 may exercise market power. Consider, for instance, that G_4 holds an FTR of, say, 120 MW between nodes 2 and 4 (denoted as $\tau_{2\to4}$). The specific profit curves, shown in Figure 4.7, break down when G_4's bid reaches the competitive level p^*; for higher values, there is no strategic behaviour anymore and the profit of G_4 is constant since G_4 is scheduled to the competitive output. If G_4 holds any of those profitable FTRs, its market power is exacerbated. Therefore, the FTR allocation process should take into account its gambling position. For this case study, consider that only market participants can bid for buying FTRs; their bids, with

Table 4.3: Opportunity cost (in $\$/MWh$) for different FTRs.

	Sink				
Source	1	2	3	4	5
1	0.0	7.205	-5.344	17.589	9.974
2	-7.205	0.0	-12.549	10.384	2.769
3	5.344	12.549	0.0	22.934	15.318
4	-17.589	-10.384	-22.934	0.0	-7.615
5	-9.974	-2.769	-15.318	7.615	0.0

nondecreasing concave benefit functions defined as $f_\ell(\tau_\ell) = \beta_\ell\tau_\ell - \gamma_\ell\tau_\ell^2$, $\forall\ell \in \mathcal{F}$, are given in Table B.3. The HPRs for all the FTR bids are computed and listed in Table 4.4. The low HPRs of G_4's bids show that their bids are more gambling than hedging positions. On the other hand, the highest values of HPRs are for τ_{12}, τ_{16}, τ_{24}, τ_1 and τ_2 which are the bids from L_2, G_3, L_3, G_1 and G_2, respectively.

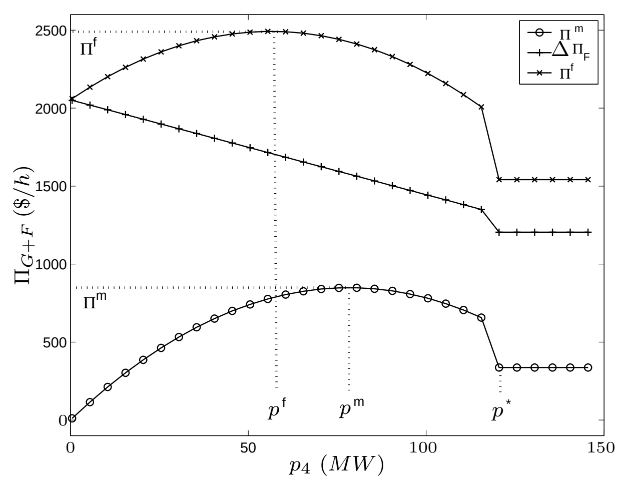

Figure 4.7: G_4's profit under different strategies.

In another simulation, the standard IEEE fourteen-node system is used to represent the transmission system [120]. It is assumed that there are 8 suppliers, each one located (and accordingly named) at nodes 1, two at node 2, 4, 5, 11, 13 and 14; there are also 10 demands, one located at each node except at nodes 1, 2, 7 and 8. Based upon a set of nodal prices –see Table 4.5, the most profitable FTRs are considered; among them are those having as source the nodes 1, 2 and 3. It is assumed that all market participants can

Table 4.4: Hedging position ratios.

ℓ	$\overline{\phi}_\ell$ $(p.u.)$	ℓ	$\overline{\phi}_\ell$ $(p.u.)$
1	0.7427	15	0.4050
2	0.7427	16	0.8608
3	0.6562	17	0.1970
4	0.1640	18	0.6966
5	0.5251	19	0.4501
6	0.5323	20	0.5922
7	0.6498	21	0.7357
8	0.6002	22	0.6187
9	0.2981	23	0.5448
10	0.2423	24	0.7438
11	0.5795	25	0.0457
12	1.0000	26	0.5422
13	0.4965	27	0.3855
14	0.0000		

bid for them. For this simulation, 288 bids were considered. For the sake of comparison, only the HPRs of bids for FTRs going from node 1 to node 5 ($\tau_{1\to5}$), and from node 3 to node 12 ($\tau_{3\to12}$) are depicted in Figure 4.8. In the first case, the FTR is defined from a low-

Table 4.5: Nodal prices in the fourteen-node system.

i	ρ_i $(\$/MWh)$	i	ρ_i $(\$/MWh)$
1	23.8837	8	46.1546
2	34.6167	9	46.3034
3	40.6550	10	46.1411
4	45.8717	11	45.7722
5	49.7545	12	48.9876
6	49.2179	13	48.8077
7	46.1546	14	47.3983

cost generation node to a high-cost demand node, and its HPRs are illustrated in Figure 4.8.a. Here, G_1 is the cheapest generator and one of the most affected by congestion; it has the highest HPR. In contrast, G_8 which is placed at node 14, has the lowest HPR. In the second case, most of the bids for the second case have low HPRs, denoting the low-hedging position this FTR provides to most of the market participants. Notice that the second FTR is defined between two load nodes. Here, the HPRs for the bids of G_8 are practically zero.

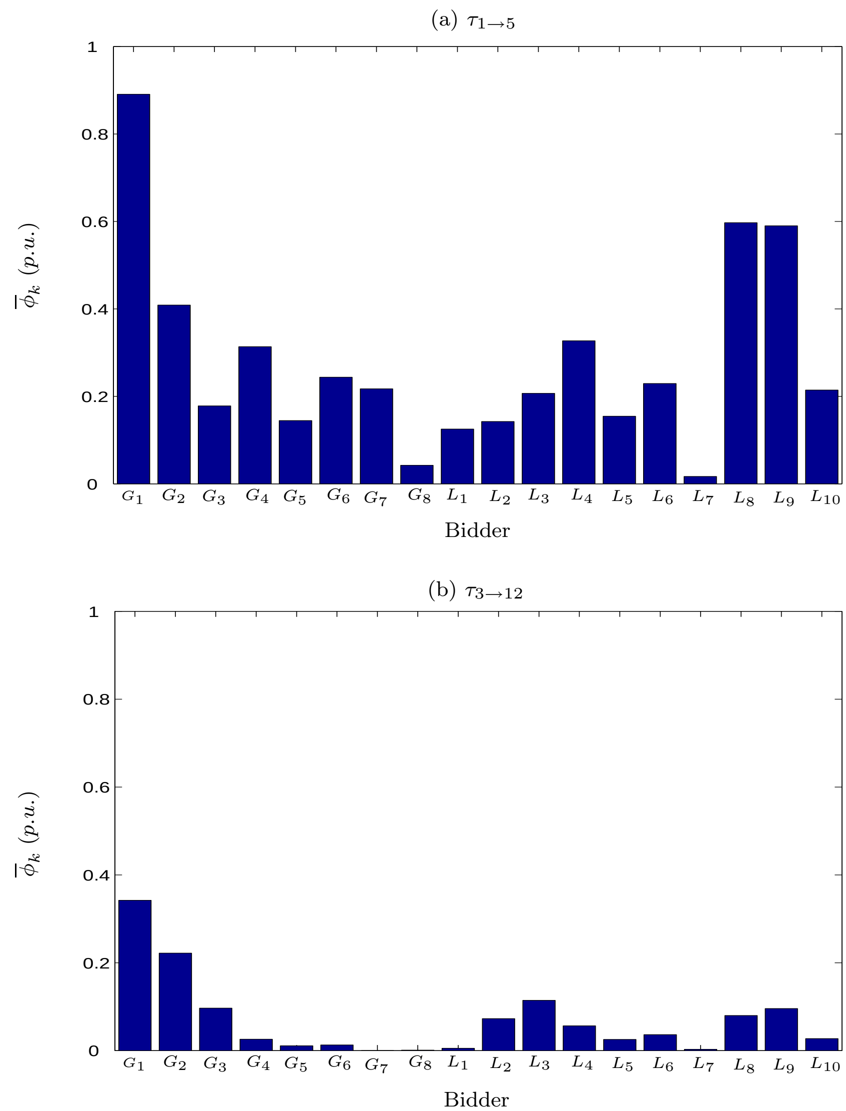

Figure 4.8: HPRs for the fourteen-node system.

4.2.3 Change in network configuration

In the operation of power systems, there are many factors or events that cause transmission lines failures. One of the classical consideration is the lost of a transmission line, called the *n-1* contingency. Since a transmission line outage produces a change in the network configuration, all the transmission usage factors will change. In order to show how changes in the network configuration affect the HPRs, they are computed under the *n-1* contingency case; a comparison of the HPRs is given in Table 4.6.

The HPRs will change as long as the network configuration does; this comes from the fact that the HPRs are based on the operation conditions. This feature allows them to capture the changing conditions of the market, and, therefore, the current positions that market participants have in the market. For instance, consider the lost of line 1-2 (contingency 1); G_4 is now placed within a load pocket, and its position to behave strategically is even stronger. All of its bids for FTRs ($\ell = 4, 10, 14, 17, 25$) have an HPR of zero. On the other hand, the highest value is for bid 24 from L_3; this load is facing the highest market price. The second highest value is for bid 22 from G_3; this generator is facing the lowest market price, and is outside the load pocket. Similar analysis can be done for the other contingencies.

4.2.4 Modified auction for FTRs

Extending the five-node example previously studied, it is now considered a daily demand profile (shown in Figure 4.9) in order to represent a historic market outcome. In this simulation, the mean values of the HPRs are based only on the hours when congestion occurs –see Figure 4.10. The FTR auction (4.15)–(4.18) is implemented as a quadratic programming problem, and solved by a primal-dual interior point method [131]. The results from both the standard and the proposed FTR auctions are summarized in Table 4.7.

As observed in Table 4.7, the inclusion of the HPRs changes the allocation of FTRs with respect to the standard FTR auction. It can be seen that the proposed HPRs screen the low-hedging position that any FTR provides to G_4, and, therefore, it can help to mitigate the room to exercise market power by G_4. See for instance, the cases for τ_{10} and τ_{14} in Table 4.7; within the standard auction, G_4 is awarded both FTRs in the amount of 120MW each; in contrast, within the modified auction, G_4 would have both FTR awards reduced

Table 4.6: HPRs for the *n-1* contingencies (p.u.).

	Contingency Line						
ℓ	None	1	2	3	4	5	6
1	0.742	0.852	0.770	0.713	0.110	0.560	0.169
2	0.742	0.403	0.770	0.414	0.074	0.532	0.092
3	0.656	0.877	0.780	0.779	0.390	0.762	0.307
4	0.131	0.0	0.214	0.200	0.0	0.232	0.0
5	0.525	0.352	0.245	0.400	0.234	0.510	0.385
6	0.532	0.438	0.088	0.509	0.182	0.564	0.110
7	0.649	0.803	0.732	0.645	0.471	0.593	0.254
8	0.600	0.614	0.356	0.522	0.624	0.598	0.615
9	0.298	0.614	0.267	0.361	0.0	0.394	0.307
10	0.193	0.0	0.503	0.408	0.093	0.367	0.000
11	0.579	0.0	0.334	0.413	0.0	0.406	0.0
12	1.000	0.0	0.532	0.770	0.624	0.805	0.307
13	0.496	0.0	0.431	0.421	0.347	0.288	0.124
14	0.0	0.0	0.0	0.0	0.0	0.0	0.0
15	0.405	0.0	0.106	0.230	0.234	0.281	0.016
16	0.860	0.693	0.503	1.000	0.579	0.931	0.417
17	0.197	0.0	0.0	0.487	0.066	0.232	0.000
18	0.696	0.474	0.324	0.441	0.385	0.747	0.473
19	0.450	0.569	0.687	0.403	0.316	0.593	0.372
20	0.592	0.875	0.728	0.622	0.719	0.777	0.865
21	0.735	0.737	0.897	0.678	0.775	0.835	1.000
22	0.618	0.929	0.699	0.561	0.707	0.754	0.730
23	0.544	0.681	0.435	0.383	0.409	0.616	0.316
24	0.743	1.000	1.000	0.865	1.000	1.000	0.692
25	0.036	0.0	0.074	0.085	0.0	0.078	0.0
26	0.542	0.0	0.059	0.408	0.0	0.367	0.214
27	0.385	0.175	0.376	0.345	0.046	0.353	0.561

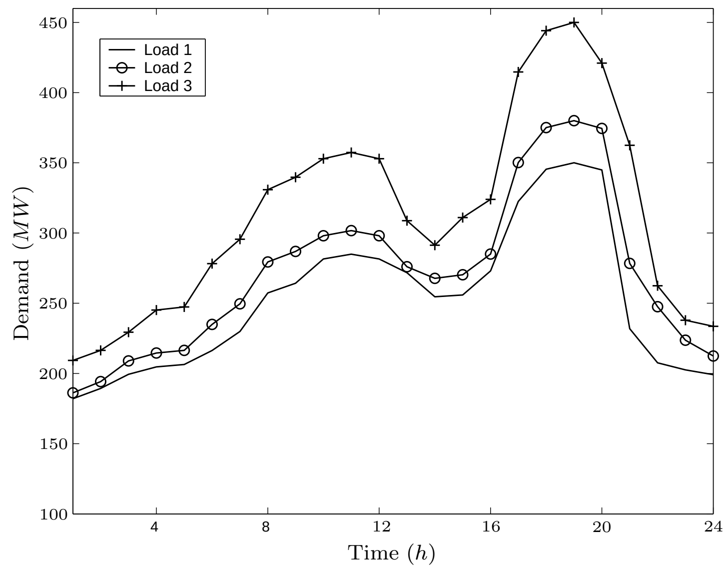

Figure 4.9: Daily demand profile.

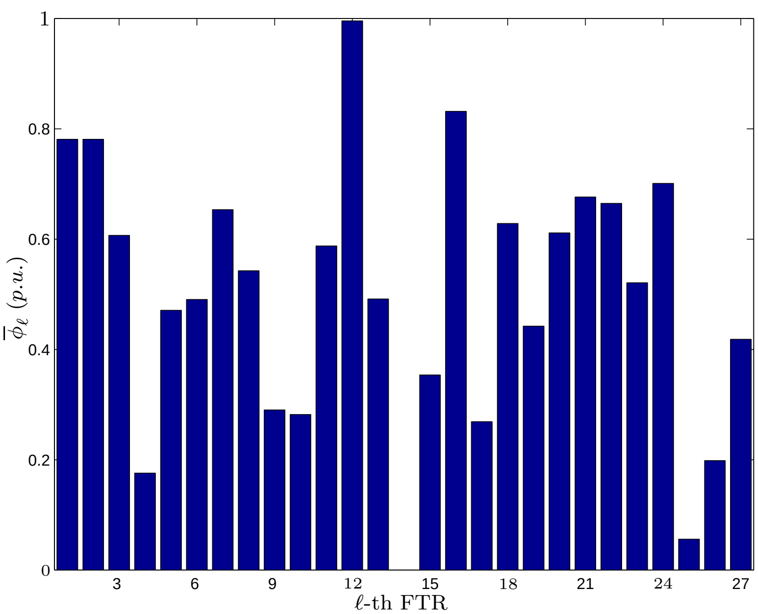

Figure 4.10: Mean values for the HPRs.

Table 4.7: Allocation of financial transmission rights (in MW).

	Standard Auction		**P**roposed Auction			
ℓ	τ_ℓ^*	$\overline{\tau}_\ell$	τ_ℓ^*	$\overline{\tau}_\ell\overline{\phi}_\ell$	τ_ℓ^* †	$\overline{\tau}_\ell$ †
1	110	110	85.88	85.88	110	110
2	100	100	78.07	78.08	100	100
3	300	300	182.01	182.01	300	300
4	0.0	120	0.0	21.08	0.0	0.0
5	0.0	300	130.73	141.27	0.0	300
6	0.0	580	0.0	284.49	0.0	580
7	0.0	110	0.0	71.88	0.0	110
8	0.0	300	0.0	162.78	0.0	300
9	0.0	300	0.0	87.06	0.0	0.0
10	120	120	33.84	33.84	0.0	0.0
11	0.0	300	176.25	176.25	130	300
12	300	300	298.68	298.68	300	300
13	110	110	54.05	54.05	110	110
14	120	120	0.0	0.0	0.0	0.0
15	190	300	106.11	106.11	300	300
16	0.0	580	0.0	482.27	0.0	580
17	0.0	150	40.33	40.33	0.0	0.0
18	0.0	300	0.0	188.46	0.0	300
19	0.0	110	0.0	48.65	0.0	110
20	0.0	580	0.0	354.49	0.0	580
21	0.0	300	0.0	202.92	0.0	300
22	0.0	580	0.0	385.58	0.0	580
23	0.0	100	0.0	52.08	0.0	100
24	166.24	300	125.91	210.27	166.23	300
25	0.0	120	6.72	6.72	0.0	0.0
26	0.0	300	10.66	59.52	0.0	0.0
27	22.38	300	125.42	125.46	32.40	300

† A threshold $\hat{\phi} = 0.35$ is introduced.

to 33.84 MW and 0 MW, respectively. Nonetheless, because of the new FTR allocation, G_4 is now awarded with 40.33 MW and 6.72 MW of τ_{17} and τ_{25}, respectively. Although G_4 still holds some FTRs, the awarded amounts are quite smaller than those of the original auction. A stronger discrimination is to introduce an HPR threshold; this avoids G_4 getting FTRs, among other low-hedging positions.

4.3 Summary

The ownership of FTRs may exacerbate the market power of some price-maker participants. The methodology proposed in this chapter can be used to screen and discriminate financial transmission rights with such potential opportunities. The methodology is based upon the use of relative hedging position ratios; these ratios comprise the network configuration, market outcomes and the participants position in the market. The proposed methodology preserves the classical auction format while some market participants are explicitly banned from obtaining either certain FTRs or more than a specified amount of them. Due to the incipient sophistication of loads, emphasis is given to the generators' side.

In addition to the proposed methodology, after FTRs have been allocated it is required that FTRs be subject to a screening in the secondary markets in order to limit those participants which were previously discriminated in the auction. On the other hand, this screening and mitigation process could be envisioned as a complement in the surveillance for market power in electricity markets.

As it's well known, an FTR scheme has a reduced liquidity; if a discrimination is introduced, like in this case, it may worsen such a liquidity. Moreover, market participants may be reluctant to any kind of intervention. What seems to be evident is that a generator should not hold an FTR which has as the sink the node/zone where the generator is located. Due to the potential complexity for carrying out any regulatory intervention on FTRs ownership, so far an alternative may be to build the FTR framework upon their allocation to other entities, such as loads or traders, rather than generators.

In fact, in order to deter market power, some markets allow non-transmission users, say, traders or arbitragers, to participate in the auctions for FTRs [63]. In this case, FTRs become investments opportunities rather than hedging instruments.

Chapter 5

Equilibrium Models of Markets for Financial Transmission Rights

Equilibrium models of markets for FTRs are presented in this chapter. These markets are formulated by using a set of equilibrium conditions for each market participant. Within the models, a conjectured congestion price response function is introduced to characterize the influence of bidders on the FTR prices. The proposed models are used to study the impact of the bidding strategies on the market for financial transmission rights. Different kinds of FTRs are modelled, such as i) obligations; ii) options; and iii) joint obligations and options. Under this framework, one-period, multi-round and multi-period markets for FTRs are formulated. The models are illustrated by a three-node power system and then extended to five- and thirty-node systems using a DC-network approximation.

This chapter is organized as follows. In §5.1, an introduction to obligations and options is given. Conjecture congestion price functions are presented in §5.2. Models of one-period markets for obligations, options, and joint obligations and options are described in §5.3, 5.4 and 5.5, respectively. Numerical examples for these cases are given in §5.6. In §5.7 and 5.8, models of multi-round and multi-period markets are formulated. Remarks in §5.9 close the chapter .

5.1 Premium *vs.* Liability

Because of the financial attributes of FTRs, the commitment of their holders is financial rather than physical; *i.e.*, an FTR holder is not required to inject/withdraw power. Due to this feature, FTRs can be considered as financial derivatives [132]. Given an FTR, $\tau_{\nu,i,j}$, going from node i to node j, its associated payoff is $\Pi{=}\tau_{\nu,i,j}(\rho_j{-}\rho_i)$. If $\rho_j{-}\rho_i{<}0$, the payoff is a charge or a liability for the holder; if $\rho_j{-}\rho_i{>}0$, the payoff is a reimbursement or premium for the holder. If both payoff directions are comprised, the FTR is an *obligation* because the contract has to be exercised even when the payoff becomes negative –see Figure 5.1. In this case, an FTR can be seen as a *forward* contract for the price of transmission congestion. Nevertheless, even negative payoffs are the right market signals to have a complete hedge and a full utilization of the system capacity [133].

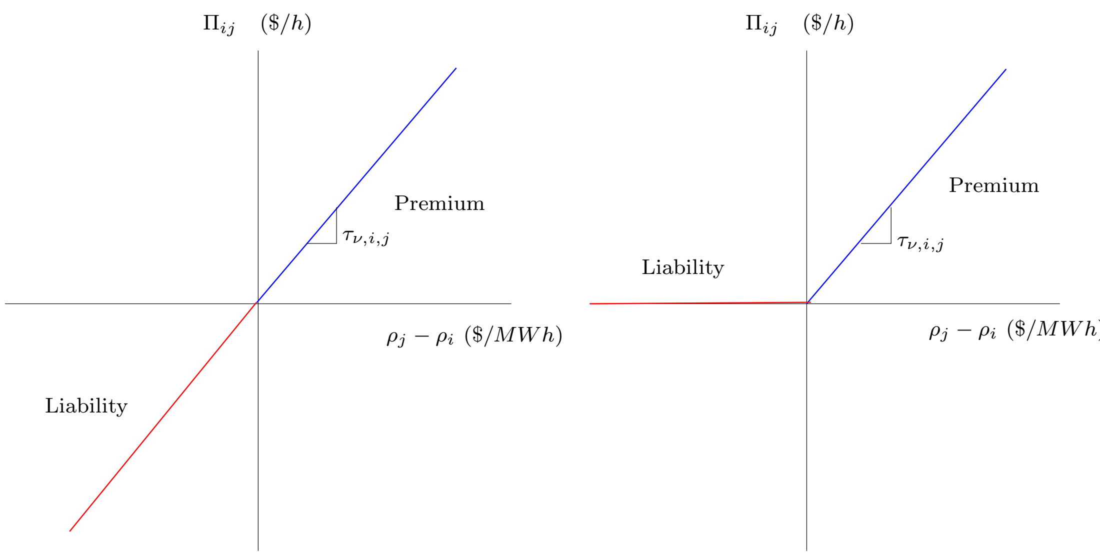

Figure 5.1: Obligation and option payoffs.

On the other hand, an FTR can be defined as an *option* so that the holder can choose when to exercise the transmission right. In this case, if $\rho_j{-}\rho_i{>}0$, the holder of the right is compensated (exercised option); however, if $\rho_j{-}\rho_i{<}0$, the holder is not charged (unexercised option). Consequently, the payoff associated with the option becomes $\Pi = \tau_{\nu,i,j} max(0, \rho_j{-}\rho_i)$. Naturally, because of their superiority, options tend to cost more than obligations. Although the options format is more advantageous for the potential holders, the issuance of options requires to put in place more restricted conditions within their allocation process.

The issuance of pure options has the drawback of offering, at most, only the maximum physical capacity of the interfaces.

An option may or may not be exercised; but this fact is not known until the energy market is cleared. Thus, in the *ex ante* issuance of options, this feature has to be taken into account. A complexity arises because the system feasibility must be preserved under any combination of exercised options. Hence, the impact of each option on the transmission constraints has to be individually recognized. This implies that any possible counterflow has to be disregarded. This constraint reduces the offered capacity for transmission rights. Nowadays, due to this burden in the implementation of markets for options, their issuance is restrained to a subset of all the potential options. This is the case of the FTR market in PJM [134].

One of the motivations to develop the flowgate model is that they are inherently options. This is because the FGRs are directional; therefore, the holder of an FGR has the right only in a single direction. In the most extreme case, where operation conditions reverse some power flows in the network, the expected and traded forward FGRs have no worth anymore. Neither FTRs nor FGRs, defined just as options, allow traders to get enough rights to schedule/hedge the full set of transactions when counterflows are required to implement their transactions. There is also a relationship between FTRs and FGRs, as pointed out in [61] and [135], which states that FTRs can be decomposed as portfolios of FGRs. If portfolios of FGRs were implemented, the FGRs as portfolio should be centrally coordinated and auctioned so that the feasibility could be preserved. Due to this difficulty, FGRs are envisioned to be useful only as financial instruments to partially hedge congestion in relatively short periods, *e.g.*, monthly issuance.

Currently, it is widely recognized that the combination of options and obligations for transmission rights can enhance the flexibility of the transmission markets; having both alternatives, traders could choose the one that works better for them [39, 64].

5.2 Influence on Congestion Prices

Within markets for transmission rights, FTRs are allocated at locational marginal prices. Since FTRs are defined to hedge congestion, losses are not considered and LMPs denote the prices to use the transmission capacity; *i.e.*, they are congestion prices. In the proposed markets formulations, congestion prices are modelled as endogenous variables within the bidders optimization problems. This implies that bidders can manipulate the congestion prices, as they are considered price-makers. This is included by means of a *conjectured congestion-price response function*.

Let us use the index $\ell \in \mathcal{F}_\nu$ to denote FTRs; while $\nu \in \Theta$ stands for the bidders index. The sets for obligation FTRs and bidders are denoted by $\mathcal{F}_\nu$ and Θ, respectively. An FTR can be defined by i) an amount of power $\tau_{\nu,\ell}$; and ii) the injection ($\xi_{\nu,\ell,i} = -1$) and withdrawal ($\xi_{\nu,\ell,i} = +1$) nodes.

Definition 4.1 (Conjectured congestion-price function [71]) Let p_i be the net power injection at node i. As the net transmission service required by bidder ν, at node i, is $\sum_\ell \xi_{\nu,\ell,i}\tau_{\nu,\ell}$, then $p_i = \sum_\ell \xi_{\nu,\ell,i}\tau_{\nu,\ell} + \sum_{h\neq\nu}\sum_\jmath \xi_{h,\jmath,i}\tau_{h,\jmath}$. Therefore, a conjectured congestion-price function can be defined as

$$\omega_{\nu,i} = \omega_i^* + A_{\nu,i}\sum_\ell \xi_{\nu,\ell,i}(\tau_{\nu,\ell} - \tau_{\nu,\ell}^*),\ \forall i \in \mathcal{I}. \tag{5.1}$$

Alternatively, a conjectured function can be defined to determine the individual impact of an FTR on the congestion prices,

$$\omega_{\nu,\ell} = \omega_{\nu,\ell}^* + A_{\nu,\ell}(\tau_{\nu,\ell} - \tau_{\nu,\ell}^*),\ \forall \ell \in \mathcal{F}_\nu, \tag{5.2}$$

where $\omega_{\nu,i}$ and $\omega_{\nu,\ell}$ are the congestion prices seen by ν at node i, and for FTR ℓ, respectively. At equilibrium, the congestion prices at node i and for FTR ν,ℓ are denoted by ω_i^* and $\omega_{\nu,\ell}^*$. $A_{\nu,i}$ and $A_{\nu,\ell}$ are the conjectured congestion-price response parameters either at node i or for FTR ℓ, respectively. The higher the parameter, the larger the prices change. For the competitive outcome, $A_{\nu,i}$, $A_{\nu,\ell} \equiv 0$. These constants can be parametrically varied to explore the impact of different expectations on price congestion changes.

Both conjectured functions (5.1) and (5.2) are a first-order approximation of bidder ν expectation of how the congestion prices, $\omega_{\nu,i}$, $\forall i$ and $\omega_{\nu,\ell}$, $\forall \ell$, will change from their equilibrium values, ω_i^*, if ν modifies its demand of transmission capacity. These conjectured congestion-price functions are an analogy to what has been used for energy markets (conjectured supply function [15]), for transmission prices manipulation (conjectured transmission price response [16]) and spinning reserve price manipulation (conjectured reserve-price function [48]).

5.3 A Market for Obligations

Throughout this chapter, the proposed FTR market formulations are based on equilibrium models; these models are built upon the complementarity conditions of every agent participating in the transmission markets. There are two kinds of agents in the market for transmission rights. On one hand, an independent entity (*e.g.*, an ISO) is in charge of allocating the transmission rights to bidders and in settling the congestion prices based on a locational marginal pricing scheme. On the other hand, there are agents which submit their bids to acquire FTRs. To complete the equilibrium model, a market clearing condition is introduced. This condition ensures that the supply of transmission rights equals the demand of them. Without any loss of generality, only the case of FTRs to be purchased by market participants is covered in this chapter.

5.3.1 FTR bidders problem

A market participant's problem is to maximize surplus from purchasing FTRs. On one side, it has a net benefit from purchasing a portfolio of FTRs throughout the system's nodes; the concave benefit function, $b(\tau_{\nu,\ell}) = \beta_{\nu,\ell}\tau_{\nu,\ell} - \gamma_{\nu,\ell}\tau^2_{\nu,\ell}$, is the maximum willingness to acquire $\tau_{\nu,\ell}$, where $\beta_{\nu,\ell}$ and $\gamma_{\nu,\ell}$ are nonnegative parameters estimated by the bidders, and to be submitted to the ISO. This function represents the expectation of ν for the avoided congestion cost if ν holds $\tau_{\nu,\ell}$ when it participates in the energy market.

On the other hand, the FTR portfolio has a cost which is paid to the FTR auctioneer. As an FTR is defined by the source and sink nodes, its opportunity cost price is given by the difference of congestion prices between such trading points, $\sum_i \omega_{\nu,i}\xi_{\nu,\ell,i}$. Although congestion prices are taken as endogenous into each bidder problem, within the equilibrium problem as a whole, the congestion prices are variables that match the equilibrium values. Therefore, the surplus-maximization problem for bidder ν can be mathematically stated as follows:

$$max \;\; \Pi_\nu = \sum_\ell \left\{ \beta_{\nu,\ell}\tau_{\nu,\ell} - \gamma_{\nu,\ell}\tau^2_{\nu,\ell} - \tau_{\nu,\ell}\sum_i \omega_{\nu,i}\xi_{\nu,\ell,i} \right\} \tag{5.3}$$

$$s.t. \;\; 0 \le \tau_{\nu,\ell} \le \overline{\tau}_{\nu,\ell}, \quad \forall \ell \in \mathcal{F}_\nu, \tag{5.4}$$

$$\omega_{\nu,i} - \omega^*_i - A_{\nu,i}\sum_\ell \xi_{\nu,\ell,i}(\tau_{\nu,\ell} - \tau^*_{\nu,\ell}) = 0, \quad \forall i \in \mathcal{I}. \tag{5.5}$$

Equation (5.4) stands for the feasible capacity of each FTR; while $\overline{\tau}_{\nu,\ell}$ is the maximum FTR capacity that bidders are willing to acquire. This parameter is estimated by bidders, and submitted to the ISO.

The Lagrangian for the problem (5.3)–(5.5) can be written as

$$\mathcal{L} = \sum_{\ell} \Big\{ \beta_{\nu,\ell}\tau_{\nu,\ell} - 2\gamma_{\nu,\ell}\tau_{\nu,\ell} - \tau_{\nu,\ell} \sum_{i} \omega_{\nu,i}\xi_{\nu,\ell,i} - \mu_{\nu,\ell}(\tau_{\nu,\ell} - \overline{\tau}_{\nu,\ell}) \Big\}, \tag{5.6}$$
$$\tau_{\nu,\ell},\ \mu_{\nu,\ell} \geq 0,\ \forall \ell \in \mathcal{F}_{\nu}.$$

where $\mu_{\nu,\ell}$ is the dual variable associated with the constraints for maximum capacity of FTR ℓ. The conjectured function (5.5) is not explicitly used as a constraint throughout the models because it is used to substitute $\omega_{\nu,i}$ into the surplus function (5.3); consequently, there is no dual variable associated with (5.5).

Proposition 5.1 *By including a conjecture congestion-price function, the surplus function for bidder ν remains concave if and only if $A_{\nu,i} \geq 0$.*

Proof. Let the surplus-maximization problem be represented with a vectorial notation. The variables and parameters are in bold letters to denote their corresponding vectors. They are defined as follows: $\boldsymbol{\beta}_{\nu} \in \mathbb{R}^{|\mathcal{F}_{\nu}|}$, $\boldsymbol{\gamma}_{\nu} = Diag(\gamma_{\nu,\ell}) \in \mathbb{R}^{|\mathcal{F}_{\nu}|x|\mathcal{F}_{\nu}|}$, $\boldsymbol{\tau}_{\nu} \in \mathbb{R}^{|\mathcal{F}_{\nu}|}$, $\boldsymbol{\omega}_{\nu} \in \mathbb{R}^{|\mathcal{I}|}$, $\boldsymbol{\omega}^{*} \in \mathbb{R}^{|\mathcal{I}|}$, $\boldsymbol{\xi}_{\nu} \in \mathbb{R}^{|\mathcal{I}|x|\mathcal{F}_{\nu}|}$, $\boldsymbol{A}_{\nu} = Diag(A_{\nu,i}) \in \mathbb{R}^{|\mathcal{I}|x|\mathcal{I}|}$.

Thus, the surplus function can be represented as

$$\Pi_{\nu} = \boldsymbol{\beta}_{\nu}^{T}\boldsymbol{\tau}_{\nu} - \boldsymbol{\tau}_{\nu}^{T}\boldsymbol{\gamma}_{\nu}\boldsymbol{\tau}_{\nu} - \big(\boldsymbol{\omega}^{*} + \boldsymbol{A}_{\nu}\boldsymbol{\xi}_{\nu}[\boldsymbol{\tau}_{\nu} - \boldsymbol{\tau}_{\nu}^{*}]\big)^{T}\boldsymbol{\xi}_{\nu}\boldsymbol{\tau}_{\nu}, \tag{5.7}$$

or equivalently,

$$\Pi_{\nu} = \boldsymbol{\beta}_{\nu}^{T}\boldsymbol{\tau}_{\nu} - \boldsymbol{\tau}_{\nu}^{T}\boldsymbol{\gamma}_{\nu}\boldsymbol{\tau}_{\nu} - \boldsymbol{\omega}^{*T}\boldsymbol{\xi}_{\nu}\boldsymbol{\tau}_{\nu} - \boldsymbol{\tau}_{\nu}^{T}\boldsymbol{B}_{\nu}\boldsymbol{\tau}_{\nu} + \boldsymbol{\tau}^{*T}\boldsymbol{B}_{\nu}\boldsymbol{\tau}_{\nu}, \tag{5.8}$$

where $\boldsymbol{B}_{\nu} = \boldsymbol{\xi}_{\nu}^{T}\boldsymbol{A}_{\nu}\boldsymbol{\xi}_{\nu}$. The matrix $\boldsymbol{\xi}_{\nu}$ stands for the incidence of FTRs across the system nodes. Notice that $\nabla^{2}_{\boldsymbol{\tau}}\Pi_{\nu} = -(2\boldsymbol{\gamma}_{\nu} + \boldsymbol{B}_{\nu})$. To ensure concavity of the surplus function, $\boldsymbol{\gamma}_{\nu}+\boldsymbol{B}_{\nu}$ has to be positive semi-definite. Because the diagonal entries of $\boldsymbol{\gamma}_{\nu}$ are nonnegative, it follows that $\boldsymbol{\gamma}_{\nu}$ is positive semi-definite. On the other hand, $\boldsymbol{B}_{\nu}$ is positive semi-definite as long as $\boldsymbol{A}_{\nu}$ is so [96]. This holds if and only if the diagonal entries of $\boldsymbol{A}_{\nu}$ are nonnegative, *i.e.*, $A_{\nu,i} \geq 0,\ \forall i$, as claimed. ■

Furthermore, by matrix properties [109], if $\gamma_{\nu,\ell} > 0,\ \forall \ell$, then $\boldsymbol{\gamma}_\nu + \boldsymbol{B}_\nu$ becomes positive definite.

Within the bidder problem, the proof of such concavity ensures that any local solution will be also a global solution.

5.3.2 ISO problem

In this equilibrium model, the ISO auctions transmission rights to efficiently allocate the scarce transmission to those who value it most. It is considered that the ISO cannot manipulate the prices of the transmission services, and, therefore, it sees $\omega_i^*,\ \forall i$ as exogenous variables. Thus, the ISO problem is to maximize the revenue from the sales of FTRs under locational marginal cost pricing. This is done by solving the following profit-maximization problem:

$$max\ \ \Pi_{ISO} = \sum_i \omega_i^* p_i, \tag{5.9}$$

where $p_i,\ \forall i$ stands for the power nodal injections; these power injections are the ISO's decisions variables which have to be feasible for the transmission system constraints. This fact implicitly ensures a simultaneous feasibility of the FTRs to be awarded.

The transmission constraints are defined by the DC power flows equations in terms of the sensitivity factors $s_{k,i}$,

$$\sum_i s_{k,i} p_i \leq \overline{z}_k^+, \quad \forall k \in \mathcal{K}, \tag{5.10}$$

$$-\sum_i s_{k,i} p_i \leq \overline{z}_k^-, \quad \forall k \in \mathcal{K}. \tag{5.11}$$

Each transmission line is modelled by means of two power flows constraints, one constraint per power-flow direction; their corresponding maximum limits are z_k^+ and z_k^-. Without any loss of generality, (5.10)–(5.11) define only thermal limits constraints, which are assumed to be convex; security constraints, such as *n-1* constraints, can be similarly modelled. The Lagrangian for the problem (5.9)–(5.11) can be written as

$$\pounds = \sum_i \omega_i^* p_i - \sum_k \Big\{ \lambda_k^+ \big(\sum_i s_{k,i} p_i - \overline{z}_k \big) + \lambda_k^- \big(-\sum_i s_{k,i} p_i - \overline{z}_k \big) \Big\}, \tag{5.12}$$

$$\lambda_k^+,\ \lambda_k^- \geq 0, \quad \forall k \in \mathcal{K},$$

$$p_i \quad free,\ \forall i \in \mathcal{I}.$$

where λ_k^+ and λ_k^- are the dual variables associated with each power flow constraint.

5.3.3 Market clearing condition

There need be a market clearing condition for consistency in the model. Such a condition is that the demand for transmission capacity required by all bidders equals the transmission capacity supplied by the ISO; *i.e.*, the set of FTRs defines the net power injection into the transmission system,

$$p_i = \sum_{\nu,\ell} \xi_{\nu,\ell,i} \tau_{\nu,\ell} \ , \quad \forall i \in \mathcal{I}. \tag{5.13}$$

This matching of supply and demand is what generates the corresponding congestion prices, ω_i^*, $\forall i$.

5.3.4 Equilibrium model

The optimality conditions for each bidder and the ISO can be gathered, together with the above market clearing condition, to compose an MLCP. Thus, the optimal allocation of FTRs is defined by the following MLCP:

- For $\tau_{\nu,\ell}, \quad \forall \ell \in \mathcal{F}_\nu, \quad \nu \in \Theta$:

$$0 \geq \beta_{\nu,\ell} - 2\gamma_{\nu,\ell}\tau_{\nu,\ell} - \sum_i \big(\omega_i^* + A_{\nu,i} \sum_h \xi_{\nu,h,i}\tau_{\nu,h}\big)\xi_{\nu,\ell,i} - \mu_{\nu,\ell} \quad \perp \quad \tau_{\nu,\ell} \geq 0 \tag{5.14}$$

- For $\mu_{\nu,\ell}, \quad \forall \ell \in \mathcal{F}_\nu, \quad \nu \in \Theta$:

$$0 \geq \tau_{\nu,\ell} - \overline{\tau}_{\nu,\ell} \quad \perp \quad \mu_{\nu,\ell} \geq 0 \tag{5.15}$$

- For $\lambda_k^+, \quad \forall k \in \mathcal{K}$:

$$0 \geq \sum_i s_{k,i} p_i - \overline{z}_k^+ \quad \perp \quad \lambda_k^+ \geq 0 \tag{5.16}$$

- For λ_k^-, $\forall k \in \mathcal{K}$:

$$0 \geq -\sum_i s_{k,i} p_i - \overline{z}_k^- \quad \perp \quad \lambda_k^- \geq 0 \tag{5.17}$$

- For p_i, $\forall i \in \mathcal{I}$:

$$0 = \omega_i^* - \sum_k s_{k,i}(\lambda_k^+ - \lambda_k^-); \quad p_i \ free \tag{5.18}$$

- For ω_i^*, $\forall i \in \mathcal{I}$:

$$0 = p_i - \sum_{\nu,\ell} \xi_{\nu,\ell,i} \tau_{\nu,\ell}; \quad \omega_i^* \quad free \tag{5.19}$$

Simultaneously solving for the primal ($\tau_{\nu,\ell}$, p_i) and dual variables ($\mu_{\nu,\ell}$, λ_k^+, λ_k^-) with prices (ω_i^*), an equilibrium point of the market for FTRs is obtained.

Definition 5.1 (Market equilibrium) Let $\boldsymbol{x}_\nu \in \mathcal{X}_\nu$ be ν's strategies; strategy $\boldsymbol{x}_\nu$ is composed by the own decision variables of ν, *i.e.*, $\boldsymbol{x}_\nu = \{\tau_{\nu,\ell}, \ \forall \ell \in \mathcal{F}_\nu\}$. Let $\boldsymbol{p} \in \mathcal{P}$ be ISO's strategies; strategy $\boldsymbol{p}$ is composed by the ISO's decisions, *i.e.*, $\boldsymbol{p} = \{p_i, \ \forall i \in \mathcal{I}\}$. Let $\boldsymbol{\omega} = \{\omega_i, \ \forall i \in \mathcal{I}\}$ be a vector of congestion prices. The vectors $\boldsymbol{x}_\nu^*$, $\forall \nu \in \mathcal{V}$, $\boldsymbol{p}^*$ and $\boldsymbol{\omega}^*$ constitute an equilibrium of the transmission market if and only if

(i) $\Pi_\nu(\boldsymbol{x}_\nu^*) \geq \Pi_\nu(\boldsymbol{x}_\nu)$, $\forall \boldsymbol{x}_\nu \in \mathcal{X}_\nu$, $\nu \in \Theta$ with market prices $\boldsymbol{\omega}^*$; *i.e.*, $\boldsymbol{x}_\nu^*$ is a solution of (5.3)–(5.5).

(ii) $\Pi_{ISO}(\boldsymbol{p}^*) \geq \Pi_{ISO}(\boldsymbol{p})$, $\forall \boldsymbol{p} \in \mathcal{P}$; *i.e.*, $\boldsymbol{y}^*$ is a solution of (5.9)–(5.11).

(iii) Given the market prices $\boldsymbol{\omega}^*$, the vectors $\boldsymbol{x}^*$ and $\boldsymbol{y}^*$ balance the supply and demand for transmission rights; *i.e.*, $\boldsymbol{x}_\nu^*$ and $\boldsymbol{y}^*$ satisfy (5.13).

A market equilibrium represents the optimal allocation of FTRs. In this work, the equilibrium model defined by the MLCP has been formulated in GAMS and solved using the PATH solver [118].

Proposition 5.2
i) The equilibrium problem (5.14)–(5.19) is equivalent to a Quadratic Programming (QP) problem.
ii) A market equilibrium point of problem (5.14)–(5.19) will be a global solution.

Proof. Using vectorial notation, the equilibrium model can be compactly written as a standard MLCP,

$$\begin{array}{cc} \mathbf{0} & \leq \\ \mathbf{0} & \leq \\ \mathbf{0} & \leq \\ \mathbf{0} & \leq \\ \hline \mathbf{0} & = \\ \mathbf{0} & = \end{array} \begin{pmatrix} -\hat{\boldsymbol{\beta}} \\ \hat{\boldsymbol{\tau}} \\ \overline{\boldsymbol{z}}^{+} \\ \overline{\boldsymbol{z}}^{-} \\ \hline \mathbf{0} \\ \mathbf{0} \end{pmatrix} + \left(\begin{array}{cccc|cc} \hat{\boldsymbol{B}} + 2\hat{\gamma} & \hat{\boldsymbol{I}} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \hat{\boldsymbol{\xi}}^T \\ -\hat{\boldsymbol{I}} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & -\boldsymbol{S} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \boldsymbol{S} & \mathbf{0} \\ \hline \mathbf{0} & \mathbf{0} & \boldsymbol{S}^T & -\boldsymbol{S}^T & \mathbf{0} & -\boldsymbol{I} \\ -\hat{\boldsymbol{\xi}} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \boldsymbol{I} & \mathbf{0} \end{array}\right) \begin{pmatrix} \boldsymbol{\tau} \\ \boldsymbol{\mu} \\ \boldsymbol{\lambda}^{+} \\ \boldsymbol{\lambda}^{-} \\ \hline \boldsymbol{p} \\ \boldsymbol{\omega}^* \end{pmatrix} \begin{array}{cccc} \perp & \boldsymbol{\tau} & \geq & \mathbf{0} \\ \perp & \boldsymbol{\mu} & \geq & \mathbf{0} \\ \perp & \boldsymbol{\lambda}^{+} & \geq & \mathbf{0} \\ \perp & \boldsymbol{\lambda}^{-} & \geq & \mathbf{0} \\ \hline & \boldsymbol{p} & & free \\ & \boldsymbol{\omega}^* & & free \end{array}$$

where

$$\hat{\boldsymbol{B}} + 2\hat{\gamma} \equiv \begin{pmatrix} \boldsymbol{B}_1 + 2\boldsymbol{\gamma}_1 & \mathbf{0} & \cdots & \mathbf{0} \\ \mathbf{0} & \boldsymbol{B}_2 + 2\boldsymbol{\gamma}_2 & \ddots & \vdots \\ \vdots & \ddots & \ddots & \mathbf{0} \\ \mathbf{0} & \cdots & \mathbf{0} & \boldsymbol{B}_{|\Theta|} + 2\boldsymbol{\gamma}_{|\Theta|} \end{pmatrix}$$

and $\boldsymbol{I}_\nu$ is the identity matrix of order $|\mathcal{F}_\nu|$,

$$\hat{\boldsymbol{I}} \equiv \begin{pmatrix} \boldsymbol{I}_1 & \mathbf{0} & \cdots & \mathbf{0} \\ \mathbf{0} & \boldsymbol{I}_2 & \ddots & \vdots \\ \vdots & \ddots & \ddots & \mathbf{0} \\ \mathbf{0} & \cdots & \mathbf{0} & \boldsymbol{I}_{|\Theta|} \end{pmatrix} \quad and \quad \hat{\boldsymbol{\xi}} \equiv \begin{pmatrix} \boldsymbol{\xi}_1, & \boldsymbol{\xi}_2, & \cdots & ,\boldsymbol{\xi}_{|\Theta|} \end{pmatrix}$$

Also let

$$\boldsymbol{\tau} \equiv \begin{pmatrix} \boldsymbol{\tau}_1 \\ \boldsymbol{\tau}_2 \\ \vdots \\ \boldsymbol{\tau}_{|\Theta|} \end{pmatrix}, \quad \boldsymbol{\mu} \equiv \begin{pmatrix} \boldsymbol{\mu}_1 \\ \boldsymbol{\mu}_2 \\ \vdots \\ \boldsymbol{\mu}_{|\Theta|} \end{pmatrix}, \quad \hat{\boldsymbol{\beta}} \equiv \begin{pmatrix} \boldsymbol{\beta}_1 \\ \boldsymbol{\beta}_2 \\ \vdots \\ \boldsymbol{\beta}_{|\Theta|} \end{pmatrix} \quad and \quad \hat{\boldsymbol{\tau}} \equiv \begin{pmatrix} \overline{\boldsymbol{\tau}}_1 \\ \overline{\boldsymbol{\tau}}_2 \\ \vdots \\ \overline{\boldsymbol{\tau}}_{|\Theta|} \end{pmatrix}$$

This model can be stated as follows:

$$0 \leq \boldsymbol{f} + \boldsymbol{F}\boldsymbol{u} + \boldsymbol{E}\boldsymbol{v} \quad \perp \quad \boldsymbol{u} \geq 0, \tag{5.20}$$

$$0 = \boldsymbol{d} + \boldsymbol{G}\boldsymbol{u} + \boldsymbol{D}\boldsymbol{v}, \quad \boldsymbol{v}\ free, \tag{5.21}$$

such that the MLCP is a combination of an LCP and linear equations. As $\boldsymbol{D}^{-1}$ exists, this problem can be defined in terms of the nonnegative variables such that the equilibrium problem is reduced to

$$0 \leq \boldsymbol{q} + \boldsymbol{M}\boldsymbol{u} \quad \perp \quad \boldsymbol{u} \geq 0, \tag{5.22}$$

where

$$\boldsymbol{q} = \boldsymbol{f} - \boldsymbol{E}\boldsymbol{D}^{-1}\boldsymbol{d} \quad and \quad \boldsymbol{M} = \boldsymbol{F} - \boldsymbol{E}\boldsymbol{D}^{-1}\boldsymbol{G}. \tag{5.23}$$

With this formulation, the market for obligation FTRs can be stated as the compact LCP form

$$\begin{matrix} \boldsymbol{0} \leq \\ \boldsymbol{0} \leq \\ \boldsymbol{0} \leq \\ \boldsymbol{0} \leq \end{matrix} \begin{pmatrix} -\hat{\boldsymbol{\beta}} \\ \hat{\boldsymbol{\tau}} \\ \overline{\boldsymbol{z}}^{+} \\ \overline{\boldsymbol{z}}^{-} \end{pmatrix} + \begin{pmatrix} \hat{\boldsymbol{B}} + 2\hat{\gamma} & \hat{\boldsymbol{I}} & \hat{\boldsymbol{\xi}}^{T}\boldsymbol{S}^{T} & -\hat{\boldsymbol{\xi}}^{T}\boldsymbol{S}^{T} \\ -\hat{\boldsymbol{I}} & \boldsymbol{0} & \boldsymbol{0} & \boldsymbol{0} \\ -\boldsymbol{S}\hat{\boldsymbol{\xi}} & \boldsymbol{0} & \boldsymbol{0} & \boldsymbol{0} \\ \boldsymbol{S}\hat{\boldsymbol{\xi}} & \boldsymbol{0} & \boldsymbol{0} & \boldsymbol{0} \end{pmatrix} \begin{pmatrix} \boldsymbol{\tau} \\ \boldsymbol{\mu} \\ \boldsymbol{\lambda}^{+} \\ \boldsymbol{\lambda}^{-} \end{pmatrix} \begin{matrix} \perp \ \boldsymbol{\tau} \geq \boldsymbol{0} \\ \perp \ \boldsymbol{\mu} \geq \boldsymbol{0} \\ \perp \ \boldsymbol{\lambda}^{+} \geq \boldsymbol{0} \\ \perp \ \boldsymbol{\lambda}^{-} \geq \boldsymbol{0} \end{matrix} \tag{5.24}$$

It follows that $\boldsymbol{M}$ has the following structure:

$$\boldsymbol{M} \equiv \begin{pmatrix} \boldsymbol{Q} & -\boldsymbol{A}^{T} \\ \boldsymbol{A} & \boldsymbol{0} \end{pmatrix} \tag{5.25}$$

where $\boldsymbol{Q} = \hat{\boldsymbol{B}} + 2\hat{\gamma}$ and

$$\boldsymbol{A} \equiv \begin{pmatrix} -\hat{\boldsymbol{I}} \\ -\boldsymbol{S}\hat{\boldsymbol{\xi}} \\ \boldsymbol{S}\hat{\boldsymbol{\xi}} \end{pmatrix} \tag{5.26}$$

In addition, let us consider the following decompositions: $\boldsymbol{u} \equiv [\boldsymbol{x},\ \boldsymbol{y}]^{T}$, where $\boldsymbol{x} = \boldsymbol{\tau}$ and $\boldsymbol{y} = [\boldsymbol{\mu},\ \boldsymbol{\lambda}^{+},\ \boldsymbol{\lambda}^{-}]^{T}$; $\boldsymbol{q} \equiv [\boldsymbol{c},\ -\boldsymbol{b}]^{T}$, where $\boldsymbol{c} = -\hat{\boldsymbol{\beta}}$ and $\boldsymbol{b} = [-\hat{\boldsymbol{\tau}},\ -\overline{\boldsymbol{z}}^{+},\ -\overline{\boldsymbol{z}}^{-}]^{T}$. Then the equilibrium problem for the obligation FTRs market is the set of KKT conditions for the following quadratic programming problem [109]:

$$\begin{aligned} min \quad & \frac{1}{2}\boldsymbol{x}^{T}\boldsymbol{Q}\boldsymbol{x} + \boldsymbol{c}^{T}\boldsymbol{x} \\ s.t. \quad & \boldsymbol{A}\boldsymbol{x} \geq \boldsymbol{b}, \\ & \boldsymbol{x} \geq \boldsymbol{0}, \end{aligned} \tag{5.27}$$

thus,

$$\begin{aligned} min \quad & \frac{1}{2}\boldsymbol{\tau}^T(\hat{\boldsymbol{B}} + \boldsymbol{2}\hat{\gamma})\boldsymbol{\tau} - \hat{\boldsymbol{\beta}}^T\boldsymbol{\tau} \\ s.t. \quad & \boldsymbol{\tau} \leq \hat{\boldsymbol{\tau}}, \\ & \boldsymbol{S}\hat{\boldsymbol{\xi}}\boldsymbol{\tau} \leq \overline{\boldsymbol{z}}^+, \\ -&\boldsymbol{S}\hat{\boldsymbol{\xi}}\boldsymbol{\tau} \leq \overline{\boldsymbol{z}}^-, \\ & \boldsymbol{\tau} \geq \boldsymbol{0}. \end{aligned} \tag{5.28}$$

The matrix $(\hat{\boldsymbol{B}} + \boldsymbol{2}\hat{\gamma})$ is positive definite (or semi-definite) because $\boldsymbol{B}_\nu + 2\boldsymbol{\gamma}_\nu$, $\forall \nu$ is so –see Proposition 5.1. This turns problem (5.28) into a convex programming problem; consequently, any local solution is also a global optimum. Moreover, when $\boldsymbol{\gamma}$, $\forall\, \nu$ is positive definite, problem (5.28) is strictly convex and there exist a unique solution [96]. ■

Proposition 5.3 *Congestion prices for obligations are defined by the active transmission constraints.*

Proof. By the complementarity condition (5.18), the congestion prices are given by

$$\omega_i^* = \sum_k s_{i,k}(\lambda_k^+ - \lambda_k^-), \quad \forall i \in \mathcal{I}. \tag{5.29}$$

These congestion prices are linear combinations of the dual variables related to the transmission constraints. However, only the dual variables of the binding transmission constraints may contribute to such prices. Recalling that the price to be charged for $\tau_{\nu,\ell}$ is the congestion price difference between source and withdrawal nodes, one gets

$$\sum_{i,k} \xi_{\nu,\ell,i} s_{i,k}(\lambda_k^+ - \lambda_k^-), \quad \forall \ell \in \mathcal{F}_\nu. \tag{5.30}$$

■

Notice that the congestion price for an obligation is negative as long as the price difference between the withdrawal and source nodes is so. This can occur, for instance, when an FTR provides counterflows.

5.4 A Market for Options

The notation used previously for obligation FTRs is preserved in this section, except the set for options which is now denoted by $\mathcal{O}$. Even though the problem for the allocation of options is the same as that for obligations, the impact of each option needs to be separately modelled.

5.4.1 FTR bidders problem

When option FTRs are to be auctioned, the surplus-maximization problem for bidder ν is the same as that for obligations. However, for the sake of modelling, the opportunity cost price for the ℓ-th option is now defined as a one-part congestion fee, $\omega_{\nu,\ell} = \sum_i \omega_{\nu,i}\xi_{\nu,\ell,i}$; thus, the bidders problem becomes

$$max \;\; \Pi_\nu = \sum_\ell \left\{ \beta_{\nu,\ell}\tau_{\nu,\ell} - \gamma_{\nu,\ell}\tau^2_{\nu,\ell} - \omega_{\nu,\ell}\tau_{\nu,\ell} \right\} \tag{5.31}$$

$$s.t. \;\; 0 \leq \tau_{\nu,\ell} \leq \overline{\tau}_{\nu,\ell}, \quad \forall \ell \in \mathcal{O}_\nu, \tag{5.32}$$

$$\omega_{\nu,\ell} - \omega^*_{\nu,\ell} - A_{\nu,\ell}(\tau_{\nu,\ell} - \tau^*_{\nu,\ell}) = 0, \quad \forall \ell \in \mathcal{O}_\nu. \tag{5.33}$$

The conjectured congestion-price function (5.33) is also defined for each FTR option. The Lagrangian for the problem (5.31)–(5.33) can be written as

$$\pounds = \sum_\ell \left\{ \beta_{\nu,\ell}\tau_{\nu,k} - \gamma_{\nu,\ell}\tau^2_{\nu,\ell} - \omega_{\nu,\ell}\tau_{\nu,\ell} - \mu_{\nu,\ell}(\tau_{\nu,\ell} - \overline{\tau}_{\nu,\ell}) \right\}, \tag{5.34}$$

$$\tau_{\nu,\ell}, \;\; \mu_{\nu,\ell} \geq 0, \quad \forall \ell \in \mathcal{O}_\nu.$$

5.4.2 ISO problem

By using the congestion prices $\omega^*_{\nu,\ell}$, the ISO profit function can be mathematically casted as

$$max \;\; \Pi_{ISO} = \sum_{\nu,\ell} \omega^*_{\nu,\ell} p_{\nu,\ell}, \tag{5.35}$$

where $p_{\nu,\ell}$ stands for the ISO decision variables. These variables can be seen as transmission capacities to be set for each transmission right. Since a transmission capacity cannot be negative, then $p_{\nu,\ell} \geq 0$.

The power flow equations are now based upon the separate contribution of FTR options. This is done by means of their Power Transfer Distribution Factors (PTDFs), $\sum_i s_{k,i}\xi_{\nu,\ell,i}$. Under any combination of options, the simultaneous feasibility of the system must be preserved. For each transmission constraint, only the options that positively impact ($\sum_i s_{k,i}\xi_{\nu,\ell,i} > 0$) on the transmission constraint are considered, but options that produce counterflows ($\sum_i s_{k,i}\xi_{\nu,\ell,i} < 0$) are disregarded [39,136]. Then the power flows are defined as

$$\sum_{\nu,\,\ell} max\big(0, \sum_i s_{k,i}\xi_{\nu,\ell,i}\big)p_{\nu,\ell} \leq \overline{z}_k^+, \quad \forall k \in \mathcal{K}, \tag{5.36}$$

$$\sum_{\nu,\,\ell} max\big(0, -\sum_i s_{k,i}\xi_{\nu,\ell,i}\big)p_{\nu,\ell} \leq \overline{z}_k^-, \quad \forall k \in \mathcal{K}. \tag{5.37}$$

The computation of the PTDFs for the options, and the direction of their contribution to the power flows can be done outside the equilibrium problem. Hence, the operator $max(\cdot,\cdot)$, in constraints (5.36) and (5.37), does not represent an extra burden to the equilibrium problem.

The Lagrangian for the problem (5.35)–(5.37) can be written as

$$\begin{aligned} \pounds = &\sum_{\nu,\,\ell} \omega^*_{\nu,\ell}p_{\nu,\ell} - \sum_k \Bigg\{ \lambda_k^+ \Big[\sum_{\nu,\ell} max\big(0, \sum_i s_{k,i}\xi_{\nu,\ell,i}\big)p_{\nu,\ell} - \overline{z}_k^+\Big] + \\ &\lambda_k^- \Big[\sum_{\nu,\,\ell} max\big(0, -\sum_i s_{k,i}\xi_{\nu,\ell,i}\big)p_{\nu,\ell} - \overline{z}_k^-\Big]\Bigg\}, \\ &\lambda_k^+, \lambda_k^- \geq 0, \quad \forall k \in \mathcal{K}, \\ &p_{\nu,\ell} \geq 0, \quad \forall \ell \in \mathcal{O}_\nu. \end{aligned} \tag{5.38}$$

5.4.3 Market clearing condition

Since each option has to be individually modelled in order to capture its impact on the transmission constraints, it is no longer possible to add up all the options to obtain net nodal power injections. Then a market clearing condition is associated with every FTR

$$p_{\nu,\ell} = \tau_{\nu,\ell}\ , \quad \forall \ell \in \mathcal{O}_\nu, \quad \nu \in \Theta. \tag{5.39}$$

Under equilibrium, the transmission capacity demanded for each FTR is matched with the transmission capacity offered by the ISO; this balancing condition generates the congestion prices at which the rights are traded.

5.4.4 Equilibrium model

Based on the Lagrangian for the bidders and ISO problems, the allocation of option FTRs is casted as the following equilibrium problem:

- For $\tau_{\nu,\ell}, \quad \forall \ell \in \mathcal{O}_\nu$:

$$0 \geq \beta_{\nu,\ell} - 2\gamma_{\nu,\ell}\tau_{\nu,\ell} - \left(\omega^*_{\nu,\ell} + A_{\nu,\ell}\tau_{\nu,\ell}\right) - \mu_{\nu,\ell} \quad \perp \quad \tau_{\nu,\ell} \geq 0 \tag{5.40}$$

- For $\mu_{\nu,\ell}, \quad \forall \ell \in \mathcal{O}_\nu$:

$$0 \geq \tau_{\nu,\ell} - \overline{\tau}_{\nu,\ell} \quad \perp \quad \mu_{\nu,\ell} \geq 0 \tag{5.41}$$

- For $p_{\nu,\ell}, \quad \forall \ell \in \mathcal{O}_\nu, \quad \nu \in \Theta$:

$$0 \geq \omega^*_{\nu,\ell} - \sum_k \left\{\lambda^+_k max(0, \sum_i s_{k,i}\xi_{\nu,\ell,i}) + \lambda^-_k max(0, -\sum_i s_{k,i}\xi_{\nu,\ell,i})\right\} \quad \perp \quad p_{\nu,\ell} \geq 0 \tag{5.42}$$

- For $\lambda^+_k, \quad \forall k \in \mathcal{K}$:

$$0 \geq \sum_{\nu,\ell} max\big(0, \sum_i s_{k,i}\xi_{\nu,\ell,i}\big)p_{\nu,\ell} - \overline{z}^+_k \quad \perp \quad \lambda^+_k \geq 0 \tag{5.43}$$

- For $\lambda^-_k, \quad \forall k \in \mathcal{K}$:

$$0 \geq \sum_{\nu,\ell} max\big(0, -\sum_i s_{k,i}\xi_{\nu,\ell,i}\big)p_{\nu,\ell} - \overline{z}^-_k \quad \perp \quad \lambda^-_k \geq 0 \tag{5.44}$$

- For $\omega^*_{\nu,\ell}, \quad \forall \ell \in \mathcal{O}_\nu, \quad \nu \in \Theta$:

$$0 = p_{\nu,\ell} - \tau_{\nu,\ell}; \quad \omega^*_{\nu,\ell} \quad free \tag{5.45}$$

Proposition 5.4 *The congestion prices for option FTRs are non-negative.*

Proof. By the complementarity condition (5.42), the congestion prices for options are

$$\omega^*_{\nu,\ell} = \sum_k \left\{ \lambda^+_k max(0, \sum_i s_{k,i}\xi_{\nu,\ell,i}) + \lambda^-_k max(0, -\sum_i s_{k,i}\xi_{\nu,\ell,i}) \right\}, \quad \forall\, p_{\nu,\ell} > 0. \qquad (5.46)$$

The congestion prices for options are linear combinations of the congestion multipliers, $\lambda^{\cdot}_k$, of only those transmission lines where $\tau_{\nu,\ell}$ has a positive impact. By definition λ^+_k, $\lambda^-_k \geq 0$, $\forall k$ and $max(\cdot,\cdot) \geq 0$; then it follows that the linear combination of congestion multipliers is nonnegative, and, consequently, $\omega^*_{\nu,\ell} \geq 0$, $\forall \nu, \ell$. ■

5.5 A Joint Market for Obligations and Options

The market model for joint obligations and options is built upon the formulation for options introduced in §5.4. For the sake of simplicity in the notation, the index ℓ is used for obligations as well as for options. The super-indices $(^f)$ and $(^o)$ stand for the obligation and option version, respectively.

5.5.1 FTR bidders problem

The FTR portfolio of bidder ν is composed by a subset of obligations $(\mathcal{F}_\nu)$ and a subset of options $(\mathcal{O}_\nu)$. The bidders objective is to maximize the surplus from its composed FTR portfolio, *i.e.*,

$$\max \sum_{\ell \in \mathcal{F}_\nu} \left\{ \beta^f_{\nu,\ell}\tau^f_{\nu,\ell} - \gamma^f_{\nu,\ell}\tau^{2f}_{\nu,\ell} - \omega^f_{\nu,\ell}\tau^f_{\nu,\ell} \right\} + \sum_{\ell \in \mathcal{O}_\nu} \left\{ \beta^o_{\nu,\ell}\tau^o_{\nu,\ell} - \gamma^o_{\nu,\ell}\tau^{2o}_{\nu,\ell} - \omega^o_{\nu,\ell}\tau^o_{\nu,\ell} \right\} \qquad (5.47)$$

$$s.t.\ 0 \leq \tau_{\nu,\ell} \leq \overline{\tau}^f_{\nu,\ell}, \quad \forall \ell \in \mathcal{F}_\nu, \qquad (5.48)$$

$$0 \leq \tau^o_{\nu,\ell} \leq \overline{\tau}^o_{\nu,\ell}, \quad \forall \ell \in \mathcal{O}_\nu, \qquad (5.49)$$

$$\omega^f_{\nu,\ell} - \omega^{*f}_{\nu,\ell} - A^f_{\nu,\ell}(\tau^f_{\nu,\ell} - \tau^{*f}_{\nu,\ell}) = 0, \quad \forall \ell \in \mathcal{F}_\nu, \qquad (5.50)$$

$$\omega^o_{\nu,\ell} - \omega^{*o}_{\nu,\ell} - A^o_{\nu,\ell}(\tau^o_{\nu,\ell} - \tau^{*o}_{\nu,\ell}) = 0, \quad \forall \ell \in \mathcal{O}_\nu. \qquad (5.51)$$

The Lagrangian for the problem (5.47)–(5.51) can be written as

$$\begin{aligned}
\pounds = &\sum_{\ell \in \mathcal{F}_\nu} \Big\{ \beta^f_{\nu,\ell}\tau^f_{\nu,\ell} - \gamma^f_{\nu,\ell}\tau^{2f}_{\nu,\ell} - \omega^f_{\nu,\ell}\tau^f_{\nu,\ell} - \mu^f_{\nu,\ell}(\tau^f_{\nu,\ell} - \overline{\tau}^f_{\nu,\ell}) \Big\} + \\
&\sum_{\ell \in \mathcal{O}_\nu} \Big\{ \beta^o_{\nu,\ell}\tau^o_{\nu,\ell} - \gamma^o_{\nu,\ell}\tau^{2o}_{\nu,\ell} - \omega^o_{\nu,\ell}\tau^o_{\nu,\ell} - \mu^o_{\nu,\ell}(\tau^o_{\nu,\ell} - \overline{\tau}^o_{\nu,\ell}) \Big\}, \qquad (5.52) \\
&\tau^f_{\nu,\ell},\ \mu^f_{\nu,\ell} \geq 0 \quad \forall \ell \in \mathcal{F}_\nu, \\
&\tau^o_{\nu,\ell},\ \mu^o_{\nu,\ell} \geq 0 \quad \forall \ell \in \mathcal{O}_\nu.
\end{aligned}$$

5.5.2 ISO problem

The ISO objective is to maximize the net profit from the allocation of transmission capacity to both obligations and options. This can be mathematically stated as

$$max \quad \Pi_{ISO} = \sum_\nu \Big\{ \sum_{\ell \in \mathcal{F}_\nu} \omega^{*f}_{\nu,\ell} p^f_{\nu,\ell} + \sum_{\ell \in \mathcal{O}_\nu} \omega^{*o}_{\nu,\ell} p^o_{\nu,\ell} \Big\}. \qquad (5.53)$$

The power flows are now defined in terms of the individual contributions of both obligations and options; *i.e.*,

$$\sum_\nu \Big\{ \sum_{\ell \in \mathcal{F}_\nu} \sum_i s_{k,i}\xi^f_{\nu,\ell,i} p^f_{\nu,\ell} + \sum_{\ell \in \mathcal{O}_\nu} max\big(0, \sum_i s_{k,i}\xi^o_{\nu,\ell,i}\big) p^o_{\nu,\ell} \Big\} \leq \overline{z}^+_k, \qquad (5.54)$$

$$\sum_\nu \Big\{ - \sum_{\ell \in \mathcal{F}_\nu} \sum_i s_{k,i}\xi^f_{\nu,\ell,i} p^f_{\nu,\ell} + \sum_{\ell \in \mathcal{O}_\nu} max\big(0, - \sum_i s_{k,i}\xi^o_{\nu,\ell,i}\big) p^o_{\nu,\ell} \Big\} \leq \overline{z}^-_k. \qquad (5.55)$$

Due to this superposition of FTRs, both obligations and options are competing for the same transmission capacity. The ISO decision variables, $p^f_{\nu,\ell}, p^o_{\nu,\ell} \geq 0$, stand for the transmission capacity to be supplied by the ISO in order to satisfy the demand of obligations and options.

The Lagrangian for the problem (5.53)–(5.55) is

$$\pounds = \sum_{\nu} \Bigg\{ \sum_{\ell \in \mathcal{F}_\nu} \omega^{*f}_{\nu,\ell} p^f_{\nu,\ell} + \sum_{\ell \in \mathcal{O}_\nu} \omega^{*o}_{\nu,\ell} p^o_{\nu,\ell} - \sum_k \lambda^+_k \Big[\sum_{\ell \in \mathcal{F}_\nu} \sum_i s_{k,i} \xi^f_{\nu,\ell,i} p^f_{\nu,\ell} + \sum_{\ell \in \mathcal{O}_\nu} max\big(0, \sum_i s_{k,i} \xi^o_{\nu,\ell,i}\big) p^o_{\nu,\ell} - \overline{z}^+_k \Big] - \sum_k \lambda^-_k \Big[- \sum_{\ell \in \mathcal{F}_\nu} \sum_i s_{k,i} \xi^f_{\nu,\ell,i} p^f_{\nu,\ell} + \sum_{\ell \in \mathcal{O}_\nu} max\big(0, -\sum_i s_{k,i} \xi^o_{\nu,\ell,i}\big) p^o_{\nu,\ell} - \overline{z}^-_k \Big] \Bigg\}, \tag{5.56}$$

$$\lambda^+_k,\ \lambda^-_k \geq 0, \quad \forall k \in \mathcal{K},$$

$$p^f_{\nu,\ell},\ p^o_{\nu,\ell} \geq 0,\ \forall \ell.$$

5.5.3 Market clearing conditions

There is a market clearing condition for obligations and another for options, *i.e.*,

$$p^f_{\nu,\ell} = \tau^f_{\nu,\ell}\ , \quad \forall \ell \in \mathcal{F}_\nu, \quad \forall \nu \in \Theta, \tag{5.57}$$

$$p^o_{\nu,\ell} = \tau^o_{\nu,\ell}\ , \quad \forall \ell \in \mathcal{O}_\nu, \quad \forall \nu \in \Theta. \tag{5.58}$$

Each market clearing condition generates the corresponding congestion prices $\omega^{*f}_{\nu,\ell}$ and $\omega^{*o}_{\nu,\ell}$. This implies that obligations and options are separately priced, even though they are simultaneously allocated. This arises from the fact that each kind –obligation or option– of FTRs has different requirements of transmission capacity, even for the same FTR.

5.5.4 Equilibrium model

The following MLCP comprises all the complementarity conditions of the ISO and bidders problems, and the market clearing conditions. The solution to this equilibrium problem simultaneously provides the optimal allocation of obligations and options.

- For $\tau^f_{\nu,\ell}, \quad \forall \ell \in \mathcal{F}_\nu, \quad \nu \in \Theta$:

$$0 \geq \beta^f_{\nu,\ell} - 2\gamma^f_{\nu,\ell} \tau^f_{\nu,\ell} - \big(\omega^{*f}_{\nu,\ell} + A^f_{\nu,\ell} \tau^f_{\nu,\ell}\big) - \mu^f_{\nu,\ell} \quad \perp \quad \tau^f_{\nu,\ell} \geq 0 \tag{5.59}$$

- For $\tau^o_{\nu,\ell}, \quad \forall \ell \in \mathcal{O}_\nu, \quad \nu \in \Theta$:

$$0 \geq \beta^o_{\nu,\ell} - 2\gamma^o_{\nu,\ell}\tau^o_{\nu,\ell} - \left(\omega^{*o}_{\nu,\ell} + A^o_{\nu,\ell}\tau^o_{\nu,\ell}\right) - \mu^o_{\nu,\ell} \quad \perp \quad \tau^o_{\nu,\ell} \geq 0 \tag{5.60}$$

- For $\mu^f_{\nu,k}, \quad \forall \ell \in \mathcal{F}_\nu, \quad \nu \in \Theta$:

$$0 \geq \tau^f_{\nu,\ell} - \overline{\tau}^f_{\nu,\ell} \quad \perp \quad \mu^f_{\nu,\ell} \geq 0 \tag{5.61}$$

- For $\mu^o_{\nu,k}, \quad \forall \ell \in \mathcal{O}_\nu, \quad \nu \in \Theta$:

$$0 \geq \tau^o_{\nu,\ell} - \overline{\tau}^o_{\nu,\ell} \quad \perp \quad \mu^o_{\nu,\ell} \geq 0 \tag{5.62}$$

- For $p^f_{\nu,\ell}, \quad \forall \ell \in \mathcal{F}_\nu, \quad \nu \in \Theta$:

$$0 \geq \omega^{*f}_{\nu,\ell} - \sum_{k,i} s_{k,i}\xi^f_{\nu,\ell,i}(\lambda^+_k - \lambda^-_k) \quad \perp \quad p^f_{\nu,\ell} \geq 0 \tag{5.63}$$

- For $p^o_{\nu,\ell}, \quad \forall \ell \in \mathcal{O}_\nu, \quad \nu \in \Theta$:

$$0 \geq \omega^{*o}_{\nu,\ell} - \sum_k \left\{ \lambda^+_k max(0, \sum_i s_{k,i}\xi^o_{\nu,\ell,i}) + \lambda^-_k max(0, -\sum_i s_{k,i}\xi^o_{\nu,\ell,i}) \right\} \quad \perp \quad p^o_{\nu,\ell} \geq 0 \tag{5.64}$$

- For $\lambda^+_k, \quad \forall k \in \mathcal{K}$:

$$0 \geq \sum_\nu \left\{ \sum_{\ell \in \mathcal{F}_\nu} \sum_i s_{k,i}\xi^f_{\nu,\ell,i}p^f_{\nu,\ell} + \sum_{\ell \in \mathcal{O}_\nu} max\big(0, \sum_i s_{k,i}\xi^o_{\nu,\ell,i}\big)p^o_{\nu,\ell} \right\} - \overline{z}^+_k \quad \perp \quad \lambda^+_k \geq 0 \tag{5.65}$$

- For $\lambda^-_k, \quad \forall k \in \mathcal{K}$:

$$0 \geq \sum_\nu \left\{ -\sum_{\ell \in \mathcal{F}_\nu} \sum_i s_{k,i}\xi^f_{\nu,\ell,i}p^f_{\nu,\ell} + \sum_{\ell \in \mathcal{O}_\nu} max\big(0, -\sum_i s_{k,i}\xi^o_{\nu,\ell,i}\big)p^o_{\nu,\ell} \right\} - \overline{z}^-_k \quad \perp \quad \lambda^-_k \geq 0 \tag{5.66}$$

- For $\omega^{*f}_{\nu,\ell}, \quad \forall \ell \in \mathcal{F}_\nu, \quad \nu \in \Theta$:

$$0 = p^f_{\nu,\ell} - \tau^f_{\nu,\ell}; \quad \omega^{f*}_{\nu,\ell} \quad free \tag{5.67}$$

- For $\omega^{*o}_{\nu,\ell}, \quad \forall \ell \in \mathcal{O}_\nu, \quad \nu \in \Theta$:

$$0 = p^o_{\nu,\ell} - \tau^o_{\nu,\ell}; \quad \omega^{o*}_{\nu,\ell} \quad free \tag{5.68}$$

Proposition 5.5 *Given an FTR in a joint market for obligations and options, the price for the option version is equal or higher than the price for the obligation version.*

Proof. Let us recall complementarity conditions (5.63) and (5.64); rearrangements yield

$$\omega^{*f}_{\nu,\ell} = \sum_k \Big\{ \lambda^+_k \sum_i s_{k,i}\xi^f_{\nu,\ell,i} + \lambda^-_k(-\sum_i s_{k,i}\xi^f_{\nu,\ell,i}) \Big\}, \quad \forall p^f_{\nu,\ell} > 0, \tag{5.69}$$

$$\omega^{*o}_{\nu,\ell} = \sum_k \Big\{ \lambda^+_k max(0, \sum_i s_{k,i}\xi^o_{\nu,\ell,i}) + \lambda^-_k max(0, -\sum_i s_{k,i}\xi^o_{\nu,\ell,i}) \Big\}, \quad \forall p^o_{\nu,\ell} > 0. \tag{5.70}$$

Since $\xi^f_{\nu,\ell,i} \equiv \xi^o_{\nu,\ell,i}$, it suffices that

$$\sum_i s_{k,i}\xi^f_{\nu,\ell,i} \leq max(0, \sum_i s_{k,i}\xi^o_{\nu,\ell,i}). \tag{5.71}$$

Consequently, it follows that

$$\sum_k \Big\{ \lambda^+_k \sum_i s_{k,i}\xi^f_{\nu,\ell,i} + \lambda^-_k(-\sum_i s_{k,i}\xi^f_{\nu,\ell,i}) \Big\} \leq \sum_k \Big\{ \lambda^+_k max(0, \sum_i s_{k,i}\xi^o_{\nu,\ell,i}) + \lambda^-_k max(0, -\sum_i s_{k,i}\xi^o_{\nu,\ell,i}) \Big\}, \tag{5.72}$$

or

$$\omega^{*f}_{\nu,\ell} \leq \omega^{*o}_{\nu,\ell}, \tag{5.73}$$

as claimed ■

This result comes from the insertion of the price components associated with negative PTDFs. Unlike obligations, negative price components are disregarded in the option price, and, hence, they do not counteract the positive price components.

5.6 Numerical Examples

For illustrative purposes, the equilibrium models have been implemented using a three-node system. This small test system allows one to understand how the FTR allocation is. Afterwards, the models are applied to a five-node system.

5.6.1 A three-node system

Let us consider the system shown in Figure 5.2. It is assumed that suppliers are placed at nodes 1 and 2, while a demand is placed at node 3. The network is modelled as lossless and with equal transmission lines' reactance; transmission lines 1-2, 1-3 and 2-3 (labelled as lines 1, 2 and 3) have limits of 90, 300 and 250 MW, respectively. Because of the network

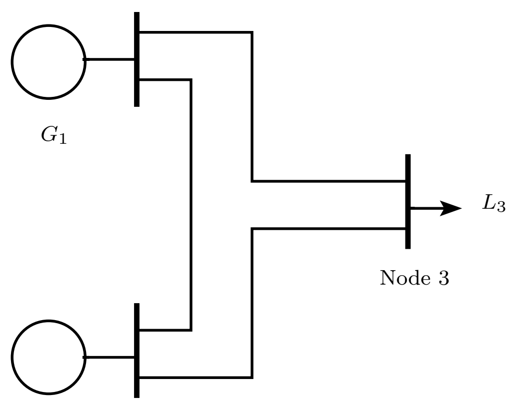

Figure 5.2: Three-node power system.

configuration, with an amount of power transferred from node 1 to node 3, two-thirds of power flow through line 2, and one-third of power flows through lines 1 and 3. Similarly, with an amount of power transferred from node 2 to node 3, two-thirds of power flow through line 3, and one-third of power flows through lines 1 and 2.

Under point-to-point financial transmission rights, two transmission rights can be naturally considered: i) an FTR for generator 1 from node 1 to node 3; and ii) an FTR for generator 2 from node 2 to node 3. These FTRs are defined by the benefit functions $B(\tau_{1,1}) = 6\tau_{1,1}$ and $B(\tau_{2,1}) = 5\tau_{2,1}$, and maximum limits $\overline{\tau}_{\nu,\ell}$=1500 MW. Due to the FTR configuration, their power flows counteract each other in transmission line 1 while their power flows reinforce each other in lines 2 and 3.

First, consider the case of obligations; in a transmission-right market, both FTRs may be issued in order to achieve the optimal allocation of transmission capacity. The counterflow that one FTR provides to the other is implicitly considered within the *obligation* version. Thus, for the issuance of obligations, it is only required that the superposition of both FTRs be feasible for the transmission system. Depending on the amount issued for each obligation, different transmission line constraints can become active. All the feasible combinations of both obligations are comprised in the monogram (shaded area) depicted in Figure 5.3. Here, the optimal FTR allocation is denoted by Π^f; it is achieved when trans-

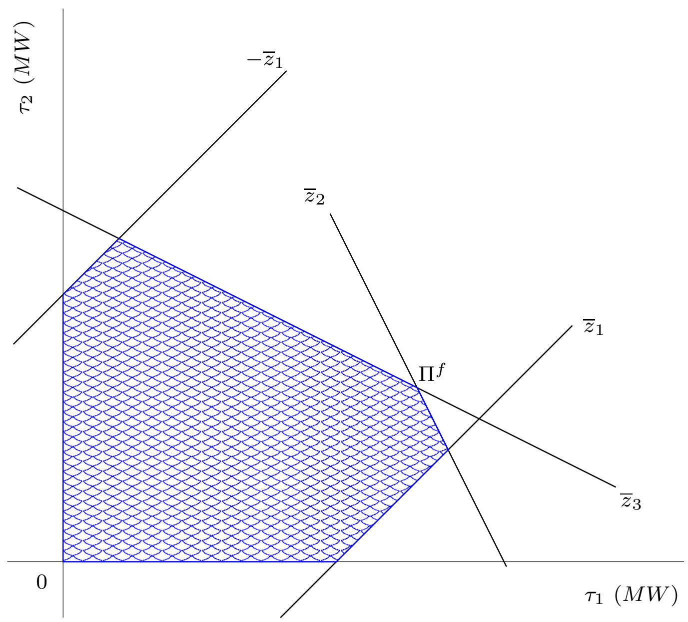

Figure 5.3: Feasible region for the issuance of obligations.

mission lines 2 and 3 are binding. The corresponding congestion multipliers are $7/MW$ and $4/MW$, respectively. In line 1, the counterflow that $\tau_{1,1}$ provides to $\tau_{2,1}$ is represented by the constraint $-\overline{z}_1$. On the other hand, the counterflow that $\tau_{2,1}$ provides to $\tau_{1,1}$ is given by the constraint $\overline{z}_1$. The positive slope of these constraints implies that if the power of one obligation is increased, providing more counterflow, the other obligation can also be increased. In contrast, the constraints for transmission lines 2 and 3 have negative slopes. This implies that if an obligation is increased, the other will have to be decreased in order to compensate the extra capacity required.

Let us now consider the issuance of options; assume that the options are defined by the same benefit functions used for obligations. Any counterflow provided by an option has to be disregarded since it may happen that such an option is not exercised. Hence, there is a reduction of the transmission capacity to be allocated, as shown in Figure 5.4. The transmission capacity of line 1 to be allocated to each option is, at most, the transmission power flow limit. The transmission constraints for line 1 in both directions are denoted by

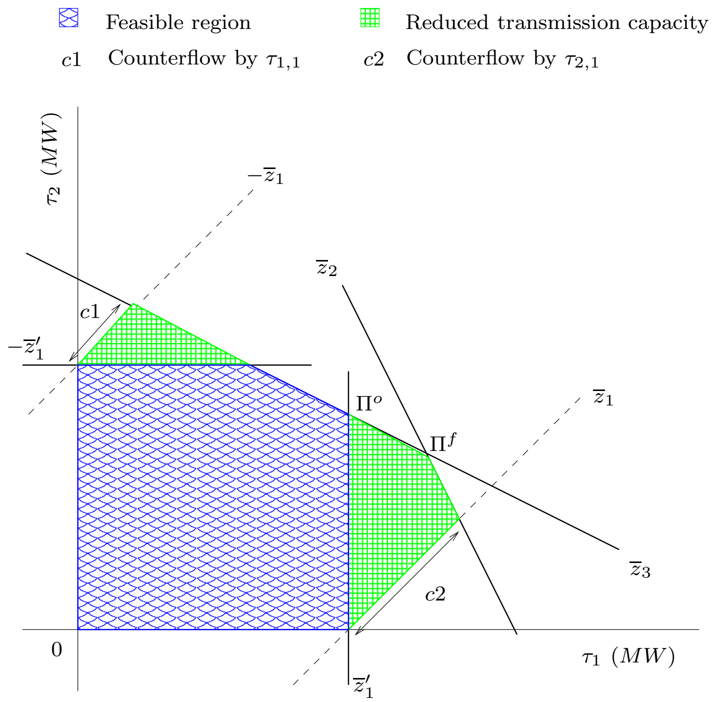

Figure 5.4: Feasible region for the issuance of options.

$\overline{z}'_1$ and $-\overline{z}'_1$. The shaded areas in both corners stand for the transmission capacity that cannot be allocated in order to preserve feasibility under the exercise of any combination of options. The optimal allocation, denoted as Π^o, occurs when transmission lines 1 and 3 are binding; their corresponding dual variables are \$10.5/MW and \$7.5/MW, respectively. The market outcomes comparison is presented in Table 5.1.

Table 5.1: Outcome comparison for different FTR markets with $A_{\nu,\ell} = 0.0$.

	Obligations		**O**ptions	
	$\tau_{1,1}$	$\tau_{2,1}$	$\tau_{1,1}$	$\tau_{2,1}$
Award (MW)	350	200	270	240
Price (\$/MW)	6	5	6	5
Cost (\$)	2,100	1,000	1,620	1,200
Benefit (\$)	2,100	1,000	1,620	1,200
Surplus (\$)	0	0	0	0
ISO Revenue (\$)	**3**,100		**2**,820	

As expected, less transmission capacity is allocated to option FTRs. This also reduces the ISO revenues from the sales of FTRs. For obligations and options, the FTR prices are the same and equal to the marginal value of bidders. The simultaneous feasibility for obligations as well as for options are shown in Figures 5.5 and 5.6.

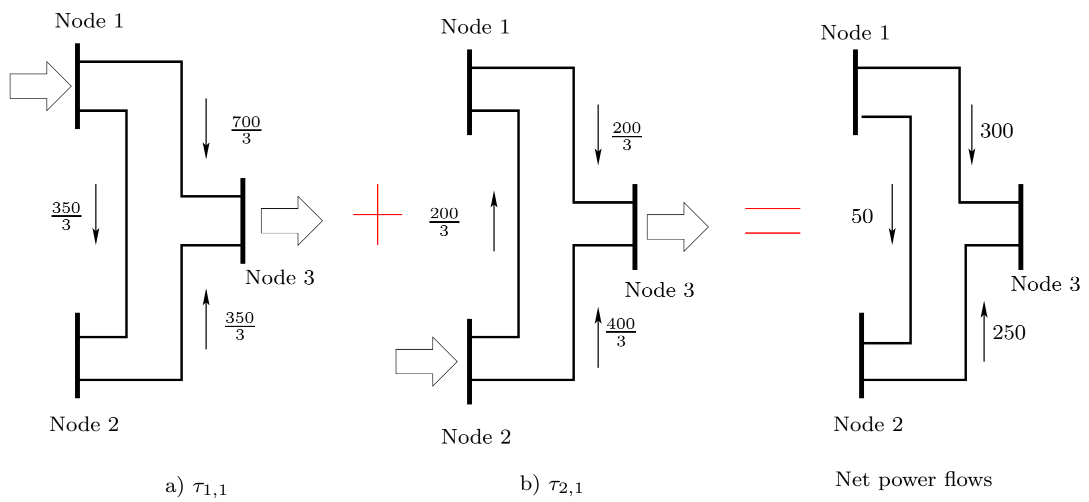

Figure 5.5: Simultaneous feasibility of obligations.

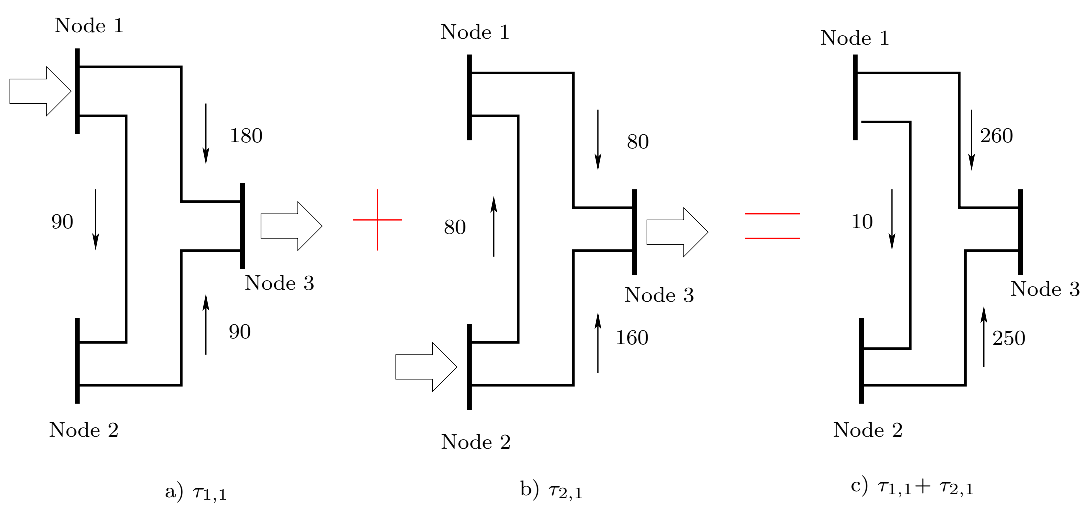

Figure 5.6: Simultaneous feasibility of options.

For obligations, both FTRs have to be considered; otherwise, there will be a violation in the transmission power limit of line 1. In contrast, for options, there can be 3 potential combinations, and for all of them the system feasibility is preserved. Case a) is when the option $\tau_{1,1}$ is exercised, and this binds the transmission constraint 1 (90 MW). Case b) is when option $\tau_{2,1}$ is exercised; this option does not bind any transmission line. However, this option cannot be further increased because if both options were exercised the outcome would be unfeasible. Finally, in Case c), both options are exercised, and the constraint of transmission line 3 (250 MW) becomes active.

In a second simulation, let us consider how changes in the bidder strategies can influence the congestion prices; for a study case, consider $A_{\nu,\ell} = 0.01$ for both FTRs, and for both obligations and options. The market outcomes are shown in Table 5.2.

Table 5.2: Outcome comparison for different FTR markets with $A_{\nu,\ell} = 0.01$.

	Obligations		**O**ptions	
	$\tau_{1,1}$	$\tau_{2,1}$	$\tau_{1,1}$	$\tau_{2,1}$
Award (MW)	350	200	270	240
Price ($/$MW$)	2.5	3	3.3	2.6
Cost ($)	875	600	891	624
Benefit ($)	2,100	1,000	1,620	1,200
Surplus ($)	1,225	400	729	576
ISO Revenue ($)	**1**,475		**1**,515	

Although the FTR bidders have the congestion prices decreased for both obligations and options, they are still awarded with the same amount of FTRs. As a result of lower congestion prices, each bidder increases its surplus, and, therefore, the ISO revenues are decreased from $3100 to $1475 in the case of obligations, and from $2820 to $1515 in the case of options. Furthermore, in the case of obligations, the congestion prices for transmission lines 2 and 3, are reduced to $2/$MW$ and 3.5MW$, respectively. Whereas for options, the congestion prices for lines 1 and 3 are reduced to $6/$MW$ and $3.9/$MW$, respectively.

The impact on the bidders' surplus of different bidding strategies is depicted in Figure 5.7. Based on these plots, one can see how the bidders' surplus is affected under different assumptions of congestion-price changes. For instance, for option $\tau_{1,1}$, in the flat region of the surplus (F), bidder 1 can freely alter the congestion prices to increase its surplus, regardless the bidding strategy of $\tau_{2,1}$. This is because option $\tau_{1,1}$ is what binds transmission constraint 1; in this line, however, the power flow from option $\tau_{2,1}$ is disregarded because

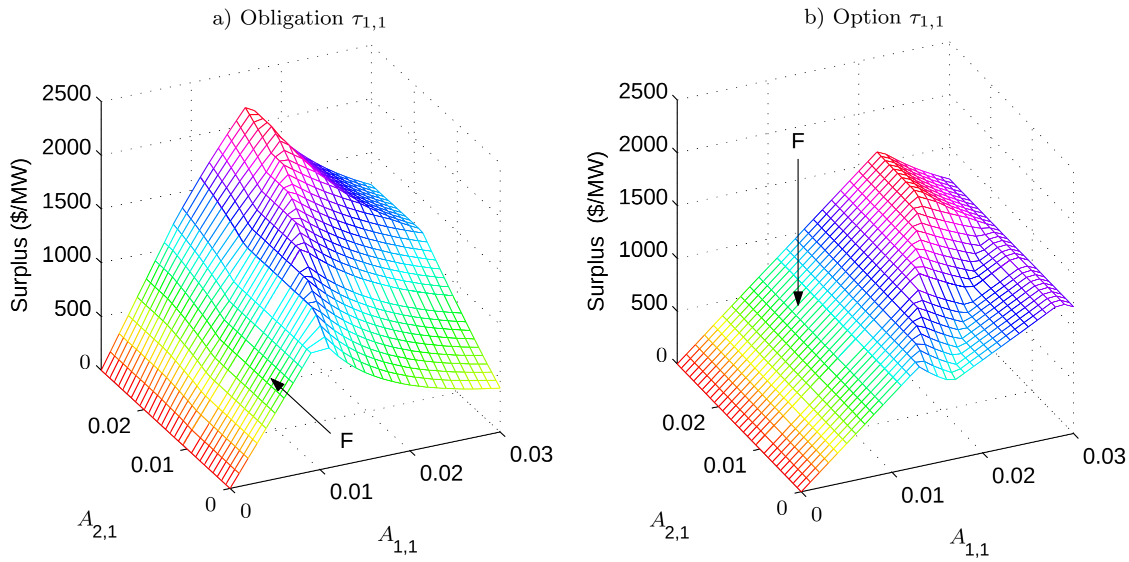

Figure 5.7: Bidders' surplus for different bidding strategies.

of its opposite direction, and, hence, it has no impact. In contrast, for obligation $\tau_{1,1}$ this also holds, but in a smaller zone. This is because the complementarity effect between obligations is taken into account to preserve feasibility.

On the other hand, the prices for transmission rights can be computed from the congestion multipliers. For instance, for obligation $\tau_{2,1}$, the price component from line 2 is (1/3)($2/$MW$), while the price component from line 3 is (2/3)($3.5/$MW$), what makes the price of $3/$MW$ for such an obligation. On the other hand, the price for option $\tau_{2,1}$ is given by the component from line 3: (2/3)($3.9/$MW$)=$2.6/MW. Although the congestion multiplier from line 1 is $6/$MW$, the power flow from this option is in opposite direction and, consequently, its corresponding price component is zero.

5.6.2 A five-node system

In this section, the proposed equilibrium models are implemented using the five-node system given in Appendix B. There are 3 bidders who have identified 9 types –obligation and option– of FTRs that could become valuable. FTRs' data are listed in Table B.4. For the issuance of both obligations and options, the competitive outcome is first computed; afterwards, it is considered the potential bidders' influence on congestion prices ($A_{\nu,\ell} = 0.025, \forall \nu, \ell$). The market outcomes for both cases are summarized in Tables 5.3–5.6.

In the case of obligations, as bidders influence the congestion prices, such prices go down. This influence of congestion prices can be seen as the bidders ability to capture some moneys from the ISO's congestion rents, an analogy to the strategic behaviour described in [9]. This also causes that bidders can have extra FTRs awarded, such as $\tau_{\nu,4}$ and $\tau_{\nu,8}$. In most of the cases, bidders benefits are not altered; however, the costs to acquire FTRs are lower, and, consequently, their corresponding surpluses are increased. Hence, all FTR bidders are better off. For the issuance of options, due to the lack of counterflows, there

Table 5.3: Awards and prices for obligations.

ν	ℓ	$A_{\nu,\ell} = 0.0$		$A_{\nu,\ell} = 0.025$	
		$\tau_{\nu,\ell}$ (MW)	$\omega^*_{\nu,\ell}$ $(\$/MW)$	$\tau_{\nu,\ell}$ (MW)	$\omega^*_{\nu,\ell}$ $(\$/MW)$
1	1	110	14.98	110	14.14
	4	120	43.01	120	39.56
	5	110	58.86	110	54.51
	9	0.0	15.84	1.48	14.95
2	1	100	14.98	100	14.14
	4	0.0	43.01	13.25	39.56
	5	120	58.86	120	54.51
	8	0.0	24.00	37.08	20.99
	9	0.0	15.84	1.52	14.95
3	1	300	14.98	300	14.14
	4	300	43.01	300	39.56
	5	190.05	58.86	176.79	54.51
	8	166.15	24.00	129.07	20.99
	9	22.55	15.84	32.77	14.95

is a different market outcome with respect to the issuance of obligations. However, the issuance of options has led to higher FTR prices, *e.g.*, for $\tau_{\nu,1}$, the obligation is priced at $\$14.98/MW$ while the option is priced at $\$19.95/MW$. In spite of the bidders influence on prices, no extra FTRs have been awarded. This is because options, at some degree, are independent across them as no option counterflow is needed for feasibility of others. This also causes the price influence to be more pronounced in the obligation case. As a result, the auctioneer has its market revenue reduced from \$54775.11 to \$50554.73 for obligations, and from \$49, 603.48 to \$47, 798.6 for options.

Table 5.4: Cost, benefit and surplus (in \$) for obligations.

		$A_{\nu,\ell} = 0.0$			$A_{\nu,\ell} = 0.025$		
ν	ℓ	$\omega^*_{\nu,\ell}\tau_{\nu,\ell}$	$B(\tau_{\nu,\ell})$	$\pi_{\nu,\ell}$	$\omega^*_{\nu,\ell}\tau_{\nu,\ell}$	$B(\tau_{\nu,\ell})$	$\pi_{\nu,\ell}$
1	1	1,648.1	2,139.5	491.3	1,555.4	2,139.5	584.0
	4	5,161.1	7,178.4	2,017.2	4,747.5	7,178.4	2,430.8
	5	6,474.5	6,569.7	95.1	5,997.1	6,569.7	572.6
	9	0.0	0.0	0.0	22.1	22.2	0.06
2	1	1,498.3	2040	541.6	1,414.0	2,040	626
	4	0.0	0.0	0.0	524.3	529.4	5.09
	5	7,063.1	7,771.2	708.0	6,542.3	7,771.2	1,228.8
	8	0.0	0.0	0.0	778.8	814.5	35.7
	9	0.0	0.0	0.0	22.8	22.9	0.06
3	1	4,495.0	7,050	2,554.9	4242.0	7050	2,807.9
	4	12,902.9	14,640	1,737.0	11,868.7	14,640	2,771.2
	5	11,186.3	11,294.7	108.3	9,638.8	10,514.0	875.2
	8	3,988.1	4,070.9	82.8	2,710.4	3,176.9	466.5
	9	357.2	359.0	1.7	490.0	520.7	30.6

5.7 A Multi-Round Market for Obligations

In order to enhance price discovery in the FTRs markets, sequential auctions can be implemented. These auctions, called *multi-round* auctions, allow market participants to adapt their successive bids. In each round, only a portion of the transmission capacity is auctioned. FTRs awarded in one round are modelled as fixed injections in subsequent rounds. The auction outcome of each round can be made available to participants before the next round takes place in order to provide for price discovery. For instance, in the PJM market [134], the transmission capacity is offered for sale in a multi-round auction consisting of four rounds. Twenty-five percent of the feasible FTR capability of the system is auctioned in each round.

Table 5.5: Awards and prices for options.

ν	ℓ	$A_{\nu,\ell} = 0.0$		$A_{\nu,\ell} = 0.025$	
		$\tau_{\nu,\ell}$ (MW)	$\omega^*_{\nu,\ell}$ $(\$/MW)$	$\tau_{\nu,\ell}$ (MW)	$\omega^*_{\nu,\ell}$ $(\$/MW)$
1	1	4.27	19.95	24.46	19.14
	4	120	49.95	120	49.80
	5	110	58.97	110	55.60
2	1	86.89	19.95	50.16	19.14
	2	18.18	59.90	96.35	57.10
	5	120	58.97	120	55.60
	6	150	64.95	150	64.55
3	1	300	19.95	167.31	19.14
	4	5.98	49.95	5.98	49.80
	5	170.67	58.97	141.74	55.60
	6	16.03	64.95	16.04	64.55

Table 5.6: Cost, benefit and surplus (in $) for options.

ν	ℓ	$A_{\nu,\ell} = 0.0$			$A_{\nu,\ell} = 0.025$		
		$\omega^*_{\nu,\ell}\tau_{\nu,\ell}$	$B(\tau_{\nu,\ell})$	$\pi_{\nu,\ell}$	$\omega^*_{\nu,\ell}\tau_{\nu,\ell}$	$B(\tau_{\nu,\ell})$	$\pi_{\nu,\ell}$
1	1	85.4	85.4	00	468.2	486.2	17.9
	4	5,994.2	7,178.4	1,184.1	5,976.2	7,178.4	1,202.1
	5	6,487.3	6,569.7	82.4	6,116.6	6,569.7	453.1
2	1	1,734.1	1,779.4	45.3	960.3	1,038.3	78.0
	2	1,089.2	1,090.0	0.8	5,502.6	5,757.9	255.3
	5	7,077.1	7,771.2	694.0	6,672.6	7,771.2	1,098.5
	6	9,742.7	10,443.7	700.9	9,682.6	10,443.7	761.1
3	1	5,987.1	7,050	1,062.8	3,203.1	4,042.9	839.8
	4	299.1	299.2	0.1	298.2	299.2	1.04
	5	10,065.5	10,152.9	87.3	7,882	8,444.5	562.5
	6	1,041.4	1,041.8	0.3	1,035.7	1,042.5	6.8

First, it is derived how the participants problem and equilibrium model are re-formulated, and then the iterative procedure is presented. The super-index $r \in \mathcal{R}$ is used throughout this section to denote each market round. In addition, an auxiliary set $\hat{\mathcal{R}} \subset \mathcal{R}$ stands for cumulative market rounds up to a current round, while $|\hat{\mathcal{R}}|$ denotes the set cardinality.

5.7.1 FTR bidders problem

Let us consider the case of a market for obligations described in §5.3. For the multi-round market, the bidders problem remains the same; all the variables and parameters are indexed for each round.

5.7.2 ISO problem

Besides indexing all variables and parameters in the ISO problem, the power-flow limits in the transmission lines are re-defined. Since transmission configuration remains the same for all rounds, the constants $s_{k,i}$ are not indexed by r. Now the transmission limits are set to be the cumulative capacity up to the current round. Thus, for the r-th round, the transmission capacity in each direction is increased by $\overline{z}_k^{+r}$ or $\overline{z}_k^{-r}$ with respect to the previous round. Then the ISO problem in the r-th round can be casted as

$$max \;\; \Pi_{ISO} = \sum_i \omega_i^{*r} p_i^r \tag{5.74}$$

$$s.t. \;\; \sum_i s_{k,i} p_i^r \leq \sum_{r=1}^{|\hat{\mathcal{R}}|} \overline{z}_k^{+r}, \quad \forall k \in \mathcal{K}, \tag{5.75}$$

$$- \sum_i s_{k,i} p_i^r \leq \sum_{r=1}^{|\hat{\mathcal{R}}|} \overline{z}_k^{-r}, \quad \forall k \in \mathcal{K}. \tag{5.76}$$

5.7.3 Market clearing condition

For this kind of market, the ISO has to take into account the FTR awards from previous rounds. Such awards represent already committed transmission capacity which is not available to be offered. In the r-th round, the transmission capacity supplied by the ISO must equal the capacity to be allocated, via FTRs, plus the awarded FTRs from previous rounds. Within the current round, the latter FTRs are modelled as fixed power injections (p_i^c),

$$p_i^r = \sum_{\nu,\,\ell} \xi_{\nu,\ell,i}^r \tau_{\nu,\ell}^r + p_i^c\ , \quad \forall i \in \mathcal{I}. \tag{5.77}$$

5.7.4 Equilibrium model

Based on the model formulation for obligations in §5.3, the derivation of the complementarity conditions for each market participant problem is straightforward. Likewise above formulations, all the complementarity and market clearing conditions are gathered to compose the following equilibrium problem:

- For $\tau_{\nu,\ell}^r, \quad \forall \ell \in \mathcal{F}_\nu^r, \quad \nu \in \Theta$:

$$0 \geq \beta_{\nu,\ell}^r - 2\gamma_{\nu,\ell}^r \tau_{\nu,\ell}^r - \sum_i \big(\omega_i^{*r} + A_{\nu,i} \sum_h \tau_{\nu,h}^r \xi_{\nu,h,i}^r\big) \xi_{\nu,\ell,i}^r - \mu_{\nu,\ell}^r \quad \perp \quad \tau_{\nu,\ell}^r \geq 0 \tag{5.78}$$

- For $\mu_{\nu,\ell}^r, \quad \forall \ell \in \mathcal{F}_\nu^r, \quad \nu \in \Theta$:

$$0 \geq \tau_{\nu,\ell}^r - \overline{\tau}_{\nu,\ell}^r \quad \perp \quad \mu_{\nu,\ell}^r \geq 0 \tag{5.79}$$

- For $p_i^r, \quad \forall i \in \mathcal{I}$:

$$0 = \omega_i^{*r} - \sum_k s_{k,i}(\lambda_k^{+r} - \lambda_k^{-r}); \quad p_i^r \ \ free \tag{5.80}$$

- For $\lambda_k^+, \quad \forall k \in \mathcal{K}$:

$$0 \geq \sum_i s_{k,i} p_i^r - \sum_{r=1}^{|\hat{\mathcal{R}}|} \overline{z}_k^{+r} \quad \perp \quad \lambda_k^{+r} \geq 0 \tag{5.81}$$

- For $\lambda_k^-, \quad \forall k \in \mathcal{K}$:

$$0 \geq -\sum_i s_{k,i} p_i^r - \sum_{r=1}^{|\hat{\mathcal{R}}|} \overline{z}_k^{-r} \quad \perp \quad \lambda_k^{-r} \geq 0 \tag{5.82}$$

- For $\omega_i^{*r}, \quad \forall i \in \mathcal{I}$:

$$0 = p_i^r - \sum_{\nu,\ell} \xi_{\nu,\ell,i}^r \tau_{\nu,\ell}^r - p_i^c; \quad \omega_i^{*r} \quad free \tag{5.83}$$

5.7.5 Iterative procedure

An outline of the multi-round market for FTRs is shown next.

Algorithm 1 Multi-round market for FTRs.

STEP 0	Initialization:
	Set $p_i^c = 0,\ \forall i \in \mathcal{I}$.
	Set $r = 1$.
STEP 1	Compute an equilibrium:
	Set $\lvert\hat{\mathcal{R}}\rvert = r$.
	Solve (5.78)–(5.83) for $\tau_{\nu,\ell}^r$.
STEP 2	Update awarded transmission capacity:
	Compute the awarded transmission capacity $\sum_{\nu,\ell} \xi_{\nu,\ell,i}^r \tau_{\nu,\ell}^r,\ \forall i \in \mathcal{I}$.
	Update the committed power capacity $p_i^c \leftarrow p_i^c + \sum_{\nu,\ell} \xi_{\nu,\ell,i}^r \tau_{\nu,\ell}^r,\ \forall i \in \mathcal{I}$.
STEP 3	Finalization:
	If $r = \lvert\mathcal{R}\rvert$, stop; otherwise, $r \leftarrow r + 1$ and return to STEP 1.

Similar formulations can be derived for the cases of pure options, and joint obligations and options.

5.7.6 Numerical examples

A three-node system

Given the three-node example in §5.6.1, consider an extension of a four-round market for obligations. The benefit function parameters for both FTRs are listed in Table 5.7. In each round, the transmission capacity for lines 1, 2 and 3 are 22.5, 75 and 62.5 MW, respectively.

Table 5.7: Benefit function parameters (in $\$/MW$) for FTRs.

		Round			
ν	ℓ	$\beta_{\nu,\ell}^1$	$\beta_{\nu,\ell}^2$	$\beta_{\nu,\ell}^3$	$\beta_{\nu,\ell}^4$
1	1	6	5.5	4.7	4.6
2	1	5	5	4.9	4.6

The market outcomes are summarized in Table 5.8. Due to the FTRs and the network configuration are the same in each round, the market outcome in each round is of the same kind. The FTR awards are the same through the rounds, and equal to 25% of that of the one-period market outcome from §5.6.1. However, the FTR prices are different in each round because the FTRs value, given by their benefit function, is different among rounds.

Table 5.8: Multi-round market outcome.

r	ν, ℓ	$\tau^r_{\nu,\ell}$ (MW)	Price $(\$/MW)$	Cost (\$)	ISO Rev. (\$)
1	1,1	87.5	6	525	775
	2,1	50	5	250	
2	1,1	87.5	5.5	481.2	731.2
	2,1	50	5	250	
3	1,1	87.5	4.7	411.2	656.2
	2,1	50	4.9	245	
4	1,1	87.5	4.6	402.5	632.5
	2,1	50	4.6	230	
Net ISO Revenue (\$)					2795

A thirty-node system

The proposed model has been implemented in the standard IEEE-based test power system of 30 nodes and 41 transmission lines [120]. Twenty FTRs are considered to be issued; they are shown in Table 5.9.

Table 5.9: Source and sink nodes for FTRs.

ℓ	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
	2	2	2	9	8	15	13	18	23	23	25	25	30	2	15	18	8	18	1	27
	1	4	5	10	7	14	16	16	20	25	27	28	28	15	7	30	20	5	29	5

For Nine FTR portfolios, the parameters of the benefit functions are randomly generated, and they are grouped into sets of three portfolios. Each set is defined with FTRs of high, medium and low value. The multi-round market outcome is given in Tables 5.10–5.11.

Table 5.10: FTR prices (in $\$/MW$). Multi-round market.

ℓ	1	2	3	4	5	6	7	8	9	10	11	12	13
r=1	15.8	17.2	42.4	19.2	17.5	16.4	16.8	41.9	40.1	15.7	15.3	17.4	17.1
r=2	18.4	18.9	42.5	17.5	20.0	15.0	16.4	40.3	56.4	18.1	19.4	19.2	14.9
r=3	17.0	17.9	43.9	17.4	16.6	18.6	19.1	49.7	40.0	18.2	17.5	18.6	15.2
r=4	15.1	19.2	42.4	15.0	15.1	16.4	16.4	43.5	39.6	16.9	15.2	17.5	18.1

ℓ	14	15	16	17	18	19	20
r=1	16.2	18.7	16.0	16.5	48.3	16.9	17.3
r=2	14.1	14.7	16.1	35.8	47.9	18.9	16.6
r=3	18.3	16.2	23.5	13.9	54.0	14.2	16.3
r=4	17.7	15.7	17.3	15.6	49.1	16.3	15.9

Table 5.11: FTR awards (in MW). Multi-round market.

ν	1												2	
ℓ	1	4	5	6	7	10	11	12	15	17	19	20	1	2
r=1	0	0	0	0	0	0	0	0	38.6	0	0	2.8	20.4	0
r=2	0	0	0	8.2	0	7.8	14.9	0.0	32.1	0.0	6.9	5.0	22.6	8.2
r=3	19.5	24.2	7.7	0.0	10.8	5.5	9.1	5.8	0	2.8	9.9	0	0	0
r=4	20.4	0	0	0	0	0	0	4.3	0	0	0	2.8	0	0

Table 5.12: FTR awards (in MW). Multi-round market (continued).

ν	2								3						
ℓ	4	5	6	13	14	16	17	19	5	6	7	10	11	12	13
r=1	0	0	0	0	0	0	0	0	7.3	9.9	0	0	10.6	4.3	6.2
r=2	26.8	10.5	0	0	0	10	0	0	0	0	10	0	0	0	10
r=3	0	0	9.7	0.0	56.5	10	0	0	0	0	0	0	0	0	4.7
r=4	23.8	0	0	6.2	0	10	3.2	9.1	7.3	0	10.9	5.2	10.6	0	0

ν	3				4			5	6	
ℓ	14	15	17	19	4	6	10	7	16	20
r=1	55.6	0	3.2	9.1	23.8	0	5.2	10.9	100	0
r=2	44.2	0	0	0	0	0	0	0	0	0
r=3	0	0	0	0	0	0	0	0	0	1.9
r=4	55.6	38.6	0	0	0	9.9	0	0	0	0

5.8 A Multi-Period Market for Obligations

In this section, the model for obligations is extended to consider different trading periods. Since FTRs are used to hedge against congestion in the energy market, the holder is entitled to collect a share of congestion rents regardless when congestion occurs. In order to increase flexibility, FTR can be issued for different time periods. FTRs can be issued to collect congestion rents at specific time periods. For instance, there can be off-peak, on-peak and full day (off- plus on-peak) periods. For the model derived in this section, any time period, such as off- or on-peak periods, is characterized by $t \in \mathcal{T}$; while Γ is used to denote the full horizon, such as a full day. Due to the full day period overlaps with any other period t, the terms related to such a period are explicitly modelled in the formulation. All variables and parameters from §5.3 are recalled and indexed over the time periods.

5.8.1 FTR bidders problem

As long as a participant ν has the flexibility to bid for FTRs of different periods, its surplus-maximization problem across periods can be mathematically stated as follows:

$$\begin{aligned} max \quad \Pi_{\nu} = & \sum_{t} \sum_{\ell \in \mathcal{F}_{\nu}^{t}} \big(\beta_{\nu,\ell}^{t} \tau_{\nu,\ell}^{t} - \gamma_{\nu,\ell}^{t} \tau_{\nu,\ell}^{2^{t}} - \tau_{\nu,\ell}^{t} \sum_{i} \omega_{\nu,i}^{t} \xi_{\nu,\ell,i}^{t}\big) + \\ & \sum_{\ell \in \mathcal{F}_{\nu}^{\Gamma}} \big(\beta_{\nu,\ell}^{\Gamma} \tau_{\nu,\ell}^{\Gamma} - \gamma_{\nu,\ell}^{\Gamma} \tau_{\nu,\ell}^{2^{\Gamma}} - \tau_{\nu,\ell}^{\Gamma} \sum_{j} \omega_{\nu,j}^{\Gamma} \xi_{\nu,\ell,j}^{\Gamma}\big) \end{aligned} \tag{5.84}$$

$$s.t. \quad 0 \leq \tau_{\nu,\ell}^{t} \leq \overline{\tau}_{\nu,\ell}^{t}, \quad \forall \ell \in \mathcal{F}_{\nu}^{t},\ t \in \mathcal{T}, \tag{5.85}$$

$$0 \leq \tau_{\nu,\ell}^{\Gamma} \leq \overline{\tau}_{\nu,\ell}^{\Gamma}, \quad \forall \ell \in \mathcal{F}_{\nu}^{\Gamma}, \tag{5.86}$$

$$\omega_{\nu,i}^{t} = A_{\nu,i} \sum_{\ell} \xi_{\nu,\ell,i}^{t} (\tau_{\nu,\ell}^{t} - \tau_{\nu,\ell}^{*t}) + \omega_{i}^{*t}, \quad \forall i \in \mathcal{I},\ t \in \mathcal{T}, \tag{5.87}$$

$$\omega_{\nu,i}^{\Gamma} = A_{\nu,i} \sum_{\ell} \xi_{\nu,\ell,i}^{\Gamma} (\tau_{\nu,\ell}^{\Gamma} - \tau_{\nu,\ell}^{*\Gamma}) + \omega_{i}^{*\Gamma}, \quad \forall i \in \mathcal{I}. \tag{5.88}$$

5.8.2 ISO problem

The ISO problem is to maximize the revenue from the sales of FTRs through the trading periods. This is done by solving the following profit-maximization problem:

$$max \ \ \Pi_{ISO} = \sum_{t,i} \omega_{i}^{*t} p_{i}^{t} + \sum_{i} \omega_{i}^{*\Gamma} p_{i}^{\Gamma}. \tag{5.89}$$

For any combination of FTRs to be chosen, there is a need to ensure a simultaneous feasibility; thus, the ISO's decisions (p_i^t, p_i^Γ) have to be feasible for the transmission system constraints at any trading period, *i.e.*,

$$\sum_{i} s_{k,i}(p_{i}^{t} + p_{i}^{\Gamma}) \leq \overline{z}_{k}^{+}, \quad \forall k \in \mathcal{K},\ t \in \mathcal{T}, \tag{5.90}$$

$$- \sum_{i} s_{k,i}(p_{i}^{t} + p_{i}^{\Gamma}) \leq \overline{z}_{k}^{-}, \quad \forall k \in \mathcal{K},\ t \in \mathcal{T}. \tag{5.91}$$

At period t, the net power injections in the system are composed by both the nodal power injection p_i^t, corresponding to the current time period t, and the nodal power injection from the full horizon p_i^Γ. Therefore, FTRs of period t are going to be competing for the transmission capacity with FTRs of period Γ.

5.8.3 Market clearing condition

At each period, the demand for transmission services, required by all bidders, must equal the transmission capacity supplied by the ISO, *i.e.*,

$$p_i^t = \sum_{\nu,\ell} \xi_{\nu,\ell,i}^t \tau_{\nu,\ell}^t \ , \quad \forall i \in \mathcal{I},\ t \in \mathcal{T}, \tag{5.92}$$

$$p_i^\Gamma = \sum_{\nu,\ell} \xi_{\nu,\ell,i}^\Gamma \tau_{\nu,\ell}^\Gamma \ , \quad \forall i \in \mathcal{I}. \tag{5.93}$$

5.8.4 Equilibrium model

The optimal allocation of FTRs can be characterized by the following MLCP:

- For $\tau_{\nu,\ell}^t, \quad \forall \ell \in \mathcal{F}_\nu^t,\ \nu \in \Theta,\ t \in \mathcal{T}$:

$$0 \geq \beta_{\nu,\ell}^t - 2\gamma_{\nu,\ell}^t \tau_{\nu,\ell}^t - \sum_i \big(\omega_i^{t^*} + A_{\nu,i} \sum_h \tau_{\nu,h}^t \xi_{\nu,h,i}^t\big) \xi_{\nu,\ell,i}^t - \mu_{\nu,\ell}^t \quad \perp \quad \tau_{\nu,\ell}^t \geq 0 \tag{5.94}$$

- For $\tau_{\nu,\ell}^\Gamma, \quad \forall \ell \in \mathcal{F}_\nu^\Gamma,\ \nu \in \Theta$:

$$0 \geq \beta_{\nu,\ell}^\Gamma - 2\gamma_{\nu,\ell}^\Gamma \tau_{\nu,\ell}^\Gamma - \sum_i \big(\omega_i^{\Gamma^*} + A_{\nu,i} \sum_h \tau_{\nu,h}^\Gamma \xi_{\nu,h,i}^\Gamma\big) \xi_{\nu,\ell,i}^\Gamma - \mu_{\nu,\ell}^\Gamma \quad \perp \quad \tau_{\nu,\ell}^\Gamma \geq 0 \tag{5.95}$$

- For $\mu_{\nu,\ell}^t, \quad \ell \in \mathcal{F}_\nu^t,\ \nu \in \Theta,\ t \in \mathcal{T}$:

$$0 \geq \tau_{\nu,\ell}^t - \overline{\tau}_{\nu,\ell}^t \quad \perp \quad \mu_{\nu,k}^t \geq 0 \tag{5.96}$$

- For $\mu_{\nu,\ell}^\Gamma, \ell \in \mathcal{F}_\nu^\Gamma,\ \nu \in \Theta$:

$$0 \geq \tau_{\nu,\ell}^\Gamma - \overline{\tau}_{\nu,\ell}^\Gamma \quad \perp \quad \mu_{\nu,k}^\Gamma \geq 0 \tag{5.97}$$

- For $p_i^t, \quad \forall i \in \mathcal{I},\ t \in \mathcal{T}$:

$$0 = \omega_i^{*t} - \sum_k s_{k,i}(\lambda_k^{t+} - \lambda_k^{t-}); \quad p_i^t \ \ free; \tag{5.98}$$

- For $p_i^\Gamma, \quad \forall i \in \mathcal{I}$:

$$0 = \omega_i^{*\Gamma} - \sum_{t,k} s_{k,i}(\lambda_k^{t+} - \lambda_k^{t-}); \quad p_i^\Gamma \ \ free \tag{5.99}$$

- For λ_k^{t+}, $\quad \forall k \in \mathcal{K},\, t \in \mathcal{T}$:

$$0 \geq \sum_i s_{k,i}(p_i^t + p_i^\Gamma) - \overline{z}_k^+ \quad \perp \quad \lambda_k^{t+} \geq 0 \tag{5.100}$$

- For λ_k^{t-}, $\quad \forall k \in \mathcal{K},\, t \in \mathcal{T}$:

$$0 \geq -\sum_i s_{k,i}(p_i^t + p_i^\Gamma) - \overline{z}_k^- \quad \perp \quad \lambda_k^{t-} \geq 0 \tag{5.101}$$

- For ω_i^{*t}, $\quad \forall i \in \mathcal{I},\, t \in \mathcal{T}$:

$$0 = p_i^t - \sum_{\nu,\ell} \xi_{\nu,\ell,i}^t \tau_{\nu,\ell}^t; \quad \omega_i^{*t} \, free \tag{5.102}$$

- For $\omega_i^{*\Gamma}$, $\quad \forall i \in \mathcal{I}$:

$$0 = p_i^\Gamma - \sum_{\nu,\ell} \xi_{\nu,\ell,i}^\Gamma \tau_{\nu,\ell}^\Gamma; \quad \omega_i^{*\Gamma} \, free \tag{5.103}$$

Simultaneously solving for the primal ($\tau_{\nu,\ell}^t$, $\tau_{\nu,\ell}^\Gamma$, p_i^t, p_i^Γ) and dual variables ($\mu_{\nu,\ell}^t$, $\mu_{\nu,\ell}^\Gamma$, λ_k^+, λ_k^-) with prices (ω_i^{*t}, $\omega_i^{*\Gamma}$), gives an equilibrium point for the FTRs' awards.

Proposition 5.6 *In a multi-period market for FTRs, the congestion prices for the full day period are composed by the congestion prices of the off- and on-peak periods.*

Proof. Recalling equilibrium conditions (5.98) and (5.99), the congestion prices for periods t and Γ are

$$\omega_i^{*t} = \sum_k s_{k,i}(\lambda_k^{t+} - \lambda_k^{t-}), \quad \forall i \in \mathcal{I},\, t \in \mathcal{T}, \tag{5.104}$$

$$\omega_i^{*\Gamma} = \sum_{t,k} s_{k,i}(\lambda_k^{t+} - \lambda_k^{t-}), \quad \forall i \in \mathcal{I}. \tag{5.105}$$

Introducing (5.104) into (5.105), it holds that

$$\omega_i^{*\Gamma} = \sum_t \omega_i^{*t}, \quad \forall i \in \mathcal{I}, \tag{5.106}$$

as claimed ■

From this proof, it is straightforward to conclude that given the same FTR for all time periods, the FTR price in the full day is the sum of the prices from off- and on-peak periods. Consequently, the full day price is not less than the price of either the off- or on-peak periods.

5.8.5 Numerical examples

A three-node system

The test system described in §5.6.1 is extended to a multi-period market case. Three periods in the market are considered: i) off-peak; ii) on-peak; and iii) full-day. The linear parameters of the FTR benefit functions are given in Table 5.13. The maximum value for all FTRs is of 1500 MW.

Table 5.13: Linear parameter values ($\beta_{\nu,\ell}$) of the benefit functions.

ν, ℓ	Off-peak	On-peak	Full-day
$1, 1$	2	4.5	6
$2, 1$	2	2	5

As less congestion can be expected in the off-peak period, the corresponding benefit functions characterize a low willingness to acquire FTRs. On the other side, FTRs can become more valuable in the on-peak period as more congestion may arise. Furthermore, FTRs for the full day can have a higher expectation than those of the on-peak period. This is because with full-day FTRs the holder will collect moneys from both off- and on-peak periods. The results for this case are summarized in Table 5.14.

In the multi-period market, the FTRs of all periods are simultaneously allocated. Since full-day FTRs overlap with the off-peak and on-peak FTRs, their prices become inter-related. For this case, the optimal solution is to allocate FTRs in both the off- and on-peak periods. The allocation in off- and on-peak periods is the same as if every period were independently cleared. However, no right is allocated to the full-day period. This is because of the low evaluation to acquire such FTRs in comparison to the composed value of off- and on-peak FTRs. The marginal values of full-day FTRs are \$6/MW and \$5/MW; nonetheless, the clearing prices for them are \$6.5/MW and \$6/MW, which are the sum of

Table 5.14: Multi-period market outcome. Three-node system.

	Off-peak		**O**n-peak		**F**ull-day	
	$\tau_{1,1}$	$\tau_{2,1}$	$\tau_{1,1}$	$\tau_{2,1}$	$\tau_{1,1}$	$\tau_{2,1}$
Award (MW)	350	200	350	200	0	0
Price ($\$/MW$)	2	2	4.5	4.0	6.5	6.0
Cost ($)	700	400	1575	800	0	0
Benefit($)	700	400	1575	800	0	0
Surplus($)	0	0	0	0	0	0
ISO Revenue ($)	**1**100		**2**375		**0**	

the clearing prices from the off- and on-peak periods.

A thirty-node system

The thirty-node system from §5.7.6 is used here for a multi-period market case. There are twenty FTRs and nine FTR portfolios; this makes a total of 540 FTRs in the market. For each time period, the parameter values of the benefit functions are randomly generated. The prices and awards for FTRs are listed in Tables 5.15 and 5.16.

Table 5.15: FTR prices (in $\$/MW$). Multi-period market.

ℓ	1	2	3	4	5	6	7	8	9	10
Off-peak	6.6	7.6	16.8	6.8	7.2	7.2	8.9	21.3	19.2	7.5
On-peak	11.8	12.9	32.5	14.1	15.0	12.8	12.3	35.7	29.0	12.3
Full-day	18.4	20.5	49.3	20.9	22.3	20.1	21.2	57.1	48.2	19.9

Two features can be highlighted for this example. First, for all FTRs, the sum of prices

ℓ	11	12	13	14	15	16	17	18	19	20
Off-peak	7.2	8.2	7.8	6.7	6.0	7.6	8.8	21.3	8.8	6.8
On-peak	11.5	13.9	13.2	12.6	15.6	16.9	10.9	41.3	11.8	13.5
Full-day	18.8	22.1	21.1	19.3	21.7	24.5	19.7	62.7	20.6	20.4

of off-peak and on-peak periods equals the prices of the full-day period. Second, when FTRs are to be allocated to full-day FTRs, no capacity is allocated to off- or on-peak periods; meanwhile, the capacity allocated to off-peak FTRs is the same capacity allocated to on-peak FTRs.

Table 5.16: FTR awards (in MW). Multi-period market.

ν	1					2	3			
ℓ	7	10	11	15	20	1	4	5	6	12
Off-peak	0	0	0	0	0	0	95.3	0	0	0
On-peak	0	0	0	154.4	11.2	0	0	0	39.8	0
Full-day	43.8	21.1	42.5	0	0	81.7	0	29.3	0	17.4

ν	3			4	5	7			
ℓ	13	16	17	19	14	4	6	15	20
Off-peak	0	40	0	0	0	0	39.8	154.4	11.2
On-peak	0	40	0	0	0	95.4	0	0	0
Full-day	24.8	0	12.9	36.5	222.5	0	0	0	0

5.9 Summary

An alternative framework for modelling competition in markets for financial transmission rights of power systems has been presented. The proposed models are based upon equilibrium conditions for all the agents that participate in the transmission market. The conjectured congestion-price response function gives the flexibility to analyze the influence on prices by specific FTRs. Within this unified framework, different markets for FTRs can be easily modelled, and the interactions of different kind of FTRs can be analyzed. Due to the convexity nature of the formulation, the proposed framework is computationally tractable for larger systems.

The models proposed in this chapter can be useful as a mechanism to study bidding strategies and their impact on congestion prices. Through numerical examples, it has been shown that bidders can reduce the congestion prices they face without decreasing the amount of awarded FTRs, and that bidders can more easily impact congestion prices of options. However, for the same degree of manipulation, the impact on congestion prices is stronger for the case of obligations. This arises because obligations are more inter-related among them through counterflows. This also suggests that markets for options can be more susceptible to price manipulation since bidders can individually impact transmission constraints.

The introduction of new products, such as auction revenue rights, may create extra incentives for bidders, and produce more complex interaction within markets. The proposed framework can be useful to study these further developments.

Chapter 6

Conclusions

6.1 Summary of Contributions

Due to the complexity of electricity markets, there has been an increasing need to elaborate more sophisticated models for the proper analysis of competition and market efficiency. The research in this thesis has been focused on the development of more comprehensive models for competition in the markets for energy, spinning reserve and financial transmission rights. These models provide more realism to the incentives and impact of participants strategies in the markets.

In Chapter 3, an equilibrium model for imperfect competition has been developed. Such a model is a direct extension of a model described in [18]. In this research direction, the following works have been conducted:

- In contrast to most previous static models, a dynamic model for oligopolistic competition in energy markets has been developed in this work; temporal constraints and energy limits have been included. Through numerical examples, the impact of these constraints has been illustrated. Such kind of model and analysis has not been explicitly performed in previous works.

- A joint market for energy and spinning reserve has been proposed. In various real-world markets, a joint optimization of products is carried out. Thus, an integrated approach for modelling competition is a better representation of actual markets. Furthermore, the strategic behaviour of suppliers in the spinning reserve market is here addressed. This is included by means of a conjecture spinning reserve price function.

- An analytical derivation to quantify the opportunity cost between energy and spinning reserve has been carried out. Even for large systems, this derivation allows one to identify which elements are impacting the opportunity cost, and how both energy and spinning reserve prices are affecting each other. This formulation defines the opportunity cost for a whole range of strategy behaviors, ranging from the Cournot to the competitive one. No previous mathematical derivation has been previously carried out in this direction.

- Financial transmission rights have been included in the models of imperfect competition in order to analyze their impact on the market. In contrast to previous works applied to small systems, the proposed model can be easily applied to large power systems where there are GenCos with many generation units and portfolios of FTRs.

In Chapter 4, the impact of FTRs on the strategic behaviour of generators is extended. Screening and mitigation of the exacerbation of market power due to the ownership of financial transmission rights have been studied. The contributions made in this direction are as follows:

- An index, called Hedging Position Ratio, has been proposed to determine the potential hedging level of each FTR bid. This index is based upon the actual impact that each participant has on the energy market, and upon the potential impact that would have with the FTR bid. Numerical examples show that this kind of ratio may identify the gambling opportunities that some FTRs can provide to their holders to exacerbate market power.

- A modified market for financial transmission rights is presented. Unlike actual markets, bids for the maximum amount of FTRs to be contracted are modified accordingly to the hedging position that such FTRs would provide to their holders. In this way, critical FTR bids are left out or reduced in the allocation process.

- Through the inclusion of contingencies, the effect of changes in the network configuration on the hedging position ratios has been presented with a numerical example.

It is shown that even with such changes, the hedging ratios identify critical FTRs as these ratios are based on the system network conditions.

In Chapter 5, equilibrium models of markets for financial transmission rights are presented. These models are introduced with the goal to provide insights on the impact that bidders can have on the congestion prices. Contributions in this chapter are as follows:

- A unified modelling framework of markets for: i) options, ii) obligations and iii) joint obligations and options is presented. In addition, equilibrium models have been formulated for multi-period and multi-round markets for obligations. Based on the latter models, extensions for options , and joint obligations and options are straightforward.
- Two kinds of conjectured congestion-price response functions have been introduced to characterize the effect on congestion prices from bidders strategies.
- Based on equilibrium conditions, it is shown that: i) active transmission constraints define the prices for transmission rights, ii) the prices for options are non-negative, iii) the price of an option is equal to or higher than the price for an obligation, for the same FTR, and iv) the price of an obligation in the full horizon period is the sum of all period prices.
- The effect of bidding strategies on congestion prices is presented through numerical examples. It is shown that the bidders' influence on market prices is higher with obligations than it is with options. However, with options, bidders can more individually impact the congestion prices.

None of the equilibrium models here presented has been previously formulated.

6.2 Directions for Future Research

In regard to future work on imperfect competition that is described in Chapter 3, the following directions are proposed:

- Based on the conjecture formulation, market power analysis can be extended to capacity markets.

- New schemes for provision of reserves in a locational basis are being proposed [41]. However, transmission constraints restrain the access not only to energy but also to reserves. The study of market power within a region-based scheme to provide reserves should be pursued.

- The proposed model is based on thermal generation units. A direct extension is the inclusion of hydro generation units. In addition, a further step in the analysis of oligopoly markets is to include the transmission-system losses.

- Models to consider cases of incomplete information are now being considered [137]. Although these models require assumptions on the kind of incomplete information, comparisons between the cases of complete information and those with different assumption of incomplete information should be pursued.

- Since trades of power can be done either through the Pool or by Bilateral Contracts (BC), a mixed Pool/BC model for oligopoly is an issue worth of research.

In regard to future work on models and markets for transmission rights presented in Chapter 4 and 5, the following directions are proposed:

- Markets for obligations and options with multi-rounds and multi-periods are becoming the standard framework for trading FTRs. However, the implementation of this kind of market faces computational challenges. The computational burden imposed by the introduction of options demands alternative approaches; models based on mixed integer techniques may be a worthy option to be explored.

- The proposed equilibrium models and analysis can be extended to the case of FGRs, and the joint issuance of FTRs and FGRs. This would allow one to identify the potential impact of bidders strategies on very specific transmission constraints, which may become a concern in major interfaces.

- The ownership of auction revenue rights may give extra incentives for strategic bidding in markets for FTRs [68]. The proposed equilibrium framework should be extended to account for ARRs ownership into bidders strategies. Unlike the analytical procedure for a two-node system in [68], the equilibrium model version could be used for larger systems.

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Appendix A

Power Distribution Factors

Throughout this thesis, different kinds of power distribution factors have been used; they are next defined.

Definition A.1 (Sensitivity Factors) Let us consider a vector of power injections $\boldsymbol{p} = [p_1, p_2, ..., p_s, ... p_{\mathcal{I}}]^T$, noting that p_s is the injection at the slack node. The real power flow z_{ij}, through any transmission line between nodes i and j, can be defined in terms of nodal power injections by using the sensitivity (power distribution) factors. These factors quantify the change in a line power flow, Δz_{ij}, by a change of power injection at a specific node, subject to its respective balance adjustment at the slack node, *i.e.*,

$$\langle \boldsymbol{s}_{ij} , \Delta \boldsymbol{p} \rangle = \Delta z_{ij}, \tag{A.1}$$

with

$$\langle \boldsymbol{e} , \Delta \boldsymbol{p} \rangle = 0. \tag{A.2}$$

Based on DC power flows, the l-th sensitivity factor, $s_{ij,l}$, is computed as follows [125]:

$$s_{ij,l} = \frac{X_{il} - X_{jl}}{x_{ij}}, \tag{A.3}$$

where X_{il} and X_{jl} are elements of the *reactance matrix*, and x_{ij} is the reactance for the line $ij \in \mathcal{K}$.

Definition A.2 (Power Transfer Distribution Factors [PTDFs]) Given a point-to-point transaction ℓ (with r and t denoting the injection and withdrawal nodes, respectively), the Power Transfer Distribution Factors (PTDFs), $\varrho_{ij,\ell}$, (corresponding to the transmission line between nodes i and j) can be computed as the superposition of two transactions: one between the injection and slack nodes, and another between the slack and withdrawal nodes; thus, based on the sensitivity factors, it is given by

$$\varrho_{ij,\ell} = s_{ij,r} - s_{ij,t}. \tag{A.4}$$

Given 1MW to be transmitted from r to t, a PTDF $\varrho_{ij,\ell}$ stands for the fraction of the power that will flow through the transmission line ij.

Definition A.3 (Generalized Generation and Load Distribution Factors [127]) The acronyms GGDFs and GLDFs are used in this thesis to denote the Generalized Generation Distribution Factors and the Generalized Loads Distribution Factors.

In the case of GGDFs, a power flow $\boldsymbol{z}_{ij}$ can be expressed as a linear combination of generations, *i.e.,*

$$\boldsymbol{z}_{ij} = \langle\, \boldsymbol{\varphi}_{ij}\,,\, \boldsymbol{g}\,\rangle. \tag{A.5}$$

A new power flow $\hat{\boldsymbol{z}}_{ij}$ is produced by an increase in generation $\Delta\, g_l$, at a node l, with a compensation at the slack node, *i.e.,*

$$\hat{\boldsymbol{z}}_{ij} = \langle\, \boldsymbol{\varphi}_{ij}\,,\, \boldsymbol{g}\,\rangle + (\varphi_{ij,l} - \varphi_{ij,s})\,\Delta\, g_l. \tag{A.6}$$

From (A.5), it becomes

$$\hat{\boldsymbol{z}}_{ij} - \boldsymbol{z}_{ij} = (\varphi_{ij,l} - \varphi_{ij,s})\,\Delta\, g_l. \tag{A.7}$$

A change in generation, at node l, can also be given in terms of sensitivity factors, *i.e.,*

$$s_{ij,l}\Delta g_l = \Delta \boldsymbol{z}_{ij} = \hat{\boldsymbol{z}}_{ij} - \boldsymbol{z}_{ij}. \tag{A.8}$$

By comparing (A.7) and (A.8) one gets

$$\varphi_{ij,l} - \varphi_{ij,s} = s_{ij,l}. \tag{A.9}$$

By transferring all the generation to the slack node and using superposition, (A.8) is

$$\hat{\boldsymbol{z}}_{ij} - \boldsymbol{z}_{ij} = -\langle\, \boldsymbol{s}_{ij}\,,\, \boldsymbol{g}\,\rangle. \tag{A.10}$$

Since

$$\hat{\boldsymbol{z}}_{ij} = \varphi_{ij,s} g_s, \tag{A.11}$$

where

$$g_s = \langle\, \boldsymbol{e}\,,\, \boldsymbol{g}\,\rangle. \tag{A.12}$$

Thus, (A.10) becomes

$$\varphi_{ij,s} g_s = \boldsymbol{z}_{ij} - \langle\, \boldsymbol{s}_{ij}\,,\, \boldsymbol{g}\,\rangle. \tag{A.13}$$

The (GGDF) transmission usage factor $\varphi_{ij,s}$ is given by

$$\varphi_{ij,s} = \frac{\boldsymbol{z}_{ij} - \langle\, \boldsymbol{s}_{ij}\,,\, \boldsymbol{g}\,\rangle}{\langle\, \boldsymbol{e}\,,\, \boldsymbol{g}\,\rangle}. \tag{A.14}$$

A similar procedure can be applied to derive the GLDFs.

Appendix B

Test Systems Data

B.1 Five-node System Data

For this test system, transmission lines' data are listed in Table B.1. Suppliers are modelled with cost functions defined as $c_i(p_i) = \beta_i p_i + \gamma_i p_i^2$, where β_i and γ_i are parameters. Supply bids are given in Table B.2 The inelastic loads at nodes 2, 4 and 5 are 300 MW.

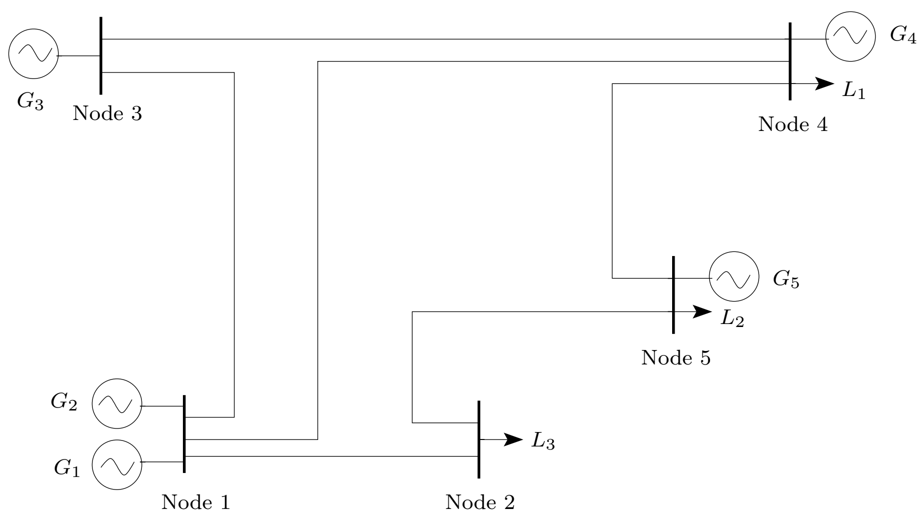

Figure B.1: Five-node power system.

Table B.1: Transmission lines data. Five-node system.

Line k	Source i	Sink j	x_{ij} (p.u.)	$\overline{z}_i$ (MW)
1	1	2	.0281	400
2	1	3	.0064	400
3	1	4	.0304	250
4	2	5	.0108	350
5	3	4	.0297	240
6	4	5	.0297	240

Table B.2: Suppliers bids data. Five-node system.

Supplier i	$\underline{p}_i$ (MW)	$\overline{p}_i$ (MW)	β_i $(\$/MW)$	γ_i $(\$/MW^2)$
1	0	110	14	.0140
2	0	100	15	.0140
3	0	600	10	.0025
4	0	150	30	.0250
5	0	520	30	.0250

Table B.3: Financial transmission rights bids.

ℓ	Bidder	Source r	Sink t	β_ℓ ($\$/MW$)	γ_ℓ ($\$/MW^2$)	$\overline{\tau}_\ell$ (MW)
1	G_1	1	2	20	0.0050	110
2	G_2	1	2	21	0.0060	100
3	L_3	1	2	25	0.0050	300
4	G_4	1	4	50	0.0050	120
5	L_1	1	4	60	0.0025	300
6	G_3	1	4	45	0.0050	580
7	G_1	1	5	30	0.0035	110
8	L_2	1	5	35	0.0035	300
9	L_3	1	5	35	0.0025	300
10	G_4	2	5	60	0.0015	120
11	L_3	2	5	40	0.0040	300
12	L_2	2	5	50	0.0040	300
13	G_1	2	4	60	0.0025	110
14	G_4	2	4	65	0.0020	120
15	L_2	2	4	60	0.0030	300
16	G_3	3	4	60	0.0020	580
17	G_4	3	4	70	0.0025	150
18	L_1	3	4	65	0.0015	300
19	G_1	3	5	40	0.0050	110
20	G_3	3	5	45	0.0025	580
21	L_2	3	5	42	0.0040	300
22	G_3	3	2	20	0.0020	580
23	G_2	3	2	22	0.0010	100
24	L_3	3	2	25	0.0030	300
25	G_4	5	4	15	0.0040	120
26	L_3	5	4	15	0.0035	300
27	L_1	5	4	16	0.0035	300

Table B.4: Benefit function parameters for FTRs.

ν	ℓ	Source	Sink	β_ℓ ($/MW$)	γ_ℓ ($/MW^2$)	$\overline{\tau}_\ell$ (MW)
1	1	1	2	20	0.0050	110
	2	1	4	50	0.0050	120
	3	1	5	30	0.0035	110
	4	2	5	60	0.0015	120
	5	2	4	60	0.0025	110
	6	3	4	60	0.0020	580
	7	3	5	40	0.0050	110
	8	3	2	20	0.0020	580
	9	5	4	15	0.0040	120
2	1	1	2	21	0.0060	100
	2	1	4	60	0.0025	300
	3	1	5	35	0.0035	300
	4	2	5	40	0.0040	300
	5	2	4	65	0.0020	120
	6	3	4	70	0.0025	150
	7	3	5	45	0.0025	580
	8	3	2	22	0.0010	100
	9	5	4	15	0.0035	300
3	1	1	2	25	0.0050	300
	2	1	4	45	0.0050	580
	3	1	5	35	0.0025	300
	4	2	5	50	0.0040	300
	5	2	4	60	0.0030	300
	6	3	4	65	0.0015	300
	7	3	5	42	0.0040	300
	8	3	2	25	0.0030	300
	9	5	4	16	0.0035	300

B.2 Thirty-node System Data

The transmission data are listed in Table B.5, while the suppliers and loads data are given in Tables B.6 and B.7, respectively. For this case, the supply bids are linear and all the loads are considered inelastic.

Table B.5: Transmission lines data. Thirty-node system.

Line k	Source i	Sink j	x_{ij} (p.u.)	$\overline{z}_i$ (MW)	Line k	Source i	Sink j	x_{ij} (p.u.)	$\overline{z}_i$ (MW)
1	1	2	0.0575	125	22	12	13	0.1400	75
2	1	3	0.1852	125	23	12	14	0.2559	30
3	2	4	0.1737	50	24	12	15	0.1304	30
4	3	4	0.0379	100	25	12	16	0.1987	30
5	2	5	0.1983	70	26	14	15	0.1997	20
6	2	6	0.1763	75	27	16	17	0.1932	20
7	4	6	0.0414	100	28	15	18	0.2185	20
8	4	12	0.2560	50	29	18	19	0.1292	20
9	5	7	0.1160	75	30	19	20	0.0680	20
10	6	7	0.0820	125	31	15	23	0.2020	20
11	6	8	0.0420	30	32	21	22	0.0236	30
12	6	9	0.2080	75	33	22	24	0.1790	20
13	6	10	0.5560	30	34	23	24	0.2700	20
14	6	28	0.0599	20	35	24	25	0.3292	20
15	8	28	0.2000	30	36	25	26	0.3800	20
16	9	11	0.2080	40	37	25	27	0.2087	20
17	9	10	0.1100	75	38	27	29	0.4153	20
18	10	20	0.2090	30	39	27	30	0.6027	20
19	10	17	0.0845	30	40	28	27	0.3960	50
20	10	21	0.0749	30	41	29	30	0.4530	20
21	10	22	0.1499	30					

Table B.6: Suppliers bids data. Thirty-node system.

Supplier i	Node	$\underline{p}_i$ (MW)	$\overline{p}_i$ (MW)	β_i (\$/MW)
1	1	0	150	30.0
2	2	0	175	28.5
3	3	0	50	25.0
4	8	0	100	30.0
5	11	0	100	30.0
6	13	0	125	37.0
7	14	0	200	40.0
8	17	0	50	35.0
9	18	0	100	32.0
10	22	0	150	40.0
11	23	0	150	37.0
12	27	0	120	45.0
13	7	0	75	45.0

Table B.7: Demand data. Thirty-node system.

Demand i	Node	$\overline{d}_i$ (MW)		Demand i	Node	$\overline{d}_i$ (MW)
1	3	100		11	18	30
2	5	75		12	19	10
3	7	50		13	20	35
4	8	20		14	21	20
5	10	50		15	23	50
6	12	30		16	24	5
7	13	15		17	25	10
8	14	50		18	26	10
9	15	30		19	27	50
10	16	5		20	29	20

B.3 57-node System Data

Table B.8: Generation units data. 57-node system.

ν	h	i	$\overline{g}_{\nu,i,h}$	$\alpha_{\nu,i,h}$	$\beta_{\nu,i,h}$	$\overline{\Delta}\, g_{\nu,i,h}$	$\underline{\Delta}\, g_{\nu,i,h}$
			(MW)	$(\$/MWh)$	$(\$/MWh)$	(MW/h)	(MW/h)
1	1	1	680	3	10	40	40
	2	3	250	8	25	40	40
	3	9	100	12	35	40	40
2	1	2	200	10.5	32	40	40
	2	6	400	6.5	20	40	40
	3	8	650	3.5	11	40	40
3	1	12	510	4	13	40	40
	2	15	375	6	18	40	40
4	1	25	175	10	24	40	40
	2	29	200	7	24	40	40
5	1	38	150	9	27.5	40	40
	2	49	300	6.5	20	40	40
6	1	54	380	4	12	40	40
7	1	17	375	6	24	40	40
	1	32	220	10	38	40	40

Table B.9: Peak demands for the 57-node system (in MW).

i	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
d_i	105	52.5	70	0	44.5	131.25	0	175	140	61.25	0	87.5	66.5	54.25	56

i	16	17	18	19	20	21	22	23	24	25	26	27	28	29	30
d_i	79	56	64.75	57.75	78.75	0	0	78.75	0	78.75	0	70	87.5	47.5	60

i	31	32	33	34	35	36	37	38	39	40	41	42	43	44	45
d_i	101.5	52.5	70	0	105	0	0	77	0	0	64.75	42.25	43.75	61.25	0

i	46	47	48	49	50	51	52	53	54	55	56	57
d_i	0	52.5	0	84	56	49	61.25	70	71.75	82.25	66.5	82.25

B.4 118-node System Data

Table B.10: Generation units data. 118-node system.

ν	h	i	$\beta_{\nu,i,h}$	$\overline{g}_{\nu,i,h}$
			($\$/MWh$)	(MW)
1	1	1	25	200
	1	12	30	285
	1	49	15	400
	1	59	17	350
	1	65	12	590
	1	69	8	820
	1	89	9	700
2	1	10	12	550
	1	25	16	420
	1	26	14	500
	1	46	24	220
	1	54	23	250
	1	61	16	360
	1	66	16	590
	1	80	10	675
	1	100	15	350
3	1	4	25	220
	1	6	26	200
	1	8	26	200
4	1	15	25	200
	1	18	25	200
	1	19	26	200
	1	24	28	200
5	1	27	26	200
	1	31	25	200
	1	113	27	200
	1	116	30	200

Table B.11: Generation units data (Continued).

ν	h	i	$\beta_{\nu,i,h}$	$\overline{g}_{\nu,i,h}$
			$(\$/MWh)$	(MW)
	1	32	24	200
6	1	34	24	200
	1	90	28	200
	1	36	27	200
7	1	85	25	200
	1	87	28	200
	1	40	28	200
8	1	42	28	200
	1	55	25	200
	1	62	26	200
9	1	56	24	200
	1	111	26	200
10	1	70	35	200
	1	72	35	200
	1	73	32	200
11	1	74	35	200
	1	76	34	200
	1	77	25.5	200
	1	91	27	200
12	1	103	32	240
	1	104	26	200
	1	92	26	200
13	1	99	26.5	220
	1	111	26	250
	1	105	22	200
14	1	107	25	200
	1	110	23	200

Table B.12: Demands for the 118-node system (in MW).

i	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16
d_i	76.5	30	58.5	45	0	78	28.5	0	0	0	105	70.5	51	21	135	37.5

i	17	18	19	20	21	22	23	24	25	26	27	28	29	30	31	32
d_i	16.5	90	67.5	42	21	15	10.5	0	0	0	93	25.5	36	0	64.5	88.5

i	33	34	35	36	37	38	39	40	41	42	43	44	45	46	47
d_i	34.5	88.5	49.5	46.5	0	0	40.5	30	55.5	55.5	27	24	79.5	42	51

i	48	49	50	51	52	53	54	55	56	57	58	59	60	61
d_i	30	130.5	25.5	25.5	27	34.5	169.5	94.5	81	18	18	415.5	117	0

i	62	63	64	65	66	67	68	69	70	71	72	73	74	75	76	77
d_i	115	0	0	0	58.5	42	0	0	99	0	0	0	102	70.5	102	91.5

i	78	79	80	81	82	83	84	85	86	87	88	89	90	91	92
d_i	106.5	58.5	195	0	81	30	16.5	36	31.5	0	72	0	117	0	97.5

i	93	94	95	96	97	98	99	100	101	102	103	104	105	106	107
d_i	18	45	63	57	22.5	51	0	55.5	33	15	34.5	57	46.5	64.5	42

i	108	109	110	111	112	113	114	115	116	117	118
d_i	15	12	58.5	0	37.5	0	12	33	0	30	49.5

Table B.13: Financial transmission rights portfolios (in MW).

ν	1				2				3	4			5		
m	49	65	69	69	54	61	66	80	–	24	19	10	27	31	116
ℓ	70	74	75	29	70	76	11	118	–	12	6	15	19	76	118
$\overline{\tau}_{\nu,m,\ell}$	140	100	85	200	170	150	120	75	–	100	50	30	60	62	56

ν	6	7	8		9	10	11		12	13	14	
m	32	85	55	40	112	–	77	67	91	99	105	110
ℓ	4	74	60	35	117	–	74	73	82	74	117	118
$\overline{\tau}_{\nu,m,\ell}$	40	65	48	50	68	–	50	45	80	35	85	35